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A Dynamic Yield Curve Model with Stochastic Volatility and Non-Gaussian Interactions: An Empirical Study of Non-standard Monetary Policy in the Euro Area

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Abstract

We develop an econometric methodology for the study of the yield curve and its interactions with measures of non-standard monetary policy during possibly turbulent times. The yield curve is modeled by the dynamic Nelson-Siegel model while the monetary policy measurements are modeled as non-Gaussian variables that interact with latent dynamic factors, including the yield factors of level and slope. Yield developments during the financial and sovereign debt crises require the yield curve model to be extended with stochastic volatility and heavy tailed disturbances. We develop a flexible estimation method for the model parameters with a novel implementation of the importance sampling technique. We empirically investigate how the yields in Germany, France, Italy and Spain have been affected by monetary policy measures of the European Central Bank. We model the euro area interbank lending rate EONIA by a log-normal distribution and the bond market purchases within the ECB's Securities Markets Programme by a Poisson distribution. We find evidence that the bond market interventions had a direct and temporary effect on the yield curve lasting up to ten weeks, and find limited evidence that purchases changed the relationship between the EONIA rate and the term structure factors.

1 Introduction

We develop a nonlinear non-Gaussian modeling framework for analyzing the relationship between government bond yields and monetary policy measures during possibly turbulent

times such as a financial or sovereign debt crisis. For this purpose we generalize the yield curve model with macro-economic variables of Diebold, Rudebusch & Aruoba (2006) by introducing stochastic volatility and non-Gaussian interaction variables. To estimate the parameters of the resulting non-Gaussian dynamic factor model we propose a simulation based estimation method that is based on a novel implementation of the importance sampling technique that accounts for mean and variance effects separately. We adopt the model to empirically investigate the joint dynamics of euro area yield data and monetary policy measures as conducted by the European Central Bank (ECB) during 2004-2012.

Our starting point for modeling the government bond yields is the dynamic Nelson-Siegel (DNS) model; see Nelson & Siegel (1987) and Diebold & Rudebusch (2012). The DNS model describes the term structure of the yields by three common dynamic factors that are labeled level, slope and curvature. The model is typically able to explain a large part of the variance that is observed in the government bond yields and has good forecasting properties; see Diebold & Li (2006).

We are interested in studying the interaction between the level, slope and curvature factors of the term structure model and monetary policy measurements. We model the observations that are related to the monetary policy by appropriate non-Gaussian densities that are defined conditional on a set of latent dynamic factors. For example, we model the euro area EONIA interbank lending rate by the log-normal density to accommodate the property that it can be close to zero for an extended period of time but does not go negative during our sample period. Also, direct interventions in bond markets, such as those conducted within the ECB's Securities Markets Programme (SMP) during 2010-12, are modeled by the Poisson distribution. The latent term structure and monetary policy factors are jointly modeled by a vector autoregressive process. The joint model allows us to conduct inference regarding the contemporaneous and medium term interactions between the monetary policy measures and the yield curve.

During the recent financial and sovereign-debt crisis (2008-2012) in Europe sovereign bond yields of several euro area countries fluctuated heavily. Countries for which we observe these features are Italy and Spain, and to a lesser extend also for Germany and France. To capture the deviations from the standard normal distribution for the yields we extend the DNS model by allowing for heavy tailed errors and time-varying variances. In particular,

2

we specify the error term of the observation equation of the DNS model by the multivariate Student's t distribution where the variance matrix is driven by a common dynamic factor. The common factor structure is specified in a similar way as in Jungbacker & Koopman (2006) and Carriero, Clark & Marcellino (2013), who both consider Gaussian models. The resulting joint model is non-Gaussian with time-varying factor structures for both the mean and the variance.

Parameter estimation is non-trivial for the joint model. The non-Gaussian densities for the yields and the monetary policy variables in combination with the latent stochastic factor structures prohibit closed form solutions for the likelihood. Instead, we express the likelihood as a high dimensional integral and develop a novel implementation of the importance sampling technique for its evaluation. In particular, we draw M samples for the latent volatility process from an appropriate importance density. For each sampled volatility path we construct an importance density to integrate out the latent term structure and monetary policy factors. The importance densities for the mean factors are thus conditional on the sampled volatility paths. The construction of the importance densities is adopted from Shephard & Pitt (1997), Durbin & Koopman (1997) and Jungbacker & Koopman (2007). The conditional importance sampling approach is stable and satisfies standard convergence criteria as formulated by Geweke (1989).

We propose extensions in four directions of research. First, we allow for non-Gaussian densities for the monetary policy variables that interact with the yield curve. Linear specifications for incorporating macroeconomic and monetary policy variables in term structure models have been considered in Ang & Piazzesi (2003), Dewachter & Lyrio (2006), Diebold et al. (2006), Hordahl, Tristani & Vestin (2006), Rudebusch & Wu (2008), Ludvigson & Ng (2009) and Koopman & van der Wel (2013). Second, we extend the dynamic Nelson Siegel model to allow for time-varying variances and heavy-tailed errors; see also Diebold & Li (2006), Diebold et al. (2006), Diebold, Li & Yue (2008), Koopman, Mallee & van der Wel (2010), Christensen, Diebold & Rudebusch (2011), Cakmakli (2011) and Diebold & Rudebusch (2012). Third, we provide a Monte Carlo maximum likelihood method for the estimation of parameters for non-Gaussian models with dynamic factor structures for both the mean and the variance. From a Bayesian perspective, estimation methods for these types of models have been considered in Chib, Nardari & Shephard (2006), Lopes & Car-

valho (2007) and Chib, Omori & Asai (2009). Fourth, by result, we provide an alternative method for studying the impact of possibly non-standard monetary policy measures on the yield curve. Recently many studies have aimed to quantify the effect of central bank asset purchase programs on the yields; see Krishnamurthy & Vissing-Jorgensen (2011), D' Amico & King (2012), D' Amico, English, Lopez-Salido & Nelson (2012), Wright (2012), De Pooter, Martin & Pruitt (2013) and Eser & Schwaab (2013). Typically, these studies focus on estimating the contemporaneous effects of asset purchase announcements. Our approach allows for the estimation of both short term and longer term effects, see Wright (2012).

The general framework is illustrated by analyzing the interaction between the monetary policy of the ECB and the yield curves of four large euro area countries; France, Germany, Italy and Spain. First, we show that during the financial and sovereign debt crises the response of sovereign bond yield curve factors to changes in the EONIA, a euro area overnight interbank lending rate and proxy of the ECB's monetary policy stance, was different (impaired) for at least some countries in the euro area.

This observation provides one rationale for non-standard monetary policy measures. Second, we investigate the role of the ECB's bond market purchases within the SMP. In principle, such asset purchases can impact yields in two interrelated ways. First, outright purchases *directly* add liquidity and depth to impaired secondary markets. As a result, required liquidity risk premia should fall and prices are supported. In addition, portfolio balance (stock) effects and certain signalling effects may further support prices, see D' Amico & King (2012), De Pooter et al. (2013) and Eser & Schwaab (2013). Second, purchases may also help *indirectly* in restoring the transmission of the common EONIA interbank rate into government yield curves, see González-Páramo (2011). For example, purchases may send a signal that the ECB is willing to implement unprecedented policies and is likely to aggressively support solvent but illiquid banks in stressed SMP countries. This contributes towards making private interbank funding at low rates available to such banks. We find that the purchases have a direct temporary effect on the term structure factors. The instantaneous effect on the level factor of both Italy and Spain is negative, while results are mixed for the slope factor. For Italy the yield reducing effect becomes positive relatively quickly (2 weeks), whilst for Spain the yield reducing effect of the purchases dies out more slowly (10 weeks). We also present limited and relatively more tentative evidence that asset purchases changed the joint

4

dynamics of the interbank lending rate (EONIA) and the term structure factors.

The remainder of the paper is organized as follows. Section 2 formally describes the joint model for the yield curve and the monetary policy. Section 3 develops our Monte Carlo maximum likelihood method for estimating the parameters of the joint model. We present our empirical study for euro area yield curves and monetary policy in Section 4. Section 5 summarizes our findings and discusses some directions for future research.

2   Statistical model

We consider the dynamic Nelson-Siegel yield curve model that is extended with macroeconomic variables in Diebold et al. (2006). We extend their general modeling framework in two directions. First we introduce stochastic volatility processes for the dynamic scaling of the measurement disturbances that may come from heavy tailed distributions. The recent financial crises have shown that constant variance and Gaussian assumptions are not necessary applicable for modeling interest rates and related variables. Second we extend the model with variables that are associated with (non-standard) monetary policy, financial measurement and sovereign-debt crisis variables. The characteristics of these variables are different from the more usual macroeconomic variables. Although linear and Gaussian assumptions can be questioned for macroeconomic variables as well, they are certainly not applicable for many monetary and financial variables. Hence the need for a more general modeling framework. In Section 3 we discuss parameter estimation, signal extraction and and how we account for the non-Gaussian and nonlinear extensions in our model.

2.1   The dynamic Nelson-Siegel yield curve model

For a cross section of yields y_{τ_i}, where τ_i is the maturity of the ith yield, the Nelson & Siegel (1987) model is given by

$$y_{\tau_i} = \beta_1 + \beta_2 \left(\frac{1 - e^{-\lambda \tau_i}}{\lambda \tau_i} \right) + \beta_3 \left(\frac{1 - e^{-\lambda \tau_i}}{\lambda \tau_i} - e^{-\lambda \tau_i} \right) + \epsilon_{\tau_i}, \tag{1}$$

for $i = 1, \ldots, N$, where β_1, β_2, β_3 and λ are the deterministic model parameters and where ϵ_{τ_i} is the disturbance term. Model (1) provides a parsimonious representation for describing

a potentially large cross section of yields with different maturities τ_i. The model parameters can be estimated by combining a grid search over λ with ordinary least squares regressions for β_1, β_2 and β_3. This model is used up to minor changes on a daily basis at many financial investment agencies and central banks.

Diebold & Li (2006) show that the parameters β_1, β_2 and β_3, when viewed in a sequence for different consecutive time periods, can be interpreted as latent dynamic factors. In particular, when denoting the yields for time period t by $y_{\tau_i,t}$ we obtain the model

$$y_{\tau_i,t} = \beta_{1,t} + \beta_{2,t}\left(\frac{1 - e^{-\lambda\tau_i}}{\lambda\tau_i}\right) + \beta_{3,t}\left(\frac{1 - e^{-\lambda\tau_i}}{\lambda\tau_i} - e^{-\lambda\tau_i}\right) + \epsilon_{\tau_i,t}, \tag{2}$$

for $t = 1, \ldots, T$, where $\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$ are time-varying versions of β_1, β_2 and β_3 and $\epsilon_{\tau_i,t}$ is the time t disturbance term that is assumed independent over t. The latent factors $\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$ are referred to as level, slope and curvature. The parameters of model (2) can also be obtained by a grid search over λ in combination with regression estimates for $\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$. In a second step, the estimated series $\beta_{1,t}$, $\beta_{2,t}$ and $\beta_{3,t}$ are typically modeled by independent autoregressive processes or a single vector autoregressive process.

A further advancement is presented in Diebold et al. (2006) who show that the dynamic Nelson-Siegel model (2) can be conveniently expressed in state space form. The measurement equation for $y_t = (y_{\tau_1,t}, \ldots, y_{\tau_N,t})'$ is given by

$$y_t = \Lambda f_t + \epsilon_t, \qquad \epsilon_t \sim N(0, \Omega), \qquad t = 1, \ldots, T, \tag{3}$$

where Λ is the $N \times 3$ loading matrix, $f_t = (\beta_{1,t}, \beta_{2,t}, \beta_{3,t})'$ is the vector of latent dynamic factors and $\epsilon_t = (\epsilon_{\tau_1,t}, \ldots, \epsilon_{\tau_N,t})'$ is the disturbance vector. The disturbance vector is normally distributed with diagonal variance matrix Ω. The structure for the loading matrix Λ is inherited from the Nelson & Siegel (1987) model and is given by

$$\Lambda = (\lambda_1, \ldots, \lambda_N)', \quad \lambda_i = \left(1, \left(\frac{1 - e^{-\lambda\tau_i}}{\lambda\tau_i}\right), \left(\frac{1 - e^{-\lambda\tau_i}}{\lambda\tau_i} - e^{-\lambda\tau_i}\right)\right)', \quad i = 1, \ldots, N.$$

When the term structure factors f_t follow a vector autoregressive process of order one the linear state space form is completed. In particular, the transition equation of the state space

model is given by

$$(f_t - \mu_f) = \Phi(f_{t-1} - \mu_f) + \eta_t, \qquad \eta_t \sim N(0, \Sigma_\eta), \tag{4}$$

where μ_f is the mean vector, Φ is the autoregressive matrix and η_t is the disturbance term that is normally distributed with mean zero and variance matrix Σ_η. The initial state is assumed normally distributed with mean μ_f and variance V, where V satisfies $V = \Phi V \Phi' + \Sigma_\eta$. The matrices Φ and Σ_η can be fully parameterized or have only diagonal elements different from zero, depending on whether structural analysis or forecasting is the goal. The linear state space form enables the use of the Kalman filter and smoother to compute the predicted, filtered and smoothed estimates for the latent factors f_t. The unrestricted parameters in Φ, Σ_η, Λ and Ω are estimated by likelihood based methods, where the likelihood is computed from the prediction error decomposition that is provided by the Kalman filter. The Kalman filter based estimation methods provide efficiency gains when compared to the two-step estimation method discussed above.

We refer to the model in equations (3) and (4) as the dynamic Nelson-Siegel (DNS) model. The model is a regular linear state space model for which all the methodology that is discussed in Durbin & Koopman (2012, Part 1) applies. In the next two sections we modify and extend the DNS model to (a) to allow for time-varying volatility and heavy tailed distributions and (b) to allow for non-Gaussian variables that interact with the term structure factors.

2.2 Stochastic volatility and heavy tailed disturbances

We modify the assumptions for the distribution of the error term in the observation equation (3) of the dynamic Nelson-Siegel model to allow for a heavy tailed distribution and stochastic volatility. Two key reasons for these extensions are formulated next.

First, times of financial crisis typically imply changing levels of volatility and the occurrence of extreme yield changes; see also Koopman et al. (2010). Sudden and large changes in bond yields can occur, in particular in illiquid markets, when there are unexpected news, for example, about fiscal policy variables, rating downgrades, emergency initiatives, or other political events. Furthermore, even in cases for which there are not many extreme events,

heavy tailed error distributions such as the Student's t distribution can greatly improve the accuracy for the factor estimates (see Mesters & Koopman (2014)) and stabilize importance sampling methods for stochastic volatility (see Koopman, Shephard & Creal (2009)).

Second, term structure models are typically considered for monthly yield data. During the recent financial crisis some of the monetary policy interventions have been operating and changing on a daily or weekly basis. Studying the impact of these interventions on a lower frequency is difficult as many other shocks also affect the yields. When modeling the yields at higher frequencies, correlation and clustering naturally occurs in the variance of the yields. This type of correlation is typically ignored at the monthly frequency, but becomes hard to ignore at higher frequencies; see the discussion in Diebold & Rudebusch (2012). In our extended model we explicitly model the time-varying volatility in the error term.

We replace the observation equation for the yields, given in (3), by

$$y_t = \Lambda f_t + e_t, \qquad e_t \sim t(0, \Sigma_t, \nu), \qquad t = 1, \ldots, T, \tag{5}$$

where $t(0, \Sigma_t, \nu)$ denotes the Student's t density with mean zero, variance matrix Σ_t and degrees of freedom ν. Similar as for the mean process Λf_t of the yields, we assume a factor structure for the variance process. We follow Carriero et al. (2013) and consider the one-factor structure given by

$$
\begin{aligned}
\Sigma_t &= \operatorname{diag}(\sigma_{1,t}^2, \ldots, \sigma_{N,t}^2), & \sigma_{i,t}^2 &= w_i^2 \exp h_t, \\
(h_t - \mu_h) &= \gamma(h_{t-1} - \mu_h) + \zeta_t, & \zeta_t &\sim N(0, \sigma_\zeta^2),
\end{aligned}
\tag{6}
$$

where w_i^2 is the maturity-specific loading for the log variance process h_t, which follows an autoregressive process of order one, where μ_h is the mean, γ is the autoregressive coefficient and ζ_t is the disturbance term with mean zero and variance σ_ζ^2.

To identify a rotation of the factor structure we normalize the volatility process h_t, such that $\operatorname{Var}(h_t) = 1$, and we restrict $w_1 = 1$. We assume that the innovations ζ_t are independent of the term structure innovations η_t. This assumption is in line with the empirical results reported in Andersen & Benzoni (2010).

2.3 The interaction of non-Gaussian variables

The level, slope and curvature factors f_t provide an abstract description of the term structure of the yields. In general there is interest in incorporating explanatory variables in the term structure model and studying their interaction with the term structure factors. We are interested in both (i) the effect of the explanatory variables on the term structure factors and (ii) the effect of the term structure factors on the explanatory variables. When the explanatory variables follow a linear time series process we can include them in the term structure model using the methods described in Diebold et al. (2006). In particular, let x_t denote the $m \times 1$ vector of explanatory variables. We can include x_t in the vector autoregressive process for the term structure factors as follows:

$$\begin{pmatrix} f_t - \mu_f \\ x_t - \mu_x \end{pmatrix} = H \begin{pmatrix} f_{t-1} - \mu_f \\ x_{t-1} - \mu_x \end{pmatrix} + \xi_t, \qquad \xi_t \sim N(0, Q), \tag{7}$$

where the top left hand blocks of H and Q contain Φ and Σ_η, respectively. The off-diagonal elements can be used to study interactions among the term structure factors and the explanatory variables.

In this paper the variables x_t are related to monetary policy during crisis times and cannot be argued to follow a linear Gaussian time series model. Therefore we need to modify the approach of Diebold et al. (2006). In particular, we model the explanatory variables by non-Gaussian densities that are defined conditional on a set of latent dynamic factors θ_t.

We define

$$x_{j,t} \sim p_j(x_{j,t}|\theta_t), \qquad j = 1, \ldots, m, \tag{8}$$

where the density $p_j(\cdot|\cdot)$ can be different for each j. The factors for the explanatory variables are jointly modeled with the factors for the term structure in

$$(\alpha_t - \mu_\alpha) = H(\alpha_{t-1} - \mu_\alpha) + \xi_t, \qquad \xi_t \sim N(0, Q), \tag{9}$$

where $\alpha_t = (f_t', \theta_t')'$ and the off diagonal blocks of H and Q capture the interaction between the factors θ_t and f_t. Examples for non-Gaussian monetary policy variables that we consider in the empirical illustration are given below. In principal, a large number of mixed-

measurement explanatory variables can be included in our model.

Example 1: Interbank lending rate

The euro overnight index average (EONIA) is a benchmark rate for overnight unsecured interbank loans in the euro area. It is closely related to ECB monetary policy rates and therefore proxy its monetary policy stance. During the recent financial crisis the EONIA interbank rate declined to close to zero.[1] Modeling the interbank lending rate without this non-linearity leads to incorrect inference regarding the effect of interbank rates on the yield curve. Several densities can be considered that are able to incorporate this non-linearity. A convenient choice is the log-normal density, which is able to mimic the observation that the interbank rate has never actually reached zero but has been low for an extended period and cannot become negative. Let $x_{1,t}$ denote the interbank rate. The conditional log density for $x_{1,t}$ given $\theta_{1,t}$ is given by

$$\log p_1(x_{1,t}|\theta_{1,t}) = -\log x_{1,t}\sigma\sqrt{2\pi} - \frac{(\ln x_{1,t} - \theta_{1,t})^2}{2\sigma^2}, \tag{10}$$

where $\theta_{1,t}$ is the log mean scale parameter and σ is the shape parameter.

Example 2: bond market purchases

A second example of the monetary policy of the ECB are the bond market purchases that were conducted under the Securities Markets Programme. Within this program the ECB bought government bonds at different maturities at different points in time in secondary markets. We model the purchases amount $x_{2,t}$ by a Poisson distribution with time-varying log intensity $\theta_{2,t}$. Purchase amounts are non-negative, and zero when the program is inactive. The conditional log density can be expressed by

$$\log p_2(x_{2,t}|\theta_{2,t}) = x_{2,t}\theta_{2,t} - \exp\theta_{2,t} - \log x_{2,t}!. \tag{11}$$

The empirical section of this paper provides more details for the asset purchase program.

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[1]The EONIA rate does not go negative as banks would prefer to hold reserves at the central bank than to lend funds at a negative interest rate, provided that the central bank reserves are remunerated at a positive or zero interest rate.

2.4   Discussion

A multitude of extensions of this baseline model are possible in principle. First, as an alternative to changing the distribution of ϵ_t in the measurement equation (3) it would be possible to change the distribution of η_t in the state equation (4) instead; see for example Cakmakli (2011). Changes in yields would then likely be captured by more volatile term structure factors and monetary policy signals. Second, time varying volatility in both the measurement and state equation could be considered, as implemented by Stock & Watson (2007) and Shephard (2013) in the context of univariate inflation models. Third, our joint model could be extended to include correction terms that make it arbitrage free, see Christensen et al. (2011). Such no-arbitrage restrictions are likely to be satisfied in well-functioning markets during non-crisis times. Fourth, the country-specific yield curves could be combined in a regional model of the euro area, as in the global model of Diebold et al. (2008). We leave these interesting extensions for future research.

The robust joint model for the term structure and the monetary policy is given by equations (5), (6), (8) and (9). The model parameters are summarized in the vector ψ, which contains the parameters pertaining to the extended dynamic Nelson Siegel model as well as the parameters for the monetary policy model.

3   Estimation method

While the extensions for the dynamic Nelson Siegel model that we propose in Sections 2.2 and 2.3 are easy to motivate, they do have the consequence that we can no longer use standard Kalman filter methods for parameter estimation and for the extraction of the latent stochastic factors. For example, by changing the Gaussian density to the Student's t density with time-varying volatility in (5) and adding the non-Gaussian monetary policy variables we can no longer express the likelihood in closed form. Therefore, we need to resort to simulation methods for parameter estimation.

3.1 Importance sampling

We summarize the observations for the yields and the monetary policy in the vector $z = (y', x')'$, where $y = (y_1', \ldots, y_T')'$ and $x = (x_1', \ldots, x_T')'$. The loglikelihood for observation vector z is defined by $\ell(\psi; z) = \log p(z; \psi)$, where $p(z; \psi)$ denotes the joint density of the observations for a given parameter vector ψ. In the remainder of this section we drop the dependence on parameter vector ψ for notational convenience and define $\log p(z) = \log p(z; \psi)$.

The complete joint density of the endogenous variables is given by $p(z, \alpha, h)$. It follows that we must integrate both h and α from the complete joint density to obtain the marginal likelihood. We approach this problem sequentially starting with the integral over h. In particular, we can the express the marginal density $p(z)$ by

$$p(z) = \int_h p(z|h)p(h) \, \mathrm{d}h, \tag{12}$$

where $p(h)$ is implied by model (6) and $p(z|h)$ is unknown in closed form. More specifically, $p(z|\alpha, h)$ is defined in closed form by the joint model given in Section 2, but the integral over α cannot be evaluated analytically since $p(z|\alpha, h)$ has non-Gaussian features.

The integral in (12) is high dimensional and we use the importance sampling technique (see Ripley (1987)) to rewrite the integral as

$$p(z) = \int_h \frac{p(z|h)p(h)}{g_1(h|z)} g_1(h|z) \, \mathrm{d}h = g_1(z) \int_h \frac{p(z|h)}{g_1(z|h)} g_1(h|z) \, \mathrm{d}h, \tag{13}$$

where $g_1(h|z)$ is the importance density and the second equality follows as we impose $g_1(h) \equiv p(h)$.

Next, we outline the construction of the importance density $g_1(h|z)$. In general $g_1(h|z)$ should be proportional to $p(z, h)$, easy to sample from and easy to compute. Meeting the first requirement is complicated for the joint model since $p(z|h)$ cannot be expressed in closed form. Our strategy is as follows. We linearize the joint model $p(z|h, \alpha)$ and then integrate out α analytically from the linearized model. Based on this approximating linearized model, which only depends on h, we construct the importance density $g_1(h|z)$ using the methods developed in Shephard & Pitt (1997), Durbin & Koopman (1997) and Jungbacker & Koopman

(2007). The details for the construction are given in Appendix A.

After the importance density is constructed we may approximate the integral in (13) by Monte Carlo simulation. In particular, the Monte Carlo approximation for the likelihood is given by

$$\hat{p}(z) = g_1(z) \ M^{-1} \sum_{i=1}^{M} \frac{p(z|h^{(i)})}{g_1(z|h^{(i)})}, \tag{14}$$

where the samples $h^{(i)}$ are drawn from $g_1(h|z)$, for $i = 1, \ldots, M$.

To evaluate the non-Gaussian densities $p(z|h^{(i)})$ in (14), for $i = 1, \ldots, M$, we also consider an importance sampling approach. In particular, for each given $h^{(i)}$ we may write

$$
\begin{aligned}
p(z|h^{(i)}) &= \int_{\alpha} p(z|\alpha; h^{(i)}) p(\alpha) \ \mathrm{d}\alpha \\
&= \int_{\alpha} \frac{p(z|\alpha; h^{(i)}) p(\alpha)}{g_2(\alpha|z; h^{(i)})} g_2(\alpha|z; h^{(i)}) \ \mathrm{d}\alpha, \\
&= g_2(z) \int_{\alpha} \frac{p(z|\alpha; h^{(i)})}{g_2(z|\alpha; h^{(i)})} g_2(\alpha|z; h^{(i)}) \ \mathrm{d}\alpha, \tag{15}
\end{aligned}
$$

where $g_2(\alpha|z; h^{(i)})$ is the second importance density, which is constructed given z and the sampled volatility path $h^{(i)}$. The first equality in (15) follows as α and h are considered independent and the third equality follows as we impose $g_2(\alpha) = p(\alpha)$. The density $p(z|\alpha; h^{(i)})$ includes mixed-measurements. For example, $p(y|\alpha; h^{(i)})$ is considered to be equal to the Student's t density and $p(x|\alpha; h^{(i)})$ is equal to a variety of densities; see Section 2.3. The construction of importance densities for mixed-measurement non-Gaussian time series is considered in Koopman, Lucas & Schwaab (2012) and we follow their approach. The details for the construction of $g_2(\alpha|z; h^{(i)})$ are given in Appendix A.

A Monte Carlo average for the integral in (15) is given by

$$\hat{p}(z|h^{(i)}) = g_2(z) \ M^{-1} \sum_{j=1}^{M} \frac{p(z|\alpha^{(j)}, h^{(i)})}{g_2(z|\alpha^{(j)}; h^{(i)})}, \tag{16}$$

where the samples $\alpha^{(j)}$ are drawn from $g_2(\alpha|z; h^{(i)})$ for $j = 1, \ldots, M$. The estimate $\hat{p}(z|h^{(i)})$ replaces $p(z|h^{(i)})$ in (13).

Convergence of $\hat{p}(z)$ to $p(z)$ is guaranteed by the law of large numbers. The convergence

rate depends on the variance of the ratios

$$w_1^{(i)} = \frac{\hat{p}(z|h^{(i)})}{g_1(z|h^{(i)})} \quad \text{and} \quad w_2^{(j)}(h^{(i)}) = \frac{p(z|\alpha^{(j)}; h^{(i)})}{g_2(z|\alpha^{(j)}; h^{(i)})}, \tag{17}$$

for $i = 1, \ldots, M$ and $j = 1, \ldots, M$. The variances of the sequences $w_1^{(i)}$ and $w_2^{(j)}(h^{(i)})$ must be finite in order to guarantee a $\sqrt{M}$ convergence rate; see Geweke (1989). For $w_2^{(j)}(h^{(i)})$ this must hold for each sampled path $h^{(i)}$. Thus, in total for $M + 1$ sequences of weights the variances must exist. In our empirical applications we test for this using the extreme value based tests of Koopman et al. (2009).

4 The empirics of non-standard monetary policy in the euro area

We adopt our extended model for sovereign yields and monetary policy measurements to data from four large countries in the euro area: Germany, France, Italy and Spain. We aim to study the interaction between the country-specific yield curves and monetary policy measurements of the European Central Bank. We include a euro overnight interbank lending rate (EONIA) as a proxy of its monetary policy stance, and later also consider bond market purchases that were conducted within the Securities Market Programme (SMP) in the Italian and Spanish debt markets.[2] We consider the confidential country-level breakdown of the bond purchases in our analysis.

4.1 Data

4.1.1 Yield data

Our empirical study is based on data for euro area sovereign bond yields. We construct zero-coupon yields using the method discussed in Brousseau (2002), which in turn is closely related to the Fama & Bliss (1978) procedure. We refer to Brousseau (2002) for a detailed discussion

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[2]At the end of 2012 the ECB held €99.0bn in Italian sovereign bonds and €43.7bn in Spanish debt that was acquired within the SMP. In addition it held positions in Portuguese (€21.6bn), Irish (€13.6bn), and Greek (€30.8bn) debt securities, see the ECB (2013) annual report. No SMP purchases were made in the bond markets of France and Germany.

of the construction method. We construct panels of zero-coupon yields for Germany, France, Italy and Spain. Each panel consists of yields for $N = 10$ maturities from 1 January 2004 up to 31 December 2012. The 10 maturities are evenly spread between 1 and 10 years. We consider weekly observations, which are obtained by taking every Friday end-of-day yield.

Figure 1 presents a subset of the yield data. The yields suggest the presence of an underlying factor structure. Although the yield series vary heavily over time for each of the maturities, a strong common pattern is apparent in the series. For most days, the yield curve is an upward sloping function of time to maturity. Two instances of an almost inverted yield curve can be detected, however. For all countries an inverted curve occurred, or nearly occurred, in 2008 corresponding to the bankruptcy of Lehman Brothers, a significant event during the global financial crisis. For Italy and Spain a second period of an almost inverted curve occurred in 2012 during a particularly intense phase of the euro area sovereign debt crisis.

After 2010 the overall trend for French and German yields is downwards, whereas the trend for Italian and Spanish yields is upwards. This pattern is consistent with capital flows from the latter stressed countries to the former non-stressed countries during the sovereign-debt crisis. Shorter term maturities are more volatile than longer term maturities for all four countries.

We distinguish four different sampling periods in our empirical analysis: 1. full (2004-1 until 2012-52), 2. pre-crisis (2004-1 until 2007-52), 3. financial crisis (2008-1 until 26-2010) and 4. sovereign-debt crisis (2010-27 until 2012-52). Tables 1 and 2 provide descriptive statistics for the yields. For the 1, 5 and 10 year maturities we report the mean, standard deviation, skewness, kurtosis, minimum and maximum statistics. The summary statistics confirm that the yield curves tend to be upward sloping and that volatility is lower for the rates with longer time to maturity.

In the pre-crisis period all the sample moments are approximately similar for all countries. The skewness and kurtosis statistics indicate that the yields are almost normally distributed. During the financial crisis sample the variance increases for the 1 and 5 year maturities for all countries, while the variance for the 10 year maturity yields remains approximately unchanged. The level of the lower maturities decreases by almost 400 basis points for all countries. Both the sample skewness and kurtosis statistics increase. Finally, during the

Country	Sample	Maturity	Mean	Sd	Skew	Kurt	Min	Max
France	*Full*	1	2.03	1.35	0.28	1.78	0.05	4.57
		5	2.93	0.87	-0.34	2.64	0.73	4.72
		10	3.64	0.56	-0.56	2.97	2.10	4.70
	Pre	1	2.92	0.81	0.37	1.51	1.90	4.35
		5	3.45	0.49	0.07	2.22	2.52	4.51
		10	3.88	0.38	-0.23	2.02	3.11	4.60
	Fin	1	1.96	1.48	0.60	1.64	0.36	4.57
		5	3.08	0.72	0.58	2.27	1.82	4.72
		10	3.87	0.38	0.29	2.45	3.04	4.70
	Sov	1	0.68	0.45	0.22	1.91	0.05	1.48
		5	1.94	0.60	-0.23	2.18	0.73	3.01
		10	3.02	0.45	-0.27	2.06	2.10	3.81
Germany	*Full*	1	1.95	1.40	0.21	1.73	-0.07	4.51
		5	2.70	1.08	-0.53	2.41	0.33	4.59
		10	3.32	0.83	-0.85	2.84	1.27	4.53
	Pre	1	2.91	0.80	0.38	1.53	1.89	4.35
		5	3.44	0.48	0.06	2.24	2.53	4.49
		10	3.84	0.37	-0.28	2.11	3.09	4.53
	Fin	1	1.89	1.47	0.64	1.70	0.26	4.51
		5	2.86	0.76	0.59	2.31	1.53	4.59
		10	3.46	0.48	0.41	2.25	2.54	4.52
	Sov	1	0.47	0.48	0.64	2.14	-0.07	1.45
		5	1.34	0.72	0.34	1.84	0.33	2.80
		10	2.25	0.64	0.26	1.78	1.27	3.49

Table 1: Descriptive statistics for the euro area yield curve data.

Country	Sample	Maturity	Mean	Sd	Skew	Kurt	Min	Max
Italy	*Full*	1	2.63	1.11	0.39	2.54	0.77	6.45
		5	3.74	0.77	1.08	4.75	2.61	7.10
		10	4.47	0.68	1.20	4.82	3.31	6.96
	Pre	1	2.94	0.80	0.36	1.49	1.91	4.35
		5	3.55	0.50	0.04	2.10	2.63	4.60
		10	4.07	0.38	-0.31	2.01	3.31	4.77
	Fin	1	2.22	1.38	0.56	1.60	0.77	4.66
		5	3.54	0.63	0.48	2.16	2.69	5.00
		10	4.42	0.28	0.48	2.40	4.02	5.09
	Sov	1	2.54	1.06	1.52	5.39	1.34	6.45
		5	4.24	1.00	0.50	2.84	2.61	7.10
		10	5.15	0.79	0.26	2.43	3.79	6.96
Spain	*Full*	1	2.70	1.07	0.05	2.26	0.67	5.92
		5	3.75	0.78	0.90	3.96	2.53	6.98
		10	4.43	0.84	1.05	3.51	3.13	7.10
	Pre	1	2.92	0.81	0.39	1.52	1.89	4.37
		5	3.46	0.50	0.05	2.20	2.53	4.53
		10	3.89	0.38	-0.21	2.01	3.13	4.60
	Fin	1	2.18	1.38	0.51	1.59	0.67	4.54
		5	3.42	0.55	0.57	2.29	2.70	4.74
		10	4.20	0.25	0.63	2.45	3.85	4.81
	Sov	1	2.86	0.88	0.81	3.83	1.30	5.92
		5	4.52	0.80	0.31	3.25	3.01	6.98
		10	5.52	0.71	-0.19	2.86	4.08	7.10

Table 2: Descriptive statistics for the euro area yield curve data.

sovereign-debt crisis period large differences appear between France and Germany on the one hand and Italy and Spain on the other hand. The yields of France and Germany reach their lowest point for the entire sample and the variance decreases towards values that are observed for the pre-crisis sample. In contrast, the yields of Italy and Spain reach their respective peaks and the variance of the higher maturities increases.

We draw two main conclusions from our inspection of the yield data. First, large within-country differences exist in the time series properties across sub-periods for all four countries. Both the mean and the variance of the yields clearly change over time. Second, cross-country differences are relatively small prior to the sovereign-debt crisis, and become sizeable thereafter. During the sovereign-debt crisis the yields of France and Germany approximately return to their pre-crisis properties, whereas the yields for Italy and Spain continue to increase and become even more volatile.

4.1.2 Monetary policy measurements: EONIA rate and asset purchases

The primary objective of the European Central Bank is to maintain price stability within the euro area. In its pursuit of price stability, it aims to maintain inflation rates below, but close to, an inflation target of 2% over the medium term. We distinguish between standard (interest rate) and non-standard monetary policy measures (such as outright government bond purchases) that are employed to achieve the primary objective.

In our empirical analysis we include the weekly average of the EONIA rate. The EONIA rate closely tracks the ECB's monetary policy rates, and is therefore useful as a proxy of its monetary policy stance. The weekly average is taken to avoid outliers that may occur when financial institutions target their respective reserve requirements. Figure 1 contains our weekly average EONIA rate. The EONIA rate is approximately equal to the respective 1 year sovereign yield for France and Germany during the entire sample from 2004-2012. For Italy and Spain, the EONIA rate is also close to the 1 year yield until early 2010, and then diverge markedly. EONIA is mostly below 1% from October 2008 onwards, and varies between 0.1% and 0.5% in 2012.

In May 2010 the ECB's Governing Council decided to conduct asset purchases in some euro area government bond markets in order to mitigate impairments to the monetary transmission mechanism by addressing the mal-functioning of certain government bond markets
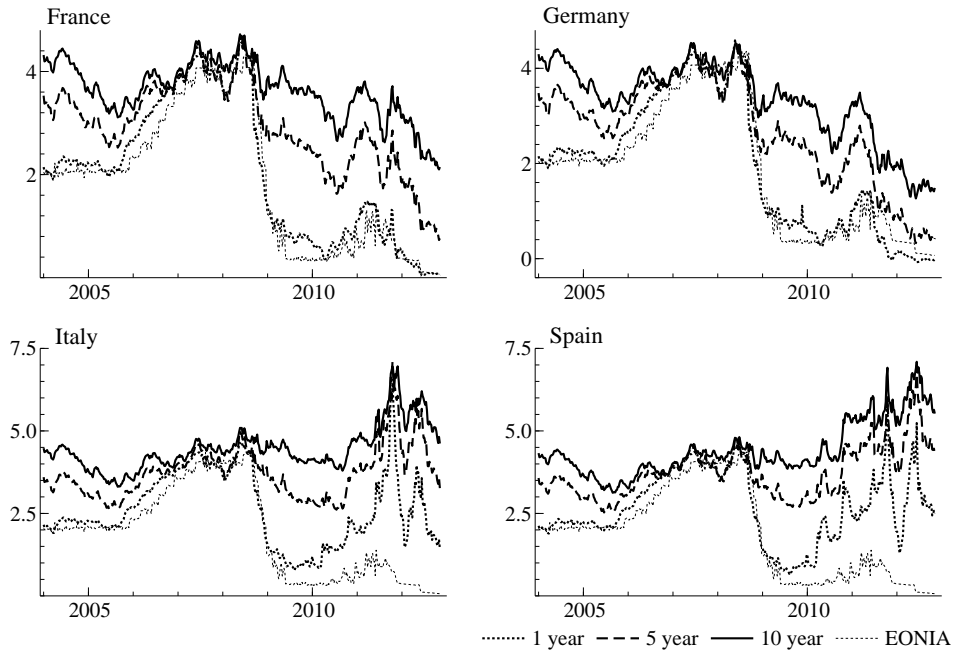
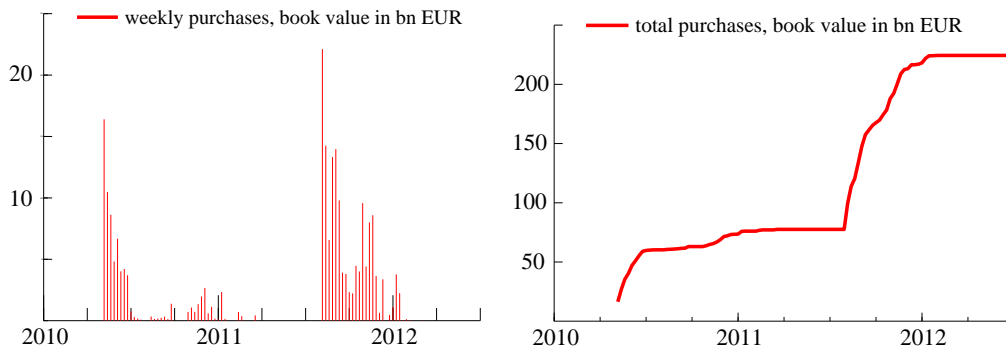Figure 1: Selection of zero-coupon government bond yields for selected euro area countries

Figure 2: Summary of the bond market purchases

in stressed countries, see for instance González-Páramo (2011) and ECB (2014). Implicit in the concept of impaired markets is the notion that government bond yields can be unjustifiably high and illiquid, see Constâncio (2011). The SMP consists of interventions in the form of outright secondary market purchases. Almost all purchases pertain to bonds with maturities between 2 and 10 years. We refer to Giannnone, Lenza, Pill & Reichlin (2011), Eser, Amaro, Iacobelli & Rubens (2012), and ECB (2014) for details.

Figure 2 presents the total weekly settled amount of all bond purchases within the SMP in billion euro. Approximately €214 billion (bn) of government bonds were acquired from 2010 to early 2012. The purchases in the beginning of 2010 were related to Greek, Portuguese and Irish bonds. The SMP was extended to include Spain and Italy from 08 August 2011 until late January 2012 (24 weeks). Clearly, the bond market purchases are non-negative, integer-valued, and often zero when the program is inactive. In our analysis we consider the average country-specific weekly amounts at par value.

4.2 Interest rate monetary policy in calm and storm

This section discusses the estimation results for the joint model for the yields and the EONIA rate. Non-standard monetary policy measures such as bond purchases are added in Section 4.3. The yields are modeled by the dynamic Nelson-Siegel model with Student's t errors and factor stochastic volatility; see equation (6). For simplicity we focus on term structure models with only two factors (level and slope). As usual, the first two factors capture almost all of the systemic variation in the yield data; see also Diebold et al. (2008).

The EONIA rate is modeled by the log normal distribution where the log mean $\theta_{1,t}$ is updated jointly with the term structure factors; see Section 2.3. For this model we estimate the parameter vector ψ for the four countries and distinguish the four sample periods introduced in Section 4.1. We fix the degrees of freedom $\nu = 10$ to allow for heavy tails in the errors. Experiments with lower degrees of freedom gave similar results. The other parameters are estimated using the simulation methods that are developed in Section 3. We use $M = 100$ draws from the importance densities to approximate the likelihood. For each model that we estimate we check the variance in the importance sampling weights to ensure $\sqrt{M}$ convergence for the Monte Carlo likelihood in (14).

4.2.1 Parameter and factor estimates

The parameter estimates that relate to matrices H and Q are shown in Tables 3 and 4. The matrices H and Q capture the lagged and contemporaneous interactions, respectively, between the term structure and monetary policy factors. In particular, the far right column and the bottom row capture the interactions between the level and slope factors and the (log mean of) the EONIA rate.

For the pre-crisis sample (the 'calm') the EONIA rate has a positive lagged effect on the level factor. This holds for all countries and the coefficients for the pre-crisis sample range between 0.06 and 0.09. The lagged interaction between the slope factor and the interbank rate is small and the contemporaneous interaction is close to zero for the pre-crisis sample. As a result, a decrease in short term interbank rates is mainly transmitted into the term structure by lowering the level factor. Statistical significance is low, which may reflect our use of weekly data in sub-samples.

During the financial crisis the transmission of the interbank rate into the level factor is reduced for all countries. The lagged interaction coefficients become zero or even negative. The volatility in the factors increases significantly, in particular for the slope factors. The interaction between the level and slope factors becomes significantly negative for all four countries, i.e., an increasing level factor is associated with a decreasing slope factor in this period. This may reflect the central bank's aggressive cuts of its main policy rate during this time from 4.25% to 1%, while long term sovereign yields remained at elevated levels.

During the sovereign debt crisis, pronounced cross-country differences emerge. The EONIA rate is negatively related to the level factor for Italy and Spain. Short term sovereign yields decrease, reflecting in part additional monetary policy rate cuts during 2012 to 0.5%, while medium and long term sovereign yields rise, probably due to growing sovereign credit risk concerns, see ECB (2014). Arguably, the very low interbank rates are not transmitted adequately along the yield curve for these two countries. This is in contrast to Germany and France, where the lagged effects of the EONIA rate on the level factor is positive for France, and only slightly negative for Germany.

Figure 3 presents the smoothed means for the term structure factors and monetary policy factor together with the sample split points. The smoothed estimates are computed as

Table 3: Parameter estimates for the interaction between the euro area yield curves and the interbank rate.

Country	Sample	H			Q		
France	*Full*	0.98 (0.02)	-0.01 (0.01)	0.01 (0.01)	0.01 (0.00)	0.03 (0.00)	0.01 (0.00)
		-0.02 (0.02)	0.99 (0.01)	0.00 (0.02)	-0.01 (0.00)	0.00 (0.00)	0.00 (0.00)
		0.02 (0.01)	0.02 (0.01)	0.97 (0.01)	-0.00 (0.00)	0.00 (0.00)	0.01 (0.00)
	Pre	0.99 (0.03)	-0.01 (0.02)	0.07 (0.08)	0.01 (0.00)	0.01 (0.00)	
		-0.05 (0.03)	0.97 (0.03)	-0.00 (0.09)	-0.01 (0.01)	0.00 (0.00)	
		0.01 (0.00)	0.01 (0.00)	0.96 (0.01)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
	Fin	0.87 (0.06)	-0.03 (0.02)	0.01 (0.03)	0.02 (0.00)	0.06 (0.01)	
		0.02 (0.13)	1.02 (0.04)	-0.04 (0.05)	-0.02 (0.00)	0.00 (0.00)	
		-0.01 (0.04)	0.03 (0.01)	0.94 (0.02)	-0.00 (0.00)	0.00 (0.00)	0.01 (0.00)
	Sov	0.92 (0.04)	-0.00 (0.03)	0.03 (0.04)	0.02 (0.00)	0.05 (0.01)	
		0.02 (0.06)	0.95 (0.05)	-0.00 (0.05)	-0.02 (0.00)	0.01 (0.00)	
		0.08 (0.05)	0.10 (0.04)	0.90 (0.04)	-0.00 (0.00)	0.01 (0.00)	0.02 (0.01)
Germany	*Full*	0.99 (0.01)	0.01 (0.01)	-0.01 (0.02)	0.02 (0.00)	0.03 (0.00)	
		0.00 (0.01)	0.99 (0.01)	-0.00 (0.02)	-0.02 (0.00)	0.00 (0.00)	
		0.03 (0.01)	0.02 (0.00)	0.97 (0.01)	-0.00 (0.00)	0.00 (0.00)	0.01 (0.00)
	Pre	0.99 (0.03)	-0.00 (0.02)	0.06 (0.08)	0.01 (0.00)	0.01 (0.00)	
		-0.05 (0.04)	0.97 (0.03)	0.01 (0.10)	-0.01 (0.01)	0.00 (0.00)	
		0.01 (0.00)	0.01 (0.00)	0.96 (0.01)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
	Fin	0.81 (0.05)	0.02 (0.01)	-0.05 (0.02)	0.02 (0.00)	0.06 (0.01)	
		0.17 (0.10)	1.00 (0.02)	0.01 (0.04)	-0.02 (0.00)	0.00 (0.00)	
		0.04 (0.04)	0.03 (0.01)	0.95 (0.02)	-0.00 (0.00)	0.00 (0.00)	0.01 (0.00)
	Sov	0.98 (0.03)	0.02 (0.04)	-0.01 (0.03)	0.04 (0.01)	0.06 (0.01)	
		0.03 (0.04)	0.95 (0.05)	-0.00 (0.04)	-0.04 (0.01)	0.00 (0.00)	
		0.07 (0.03)	0.06 (0.03)	0.95 (0.03)	0.00 (0.00)	0.00 (0.00)	0.02 (0.01)

Country	Sample	H			Q		
Italy	*Full*	$1.02_{0.01}$	$0.03_{0.01}$	$-0.01_{0.01}$	$0.02_{0.00}$		
		$-0.09_{0.02}$	$0.94_{0.01}$	$0.02_{0.02}$	$-0.02_{0.00}$	$0.07_{0.01}$	
		$-0.02_{0.01}$	$-0.01_{0.00}$	$0.99_{0.01}$	$-0.00_{0.00}$	$0.00_{0.00}$	$0.01_{0.00}$
	Pre	$0.99_{0.04}$	$-0.00_{0.04}$	$0.07_{0.12}$	$0.01_{0.00}$		
		$-0.04_{0.05}$	$0.97_{0.04}$	$-0.03_{0.13}$	$-0.01_{0.04}$	$0.02_{0.01}$	
		$0.01_{0.00}$	$0.01_{0.00}$	$0.95_{0.01}$	$-0.00_{0.04}$	$0.00_{0.00}$	$0.00_{0.00}$
	Fin	$0.98_{0.09}$	$-0.00_{0.05}$	$0.00_{0.05}$	$0.04_{0.01}$		
		$-0.23_{0.15}$	$0.86_{0.09}$	$0.04_{0.08}$	$-0.06_{0.01}$	$0.12_{0.02}$	
		$-0.05_{0.04}$	$0.01_{0.02}$	$0.94_{0.02}$	$-0.00_{0.00}$	$0.00_{0.00}$	$0.01_{0.00}$
	Sov	$0.95_{0.02}$	$0.05_{0.02}$	$-0.03_{0.03}$	$0.03_{0.01}$		
		$-0.05_{0.06}$	$0.87_{0.04}$	$0.16_{0.07}$	$-0.01_{0.01}$	$0.20_{0.03}$	
		$-0.04_{0.02}$	$0.00_{0.01}$	$0.98_{0.02}$	$-0.00_{0.00}$	$-0.00_{0.01}$	$0.02_{0.00}$
Spain	*Full*	$0.99_{0.01}$	$0.00_{0.01}$	$-0.00_{0.01}$	$0.02_{0.01}$		
		$-0.04_{0.02}$	$0.97_{0.01}$	$0.02_{0.02}$	$-0.00_{0.00}$	$0.05_{0.01}$	
		$-0.02_{0.01}$	$0.00_{0.01}$	$0.98_{0.01}$	$-0.00_{0.00}$	$-0.00_{0.00}$	$0.01_{0.00}$
	Pre	$0.98_{0.03}$	$-0.01_{0.03}$	$0.09_{0.08}$	$0.01_{0.00}$		
		$-0.03_{0.03}$	$0.99_{0.03}$	$-0.05_{0.09}$	$-0.00_{0.01}$	$0.01_{0.00}$	
		$0.01_{0.00}$	$0.01_{0.00}$	$0.96_{0.01}$	$-0.00_{0.00}$	$0.00_{0.00}$	$0.00_{0.00}$
	Fin	$0.91_{0.05}$	$-0.01_{0.02}$	$-0.02_{0.03}$	$0.02_{0.00}$		
		$-0.01_{0.15}$	$0.99_{0.04}$	$-0.02_{0.06}$	$-0.03_{0.01}$	$0.11_{0.02}$	
		$-0.01_{0.03}$	$0.03_{0.01}$	$0.93_{0.02}$	$-0.00_{0.00}$	$0.00_{0.00}$	$0.01_{0.00}$
	Sov	$0.93_{0.03}$	$0.02_{0.02}$	$-0.04_{0.04}$	$0.05_{0.001}$		
		$0.01_{0.06}$	$0.86_{0.04}$	$0.17_{0.07}$	$-0.01_{0.01}$	$0.15_{0.03}$	
		$-0.04_{0.02}$	$-0.01_{0.02}$	$0.98_{0.03}$	$0.00_{0.00}$	$-0.01_{0.01}$	$0.02_{0.01}$

Table 4: Parameter estimates for the interaction between the euro area yield curves and the interbank rate.
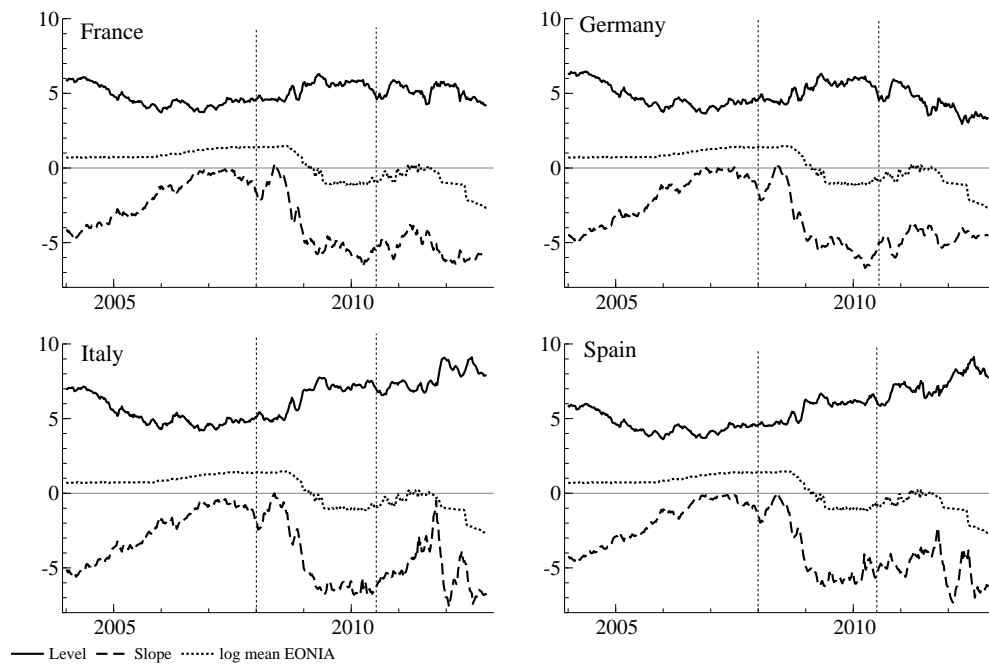
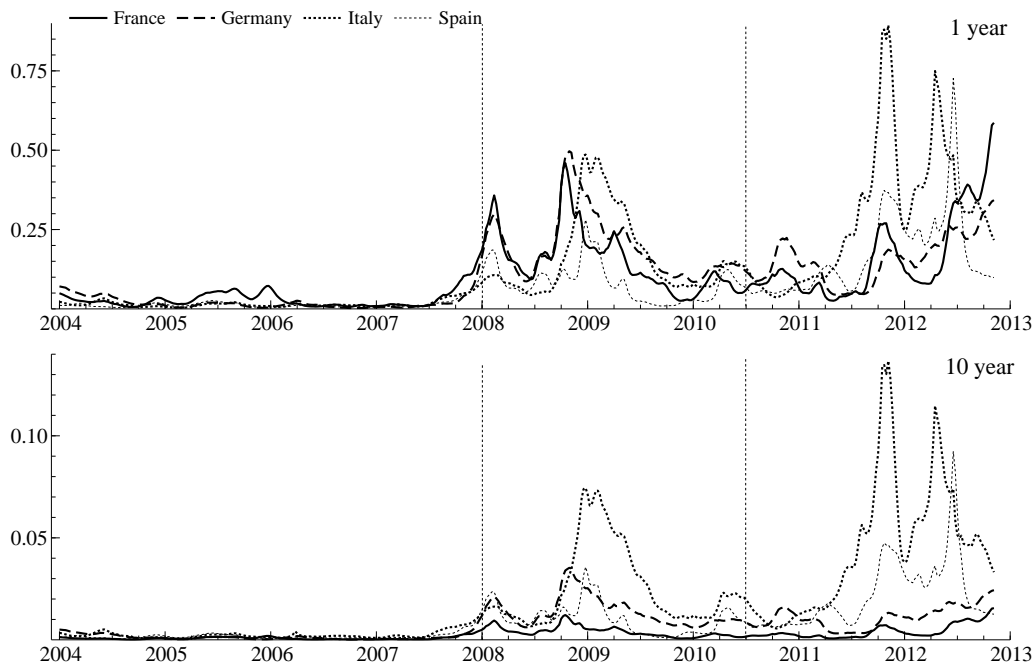Figure 3: Smoothed mean factors for selected euro area countries

Figure 4: Selection of smoothed volatility processes for selected euro area countries

discussed in Appendix B using the full sample parameter estimates and as reported in Tables 3 and 4. The factor estimates are as expected. For all countries the pre-crisis sample is characterised by decreasing level factors and increasing slope factors. During the financial crisis the volatility in the term structure factors increases and the slope factors decrease rapidly, along with the common log EONIA rate factor. During the sovereign-debt crisis the level factors of France and Germany are declining, whereas the level factors of Italy and Spain are increasing.

Figure 4 presents the smoothed volatility processes for the 1 year and 10 year maturities ($w_1^2 \exp h_t$ and $w_{10}^2 \exp h_t$). The volatility paths of the 1 year maturities are relatively similar across the four countries. There is little volatility prior to 2008, after which the volatility process increases and finally peaks during the most intense phase of the financial and sovereign-debt crisis in 2011-12. The cross-country differences are more pronounced for the respective 10 year maturities. For example, the volatility for Italian and Spanish bonds reaches their respective peaks during the sovereign-debt crisis, whereas the volatility for French and German yields remains relatively low.

4.2.2 Impulse response functions

This section investigates the implied dynamics of the parameter estimates in Tables 3 and 4 by computing generalized impulse response functions as suggested in Pesaran & Shin (1998). The impulse responses, strictly speaking, do not identify a causal relationship between the terms structure and monetary policy factors. However, they are informative when we compare the different dynamic responses across different sample periods.

Figures 5–8 show the impulse response functions that correspond to standardized shocks to the level and slope factors and the log mean of EONIA. Generally, the impulse responses are approximately similar across countries during the pre-crisis and financial crisis sub-samples, and differ substantially within each country across the three sub-samples considered. We focus our discussion mainly on the effects of an unexpected change in the log mean EONIA rate.

For the pre-crisis sample the impulse responses are approximately similar for all countries. A positive shock to the log EONIA rate is transmitted into the level factor fairly persistently and over a long time period. Also the slope factor shows a positive response which becomes
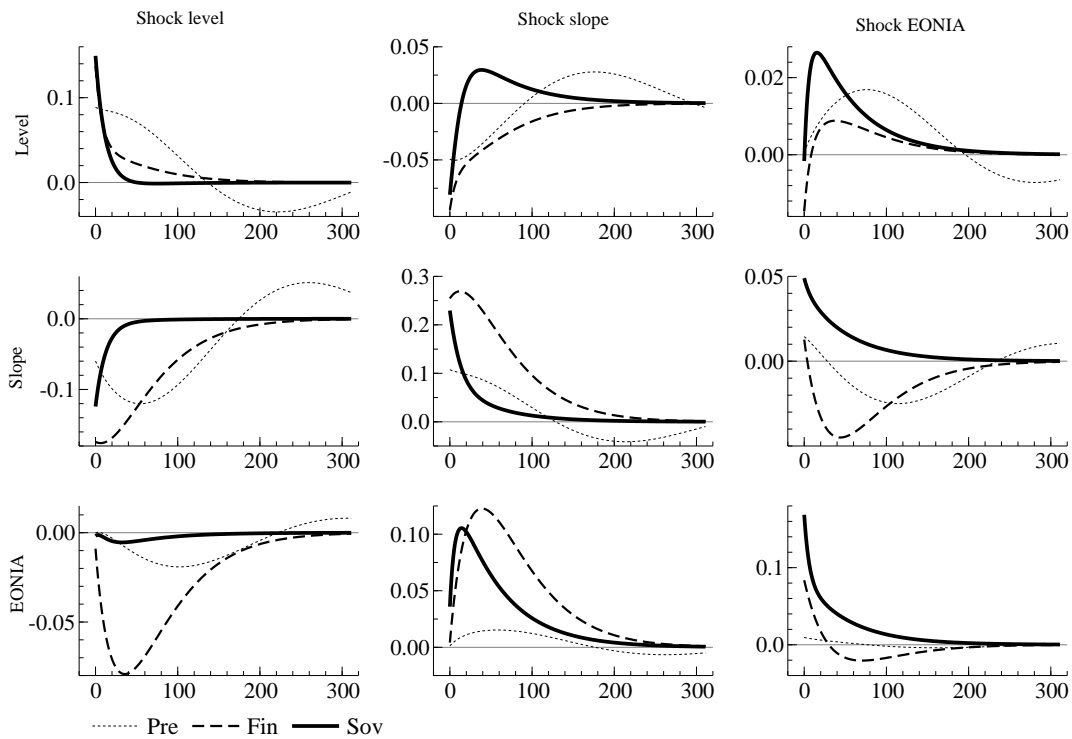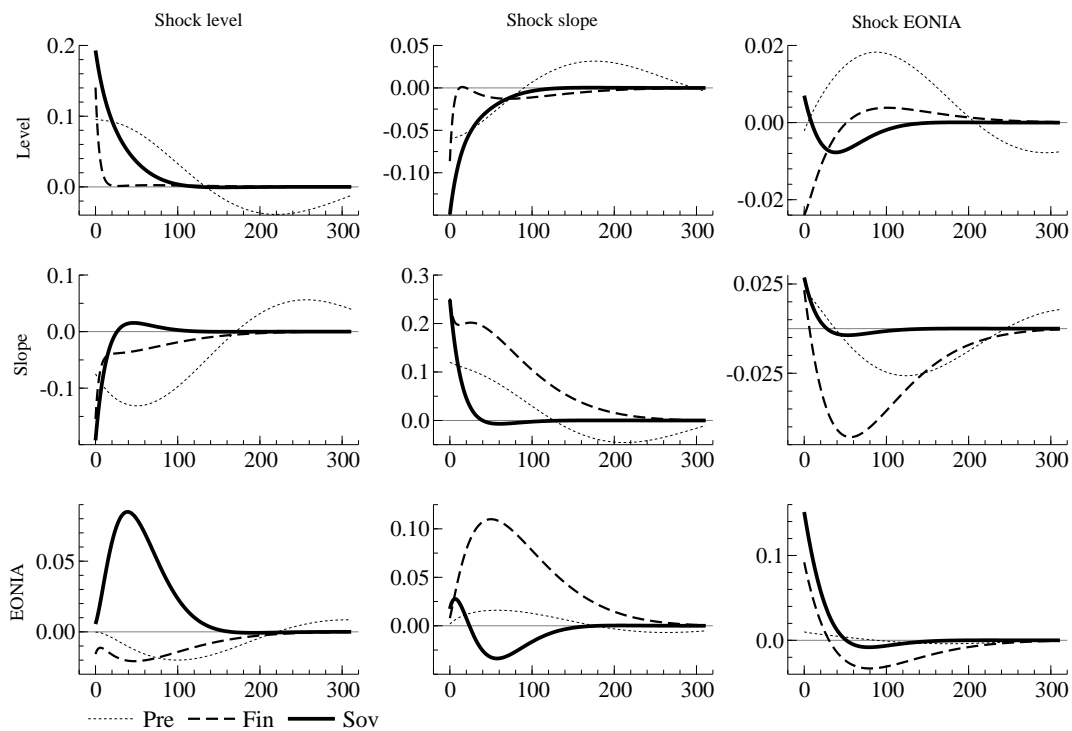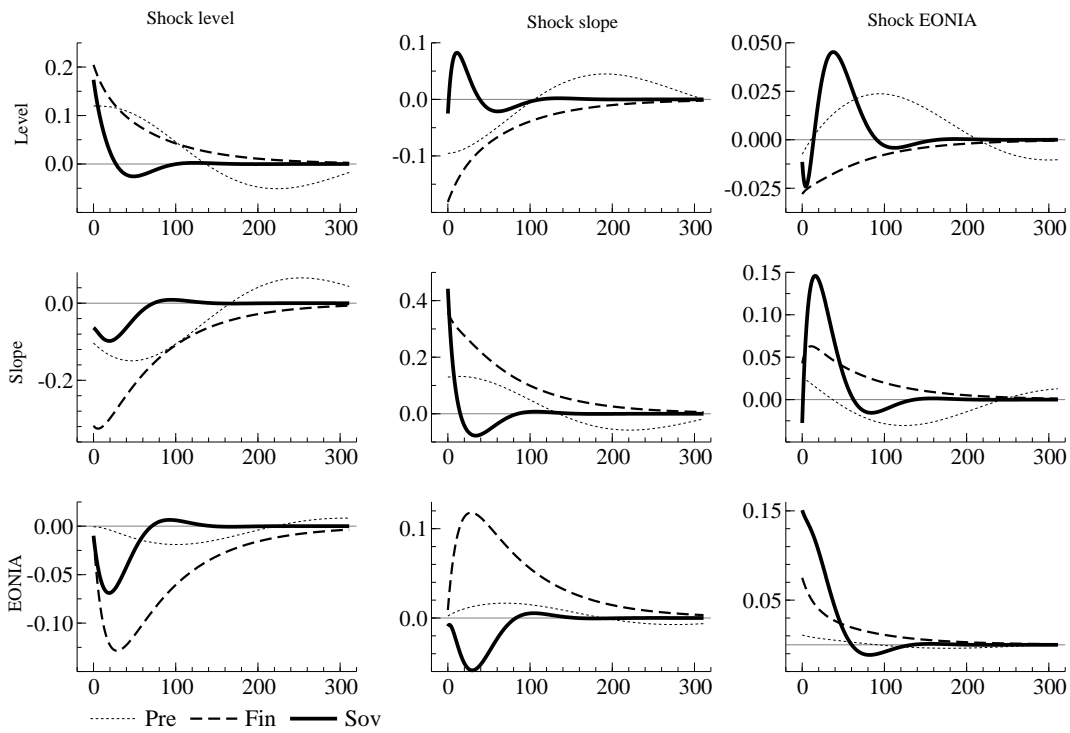
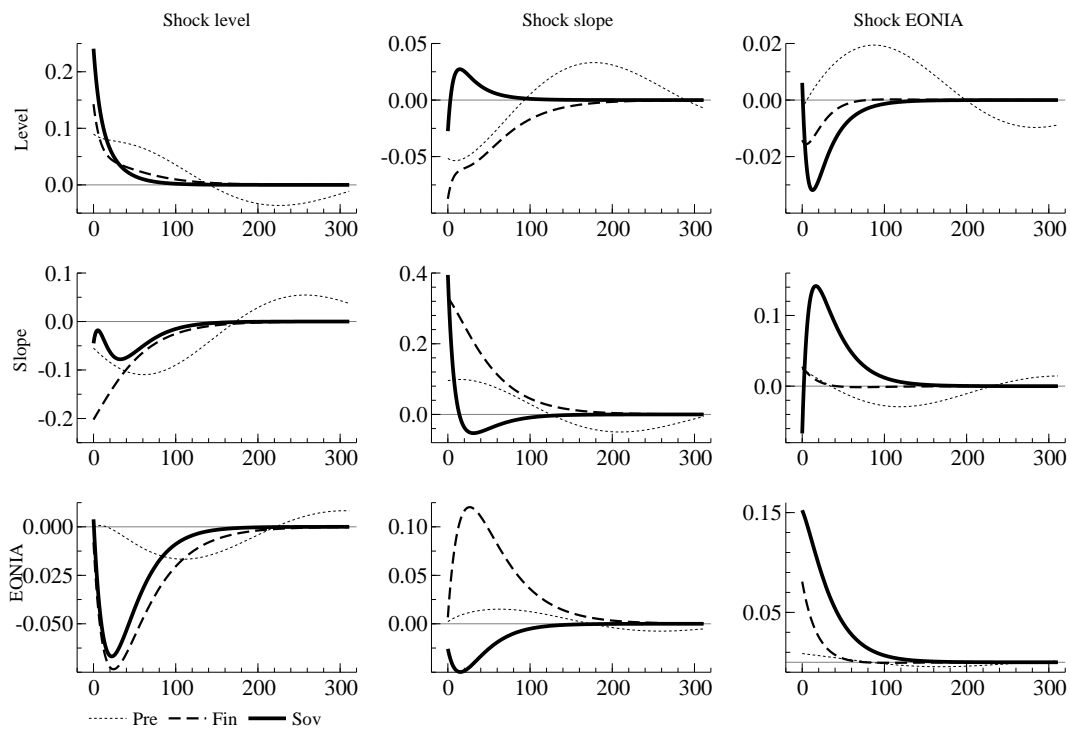Figure 5: France

Figure 6: Germany

Figure 7: Italy

Figure 8: Spain

negative only after approximately one year. As a result, the monetary policy shock is slowly absorbed in the level and slope factors for all countries.

During the financial crisis the impulse responses change in direction and persistence. A positive shock to the log EONIA rate now leads to a decreasing level factor for all countries. For France and Germany the shock eventually becomes positive, whereas for Italy and Spain it converges to zero without becoming positive. The response of the slope factor for France and Germany is initially positive but then decreases more rapidly. This suggests that an EONIA rate shock is transmitted only into the short end of the yield curve, and that its effect dies out quickly. We conclude that the financial crisis changed the statistical relationship between the EONIA rate and the term structure factors when compared to the pre-crisis period. For France, Germany and Italy the transmission appears to occur mainly via the slope factor instead of the level factor.

During the sovereign-debt crisis, differences across countries are most apparent. A positive shock to the log EONIA rate has positive effects on the level and slope factors for France and Italy, but its effects are in the opposite direction for Spain. A positive shock to the slope factor coincides with a rising level factor for France and Germany, but has less influence on the level factors of Italy and Spain. We conclude that monetary policy transmission from interbank rates into sovereign yield curves differs markedly across countries during the sovereign debt crisis, and that it is qualitatively different from the previous two sub-samples. Stark differences in monetary policy transmission from the single monetary policy rate to key interest rates in individual countries within the euro area during turbulent times has been one rationale for the adoption of non-standard monetary policy measures, see ECB (2014).

4.3 Standard and non-standard monetary policy measures

This section considers asset purchases of Italian and Spain debt securities as conducted within the ECB's SMP as a second monetary policy measurement. The weekly average of the respective purchases is modeled by a Poisson distribution with a time-varying intensity parameter. The log intensity of the Poisson distribution $\theta_{2,t}$ is jointly updated with the term structure factors and the log mean of the EONIA rate. For parameter estimation we employ

Italy	H	$0.83_{0.09}$	$0.04_{0.02}$	$-0.24_{0.08}$	$0.02_{0.01}$
		$-0.02_{0.16}$	$0.87_{0.04}$	$0.24_{0.29}$	$-0.00_{0.02}$
		$-0.22_{0.07}$	$-0.00_{0.02}$	$0.66_{0.11}$	$0.02_{0.01}$
		$-0.60_{0.84}$	$0.03_{0.11}$	$0.23_{1.58}$	$1.00_{0.11}$
	Q	$0.03_{0.01}$			
		$-0.01_{0.01}$	$0.19_{0.04}$		
		$-0.00_{0.00}$	$-0.00_{0.01}$	$0.02_{0.00}$	
		$-0.03_{0.02}$	$0.05_{0.09}$	$-0.07_{0.11}$	$0.44_{1.34}$
Spain	H	$0.98_{0.06}$	$-0.00_{0.03}$	$-0.07_{0.06}$	$0.01_{0.01}$
		$-0.32_{0.15}$	$0.98_{0.05}$	$0.41_{0.15}$	$-0.09_{0.02}$
		$-0.05_{0.04}$	$-0.01_{0.02}$	$0.99_{0.03}$	$-0.00_{0.01}$
		$-1.41_{0.61}$	$0.49_{0.17}$	$0.60_{0.40}$	$0.70_{0.08}$
	Q	$0.05_{0.01}$			
		$0.01_{0.01}$	$0.09_{0.02}$		
		$0.00_{0.00}$	$-0.01_{0.01}$	$0.02_{0.00}$	
		$-0.02_{0.04}$	$-0.03_{0.05}$	$0.03_{0.04}$	$0.20_{0.17}$

Table 5: Parameter estimates for the interaction between the euro area yield curves and the monetary policy factors. The ordering in the matrices is as follows: level factor, slope factor, log mean EONIA, and log purchase intensity.

yield data from our sovereign debt crisis sub-sample from mid-2010 to December 2012.

In principle, purchase interventions can impact yields in different interrelated ways. First, outright purchases add liquidity and depth to impaired secondary markets. At a minimum, required liquidity risk premia should fall and prices should be supported as a result. In addition, local supply effects in segmented markets as well as certain signaling effects may further support prices, see D' Amico & King (2012), De Pooter et al. (2013), and Eser & Schwaab (2013). Second, purchases may also indirectly affect the relationship between proxies of the monetary policy stance such as the EONIA rate and the sovereign yield curve factors, see González-Páramo (2011) and Constâncio (2011). This is the case, for example, if purchases were to contribute towards making interbank funding at low rates available to illiquid banks in stressed SMP countries. Non-standard monetary policies, though primarily targeted towards improving market depth and liquidity in volatile government bond markets, would then contribute towards making conventional interest rate policies more effective.

Table 5 reports the parameter estimates for the H and Q matrices. The bottom left element in the Q matrix shows that the log purchasing intensity is negatively correlated with

the level factor for Italy and Spain. Only the Spanish correlation is significant, however; this may reflect the fact that only in a limited number of weeks (less then 25) purchases were made in each debt market. This finding provides some tentative support that the outright purchases had a direct instantaneous effect on the level factor in Italy, and the level and slope factor in Spain. The bottom rows of the H matrices report that the lagged effects of the purchasing intensity are mean reverting for Italy and Spain. This indicates that the effect from the negative instantaneous correlation is temporary.

Figure 9 plots the respective generalized impulse response functions. These response functions correspond to a standardized shock to the level factor, slope factor, log mean EONIA rate, and log purchasing intensity, respectively. The instantaneous effect of an unexpected shock to the log purchasing intensity is negative for the level factor for both Italy and Spain. For Italy the effect becomes positive quickly, however, whereas for Spain it contributes to gradually lowering the level factor. Overall, this suggests that the effects of asset purchases on the term structure factors may be quite different across countries, possibly due to differences in the public policy response to the programme.

We find mixed evidence that the asset purchase interventions changed the relationship between EONIA rates to the term structure factors. For Italy, the interaction between the EONIA rate and the term structure factors changes when the purchases are included in the model. In Table 4, the lagged effect of the log mean of the EONIA rate on the level factor is -0.03 and not significant. When the purchases are included, the effect becomes significant with magnitude -0.24. However, this suggests that an unexpected decrease in the EONIA rate would lead to an increase in the Italian level factor. The effect on the slope factor remains positive when the purchases are included, but is not significant anymore. Such changes in the joint dynamics between the EONIA rate and the term structure factors are less strong for Spain. Comparing the third column of Figure 9 with the right columns in Figures 5 and 8 suggests only minor differences in the transmission of the common EONIA rate into the country-specific term structure factors.

Finally, Figure 10 shows the effect of a shock to the log purchasing intensity on the yields by combining the effect on the level and slope factors using the estimated Nelson-Siegel loading matrix. We show the effect on the 1, 5 and 10 year maturity yields. The differences between Italy and Spain are again large. For Italy the response becomes positive quickly
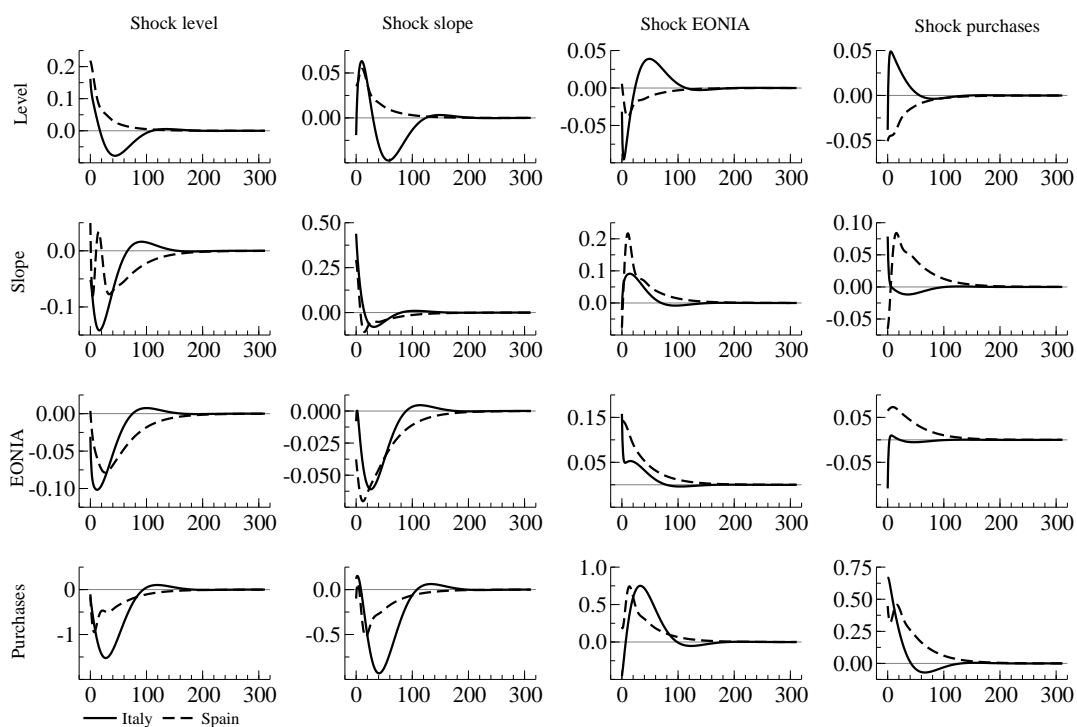
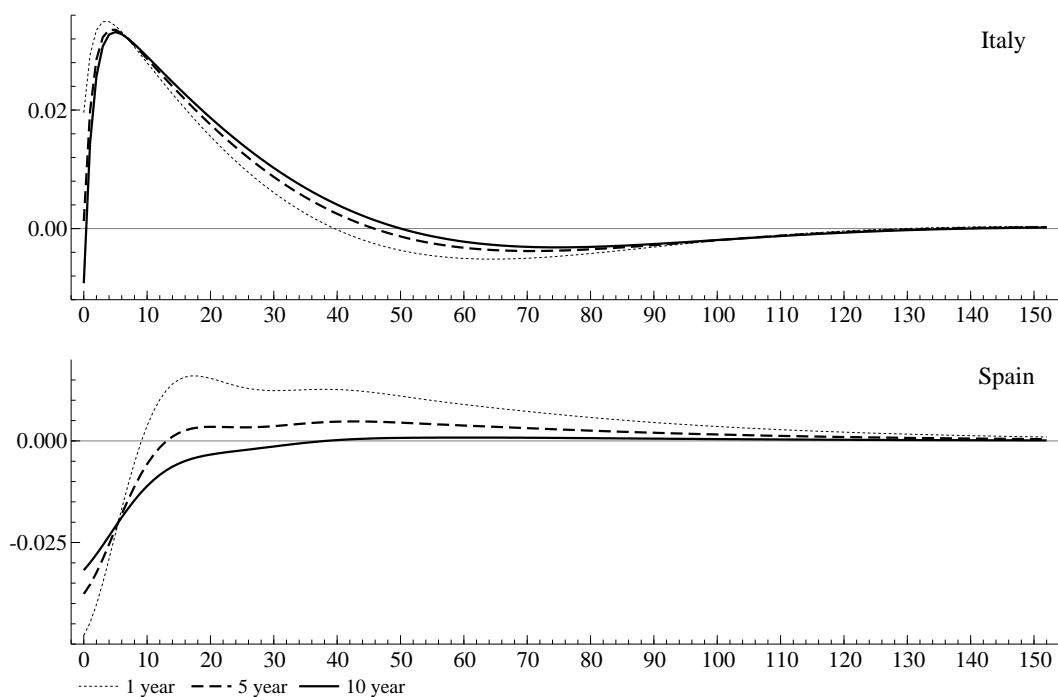Figure 9: Impulse responses for models including purchases

Figure 10: Impulse responses for the yields of standardized shocks to the log purchasing intensity

(2 weeks) whereas for Spain the shock remains negative for an extended number of weeks (10 weeks) and dies out more slowly. Due to a low number of observations for the SMP purchases we need to remain careful in our conclusions. We tentatively conclude that (i) the instantaneous impact of asset purchases undertaken within the SMP in Italy and Spain on the level of yields was likely negative, (ii) that the instantaneous impact is temporary, and that (iii) the persistence of the yield impact of asset purchases differs across the two countries considered in this study.

5 Conclusion

We have developed a nonlinear non-Gaussian modeling framework for analyzing the relationship between government bond yields and monetary policy during turbulent times. The government bond yields are modeled by an extended dynamic Nelson-Siegel model where the observations errors are modeled by the Student's t density with time-varying factor stochastic volatility. The monetary policy measurements are modeled by appropriate non-Gaussian densities that are defined conditional on a set of latent dynamic factors.

For the estimation of the joint model we have developed a simulation based estimation method that is based on the importance sampling technique. The feasibility of the method is due to the construction of conditional importance densities, which sample the term structure and monetary policy factors conditional on samples of the volatility factor.

The empirical application to euro area sovereign bond markets and the monetary policy measures of the ECB highlights the relevance and flexibility of the modeling framework. In this context we discussed the interaction between the term structure factors on the one hand and proxies of the monetary policy stance as well as bond purchases for several large countries in the euro area.

Appendix A

In this appendix we detail the construction of the importance densities $g_1(h|z)$ and $g_2(\alpha|z; h^{(i)})$ that are needed for the evaluation of the Monte Carlo likelihood in Section 3.

Constructing $g_1(h|z)$

We start with the construction of $g_1(h|z)$. We aim to choose $g_1(h|z)$ to follow a Gaussian distribution with mean equal to the mode of $p(h|z)$ and variance equal to the curvature around the mode. To achieve this we construct two instrumental linear Gaussian models. First, the linear Gaussian approximating model for the mean factors $\alpha_t = (f_t', \theta_t')'$ is given by

$$
\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} c_{1,t} \\ c_{2,t} \end{bmatrix} + \begin{bmatrix} \Lambda & 0 \\ 0 & I_k \end{bmatrix} \begin{bmatrix} f_t \\ \theta_t \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix} \tag{18}
$$

with

$$
\begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix} \sim NID \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} D_{1,t} & 0 \\ 0 & D_{2,t} \end{bmatrix} \right),
$$

where $c_{1,t} = 0$ and $D_{1,t}$ is diagonal with elements $d_{1,t,i}^2$, for $i = 1, \ldots, N$ on the main diagonal. The elements $d_{1,t,i}^2$ are given by

$$
d_{1,t,i}^2 = \frac{1}{\nu + 1} \left[(\nu - 2)\omega_i^2 \exp(h_t) + (y_{i,t} - \lambda_i' f_t)^2 \right], \tag{19}
$$

which follows from linearizing the Student's t density around its mode using the first derivative; see Durbin & Koopman (2012, Section 10.8.1). Further, $c_{2,t}$ and $D_{2,t}$ are found by solving

$$
\frac{\partial \log p(x_t|\theta_t)}{\partial \theta_t} = \frac{\partial \log g(x_t|f_t, \theta_t)}{\partial \theta_t}, \qquad \frac{\partial^2 \log p(x_t|\theta_t)}{\partial \theta_t \partial \theta_t'} = \frac{\partial^2 \log g(x_t|f_t, \theta_t)}{\partial \theta_t \partial \theta_t'}, \tag{20}
$$

for $t = 1, \ldots, T$, where $g(x_t|f_t, \theta_t)$ has mean $c_{2,t} + \theta_t$ and diagonal variance matrix $D_{2,t}$ (see (18)) and $p(x_t|\theta_t)$ is a mixture of non-Gaussian densities; see Section 2.3.

By construction, the likelihood for the approximating model (18) given h is given by

$$
\log g(z|h) = \text{const} - \frac{1}{2} \sum_{t=1}^{T} \log |F_t| + v_t' F_t^{-1} v_t,
$$

where

$$
F_t = \begin{bmatrix} \Lambda & 0 \\ 0 & I_k \end{bmatrix} P_t \begin{bmatrix} \Lambda & 0 \\ 0 & I_k \end{bmatrix}' + \begin{bmatrix} D_{1,t} & 0 \\ 0 & D_{2,t} \end{bmatrix},
$$

and

$$v_t = z_t - \begin{bmatrix} \Lambda & 0 \\ 0 & I_k \end{bmatrix} a_t,$$

where $a_t = \mathrm{E}_g(\alpha_t | y_{t-1}, \ldots, y_1)$ and $P_t = \mathrm{Var}_g(\alpha_t | y_{t-1}, \ldots, y_1)$, which are both computed by applying the Kalman filter to model (18). We notice that h_t only enters the likelihood of model (18) via $D_{1,t}$

The approximating model for the log variance h_t is given by

$$\tilde{z}_t = h_t + v_t \qquad v_t \sim NID(0, b_t^2), \tag{21}$$

where $\tilde{z}_t$ and b_t^2 are obtained by the Gauss Newton type Algorithm A that is given below. The importance density $g_1(h|z)$ is based on the linear Gaussian model (21). Samples $h^{(i)} \sim g_1(h|z)$ are drawn by applying the simulation smoothing methods of Frühwirth-Schnatter (1994), Carter & Kohn (1994), de Jong & Shephard (1995) and Durbin & Koopman (2002).

Algorithm A

1. Initialize $h = h^*$ and $\alpha = \alpha^*$;

2. Given h^* and α^*; compute $c_{1,t}$, $c_{2,t}$, $D_{1,t}$ and $D_{2,t}$ from (19) and (20) for $t = 1, \ldots, T$;

3. Apply the Kalman filter to model (18) to obtain v_t and F_t;

4. Compute $\tilde{z}_t = h_t^* - \left[\frac{\partial^2 \log g(z|h^*)}{\partial h_t^* \partial h_t^{*\prime}} \right]^{-1} \frac{\partial \log g(z|h^*)}{\partial h^*}$ and $b_t^2 = - \left[\frac{\partial^2 \log g(z|h^*)}{\partial h_t^* \partial h_t^{*\prime}} \right]^{-1}$;

5. Update h^* by computing $\mathrm{E}_{g_1}(h|\tilde{z})$ by applying the Kalman filter smoother to model (21);

6. Update α^* by computing $\mathrm{E}_g(\alpha|z)$ using the smoothing recursions given the output of the Kalman filter in step (iii) ;

7. Iterate between (2) and (6) until convergence.

The derivatives in step (4) are computed using the methods developed by Koopman & Shephard (1992). In particular, we use their derivations to compute the derivative with respect to $D_{1,t}$ and we use the chain rule to take the derivative with respect to h_t.

The intuition behind the construction of $g_1(h|z)$ is as follows. We linearize the model $p(z|\alpha, h)$ to obtain the model $g(z|\alpha, h)$, for which an instrumental basis is given by (18), and integrate α from this linear model using the Kalman filter. The resulting model implied by $g(z|h)$ is again nonlinear in h. We construct $g_1(h|z)$ such that its mean is equal to the mode of $g(h|z)$. This is done in step (4) where we construct the Laplace approximation of $g(z|h)$; see Jungbacker & Koopman (2007).

Constructing $g_2(\alpha|z; h^{(i)})$

We choose $g_2(\alpha|z; h^{(i)})$ to follow a Gaussian distribution with mean equal to the mode of $p(\alpha|z; h^{(i)})$ and variance equal to the curvature around the mode. Given the sampled path $h^{(i)}$ the linear Gaussian model (18) only depends on the mean vector α. This model with $h = h^{(i)}$ serves as an instrumental basis for obtaining the mode of $p(\alpha|z; h^{(i)})$. The following algorithm can be used to obtain the mode.

Algorithm B

1. Initialize $\alpha = \alpha^*$;

2. Given $h^{(i)}$ and α^*; compute $c_{1,t}$, $c_{2,t}$, $D_{1,t}$ and $D_{2,t}$ from (19) and (20) for $t = 1, \dots, T$;

3. Update α^* by computing $\mathrm{E}_{g_2}(\alpha|z)$ by applying the Kalman filter smoother to model (21);

4. Iterate between (2) and (3) until convergence.

The Algorithm B is run for every sample $h^{(i)}$ and samples $\alpha^{(j)} \sim g_2(\alpha|z; h^{(i)})$ drawn by applying the simulation smoothing methods of Frühwirth-Schnatter (1994), Carter & Kohn (1994), de Jong & Shephard (1995) and Durbin & Koopman (2002).

Appendix B

In this Appendix we detail the computation of the conditional expectations $E(h|z)$ and $E(\alpha|z)$. From the definition it follows that

$$
\begin{aligned}
E(h|z) &= \int_h hp(h|z) \, \mathrm{d}h \\
&= \int_h h \frac{p(h|z)}{g_1(h|z)} g_1(h|z) \, \mathrm{d}h \\
&= \frac{g_1(z)}{p(z)} \int_h h \frac{p(z|h)}{g_1(z|h)} g_1(h|z) \, \mathrm{d}h \\
&= \frac{\int_h h \frac{p(z|h)}{g_1(z|h)} g_1(h|z) \, \mathrm{d}h}{\int_h \frac{p(z|h)}{g_1(z|h)} g_1(h|z) \, \mathrm{d}h},
\end{aligned}
$$

which can be estimated by

$$
\hat{E}(h|z) = \frac{\sum_{i=1}^M h^{(i)} \frac{p(z|h^{(i)})}{g_1(z|h^{(i)})}}{\sum_{i=1}^M \frac{p(z|h^{(i)})}{g_1(z|h^{(i)})}},
$$

where the samples $h^{(i)}$ are drawn from $g_1(h|z)$. The construction of $g_1(h|z)$ is discussed in Appendix A and $p(z|h^{(i)})$ is evaluated as discussed in Section 3.1.

The conditional expectation for the factors is given by

$$
\begin{aligned}
E(\alpha|z) &= \int_\alpha \alpha p(\alpha|z) \, \mathrm{d}\alpha \\
&= \int_\alpha \alpha \int_h p(\alpha|h, z) p(h|z) \, \mathrm{d}h \, \mathrm{d}\alpha \\
&= \int_\alpha \alpha \int_h \frac{p(\alpha|h, z) p(h|z)}{g_2(\alpha|h, z) g_1(h|z)} g_2(\alpha|h, z) g_1(h|z) \, \mathrm{d}h \, \mathrm{d}\alpha \\
&= \frac{g_1(z)}{p(z)} \int_\alpha \alpha \int_h \frac{p(\alpha|h, z) p(z|h)}{g_2(\alpha|h, z) g_1(z|h)} g_2(\alpha|h, z) g_1(h|z) \, \mathrm{d}h \, \mathrm{d}\alpha.
\end{aligned}
$$

Under the assumption that we may switch the order of integration we obtain

$$
\begin{aligned}
E(\alpha|z) &= \frac{g_1(z)}{p(z)} \int_\alpha \alpha \int_h \frac{p(\alpha|h, z) p(z|h)}{g_2(\alpha|h, z) g_1(z|h)} g_2(\alpha|h, z) g_1(h|z) \, \mathrm{d}h \, \mathrm{d}\alpha \\
&= \frac{g_1(z)}{p(z)} \int_h \frac{p(z|h)}{g_1(z|h)} \int_\alpha \alpha \frac{p(\alpha|h, z)}{g_2(\alpha|h, z)} g_2(\alpha|h, z) \, \mathrm{d}\alpha \, g_1(h|z) \, \mathrm{d}h \\
&= \frac{\int_h \frac{p(z|h)}{g_1(z|h)} \int_\alpha \alpha \frac{p(\alpha|h,z)}{g_2(\alpha|h,z)} g_2(\alpha|h, z) \, \mathrm{d}\alpha \, g_1(h|z) \, \mathrm{d}h}{\int_h \frac{p(z|h)}{g_1(z|h)} \int_\alpha \frac{p(\alpha|h,z)}{g_2(\alpha|h,z)} g_2(\alpha|h, z) \, \mathrm{d}\alpha \, g_1(h|z) \, \mathrm{d}h},
\end{aligned}
$$

which can be approximated by

$$\hat{E}(\alpha|z) = \frac{\sum_{i=1}^{M} \frac{p(z|h^{(i)})}{g_1(z|h^{(i)})} \left[M^{-1} \sum_{j=1}^{M} \alpha^{(j)} \frac{p(\alpha^{(j)}|h^{(i)},z)}{g_2(\alpha^{(j)}|h^{(i)},z)} \right]}{\sum_{i=1}^{M} \frac{p(z|h^{(i)})}{g_1(z|h^{(i)})} \left[M^{-1} \sum_{j=1}^{M} \frac{p(\alpha^{(j)}|h^{(i)},z)}{g_2(\alpha^{(j)}|h^{(i)},z)} \right]},$$

where the samples $h^{(i)}$ are drawn from $g_1(h|z)$ and the samples $\alpha^{(j)}$ from $g_2(\alpha|h^{(i)}, z)$. The construction of both importance densities is discussed in Appendix A.

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