Cyclical Attention to Saving*

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Abstract

This paper explores the business cycle implications of limited household attention to choosing between different savings products. First, I document substantial dispersion in the interest rates offered by retail banks in the UK, even among products with extremely similar characteristics. Using a novel combination of data I show that households move up within the distribution of interest rates available, on average choosing products closer to the highest interest rates in the market, during macroeconomic contractions. I show that endogenous household attention decisions can explain this result, as in contractions the marginal utility of interest income is high, which encourages households to pay attention to their choice of bank. To quantify the effects of these decisions on macroeconomic aggregates, I extend a canonical New Keynesian model to include inattention to the choice between heterogeneous banks, and estimate it using my empirical findings. Variable attention amplifies the response of consumption to discount factor, technology and monetary policy shocks by 74%, 44%, and 10% respectively. Amplification from variable attention is found to account for 13% of the consumption fall observed through 2008, and policies that reduce the costs of comparing between financial products have substantial stabilization benefits.

JEL codes: D83, E32, E71

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1 Introduction

In the majority of dynamic macroeconomic models the interest rate is crucial in determining how shocks propagate through the economy, in part because it regulates the consumption of intertemporally maximising households. The interest rate is usually taken as given by households in these models, but regulators have noted that in reality savers face a range of rate-bearing products, and that they could increase the interest rate they earn on their savings by 'shopping around' for the best product (FCA, 2015).

If the extent of shopping around, or *attention* to product choice[1], is chosen by households after reviewing the likely costs and benefits of the required time and effort[2], then we should expect it to vary systematically with the business cycle, because those costs and benefits are themselves cyclical. The marginal utility of income rises when consumption falls, for example, which means that the benefit households feel from increasing the interest rate on a given stock of savings will rise. Any cyclical movements in attention will affect the propagation of shocks to consumption, by altering the response of the interest rate that households actually experience.

In this paper I therefore provide empirical evidence that attention to savings product choices is countercyclical, and show that this substantially amplifies shocks in an estimated business cycle model. I use a novel combination of detailed product-level panel data on savings products in the UK and aggregate data on how savers chose among a particular set of those products to show that households on average choose savings products with a higher interest rate relative to the distribution of offers during contractionary episodes. I then develop a novel model of endogenous attention to savings product choices, which has similar implications to the popular Burdett and Judd (1983) model of search friction-induced price dispersion, but is sufficiently tractable that it can be incorporated into a canonical New Keynesian business cycle model, and then solved and estimated using standard techniques. In the estimated model endogenous attention amplifies shocks because low consumption implies a high marginal utility of income, which encourages households to pay more attention to their savings product choices and so leads to an increase in the interest rate they face. That increases the incentives for households to save, so consumption declines further than it would have done with constant attention.

I start by documenting an important fact about savings products: on any given date banks do not all offer a common interest rate, even on products which are identical across

[1]This could also be framed in terms of search effort. I use attention here to be consistent with the model framework I develop, but I show in appendix A that a similar model with endogenous search effort without congestion externalities, as in McKay (2013), gives the same qualitative implications.

[2]Staff at the financial regulator found that shopping around decisions were indeed driven by an analysis of the costs and benefits, including time spent shopping and likely interest rate gains (Cook et al., 2002).

a wide range of non-price product features. Considering institutional details of the UK savings market, I argue that unobserved product heterogeneity is unlikely to explain the majority of this dispersion[3]. Instead, I argue that much of this interest rate dispersion persists in equilibrium because of an information friction: it is costly for households to acquire information about the set of products on offer. I obtain the relevant savings product data by digitising 14 years of monthly editions of Moneyfacts, a magazine for UK financial advisers, which is itself a justification for the information cost interpretation. Financial advisers (and indeed the Bank of England and several other regulators) would not need to pay for such a magazine if the information was easy to obtain elsewhere. The existence of interest rate dispersion among otherwise similar products is an important prerequisite for attention to savings product choices to affect the interest rate households face, and therefore for it to affect the business cycle.

To analyse household choices in savings markets, I link the product-level data with time series data from the Bank of England, which for specific sets of savings products with particular characteristics gives the average interest rate achieved on new accounts opened each month. I focus on fixed interest rate savings bonds, as these are very simple products, giving me the best chance of ruling out that decisions and rate dispersion are being driven by product idiosyncrasies or product features hidden in the Moneyfacts data. This should be viewed as a useful laboratory in which to explore household choice behaviour: none of the mechanisms for which I find evidence are specific to this market, or to the UK[4].

Using this novel combination of datasets, I show that the position of the rate households achieve within the distribution of offers they face is *countercyclical*. When the unemployment rate is high, and the level of average interest rates in the market is low[5], households on average choose products that are further up the distribution of interest rates. The average interest rate earned by households is higher relative to the low rates on offer at the 'big four' banks with the largest branch networks and market shares in other financial services, and is closer to the highest rate in the market, in such periods. When unemployment falls and interest rates rise, the interest rate that households receive

[3] As an example, I note that deposit insurance was in place throughout the period I study, so differences in bank risk should not play a significant role. I also show later that the time series patterns of household choices observed in the data do not fit with a risk-based explanation of savings choices.

[4] The mechanism through which I find variable attention to savings products affects the business cycle does not however necessarily apply in the same way to loans. I also only consider the effects of countercyclical attention to savings on consumers, leaving aside the question of what this means for the allocation of credit. I discuss these channels further in appendix B.

[5] It is not obviously the case that interest rates will be low in contractions, as that depends on what kind of shock has caused the contraction, and indeed unemployment and interest rates are not perfectly correlated in the data. The estimated model helps to understand the roles played by the two variables by allowing for distinct channels through which each could affect attention, and having the estimation decide which explains the most variation in household choices.

on average falls away from the top end of the market, and gets closer to the (on average) lower rates at the largest banks.

To understand the impact of this pattern on the business cycle, and to explore policy implications, I then construct a structural model to explain my empirical results. In particular, I show that the countercyclical position of household interest rates within the distribution of offers can be explained by a model in which households face costs of processing information about which bank they ought to use for their savings, following the rational inattention framework in Matějka and McKay (2015). After a contractionary shock consumption falls, and so the marginal utility of income rises. This causes households to pay more attention to their savings choices, and so they more reliably choose the banks offering the highest interest rates in the market, as seen in the data. I also show that several other potential explanations of the empirical results do not survive scrutiny using other sources of data on the UK retail savings industry.

The model I develop, in which interest rate dispersion is endogenously determined by heterogeneous profit maximising banks facing imperfectly attentive households, is sufficiently tractable that I am able to embed it in an otherwise standard New Keynesian DSGE model, which I solve and estimate using standard techniques. This differs from existing macroeconomic models with limited shopping around for prices (e.g. Mckay (2013), Kaplan and Menzio (2016)), which mostly have households engaging in costly search following Burdett and Judd (1983), which outside of simple cases are not usually tractable enough to estimate. I show that the impacts of greater attention in my model on households and on interest rate setting decisions are qualitatively very similar to those of greater search effort in a Burdett-Judd model.

Interest rate dispersion in the model requires that banks face heterogeneous costs[6]. It is important, however, that the extent of interest rate dispersion generated by this cost heterogeneity is endogenous to household choices: dispersion is lower when attention is high. When attention rises bank market shares become more sensitive to their interest rates relative to their competitors, as households more accurately identify and choose the highest rates in the market. Banks offering low interest rates therefore increase their rates, and high-interest banks respond with rate increases of their own to protect their market shares, though this is partially offset by low cost banks wanting to accept a smaller increase in market share in exchange for making more profit from each saver.

This amplifies the effects of variable attention on consumption. When attention rises, households achieve higher interest rates relative to the distribution of rates on offer. This increases the interest rate experienced by households in itself, and this is then amplified

[6]This is a key difference to Burdett-Judd models, in which price dispersion arises because identical firms follow mixed strategies in pricing. Not having to solve for a mixed strategy equilibrium does, however, help a great deal with tractability.

by the fact that the rise in attention leads to a shift up in the distribution of interest rates. This effect is only partially mitigated by the fall in the dispersion of interest rates caused by the attention rise, which discourages attention from households through smaller potential gains from attention.

In the maximum likelihood estimation of the model I make use of aggregate macroeconomic data and key time series from the empirical sections of the paper. I find that the key driver of attention choices is the marginal utility of income[7]. Compared to a counterfactual model in which attention is held permanently at its steady state value, the consumption response on impact in the variable attention model to discount factor, technology and monetary policy shocks is 74%, 44% and 10% larger respectively. If attention had remained at its steady state throughout the Great Recession, the model implies that the consumption fall through 2008 would have been 13% smaller. Much of this gain from weakening the pass-through of shocks can be achieved by reducing the marginal cost of processing information about the savings products on offer each period, suggesting that recent policy initiatives from the FCA aimed at providing households with information and facilitating easy product comparisons in this market could have important benefits in reducing business cycle volatility.

Related Literature. This paper contributes to several streams of literature, on the macroeconomic effects of search/information frictions, and of deposit market frictions, and more micro-focused literature on price dispersion and consumer behaviour in markets for financial services.

There is a very large literature studying how information frictions affect business cycle fluctuations (e.g. Lucas (1972), Mankiw and Reis (2002), Angeletos and La'O (2013)). Many of these papers study frictions in how agents receive information about continuously distributed exogenous shocks[8] such as TFP or monetary policy shocks. Lucas (1972) shows, for example, that if firms cannot disentangle the information on idiosyncratic demand and the aggregate price level contained in the price they face for their own goods, then a model with no exogenous price stickiness will generate a Phillips Curve. More recently, Mackowiak and Wiederholt (2015) show that a model in which households and firms are rationally inattentive to aggregate shocks can generate plausible hump-shaped impulse responses without needing consumption habits or other forms of internal persistence often used in the DSGE literature for this purpose (e.g. in Smets and Wouters, 2007). In a related strand of literature, several papers study frictions in how agents learn

[7]Other potential drivers of attention, such as exogenous co-movement of the dispersion of interest rates with the cycle, are found to matter much less.

[8]In general, agents are assumed to have perfect knowledge of their model environment, so the distinction between tracking exogenous or endogenous variables is irrelevant, as agents can perfectly map from one to the other. See Ellison and Macaulay (2019) for an example in which the distinction does matter, and the information friction is over endogenous aggregate variables.

about the reaction functions of other agents or the relationship of endogenous model variables to shocks (Eusepi and Preston, 2011). This paper is related to these studies in that it takes seriously the idea that households have limited information about variables which are important for their decision-making, and that this friction can have business cycle implications. In contrast to the bulk of this literature, the friction I study is not over a continuously distributed shock or the parameters of a reaction function, but over the discrete choice of which bank to use for saving each period.

Specifically, I draw on the work of Matějka and McKay (2015), who show that an agent facing a discrete choice problem and information costs as in the literature on rational inattention (Sims, 2003) will use choice probabilities resembling those from a multinomial logit model, with a 'twist' to reflect the influence of prior beliefs on choices. This form of inattention has been used by Dasgupta and Mondria (2018) to study trade shocks in a model in which countries are inattentive to which country to import each good from, and by Acharya and Wee (2019) to examine information frictions in hiring decisions in a search-based model of the labour market. I extend the rational inattention literature by showing that information frictions in product choices can have substantial implications for the business cycle, and are easily incorporated into general equilibrium models since there is no role for the higher-order beliefs that commonly complicate the solution of models with imperfect information.

Another way of modelling the friction in the choice over financial products would be to use costly search, or costly shopping effort. Coibion, Gorodnichenko and Hong (2015) show that the amount of time and effort spent shopping for consumption goods increases when unemployment rises[9], echoing my findings that attention to savings product choices rises in contractions. They show that this countercyclical search effort reduces the price stickiness of the basket of goods households actually buy relative to the stickiness of an individual firm's price. The decision of how much attention to pay to choosing between savings products in this paper can be seen as an extension of their work to financial products (they specifically focus on shopping for groceries), which have particular importance for consumption decisions as they influence how households allocate their consumption over time.

Several other papers have also studied the role of search or information frictions in financial products. A large literature starting with Arrow (1987) finds that information frictions are helpful in explaining wealth inequality, as wealthier households have more incentive to process information about saving and investment choices, and so make better choices and earn higher rates of return on average than less wealthy households. Cam-

[9]Kaplan and Menzio (2016) also study a model in which unemployed households search harder for low goods prices, so in recessions the average search effort rises. They use the Burdett-Judd model of equilibrium price dispersion discussed above.

panale (2007) and Lusardi, Michaud and Mitchell (2017) show that this is quantitatively important for explaining wealth inequality, and McKay (2013) studies the importance of the wealth-search link in the welfare consequences of social security privatisation. In contrast, I focus on a model with a representative household and show that the same information friction amplifies the effect of aggregate shocks on consumption.

The literature on the importance of deposit market frictions for the business cycle goes back to Diebold and Sharpe (1990), who document significant stickiness in the pass-through from wholesale interest rates to retail deposit rates. Driscoll and Judson (2013) extend this, finding that menu costs are not a good explanation of the interest rate stickiness seen in the data. Dreschler, Savov and Schnabl (2017) find that limited pass-through to deposit rates plays a crucial role in the transmission of monetary policy shocks, through the effects on bank balance sheets. The mechanism I explore focuses on the effects of deposit frictions on households through their intertemporal consumption decisions, so should be seen as a complement to the channel discussed by Dreschler, Savov and Schnabl.

Evidence of inattention in the markets for retail financial products has been found in several market-specific studies. Martin-Oliver et al. (2009) and Branzoli (2016) show that interest rate dispersion and the incidence of choice 'mistakes' (choosing an unambiguously dominated product) respectively are less common in retail financial product markets where there is a greater incentive for consumers to pay attention to their product choice. Adams et al. (2019) conducted a large experiment in which they sent information about alternative products to savers at five retail financial institutions in the UK, and found substantial inattention to the product choices available in the market even with the prompting provided by the experiment. I contribute to this literature by exploring how that inattention varies over the business cycle, and drawing out the macroeconomic consequences of that variation.

The most closely related paper to this work is Yankov (2018), who explores the role of search frictions, which are closely related to the costs of attention studied in this paper, as a potential driver of equilibrium in deposit markets. As in my UK data, Yankov finds substantial dispersion between US banks in the interest rates they offer on certificates of deposit. He estimates a structural model based on Burdett and Judd (1983) and finds that search costs can explain the empirical dispersion. While he does consider the variation in search intensity over time, the main purpose of his paper is to examine if search frictions can account for interest rate dispersion, so I extend his work by focusing on the business cycle implications of endogenous information acquisition in deposit markets. Empirically, I differ from him by combining data on the menu of interest rates available with data on how households chose within that menu on average. This allows me to explore household choice without the structural assumptions he makes, as he only uses data on the menu of offered interest rates with no information on how savers actually chose within that distribution. My more model-free approach also suggests that costly information is important for household decisions over savings products, bolstering the argument made by Yankov.

Interestingly, however, my results on the time-variation of attention contrast sharply with his. While I find that attention is highest when interest rates are low, his estimates suggest that attention is high when rates are *high*. There are two reasons for this. Firstly, the interest rate dispersion he documents is strongly positively correlated with the level of interest rates, which is not the case in my UK data[10]. This means that incentives to search rise with interest rates in his data, but not in mine. Secondly, to keep the Burdett-Judd equilibrium tractable enough to estimate he studies a two-period model in which the saver's only source of income is interest on their assets, and where the costs of search are proportional to initial wealth. This means that consumption does not vary much in the model, so the main driver of the benefits of search is the dispersion of interest rates. To the extent that interest rates and consumption are in fact positively correlated, marginal utility will rise when interest rates fall, implying the benefits of search will rise. By developing a simpler model of endogenous interest rate dispersion based on rational inattention to discrete choices, I can tractably embed the information friction into a richer general equilibrium model, and I find that for the UK data the marginal utility of income is indeed a key driver of information choices.

The rest of the paper is organised as follows: in section 2 I detail the data sources I use, and some institutional background on savings markets in the UK. I examine this data, showing the wide and persistent dispersion in interest rates and studying household choices within that range in section 3. In sections 4 and 5 I build and estimate a New Keynesian model with endogenous attention to savings product choices to explain the empirical results and explore their implications for the propagation of business cycle shocks to consumption. Section 6 concludes.

2 Data

To study how households choose between savings products, I construct a dataset containing the choice set facing households and some summary statistics on household choices

[10] This may partly be because he looks at a range of CD's with different sizes of investment, whereas I focus on a set of products which are the same across this dimension (estimates from the regression of equation 1 in section 3 show that the size of the investment is related to the rate on offer). If the differences between rates on low and high balances grow when the level of rates rise, this could explain some of the discrepancy.

within that set. I create this dataset by combining data from two sources. To observe the choice set facing households, I digitise 13 years (1996-2009) of monthly editions of Moneyfacts, a magazine for UK financial advisers. Household choices are summarised in data on average interest rates across households on newly opened savings products each month from the Bank of England. In this section I explain the nature of these datasets, and provide some institutional background on the specific savings market I study.

2.1 Data Sources

The first dataset I use is from Moneyfacts magazine. Each month, the magazine publishes tables of the interest rates and product characteristics of the vast majority[11] of saving and credit products on offer from retail financial institutions in the UK. A key advantage of using magazine data is that it reports all dimensions of product heterogeneity which are relevant for savers[12], which means that the interest rate dispersion I find after controlling for all of these characteristics cannot be explained by explicit product differentiation. The magazine reports the full set of relevant product characteristics because it is designed for household financial advisers: if savers care about a product characteristic then financial advisers need to know about it.

Of all of the saving (and borrowing) products available in the data, I focus on a specific subset of savings products for which the product characteristics are simple and easily quantifiable: fixed interest rate savings products. This enables me to account for all relevant dimensions of product heterogeneity. In contrast, mortgages and other loans, as well as other more complicated savings products, have many more dimensions of product heterogeneity, and many products have their own idiosyncratic features, made evident by the paragraph of notes accompanying each observation in Moneyfacts. Such idiosyncrasies would make accounting for product differentiation in interest rate dispersion extremely difficult. Furthermore, Moneyfacts only reports the advertised interest rates on products: for savings products the vast majority of households receive this rate, but for loans there is a substantial amount of risk-adjustment at the level of the individual borrower, so it is not possible to cleanly identify the choice set for loans in the data.

A further advantage of studying fixed interest rate savings products is that they are mainstream savings products throughout the sample, giving me the maximum number of data points. In contrast, ISAs (another commonly used savings product in the UK) were

[11]The publishers aim to cover the universe of products and institutions, but acknowledge that they may occasionally miss a niche product from a very small provider. As I will be focusing on average household choices over a common type of savings product (fixed interest rate saving bonds), the data should contain all relevant products.

[12]For all products this includes eligibility criteria and frequency of interest payments, along with other product features.

only introduced in 1999. Further details on fixed interest rate savings products are given in section 2.2.

Household choices within this market come from the Quoted Household Interest Rate time series from the Bank of England. This reports the average interest rate earned by households each month on a subset of fixed interest rate savings products which are identical along all the dimensions of product heterogeneity identified in Moneyfacts, so it directly relates to a set of savings products which are identical except for the interest rate, and which can be easily identified in the Moneyfacts data. Importantly, the average interest rate reported is for accounts opened in that month only, not the stock of all active accounts, which would include accounts opened several months earlier when interest rates were different. The comparison of the Quoted Household Interest Rate to the set of available products in Moneyfacts is therefore only possible because the Quoted rate only includes newly opened accounts.

There are several Quoted Household Interest Rate series available for fixed rate savings products with different combinations of product characteristics. I focus on the series for products with a term of one year, an investment of £5000, and where interest is paid annually, because the Quoted Household Interest Rate series goes back to 1996 for these products, whereas the series for other combinations of features have only been published since 2009. In addition, this is one of the most common combinations of product features in the market, so my results on interest rate dispersion and household choice in section 3 are less affected by outliers than would be the case with a more niche combination of product features.

2.2 Institutional Background

Retail savings products are provided in the UK by conventional banks and building societies, which offer deposit products to fund mortgage lending[13]. Deposits at all of the institutions in the sample were covered by deposit insurance up to £35,000 throughout the period I study, substantially above the £5,000 investment size I study (I return to the issue of deposit insurance and bank risk in section 3.1.1). The largest four institutions had 74% of the market for current accounts in 2000 (Vickers, 2011)[14], and the largest branch networks. The market for savings accounts is much less concentrated, with a Herfindahl-

[13]The main differences between building societies and banks are that building societies are owned by their customers, and are more limited than banks in how much of their funding can come from wholesale money markets. For the purposes of this paper I will not distinguish between the two types of provider, as industry experts suggest it is not important for consumer choices (Maundrell, 2017). For a review of the differences see BBC (2005). As the degree of wholesale funding could be related to bank risk, I discuss this in section 3.1.1.

[14]This market share fell gradually to 64% in 2008, then rose to 77% in 2010 due to mergers in which large banks bought failing rivals (Vickers, 2011).

Hirschman Index between 20% and 30% lower than the current account market between 2000 and 2008 (Vickers, 2011). In fact in 1998 Cruickshank (2000) reports that the 'big four' banks have just 19% of savings accounts by number of accounts, but this does not account for deposit size so may not accurately reflect their market share.

Although the fixed interest rate savings products I study are a subset of all savings products, they are commonly used in the UK, so my results do not stem from a niche product serving only a small subsection of savers. The FCA (2015) found that 12% of households held these products[15], and they accounted for 20% of all cash savings balances in the UK. Indeed, the Moneyfacts data confirms that the market is large, as there are an average of 200 such products available each month in the sample. To compare with the Bank of England data I focus in on one year savings accounts with interest paid annually and an investment of £5000, and even in this sub-market there are an average of 34 products each month, and all of the major banks are present.

The key advantages of focusing on fixed interest rate products are, as mentioned above, their simplicity and that they match well with the Quoted Household Interest Rate data. In addition to these, there are two other factors which aid analysis of choices in this particular market. Firstly, product bundling is not commonplace in this market. In its 2015 report, the FCA found that 76% of savers using fixed rate bonds use an institution which is not their 'main provider of financial services'. I can also observe when there is explicit bundling in the Moneyfacts data: if, for example, a particular fixed rate product can only be purchased by someone with a current account at that bank, that is noted in the data. I do not remove the few products for which this is the case before analysing the data because they are not removed in the Quoted Rate data, but removing them does not substantially change the distribution of offered rates.

Second, the interest rate is the key product feature that matters for households in this market. As part of their 2015 report, the FCA surveyed holders of fixed interest rate bonds, who reported that the interest rate was clearly more important than all other product features for a large majority of savers in this market, and that they held fixed rate savings bonds as assets, not for transactions or any other purposes. This is important

[15]In 2006 the Wealth and Assets Survey showed that 5% of UK households held these products, and for those that did hold them they made up 31% of household asset portfolios. However this is possibly an underestimate of how many households hold these products, as many products in this market are advertised as 'fixed rate savings accounts' rather than 'fixed-term investment bonds with fixed interest rates', as they are called in the WAS. It is therefore likely that some households holding a fixed-rate bond mistakenly described it as a 'savings account' in the WAS. Banks and Tanner (1999) found that in 1998 interest-bearing accounts at banks and building societies were held by 60% of households, that mean holdings in these accounts were £5019 (close to the level of investment I focus on), and that these products accounted for 72.8% of the asset portfolios of households in possession of them. There are more recent waves of the WAS which could be studied, but these fall outside of the years of magazine data I currently have.

for my analysis, as customer service and the convenience of a large branch network are features of the savings products that I do not observe in the Moneyfacts data and cannot easily control for. That these do not seem to matter much to consumers means that this is unlikely to explain large amounts of the interest rate dispersion I find in section 3.1. The presence of a local branch is less important for these products than others because they are of a fixed maturity (I focus particularly on one year bonds), and so the saver does not need to be interacting with the bank on as regular a basis as might be the case for products which could be cut short at any time (FCA, 2015). In addition, in section 3.2 I use the interest rate that households achieve relative to the distribution of rates on offer as a measure of the 'success' of their decision making, and in section 4 I explain movements in that measure by assuming that households trade off the benefits of higher interest rates with the costs of paying enough attention to find out which banks are offering the highest rates. These exercises all assume that savers would like higher interest rates, which would be a difficult claim if other features of the savings products weighed heavily on the value households get from their saving products.

3 Empirical Results

In this section I explore household choice using the datasets described in section 2. First, I show that there is substantial heterogeneity in interest rates offered by retail banks which cannot be explained by product heterogeneity. Without interest rate dispersion, the choice of one savings product over another would have no impact on the interest rate households experience. I then construct a summary statistic for the 'success' of household choice, which measures the interest rate households actually achieved relative to the distribution of rates on offer that month. I show that on average, households more reliably choose the higher interest rate products when the average level of interest rates is low and when the unemployment rate is high.

3.1 Interest Rate Dispersion

Each month in my sample, households could achieve a wide range of different interest rates by choosing different fixed interest rate savings products from different providers. The median within-period standard deviation and interquartile range of interest rates are 60 and 75 basis points respectively, compared to a median within-period average interest rate of 521 basis points.

However, some of this dispersion is explained by the fact that these products are not all equal. They differ in the length of the bond, the minimum investment required, and the frequency with which interest is paid. In section 3.1.1 I therefore account for this product heterogeneity, and show that in all months explicit product differentiation explains no more than 53% of interest rate dispersion, and in the majority of periods it explains much less than that. I also argue that dimensions of product heterogeneity not observed in the Moneyfacts data, such as perceived bank risk, are unlikely to explain much of the remaining dispersion. I then provide evidence that limited attention is a likely cause of the remaining interest rate dispersion in section 3.1.2. This means that many savers could increase their interest income without changing any other characteristics of their saving product by switching to other providers. Increased attention to the choice of savings products would lead to this kind of switching, which is how attention affects the interest rate households experience.

3.1.1 Interest Rate Dispersion is not explained by Product Differentiation

I show that product differentiation cannot fully explain the substantial interest rate dispersion in the market for fixed interest savings products in two ways. First, for each period I regress the available interest rates on all product characteristics reported by Moneyfacts, and I find that the adjusted R^2 of this regression never exceeds 0.53, so in every month a maximum of just over half of the variation in interest rates is explained by those characteristics. The median adjusted R^2 across the sample is just 0.18. Second, I focus in on a group of products which are identical across all product characteristics except interest rates, and I show that even among these products the average standard deviation of interest rates within a period is 43 basis points (on an average interest rate of 520 basis points). Importantly for these exercises, the Moneyfacts data supplies all relevant observable dimensions of product differentiation, so the remaining interest rate dispersion must be driven by some other factor.

The first method I use to find how much of the interest rate dispersion is driven by product differentiation is to regress interest rates on all observable product characteristics each period. That is, I run the following regression each month, where X_i contains all of the product characteristics listed by Moneyfacts for product i in that month:

$$ir_i = \alpha + \beta X_i + \varepsilon_i \tag{1}$$

Across all periods, the median adjusted R^2 for this regression is 0.18, and the maximum adjusted R^2 is 0.53. At most, just over half of the variation in interest rates can therefore be explained by observable product characteristics.

The second method considers only a set of products which are identical along all dimensions of product heterogeneity recorded by Moneyfacts, so if the market was perfectly competitive (and unobserved product heterogeneity is negligible) the products considered should have the same interest rate. Within the set of products which qualify for the Quoted Household Interest Rate data (see section 2 for details) the mean within-month standard deviation of interest rates is 43 basis points, on an average interest rate of 520 basis points. In October 2000, as an example, savers could earn annual rates of return between 450 and 680 basis points at different banks on a product with identical characteristics (the standard deviation of rates that month is 44 basis points, which is the median across the sample). The histogram of these rates is plotted in figure 1. There is therefore substantial interest rate dispersion which cannot be explained by product differentiation.

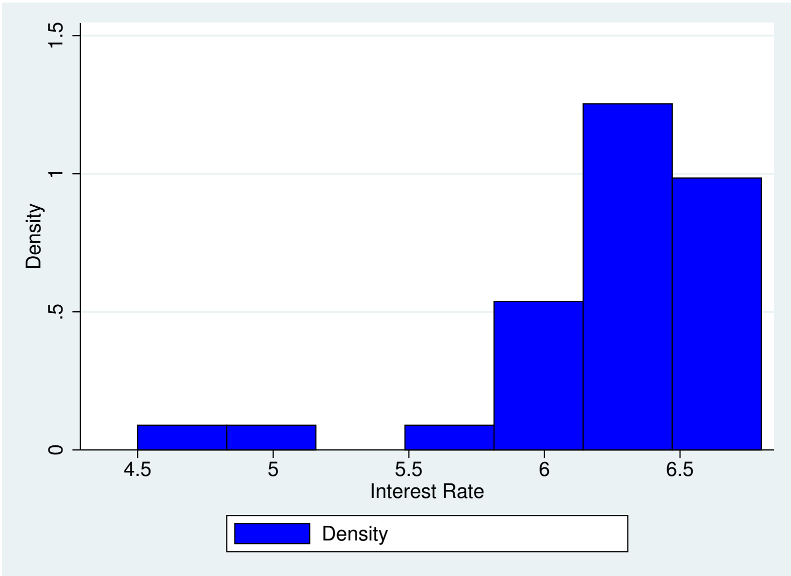


Figure 1: Histogram of annual interest rates on fixed interest rate bonds and term accounts on offer in October 2000

These two methods, however, only control for explicit product heterogeneity. It could be that other features of the product or provider are known to households, but are not part of the product features published in Moneyfacts. One such possible source of unobserved product differentiation is 'implicit bundling': if households have a preference for saving with the same institution they use for their current account, mortgage, and other financial services, then smaller providers with a smaller range of offerings in other product areas may have to pay higher interest rates on their savings to compensate savers for the lack of this convenience. The evidence collected by the regulator on this market suggests that this is not a substantial driver of interest rate heterogeneity. In their analysis of a variety of savings markets, the FCA (2015) found that the convenience of managing their money in one place was mostly important for savers choosing instant access savings accounts, which they will interact with regularly, not for the fixed term accounts studied here[16].

[16] 70% of savers who opened a fixed term account in the year before the survey was conducted cited

In keeping with this finding, the FCA also found that it is very common for savers to hold these savings products at institutions that are not their 'main provider' of financial services (see section 2.2 for further details)[17].

Another potential driver of interest rate dispersion is bank risk. If a bank is at more risk of failure, then it might have to offer savers higher interest rates to compensate them for the risk that the savers will lose their deposits. There are three reasons why this is unlikely to be the key driver of rate dispersion in this market. First, throughout the sample deposits in the UK are insured up to £35,000 (£50,000 after October 2008) per depositor per provider, which is far above the £5,000 investments I study. This removes the majority of risk to savers of bank failure, though if the insurance was not perfectly credible, or the insurer was expected to be slow in paying back deposits in failed banks, then it does not by itself completely eliminate risk as an explanation for rate dispersion. However, in a detailed study of retail deposits in the UK, Chavaz and Slutzky (2018) find that deposit rates are on average uncorrelated with a variety of measures of bank risk. As interest rate dispersion is substantial in every month of my sample, this suggests that risk is not the main driver of the dispersion. Chavaz and Slutzky do find that riskier banks offer higher interest rates when they face spikes in household attention (measured by Google searches), primarily during the 2008 financial crisis. This suggests that risk may explain why the dispersion of interest rates among similar products rises during the financial crisis in my data, but their results on the relationship between risk and deposit rates on average imply that other factors must also be at play. This is supported by the fact that including bank fixed effects in regression 1 still leaves the mean and median unexplained within-month standard deviation of interest rates at 39 and 37 basis points respectively[18].

The final reason why risk is unlikely to be the main driver of interest rate dispersion will be explored in section 3.3. I show there that the time series pattern of household choices within the set of offered interest rates is inconsistent with the bank risk explana-

the interest rate as the main factor in their choice.

[17] In fact, the mechanism studied in sections 4 and onwards could be reinterpreted as households choosing how much convenience to give up in order to achieve higher interest rates, rather than how much costly attention to pay to achieve those higher rates. The intuition and qualitative results would be identical in such a convenience-driven model.

[18] This is an inferior way of capturing risk than in Chavaz and Slutzky (2018), who use proprietary time-varying measures of bank risk from the Bank of England. Adding bank fixed effects ignores the fact that bank risk may change over time, and also removes all variation which causes a bank to offer persistently high or low rates, whether that is driven by risk or not. A model of information or search frictions would also imply that some firms offer persistently higher (lower) interest rates if they have a disproportionately small (large) weight in household prior beliefs (for details see Matějka and McKay (2012)). The regression with bank fixed costs should therefore be taken as further suggestive evidence that the Chavaz and Slutzky results apply to the fixed-rate market specifically, as well as to retail deposits in general.

tion.

There could, of course, still be other sources of unobserved product differentiation which explain the dispersion of interest rates that I have not considered here. I therefore proceed by arguing from the other side, giving reasons to believe that there are substantial costs of information/search in this market, and therefore that limited 'shopping around' could explain why interest rate dispersion persists in equilibrium.

3.1.2 Limited Attention is a plausible explanation of Interest Rate Dispersion

The presence of costly search, information, or attention has been proposed as an explanation of equilibrium price dispersion among similar products in a large number of papers, both theoretical and empirical, starting with Stigler (1961) (see Baye et al. (2006) for a review). The possibility that limited attention could explain the interest rate dispersion not accounted for by observed product differentiation is not, however, evidence in itself that households are less than fully informed about the savings products available to them. I therefore provide evidence that information costs, which lead to inattention, are in fact important in this market.

The clearest piece of evidence for the role of information costs, which would make households inattentive, comes from the FCA (and their predecessor the FSA), who regulate the market for savings products in the UK. In a study of retail financial services for the regulator, Cook et al (2002) concluded that:

> Shopping around is not cost free since consumers have to spend time and effort. The extent to which consumers shop around the market will depend on the benefits they think they can get and the costs of them doing it.

Other reports by the regulator (FSA (2000), FCA (2015)) on this market have similarly concluded that households could benefit if they searched harder for their financial products, but that such search is costly.

In addition to the remarks of the regulator, the founding of Moneyfacts, the magazine from which I obtain the data on savings products, is itself evidence that information costs are substantial in retail financial markets. Moneyfacts was expressly founded to make data on savings and loan products available to financial advisers, because until that point obtaining this information for product comparisons had been difficult (Moneyfacts (2019))[19]. This suggests that it is costly (in time, effort or money) for households to

[19] Moneyfacts (2019) claim that the idea of bringing information together to allow for financial product comparisons was 'revolutionary' at its founding in 1988. Similarly, a rival comparison service Moneysupermarket began in 1987 when the founder realised that it was very difficult for brokers to compare available mortgage deals (Hohler, 2007).

obtain this information from elsewhere: the magazine would not have been founded, and would not keep selling subscriptions, if data on the full set of available savings products was easy to find. Importantly, less than 8% of UK households employ financial advisers (Aegon, 2017) so the existence of the magazine has not itself removed the information friction which is the cause of saver inattention.

The rapid spread of comparison websites covering savings products in the early 2000s supports this evidence. The largest such comparison site in the UK, MoneySupermarket, brought in revenues in 2006 of £105 million, because very large numbers of people visited the site[20] each month to compare a variety of products, including savings products (Connon, 2007). Savers would not need to visit a comparison website if they were already fully informed about the products on offer. However, as with the founding of Moneyfacts, these websites did not reduce the cost of information to zero. It still takes time and effort to use the websites, and to process the information provided to translate it to choices. Indeed in 2019 the Financial Times ran an article about one bank's strategy for recruiting depositors titled "How Monzo is banking on customer apathy" (Kelly, 2019), indicating that savers are not fully attentive to their choices despite the availability of comparison websites.

Other authors have also concluded that inattention plays an important role in markets for retail financial products. Martin-Oliver et al. (2009) find evidence that there is less interest rate dispersion among Spanish banks in markets where households have a greater incentive to pay attention, and Branzoli (2016) finds that some consumers make the mistake of choosing a product which is strictly dominated by another product at the same bank - but that this is likely when price dispersion is higher, when the households have a greater incentive to pay attention to their choices. For the UK, Adams et al. (2019) find evidence of substantial inattention to savings product choices in a large randomised controlled trial using savers at five retail financial institutions.

Finally, I will show in sections 4 and 5 that endogenous attention decisions can explain the time series variation in how households choose from among the set of offered rates presented in the next two subsections.

[20]By the end of 2006 they had 4 million users per year (Hohler, 2007). Moneysupermarket earns revenue by charging firms each time a consumer clicks through from the site to that firm's product (Connon, 2007). The high revenues therefore reflect large numbers of households using the comparison site.

3.2 Constructing φ: a summary statistic for household choice

In this section I use the Moneyfacts and Bank of England data to study how successful households are at choosing the highest interest rate product in the market each month[21]. I do this by computing for each month the difference between the average interest rate earned by households opening new accounts and a benchmark rate, the (unweighted) mean interest rate on offer at the four largest banks, which dominate the market for current accounts and have the largest branch networks, and on average offer interest rates at the lower end of the distribution in the market. I argue that a saver not paying any attention to their choice would end up with this 'blind' rate, and any increase in the rates savers experience above this can be seen as an improvement in their choices. Normalising this difference by the standard deviation of interest rates on offer that month ensures that the measure is not mechanically affected by changes in the dispersion of interest rates.

I construct the 'no-attention' benchmark interest rate (the blind rate) to reflect a probable bias towards larger market players: small 'challenger' banks in particular are likely to be discovered only if the saver does some careful research, as they do not have large numbers of physical branches or large advertising budgets. Specifically, I construct the 'no-attention' benchmark rate by taking the average interest rate on offer from the 'big four' banks[22]. Throughout the sample period these four banks hold most of the market share in many retail banking markets, including current accounts and mortgages, and have many more branches than other banks (Office of Fair Trading, 2008). Using this as the benchmark interest rate assumes that households paying no attention to their choice of savings product are likely to go to their closest bank branch, or the bank where they hold a current account. Alternative benchmarks, such as weighting banks by their market share in previous periods, would be strongly correlated with this simple benchmark because the big four have such large stable market shares.

Figure 2 below shows the histogram of interest rates available in October 2000 on the subset of fixed interest rate savings products which appear in the Quoted Household Interest Data, with the blind interest rate shown in red and the quoted rate (the average interest rate achieved on products bought that month) shown in green. The blind rate is 106 basis points below the maximum rate that households could achieve. While they do not all get that rate, on average savers do somewhat better than they would have if they paid no attention to their choice, earning an average of 6.24% interest, 50 basis points above the blind rate[23].

[21] I deem earning a higher rate of interest as success because the savers in this market reported that the primary feature of fixed interest rate savings products that they cared about was the interest rate (FCA, 2015).

[22] These are Barclays, HSBC, Lloyds, and Royal Bank of Scotland.

[23] It is worth noting that the 'blind rate' is a simple unweighted mean of the rates at the big four

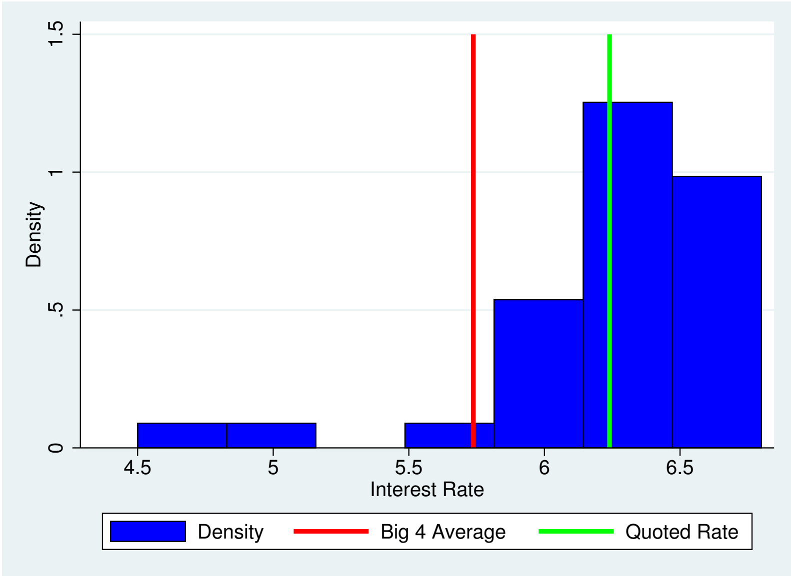


Figure 2: Histogram of annual interest rates on fixed interest rate bonds and term accounts on offer in October 2000

The statistic on household choice which I will study is the distance between the household mean and the blind rate, normalised by the standard deviation of the interest rate distribution that month. I denote the resulting statistic by φ:

$$\varphi_t := \frac{\mathbf{E}_h i_t - i_t^b}{\sigma(i_t)} \tag{2}$$

I will take φ as a summary statistic on how households chose from among a distribution of interest rates in a given month. Note that φ is homogeneous of degree zero in the levels of interest rates, so the long run decline in nominal interest rates observed by Holston et al. (2017) and others does not mechanically affect φ. If household decisions are driven by real interest rates rather than nominal rates, φ is unaffected by changes in inflation expectations for the same reason.

Although this statistic is not based on any particular model, in section 5.1 I show that there is an exact correspondence between φ and attention (as defined in the Rational Inattention literature) in my model when there are just two banks, and that attention and φ remain closely related with more banks in the market. Even in a rich partial equilibrium model with many sources of heterogeneity and variation, simulations show that φ is strongly related to attention, and is only weakly contaminated by the shape of the interest rate distribution (see appendix C).

banks, and there is some dispersion among those big banks. The highest rate on offer in October 2000 from a big four bank was 6.1%.

3.3 φ properties

The key innovation of φ relative to existing estimates of information processing in savings markets, aside from not having to rely on any specific structural modelling assumptions, is that it can be measured each month. I can therefore study how choice behaviour changes over time, at a high enough frequency to observe co-movements with aggregate variables over the business cycle. In the graphs below I plot the time series of φ, and show that it is positively correlated with the unemployment rate and negatively correlated with the level of interest rates over the business cycle.

Figure 3 below shows how φ_t varies over time.

Figure 3: φ over time, 6 month moving averages.

A substantial portion of the variation occurs at business cycle frequencies[24]. The largest falls in φ occur during the growth periods of 2004-2005 and 2006-mid 2008. Shortly after the beginning of the Great Recession in the UK mid-way through 2008, φ began to rise sharply. There was also a substantial rise in φ from July 2001 - April 2002. Although unlike the US the UK avoided recession during this period, it was a time of slowing growth, and the unemployment rate rose relative to trend.

These observations point towards a countercyclical pattern in φ, which is corroborated by figure 4. These plots show the cyclical component of φ against the cyclical components

[24] Figure 3 plots a moving average of φ to aid visualization, but even with the unsmoothed series 20% of the sample spectral mass lies between 6 and 50 quarters, the business cycle frequency domain suggested by Beaudry, Galizia and Portier (2020).

of the average interest rate in the savings market studied[25] and in unemployment[26]. Lower interest rates and higher unemployment are associated with higher φ. These relationships are strongly statistically significant: the coefficients on interest rates and unemployment in the linear best-fit lines both have p-values below 0.00001.

Furthermore, these relationships are not driven purely by extreme events. If we split the data points into before and after the run on Northern Rock[27] in September 2007, the linear relationship between interest rates and φ is not significantly different before and after this event. The relationship between φ and unemployment is significantly steeper before the crisis than after, but it remains positive and strongly significant in both sub-samples.

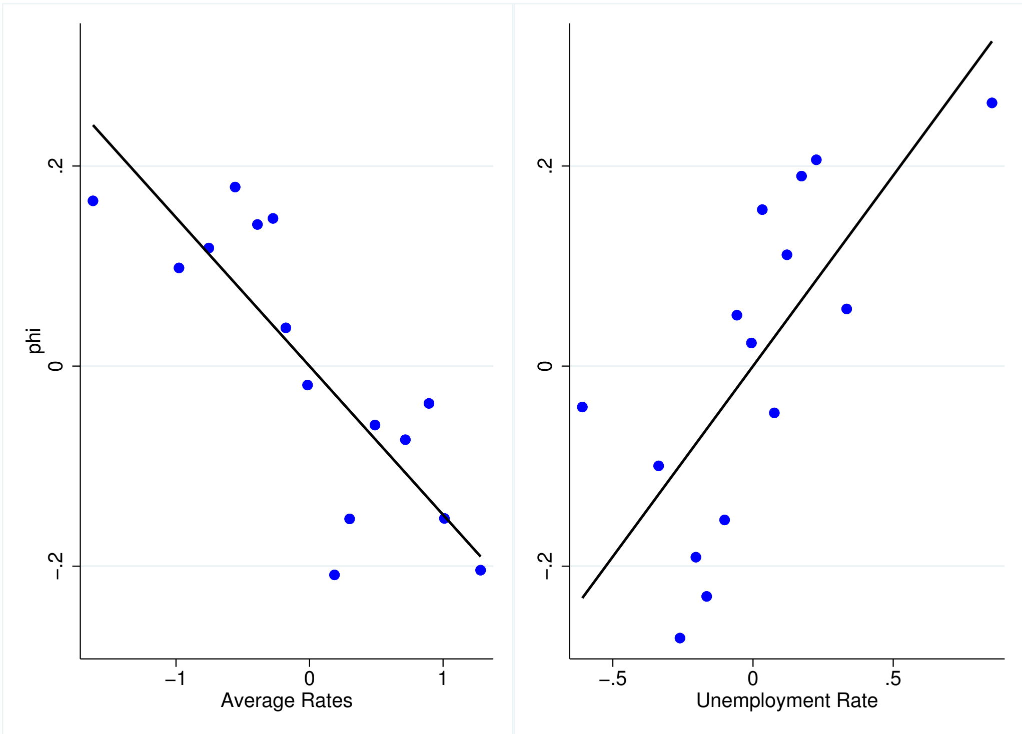


Figure 4: φ against (unweighted) average interest rates among products considered in the Quoted Household Interest Rate data and unemployment. All series are cyclical components after HP filtering. Black solid lines are from linear regressions, which give $\hat{\phi} = -0.149\hat{\bar{i}} + 2.7\text{e-}9$, $R^2 = 0.146$ and $\hat{\phi} = 0.381\hat{u} + 9.3\text{e-}10$, $R^2 = 0.170$ respectively. Blue circles are averages of φ and the regressor of interest within groups of observations, grouped by their position within the distribution of the regressor.

[25] I use the unweighted mean of the interest rates in the market here, but all rates tend to move together in this market, so using the blind rate, the Quoted Household Interest Rate, or the interest rate on one year UK treasury bills makes little quantitative difference, and no difference to the qualitative conclusions.

[26] All cyclical components are extracted using a HP filter.

[27] This was the first run on a British bank in 150 years, which marked the arrival of the crisis in US financial markets in UK retail banking.

When interest rates are high and unemployment is low, savers choose products with low interest rates, close to those offered by the big four banks. As rates fall and unemployment rises, households move up through the distribution of offered rates, more reliably choosing the higher interest rate products in the market, and so achieving higher interest rates relative to the distribution of offers than they did when average rates were high and unemployment was low. In appendix D I show that the same result comes out when I use alternative versions of φ. In particular, I show that in contractions the average interest rate achieved by households moves closer to the highest interest rate on offer in the market, as well as increasing away from the blind rate.

This result is inconsistent with the idea that interest rate dispersion, and household choices in the savings market, are driven by calculations over bank risk. If risk drives choices in this market, then we should see a flight to safety in bad times, when banks are at a greater risk of failure. Households should converge on the big banks, as these are most likely to be able to weather financial shocks, and if they did get in to trouble would be almost certain to be bailed out (they are 'too big to fail'). We should therefore see φ falling the Great Recession. In reality, as the Bank of England lowered policy rates throughout 2008 and 2009, and average interest rates in savings markets came down accordingly, and as unemployment rose, φ rose substantially. Along with the arguments presented in section 3.1.1, this suggests that risk does not explain the cyclical changes in household choice behaviour in savings markets over this period. In appendix E I show that changes in the size of the fixed-rate bond market are also unable to explain the cyclical patterns in φ_t.

Optimal household attention decisions, however, can explain these time series patterns. In recessions, consumption tends to be low, and so the marginal utility of an extra pound of interest income is high, increasing the incentives to pay attention[28]. In addition, when average rates are low in this market the dispersion of interest rates tends to be high ($Corr(\bar{i}, \sigma(i)) = -0.3$ and is significant at the 0.1% level), increasing the benefits of searching around more[29]. Finally, if there is a 'search for yield' motive, i.e. if there is something about low levels of interest rates that make households want to work harder to increase their returns, this would also encourage greater attention, and so higher φ, when

[28]Similarly, when unemployment is high the opportunity cost of time spent shopping around is low. This does not feature explicitly in the model as the cost of attention is a simple additively separable utility cost and for simplicity I keep the labour market as in the standard New Keynesian model of Gali (2008), in which there is no unemployment. However, as the model features no capital and no government spending, labour supply is perfectly correlated with consumption (to a first order approximation), so this effect is qualitatively the same as the marginal utility of income effect which is included.

[29]This correlation is partly driven by the substantial increase in interest rate dispersion during the crisis, which as mentioned in section 3.1.1 may be partly due to heightened consumer awareness of bank risk. However, this correlation remains negative and significant if I exclude the crisis periods.

average rates are low[30]. In the model in sections 4 and 5 I allow for the first two channels to operate, leaving examination of the search for yield mechanism for future work.

4 Model

Here I show that the observed countercyclicality in the success of households in choosing the best (highest interest rate) products in the market can be generated by a New Keynesian model in which households decide how much costly attention to pay to choosing between different savings products each period. There are N profit-maximising banks offering heterogeneous interest rates, and households can choose to pay a utility cost to obtain more information about the banks before choosing where to place their savings each period. 'Attention' is used to refer to the quantity of information the household chooses to pay for. Attention is countercyclical because in contractions the marginal utility of interest income is high, and the dispersion of interest rates is high[31], so the expected benefits from paying attention are large.

4.1 Households

Households choose consumption and labour as in a standard New Keynesian model, but they also choose how much attention to pay to choosing between N different banks. In equilibrium the banks all offer different interest rates (see section 4.2), and the more attention the household pays the higher the interest rate they achieve.

Specifically, I assume that there is a large representative household composed of many individuals. Each period the household decides how much each individual will consume and save, how much labour they will supply, and how much attention they will pay to the choice of savings products, to maximise expected lifetime utility. All labour and asset income is redistributed among individuals each period, so there is no inequality within the household and the utility maximisation problem is:

$$\max_{c_t, n_t, b_t, i_t^e} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \zeta_t \left(\frac{c_t^{1-\gamma}}{1-\gamma} - \frac{n_t^{1+\eta}}{1+\eta} - \mu \mathcal{I}(i_t^e) \right) \tag{3}$$

[30]The search for yield is usually mentioned in reference to financial institutions taking on more risk in order to increase their returns in low-yield environments (e.g. Martinez-Miera and Repullo, 2017). This is somewhat different from the search for yield mentioned here, as in this setting the agents searching for yield are households, and there is no change in the riskiness of their investments.

[31]This is based on the data in section 3. The pairwise correlations between the within-month standard deviation of interest rates in the products included in the Quoted Household Interest Rate data and the average interest rate in that market, and with the unemployment rate (all after HP filtering), are -0.32 and 0.26 respectively. Both correlations are significant at the 0.1% significance level.

subject to

$$c_t + b_t = \frac{b_{t-1}}{\Pi_t}(1 + i^e_{t-1}) + w_t n_t + \mathcal{D}_t - T_t \tag{4}$$

$$\mathcal{I}'(i^e_t) > 0, \mathcal{I}''(i^e_t) > 0 \tag{5}$$

I have written the budget constraint in real terms. Lower case b_t is real bond holdings, Π_t is gross inflation and w_t is the real wage. T_t are lump sum taxes and $\mathcal{D}_t$ are profits from intermediate goods firms and banks[32]. Finally, ζ_t is an AR(1) demand shock.

The novel element of this problem is the term $\mathcal{I}(i^e_t)$. This represents the amount of attention required for the household to earn an effective interest rate i^e_t on assets bought in period t (which pay off in $t+1$), and will be derived below[33]. From the household's perspective, if they pay more attention they will earn a higher rate of interest, but the interest rate gain from more attention diminishes as attention grows (these properties are expressed in condition 5). They choose how much attention to pay by balancing the expected future marginal utility of higher interest income with the costs of attention. I have modelled the costs of attention as a simple additively separable utility cost, with a constant marginal cost μ, as is common in the Rational Inattention literature (see for example Mackowiak and Wiederholt (2009)). This can be thought of as costly cognitive effort. In appendix A I show that time costs and monetary costs lead to the same qualitative conclusions.

In the maximisation problem I allow the household to directly choose the effective interest rate they face, rather than choosing the amount of attention to pay. This is equivalent to having the household choose attention directly, but the first order condition on the effective interest rate has a more readily interpretable form than a first order condition on attention.

The first order conditions comprise an Euler equation, a labour supply condition, and a first order condition on the effective interest rate:

$$c_t^{-\gamma} = \beta \mathbb{E}_t \frac{\zeta_{t+1}}{\zeta_t} \frac{(1 + i^e_t)}{\Pi_{t+1}} c_{t+1}^{-\gamma} \tag{6}$$

$$c_t^{\gamma} n_t^{\eta} = w_t \tag{7}$$

[32]In section 4.2 I show that banks charge heterogeneous interest rates because they face heterogeneous costs. I assume that these costs are redistributed back to households in this profit term. This is not important for the results, as these costs are estimated to be very small relative to consumption. When estimating an alternative model where bank costs are instead wasted, showing up in the goods market clearing condition, the qualitative results are identical and the quantitative results are very close to this version of the model.

[33]This function will depend on the distribution of interest rates offered by banks, but I have dropped this dependence from the written function to save on notation.

$$\beta b_t \mathbb{E}_t \frac{\zeta_{t+1}}{\zeta_t} \frac{c_{t+1}^{-\gamma}}{\Pi_{t+1}} = \mu \mathcal{I}'(i_t^e) \tag{8}$$

The first order condition on effective interest rates (equation 8) implies that a wealthier household will choose to process more information, and so will experience a higher interest rate. This encourages further saving through the Euler equation[34] (equation 6), but the non-concavity this implies is small enough at plausible parameter values that the first order conditions remain sufficient for utility maximisation. The proof of this is in appendix F.1.

I now turn to the derivation of $\mathcal{I}(i_t^e)$, from the decisions of individuals, who face a discrete choice Rational Inattention problem, as studied in Matějka and McKay (2015). As in the Rational Inattention literature, 'attention' in this model refers to the amount of information that each individual processes about the banks before deciding which bank to choose for their portion of the household's saving. I assume that government bonds are in positive supply, so the household saves a positive amount, which means they prefer higher interest rates to lower.

The assumption behind this problem is that the individuals have access to a wide range of information about banks. They start the period with uninformative prior beliefs, that is they have no information about which bank is offering which interest rate. If they then processed enough of the information that is potentially available to them before making their bank choice - if they paid enough attention - they would be able to precisely identify the best interest rate in the market and choose it with probability 1. However, because attention is costly, the household chooses to limit the amount of information each individual can process before choosing their bank. Intuitively, each individual could visit every bank in the market and observe their interest rate, and so correctly identify the best product in the market, but doing so requires a great deal of effort and so is prohibitively costly. I further assume that individuals cannot share information.

There are therefore two challenges facing an individual. Using terminology from Matějka and McKay (2015), an individual must decide on an *information strategy* (what kinds of information to devote their limited attention to processing) and an *action strategy* (how to translate that information into a bank choice). Formally, we can write this as the individual observing the realisation of a noisy signal about the position of banks in the distribution of interest rates before making their bank choice, but before observing that realisation they can choose the structure of the covariance between that signal and

[34]This interaction between attention and wealth implies that the model actually has two steady states, one in which all households are identical and another with two types of households, some wealthy and others at the borrowing constraint, despite those households having identical preferences and access to asset markets. As the data in section 3 is only informative about average household choices, I study the model around the representative agent steady state with no idiosyncratic household shocks, so there is never any household heterogeneity.

the true distribution of banks. The amount of information processing embodied in a particular signal structure is defined (following Sims (2003)) as the expected reduction in entropy (a measure of uncertainty) between the uninformative prior beliefs about banks, and the posterior beliefs about the rankings of banks in the interest rate distribution after observing a realisation of that signal.

Using lemma 1 from Matějka and McKay (2015), we can leave the belief distributions and signal structures in the background, and rewrite the individual's problem in terms of conditional choice probabilities[35]. The individual's maximisation problem becomes:

$$\max_{P(n|s_t)} \mathbb{E}_{s_t}(\sum_{n=1}^{N} i_{s_t}^{n} P(n|s_t)) \text{ subject to} \tag{9}$$

$$\mathcal{I}_t = \log N + \mathbb{E}_{s_t} \sum_{n=1}^{N} P(n|s_t) \log(P(n|s_t)) \tag{10}$$

I have denoted the state of the world (i.e. the ordering of banks in the interest rate distribution) as s_t. The choice variable $P(n|s_t)$ denotes the conditional probability that the individual chooses bank n given the state of the world is s_t. The individual chooses a decision rule (a set of conditional choice probabilities for each possible ranking of banks s_t) in order to maximise their expected interest rate, as the redistribution of asset income across individuals each period renders them risk neutral over interest rates. They maximise subject to the constraint that conditional choice probabilities for each bank cannot deviate too far from $\frac{1}{N}$, the choice probability that would be observed for each bank if the individual had access to no more information than their uninformative prior (in which case they would simply choose a bank at random). The more attention the household allows individuals to pay, the more their conditional choice probabilities can deviate from this uninformed level, towards the unconstrained choice rule in which $P(n|s_t) = 1$ if bank n offers the highest interest rate in state s_t, and $P(n|s_t) = 0$ otherwise[36].

Solving the individual's rational inattention problem gives a familiar multinomial logit

[35]See Matějka and McKay (2015) for a detailed discussion of this. The intuition is that it is never optimal to use limited information processing capacity on two distinct signal realisations that imply the same action, so there is a one-to-one mapping from signal realisations to actions. This means we can solve the problem by looking at actions (i.e. choice probabilities) rather than explicitly solving for signals.

[36]Note that $x \log(x)$ approaches 0 as x approaches 0 or 1, so the information processing required to implement this unconstrained rule is $\log(N)$. For all $\mathcal{I}$ below this, there must be a non-zero probability that the individual chooses a bank below the highest interest rate in the market in at least one state of the world. Since a state of the world here simply refers to an ordering of banks (I assume that each state of the world is identical except for which bank occupies each position in the rate distribution, see section 4.2 for details), and individuals have identical preferences over each state, there is no reason to set different decision rules in different states, and so the probability of a sub-optimal choice will be greater than zero for all possible orderings of banks.

choice rule[37]:

$$P(n|i_t^n, i_t^{-n}) = \frac{\exp(\frac{i_t^n}{\lambda_t})}{\sum_{k=1}^N \exp(\frac{i_t^k}{\lambda_t})} \tag{11}$$

Here I have replaced the notation for a state of the world s_t with the interest rate distribution in time t, made up of the rate offered by bank n and the rates at all of their competitors. The variable λ_t is the lagrange multiplier on the attention constraint 10 in the individual problem, or the shadow value of information. As the household increases attention, holding all else equal the constraint becomes less binding and the shadow value of information falls[38]. The mathematical details of the individual choice problem are discussed further in appendix F.

I assume that the household decides how much each individual will save before knowing whether they have chosen a bank offering a high or low interest rate. Combined with the perfect income sharing around the household, this means that all individuals save the same amount as each other in any given period, and the interest rate the household ends up facing across all of their saving equals the expected interest rate achieved by each individual's bank choice. It is this average rate that I refer to as the effective interest rate i_t^e:

$$i_t^e = \sum_{k=1}^N i_t^k P(k|i_t^k, i_t^{-k}) \tag{12}$$

Substituting out for the optimal conditional choice probabilities using equation 11, this becomes:

$$i_t^e = \frac{\sum_{k=1}^N i_t^k \exp(\frac{i_t^k}{\lambda_t})}{\sum_{k=1}^N \exp(\frac{i_t^k}{\lambda_t})} \tag{13}$$

As the shadow value of information λ_t falls (as attention increases), individuals successfully identify higher interest rate banks with a greater probability, and so the effective rate experienced by the household rises. The effective interest rate is therefore an increasing function of the probability of successfully choosing high interest savings products, and information processing $\mathcal{I}$ increases when individuals are more discriminating between banks. Therefore $\mathcal{I}'(\tilde{i}_t) > 0$. Diminishing returns to attention ensure that $\mathcal{I}''(\tilde{i}_t) > 0$.

[37] This link from Rational Inattention to the multinomial logit distribution is theorem 1 in Matějka and McKay (2015).

[38] Attention could rise without any change in λ_t however. If the dispersion of interest rates rises, then the marginal benefit of information for the individual (in terms of increasing their expected interest rate) rises, so if the shadow price of information stays constant attention will rise.

4.2 Savings Products

Attention can only change the interest rate a household experiences if there are multiple interest rates on offer in the market, and more attention helps to identify a better rate. In this section I show how I generate interest rate dispersion in the model, by assuming that households buy government bonds through banks, some of whom are more efficient than others. Inefficient (high cost) banks offer lower interest rates than their efficient competitors.

There are N banks. Each period t, each bank buys bonds from the government and sells them on to individuals, both at price 1. In the following period, the government pays the bank $1 + i_t^{CB}$ per bond bought, and the bank pays $1 + i_t^n$ to the individuals it sold to. In addition, the bank pays a transaction cost χ_t^n per bond bought.

Banks choose the interest rate they offer to individuals i_t^n to maximise profits, taking into account that the number of bonds they sell[39] will depend on how their interest rate compares with the distribution of rates offered by the other banks.

$$i_t^n = \arg\max_{\hat{i}_t^n} \Pr(\text{saver chooses } n | \hat{i}_t^n, i_t^{-n}) \cdot (i_t^{CB} - \hat{i}_t^n - \chi_t^n) \tag{14}$$

In section 4.1 I showed that the probability an individual saver chooses a bank offering i_t^n is given by:

$$\Pr(\text{saver chooses } n | i_t^n, i_t^{-n}) = \frac{\exp(\frac{i_t^n}{\lambda_t})}{\sum_{k=1}^N \exp(\frac{i_t^k}{\lambda_t})} \tag{15}$$

The variable λ_t is the Lagrange multiplier on the individual's information constraint in their bank choice problem. Given these choice probabilities, the profit maximising interest rate for bank n is given by the first order condition:

$$\left(1 - \frac{\exp(\frac{i_t^n}{\lambda_t})}{\sum_{k=1}^N \exp(\frac{i_t^k}{\lambda_t})}\right) \cdot (i_t^{CB} - i_t^n - \chi_t^n) = \lambda_t \tag{16}$$

Bank n will offer higher interest rates to individuals if the policy rate rises, or if their costs fall. They will also offer higher rates if their competitors offer higher rates, or if individuals start paying more attention to their choices (if λ_t falls).

The only part of the bank problem left to specify is the costs χ_t^n. Although the degree of individual information processing will affect the distribution of prices through λ_t, if all banks have the same costs, then in equilibrium they all offer the same interest rate regardless of household attention decisions[40] (proof in appendix F). To keep the model

[39]There is a measure 1 of individuals, so the number of bonds sold is equal to the probability an individual saver chooses that bank.

[40]This is in contrast to models of equilibrium pricing with search costs following the tradition of

tractable, I assume that each period, a ranking of banks r_t is drawn, with each bank assigned a rank $r_t^n \in \{1, 2, ..., N\}$. There is no persistence in this draw[41], so bank n receives rank r in period t with probability $1/N$. Bank n's costs χ_t^n are then a deterministic function of their rank:

$$\chi_t^n = (\tau_1 + \tau_2(i_t^{CB} - \bar{i})) \cdot (r_t^n - 1)^{\tau_3} \tag{17}$$

This means that given a policy rate i_t^{CB} and household attention choice embodied in λ_t, the distribution of interest rates is exactly pinned down. The bank at the top of the ranking has cost 0. The parameters τ_1 and τ_3 control the dispersion and skewness in bank costs, while τ_2 enables the cost distribution to react to changes in the policy rate. This is included so that the equilibrium interest rate distribution can move in response to the level of rates, as it does in the data[42]. The parameter $\bar{i}$ is the steady state nominal policy rate.

4.3 Firms

Firms produce intermediate goods using labour, and set prices subject to Rotemberg (1982) quadratic adjustment costs. A perfectly competitive final goods producer aggregates these intermediate goods with a standard CES aggregator.

The final goods producer's output Y_t is given as a function of intermediate goods $Y_t(i)$:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \tag{18}$$

Cost minimisation gives the following demand curve facing each intermediate goods producer i:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t \tag{19}$$

Burdett and Judd (1983). In those models price dispersion is an equilibrium outcome even when all firms are identical, but that result relies on there being a non-zero probability that each individual will only observe a single price offer. In this model all households consider all banks imperfectly, rather than receiving perfect information on a stochastic number of offers.

[41]This ensures that the information gathered in period t is not useful in period $t+1$. This is a simplifying assumption which is not true in the data: banks in the top decile of interest rates one month have a strong probability of remaining in that decile for the next year. However, note that the Burdett-Judd models used by others in this literature also have no persistence, as all price-setters are identical and follow mixed strategies.

[42]I allow χ_t^n to vary with the policy rate but not output, or labour supply, because the observed significant correlation between the standard deviation of interest rates and the unemployment rate in the data is almost entirely driven by movements in the level of interest rates. Regressing the standard deviation of interest rates on the average interest rate in the market and the unemployment rate yields a coefficient on the unemployment rate extremely close to 0 (p-value of 0.96), but a coefficient on the level of interest rates which is negative and significantly different from 0 (p-value of 0.009).

The price index P_t is also the final good producer's marginal cost, so this is the price offered to consumers. It is given by:

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}} \tag{20}$$

Intermediate goods firms are monopolistically competitive, and face Rotemberg (1982) quadratic adjustment costs in prices. They have a simple production function:

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \tag{21}$$

Here $N_t(i)$ is labour employed by the intermediate firms, and A_t is an AR(1) technology shock.

The firm chooses their price to maximise the expected discounted sum of profits, subject to this production function and demand:

$$\max_{P_t(i)} \mathbf{E}_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left[P_t(i)Y_t(i) - W_t N_t(i) - \frac{\psi}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 P_t Y_t \right] \tag{22}$$

subject to

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t \tag{23}$$

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \tag{24}$$

Firms are owned by households, so the stochastic discount factor $\Lambda_{t,t+1}$ can be read from the consumption Euler equation (6):

$$\Lambda_{t,t+1} = \frac{\beta}{\Pi_{t+1}} \frac{\zeta_{t+1}}{\zeta_t} (\frac{c_t}{c_{t+1}})^{\gamma} \tag{25}$$

The first order condition for intermediate firm i is:

$$\begin{aligned} &Y_t(i)(1-\epsilon) + \frac{\epsilon}{1-\alpha} \frac{W_t}{P_t(i)} \left(\frac{Y_t(i)}{A_t} \right)^{\frac{1}{1-\alpha}} \\ &- \psi \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right) \frac{P_t Y_t}{P_{t-1}(i)} + \psi \mathbf{E}_t \frac{\beta}{\Pi_{t+1}} \frac{\zeta_{t+1}}{\zeta_t} \left(\frac{c_t}{c_{t+1}} \right)^{\gamma} \left(\frac{P_{t+1}(i)}{P_t(i)} - 1 \right) \frac{P_{t+1}(i)}{P_t(i)^2} Y_{t+1} P_{t+1} = 0 \end{aligned} \tag{26}$$

All firms face the same problem and are identical, so they all choose the same prices, and so produce the same amount of output. Imposing that $Y_t(i) = Y_t$ and $P_t(i) = P_t$ we have:

$$Y_t(1-\epsilon) + \frac{\epsilon}{1-\alpha} w_t \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} - \psi(\Pi_t - 1)\Pi_t Y_t + \psi\beta \mathbf{E}_t \frac{\zeta_{t+1}}{\zeta_t}(\Pi_{t+1} - 1)\Pi_{t+1} Y_{t+1} \left(\frac{c_t}{c_{t+1}}\right)^{\gamma} = 0 \tag{27}$$

This is the Phillips curve.

The production function of intermediate producers gives us an equation for aggregate production, since all firms make identical choices, so by labour market clearing $N_t(i) = n_t$. We have:

$$Y_t = A_t n_t^{1-\alpha} \tag{28}$$

4.4 Fiscal and Monetary Policy

To ensure that households are net savers and so prefer higher interest rates I assume that the government issues a positive amount of bonds each period, and they raise lump sum taxes to cover their interest expenses. Monetary policy is set according to a standard Taylor Rule.

I assume that the stock of real bonds issued is constant at $\bar{b}$. The government raises revenue from lump sum real taxes of T_t and selling these bonds. On every nominal bond sold in period t, the government pays out $1 + i_{t+1}^{CB}$ the next period. The government budget constraint is satisfied each period. In real terms it is:

$$\frac{\bar{b}}{\Pi_t}(1 + i_t^{CB}) = \bar{b} + T_t \tag{29}$$

Market clearing for bonds implies:

$$b_t = \bar{b} \tag{30}$$

Monetary policy is conducted according to a Taylor rule:

$$1 + i_t = (1 + \bar{i}) \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\delta_\Pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\delta_Y} M_t \tag{31}$$

M_t is an AR(1) monetary policy shock.

4.5 Market Clearing

All output Y_t must be consumed in period t by households or be used to pay price adjustment costs in intermediate firms. The market clearing condition is therefore:

$$y_t = c_t + \frac{\psi}{2}(\Pi_t - 1)^2 y_t \tag{32}$$

This completes the model.

4.6 Examining the drivers and implications of attention choice

Since variable attention is the novel channel in this model, in this section I explore the effects of changing attention on other model variables, and the factors that would cause such variation.

4.6.1 Causes of attention variation

To understand how attention will respond to shocks, we need to consider the first order condition on effective interest rates (equation 8):

$$\beta b_t \mathbb{E}_t \frac{\zeta_{t+1}}{\zeta_t} \frac{c_{t+1}^{-\gamma}}{\Pi_{t+1}} = \mu \lambda_t^{-1}$$

Intuitively, the household chooses attention to equate the expected marginal utility of income in the next period (the left hand side) with the marginal costs of obtaining that extra income through paying more attention to achieve higher interest rates on their assets. It is future, rather than current, marginal utility that matters because an increase in attention in period t will mean individuals in the household make better bank decisions in period t, which will only pay off when the banks chosen pay out their interest, which is in period $t+1$. When the household expects to value income more in the next period, they therefore pay more attention in the current period in order to increase their future asset income: they choose to lower the Lagrange multiplier on individuals information constraints λ_t, allowing them to process more information before choosing their bank.

The other reason a household might pay more attention is if the marginal costs of increasing the effective interest rate fall. As discussed in section 4.1, we can interpret λ_t as the shadow price of information. If the expected future marginal utility of consumption is unchanged the household will hold this constant. If the dispersion of interest rates on offer rises, holding λ_t constant implies a rise in attention, because the 'price' of attention to the individual is constant but the expected benefits from better choices rise.

Another way to see this is to note that λ_t^{-1} is equal to $\mathcal{I}_t'(i_t^e)$. Intuitively, it is the increase in information processing required to increase the effective interest rate a small amount. Information processing is a function of choice probabilities only, so a given improvement in the probability of choosing the best interest rate in the market costs the same amount of information no matter how much better that rate is than any other. If the dispersion of interest rates rises, then a unit increase in information processing will increase the effective interest rate by more than it would have done before the rise in dispersion. The marginal information needed to increase the effective interest rate therefore falls, so the marginal cost of increasing asset income falls. As the information costs of increasing the effective interest rate are increasing and convex in i_t^e, this marginal cost is brought back up to the marginal utility of future asset income by increasing attention.

We can therefore understand the attention response to each shock by studying the responses of the marginal utility of income and the dispersion of interest rates. Using the profit maximisation conditions (equation 16) for two arbitrary banks n and m we can see that the gap between any two interest rates in the market is:

$$i_t^n - i_t^m = (\chi_t^m - \chi_t^n) - \lambda_t \sum_{j=1}^{N} \exp(\frac{i_t^j}{\lambda_t}) \Bigg(\frac{1}{\sum_{k \neq n}^{N} \exp(\frac{i_t^k}{\lambda_t})} - \frac{1}{\sum_{l \neq m1}^{N} \exp(\frac{i_t^l}{\lambda_t})} \Bigg) \tag{33}$$

Note that any movements in λ_t must be driven by the marginal utility of future income through the first order condition on i_t^e (equation 8). The only way interest rate dispersion can change independently of the marginal utility of income is therefore if the dispersion in costs χ changes. Using equation 17 we have:

$$\chi_t^m - \chi_t^n = (\tau_1 + \tau_2(i_t^{CB} - \bar{i}))\Big((r_t^m - 1)^{\tau_3} - (r_t^n - 1)^{\tau_3}\Big) \tag{34}$$

The only way this can change as shocks hit the economy is through the policy rate i_t^{CB}. The parameter τ_2 is therefore critical. If $\tau_2 = 0$, bank costs are independent of the policy rate and the only force driving attention choices is the marginal utility of future income. If $\tau_2 > 0$, the costs of all banks (except the bank with rank 1) increase when the policy rate rises, but those with higher rank draws see their costs increase by more than those with lower ranks. As those with higher ranks have higher costs to start with, this means the dispersion of interest rates rises with the policy rate. If instead $\tau_2 < 0$ then the dispersion of interest rates falls when the policy rate rises. In section 5 I find that this latter case is a better fit for the data, so for the remainder of this section I will assume we are in that parameter range. The estimation finds $\tau_2 < 0$ because the correlation between the standard deviation of interest rates in the Moneyfacts data studied in section 3 and the average level of interest rates is negative and significant.

After a positive discount factor shock, attention therefore falls. The negative $\frac{\zeta_{t+1}}{\zeta_t}$ directly reduces the importance of future income to the household. Consumption also rises both now and in future periods (assuming the shock has some persistence), further decreasing the future marginal utility of income. The policy rate rises through the Taylor Rule, so the dispersion of interest rates falls. Attention therefore falls.

The effects of monetary policy and technology shocks, however, are less straightforward. A positive monetary policy shock causes consumption to fall, so the marginal utility of future income rises if the shock is persistent. However, the policy rate rises, causing the dispersion of interest rates to fall. The two forces on attention therefore pull in opposite directions, and the overall movement in attention is ambiguous. Similarly, a persistent positive technology shock causes the marginal utility of future income to fall, but the policy rate falls so the dispersion of interest rates rises, again leading to an ambiguous sign on attention adjustment. If the marginal utility channel dominates for these shocks, then attention will rise after a monetary policy shock and fall after a technology shock. If, however, the rate dispersion channel dominates then attention will fall when $M_t > 0$ and rise when $A_t > 0$.

4.6.2 Effects on bank interest rate setting

In choosing the interest rates they will offer to individuals, banks take into account the proportion of individuals that will choose to save with them, which is affected by how the bank's rate compares to the rates of their competitors. Changes in attention therefore alter the profit maximisation conditions of the banks, and so alter the equilibrium level and distribution of interest rates. I show that an increase in attention leads to a rise in the level of interest rates in the market, and a reduction in interest rate dispersion.

Intuitively, banks choose their interest rate offer by trading off making large profits per bond (with low interest rates offered to individuals) and obtaining market share (with high interest rates). If individuals pay more attention, market share becomes more sensitive to the interest rate the bank offers relative to their competitors. Banks therefore offer higher interest rates to avoid losing large amounts of market share. Furthermore, each bank will want to increase their interest rates to keep pace with rate rises at their competitors: interest rates are strategic complements in this market.

To see this in the model equations, take the bank profit maximising condition 16 and differentiate it with respect to λ_t (derivation in appendix F):

$$\frac{di_t^n}{d\lambda_t} = \frac{1}{\lambda_t}\left[i_t^n \mathcal{S}_t^n - \lambda_t - \frac{\mathcal{S}_t^n}{1-\mathcal{S}_t^n}\Big(\sum_{j\neq n} \mathcal{S}_t^j (i_t^j - \lambda_t \frac{di_t^j}{d\lambda_t})\Big)\right] \tag{35}$$

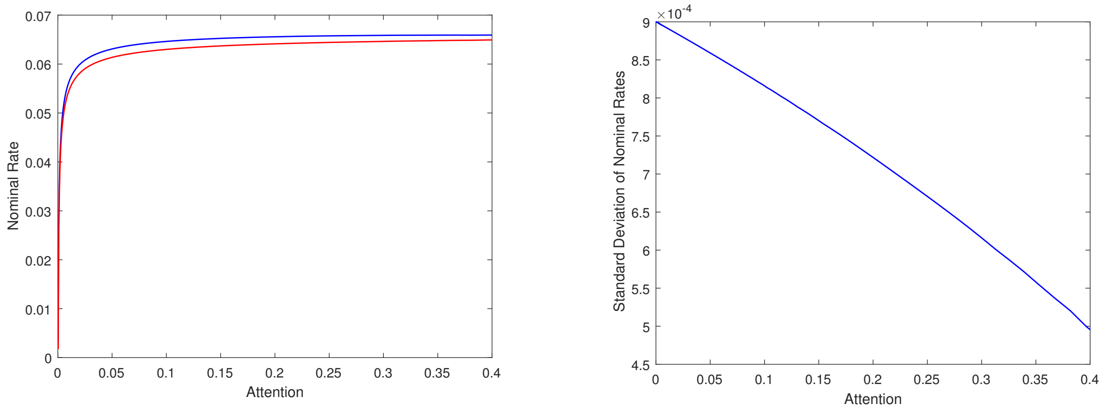
Where $\mathcal{S}_t^n = \frac{\exp(i_t^n/\lambda_t)}{\sum_k=1^N \exp(i_t^k/\lambda_t)}$ is the market share of bank n in period t.

The first term inside the square brackets is positive, pushing for higher attention (lower λ_t) to lead to lower interest rates. This comes about because banks with high interest rates will see their market share rise when attention rises if we held all interest rates constant. This provides an incentive for those banks to decrease interest rates, accept a lower market share and make more profit per bond. For banks with lower interest rates (higher costs) this effect is smaller or works in the other direction, which is reflected in the negative second term outweighing the first.

In addition to these direct effects, there is an indirect effect on the profit maximising interest rate coming from the behaviour of other banks in the market, which is summarised in the final term of equation 35. Firstly, if there are competitor banks with higher interest rates than bank n, an increase in attention will cause bank n's market share to drop, encouraging an increase in interest rates, through the same mechanism discussed for the first term. Secondly, if other banks increase their interest rates when attention rises then $\frac{di_t^j}{d\lambda_t} < 0$, which pushes $\frac{di_t^n}{d\lambda_t}$ down: interest rates are strategic complements. As competitors raise their interest rates bank n becomes less competitive and starts to lose market share, so raises their own interest rate to compete.

For this reason the interest rate rises even at the lowest cost bank in the market for most possible values of attention. The direct effects point towards this bank gaining market share when attention rises, implying they should cut their interest rate and make more profit per bond sold. However, other higher cost banks do have the incentive to raise rates, which in turn means that even the lowest cost bank raises their interest rate. In addition, the greater competition (greater elasticity of $\mathcal{S}$ to interest rates) brought about by the heightened attention reduces the dispersion of interest rates. These effects are shown in figures 5a and 5b, which show the interest rates set when there are two banks in the market, and the standard deviation of those rates, for $i^{CB} = \bar{i} = 0.0708$ and $\tau_1 = 0.0054$, the estimated values from section 5. It is only when attention gets very high (when the probability of successfully identifying the high interest rate bank exceeds 91%, substantially above the 51% in the steady state of the estimated model) that further attention increases lead to the low cost bank cutting their interest rate. As attention approaches log(2), the probability an individual correctly identifies the highest interest rate in the market approaches 1, and equilibrium interest rates approach the Bertrand equilibrium: all banks set their prices (interest rates) equal to marginal cost, except for the lowest cost bank which sets their interest rate above the next highest rate in the market by an arbitrarily small amount. This bank captures all of the market. With two banks this means that the dispersion of interest rates approaches zero[43].

[43]With more than two banks the dispersion will not approach 0 because all banks apart from the most efficient set price equal to marginal cost, and marginal costs have a non-zero dispersion. However, if we were to compute the distribution of interest rates earned by individuals, that is the distribution of rates on offer weighted by the probability of each rate being chosen by an individual, then the dispersion of this distribution would naturally approach 0 as individual information processing approached $\log(N)$, the amount needed to correctly identify the highest rate with probability 1.



(a) Equilibrium rates

(b) within-period standard deviation of rates

Figure 5: Effects of attention variation on the equilibrium interest rates with $N = 2$ banks in the market.

These are the same effects we would expect to see in the Burdett and Judd (1983) model, in which search frictions for consumers cause identical firms to follow mixed pricing strategies, leading to equilibrium price dispersion without any firm heterogeneity. This is the most common model used in the existing literature on the implications of households searching over prices for macroeconomic phenomena (see e.g. Kaplan and Menzio (2016), Yankov (2018)). Although the model used here does require heterogeneity in bank costs, the key advantage over the Burdett-Judd framework is that equilibrium interest rates can be easily found using the simple first order condition for each bank[44] (equation 16). In contrast, the Burdett-Judd mixed strategy equilibrium often needs to be found via an iterative procedure outside of simple cases[45] (see e.g. McKay (2013)), which complicates the solution of general equilibrium models with that kind of price dispersion. Estimating models with Burdett-Judd price dispersion is therefore difficult: Yankov (2018) has to assume that each household lives for only two periods, and that the only source of income variation for households over time is from variations in asset income (interest rates), in order to make the model tractable enough to estimate, and he cannot therefore use his model for business cycle counterfactuals. In contrast, my model can be solved and estimated using standard techniques, and fits in to an otherwise standard New Keynesian model, while preserving the key ways in which search effort (or attention in this model) affects the distribution of equilibrium interest rates.

[44] Alternatively, if we take $N \to \infty$ and assume that costs are drawn from a parametric distribution (rather than using the ranking-based system as in the model with finite N), we can obtain a closed-form expression for i_t^e as a function of λ_t and the parameters from the cost distribution, without needing to find the equilibrium price distribution.

[45] Kaplan and Menzio (2016) derive an analytic expression for the pdf of prices in their model assuming that households observe either one or two prices (never more), in an otherwise standard labour search model.

4.6.3 Effects on household choices

Here I show that an increase in attention leads to a fall in current consumption relative to expected future consumption, as it implies that the household faces a higher interest rate than they would if attention remained constant. This implies that any shock causing consumption to fall and attention to rise will be amplified by the change in attention. Any shock causing consumption to *rise* along with a rise in attention will be weakened by this change.

The impacts of a rise in attention are qualitatively the same as the impacts of a rise in φ.

In this model, the direction in which consumption moves after each shock is unaffected by the introduction of variable attention. That is, for any reasonable parametrisation, variable attention either dampens or amplifies the consumption response to a shock, but is not sufficient to reverse the sign of the consumption response. A positive discount factor shock $\zeta_t > 0$ or technology shock $A_t > 0$ will cause consumption to rise, while a positive monetary policy shock $M_t > 0$ will cause consumption to fall.

If attention falls (rises) after a positive discount factor or technology shock, this will therefore amplify (dampen) the consumption effect of the shock. The reverse is true of a monetary policy shock: if attention falls (rises) after a positive monetary policy shock this will dampen (amplify) the effect of the shock on consumption.

Whichever direction attention moves after these shocks, the direct effect of attention on consumption through the Euler equation described here will be amplified by two general equilibrium effects. Firstly, an increase in attention will (all else equal) cause banks to offer higher interest rates, as shown in section 4.6.2, which amplifies the effect of the attention increase on the effective interest rate experienced by the household.

The second general equilibrium effect to amplify the consumption effects of variable attention comes through the Taylor Rule for the policy rate (equation 31). If attention rises after a shock, consumption will fall relative to where it would be with fixed attention, and output and inflation will both be lower. Through the Taylor rule, the policy rate will fall, which through $\tau_2 < 0$ in the bank cost function means the dispersion of interest rates will rise. This increases the benefits of attention for the household, so attention rises further, causing a greater rise in the effective rate faced by the household.

I now estimate the model using data from section 3 and aggregate macroeconomic data. This is to understand which force dominates in determining how attention moves after technology and monetary policy shocks, and so whether attention amplifies or weakens the consumption impact of those shocks. It is also useful in order to understand the quantitative significance of variable attention for the business cycle.

5 Quantitative Model Assessment

In this section I take the model to the data. I first show that attention in the model links well to the measure φ studied in section 3. I then quantitatively evaluate the model: I set some parameters to standard values in the DSGE literature, and calibrate others to match standard long run features of the data. I estimate the remaining parameters by maximum likelihood, using macroeconomic time series for the UK and data from the empirical work in section 3. Note that while I show that φ is closely linked to $\mathcal{I}$ in the model, I do not assume an equivalence between them when using data on φ in the estimation. I simply add an equation to the model computing φ as I do in section 3. The calibration and estimation is done at a monthly frequency to allow me to use all of the data in section 3.

5.1 Attention is closely related to φ

In this section I show that an increase in attention $\mathcal{I}$ implies a higher value of φ - the average interest rate experienced by the household rises relative to the distribution of rates on offer when attention increases. This is because more attention means individuals choose the higher interest rates in the market with a greater probability, and so the effective rate faced by the household rises relative to the distribution of rates on offer when attention is increased.

In section 3 φ was introduced as a model-free concept. The result that it is related to attention in the model is therefore important to link this model with my empirical exercises. To see why the model generates a link between φ and $\mathcal{I}$, consider the model with just two banks, and define q_t as the probability an individual chooses the bank offering the higher interest rate in period t, which I will refer to as bank 1 without loss of generality. That is:

$$q_t = \frac{\exp(\frac{i_t^1}{\lambda_t})}{\exp(\frac{i_t^1}{\lambda_t}) + \exp(\frac{i_t^2}{\lambda_t})} \tag{36}$$

The assumption that individuals have uninformative priors means that if they pay no attention to bank choice they simply choose each bank with probability $1/N$, so the 'blind' rate in the model is the numerical, unweighted mean of the interest rate distribution. In this two-bank case of the model, φ therefore becomes:

$$\varphi_t = \frac{q_t i_t^1 + (1-q_t) i_t^2 - \frac{1}{2}(i_t^1 + i_t^2)}{\frac{1}{2}(i_t^1 - i_t^2)} \tag{37}$$

Simplifying this we obtain:

$$\varphi_t = \frac{q_t(i_t^1 - i_t^2) - \frac{1}{2}(i_t^1 - i_t^2)}{\frac{1}{2}(i_t^1 - i_t^2)} = 2q_t - 1 \tag{38}$$

That is, in this simple case φ_t is a linear function of the probability an individual successfully chooses the higher interest rate bank. If the individual processes no information before choosing, $q_t = 0.5$ and so $\varphi_t = 0$. If they process enough information to ensure that they always identify the high interest rate bank, $q_t = 1$ and $\varphi_t = 1$.

The information constraint (equation 5) in the two bank case can be written as an increasing concave function of q_t:

$$\mathcal{I}_t = q_t \log(q_t) + (1 - q_t)\log(1 - q_t) \tag{39}$$

Any increase in attention therefore translates to an increase in the probability an individual chooses the high interest rate bank, which leads to an increase in φ. The same intuition holds for the N bank case, though the relationship is no longer so precise. This is shown in appendix H.

5.2 Calibrated parameters

To take the model to the data, I first calibrate some parameters to standard values or long-run features of UK data, before estimating the remaining parameters in the next subsection. I restrict the estimation to the case where there are $N = 2$ banks[46], which means that the parameter τ_3 in the bank cost function is redundant. I set it to 1 without loss of generality.

Table 1 shows the calibrated parameters and their sources or targets.

Table 1: Calibrated parameters

Parameter	Value	Source/Target
β	0.9975	Average real interest rate in Moneyfacts
γ	1	Log utility
η	1	Inverse Frisch Elasticity = 1 (Gali 2008)
α	0.25	Gali (2008)
ϵ	9.67	Harrison and Oomen (2010)
ψ	2680.172	Harrison and Oomen (2010)
δ_Π	1.5	Mizen et al. (2006)
δ_Y	0.5	Mizen et al. (2006)
$\bar{b}$	1	Arbitrary

[46] I plan to return to this estimation for a larger number of banks in the next draft of this paper.

Note that the price stickiness parameter ψ is not equal to the one found in Harrison and Oomen (2010) for the UK, as their model is estimated at a quarterly frequency. To convert this for my monthly model, I find the average duration of a price in a quarterly model with Calvo pricing that would give the same Phillips Curve as the parameter in Harrison and Oomen (2010) to a first order approximation. I then convert this average duration to months, and calculate the price stickiness parameter ψ that would give the same Phillips Curve (to first order) in the monthly model.

The final parameter I calibrate is τ_1 in the bank cost function. For each parameter combination considered by the estimation, I set τ_1 such that the steady state gap between the blind rate (the unweighted mean interest rate) and the highest rate in the market (both annualised) equals the mean of this spread in the data in section 3.

5.3 Estimation

The remaining parameters not set in previous subsection are then estimated using maximum likelihood. I estimate these parameters using five data series for the UK from 1996-2009, two of which are standard macroeconomic time series (consumption and monthly CPI growth), and three which come from the empirical work in section 3 (the mean and within-month standard deviation of interest rates, and the summary statistic for household choice φ). All of these series are monthly, except for consumption. I therefore define a new model variable $c_t^q = c_t + c_{t-1} + c_{t-2}$, and allow the Kalman filter in the estimation to impute values for the missing observations for months not at the end of a quarter. For details on the data treatment see appendix I.

There are three structural shocks in the model: A_t, M_t, ζ_t. To these I add two i.i.d. measurement errors, on the standard deviation of interest rates within the month and on φ. I choose these variables because they are constructed from only one asset class, when in reality households hold assets of many types. The measurement error should help reduce the impact of shocks which are specific to the fixed-interest market from which the data is drawn, but which do not affect household portfolios more widely.

There are therefore ten parameters to estimate: the standard deviation of the five shocks, the persistence of the three structural shocks, the relationship between bank costs and the policy rate τ_2, and the cost of information μ.

The estimation returns that $\mu = 0.0379$, which importantly is significantly different from 0, so information costs do play a valuable role in helping the model explain the data. It also finds that $\tau_2 = -0.1656$, also significantly different from 0. The negative τ_2 estimate reflects the fact that some force in bank costs is needed for this model to generate the negative correlation between the level and dispersion of interest rates in the data. The full estimation results are presented in table 9 in appendix I.

5.4 Quantitative Implications of Attention

In section 4.6, I showed that variable attention amplifies the effects of discount factor shocks on consumption, but that the effect on technology and monetary policy shocks was ambiguous, because the marginal utility of consumption pushes attention in the opposite direction to the dispersion of interest rates, and so it is not clear how attention will respond to the shocks.

In the estimated model I find that the marginal utility channel dominates. That means that after a contractionary monetary policy shock, when consumption falls, households choose to increase their attention to choosing between the different banks. Households experience higher interest rates than they would have done without the attention variation, which pushes consumption down further. Similarly, a positive technology shock causes consumption to rise and so attention to fall, which amplifies the consumption response to the shock. Variable attention therefore amplifies all three of the shocks in the model. Table 2 shows the magnitude of this amplification: for each shock, it displays the elasticity of consumption to each shock in the period of impact[47] in two counterfactual models, expressed relative to the elasticities in the baseline model. The first counterfactual model is identical to the baseline, except that attention is fixed at its steady state level in every period. The second model takes the cost of information μ to 0, so every individual chooses the best bank in the market with probability 1.

Table 2: Elasticity of consumption to shocks relative to estimated benchmark.

Shock	**Variable Attention**	**φ fixed at steady state**	**$\mu = 0$**
Discount factor	1	0.265	0.769
Monetary Policy	1	0.903	0.901
Technology	1	0.560	0.912

Without variable attention, consumption is substantially less responsive to shocks than in the baseline model. The amplification is strongest for discount factor shocks, where both the marginal utility and interest rate dispersion channels increase attention. Comparing to the model with attention held at steady state, variable attention accounts for almost three quarters of the consumption effect of discount factor shocks, 44% of the effect of technology shocks, and 10% of the effect of monetary policy shocks. Reducing the information cost to zero reduces the consumption elasticity to shocks as well, but by different amounts, which is unsurprising as the steady state of the model changes with the information cost.

[47]The only persistence in this model comes from persistence in the shocks, so the relative impact elasticities are exactly the same as the relative cumulative elasticities over several periods.

Although the consumption response to shocks does not fall as far as if we could fix attention at steady state, this does show that policies that successfully reduce the difficulty of processing information and making good decisions on savings products could have non-negligible effects on consumption volatility.

Finally, I take the estimated shock series from the baseline model, and feed them into the two counterfactual models without variable attention. If attention had been fixed at steady state throughout the period, the model implies that the consumption fall between January 2008 and January 2009 would have been 12.9% smaller. If the information cost had been 0, this would have mitigated the consumption fall by 10.9%.

6 Conclusion

In this paper I have presented a novel channel through which aggregate shocks affect consumption. There is a large amount of dispersion among savings products with very similar characteristics, and households are more successful at choosing the highest interest rate products when the average level of interest rates is low and the unemployment rate is high. An improvement in household choices increases the interest rate households actually face, and so causes current consumption to fall as households postpone more consumption to future periods. Systematic variations in choice behaviour over the business cycle therefore impact the consumption response to the shocks that drive the cycle.

In an estimated model in which households are rationally inattentive to the choice between heterogeneous savings products, the marginal utility of consumption is found to be the key driver of attention choices, and so of the probabilty with which individuals successfully identify the highest interest rate product in the market. This means that variable attention amplifies the effect of aggregate shocks on consumption: if a policy-maker could have held attention at its steady-state level throughout the Great Recession consumption would have fallen by 13% less. Much of this benefit could have been achieved by reducing the cost of information, so this analysis suggests that policies aimed at making it easier for households to 'shop around' for financial products could have beneficial effects on business cycle volatility.

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A The core mechanism in alternative models

Here I show that the main mechanism of the inattention model laid out in section 4 is also present in a broad class of models in which households can pay a cost to increase the interest rate they face. This includes a model with frictional search for savings products, as in McKay (2013). For simplicity I assume an exogenously fixed distribution of interest rates here. As discussed in section 1, it is possible to endogenise this in a frictional search model as a mixed-strategy equilibrium for banks: this is the idea in Burdett and Judd (1983). However, that mixed strategy equilibrium often has to be solved for numerically, making it too intractable for rich business cycle analysis. In appendix F I show that the equilibrium interest rate distribution in the inattention model (which is endogenous to household attention decisions) shares many of the well-known properties of the equilibrium price dispersion obtained in Burdett-Judd models.

Consider an infinitely lived household who chooses consumption and saving each period (leaving labour supply and other choices in the background for simplicity) to maximise expected lifetime utility subject to a standard budget constraint (income comes from an endowment y_t and asset income). This household problem is set out below; the only difference to the familiar textbook problem is that households can also choose in period t to pay a cost to increase the interest rate they face i_t^e. That is, to achieve a given interest rate they must pay a cost $C(i_t^e)$, where C is an increasing convex function. I will consider two alternative specifications for this cost, one in which the cost is a utility cost which is additively separable from consumption in the utility function, and another in which it is a monetary cost entering the budget constraint. The utility cost specification could be thought of as time or effort spent searching for products, while the monetary cost would be paying an advisor or intermediary to search on their behalf. The specification in use is determined by the binary variable ϕ: when $\phi = 0$ the cost is a utility cost, when $\phi = 1$ we are studying the monetary cost specification.

$$\max_{c_t, b_t, i_t^e} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \big[u(c_t) - (1-\phi)C(i_t^e)\big] \tag{40}$$

subject to

$$c_t + b_t + \phi C(i_t^e) = y_t + b_{t-1}(1 + i_{t-1}^e) \tag{41}$$

We obtain a familiar consumption Euler equation, and a first order condition on i_t^e:

$$u'(c_t) = \beta(1 + i_t^e)\mathbb{E}_t u'(c_{t+1}) \tag{42}$$

$$(1-\phi)C'(i_t^e) + \phi u'(c_t)C'(i_t^e) = \beta b_t \mathbb{E}_t u'(c_{t+1}) \tag{43}$$

The marginal utility of increasing the interest rate is equal to the marginal utility of future income multiplied by the stock of assets over which the interest rise will have an effect. This is intuitive, since a rise in the interest rate today constitutes a rise in asset income in the following period. The household simply equates this marginal utility with the marginal cost of achieving the rise in interest rates. With a diminishing marginal utility of consumption, when expected future consumption falls the marginal utility of higher interest rates rises. In the utility cost specification households will respond by paying to increase their interest rate (which as C is convex leads to a rise in $C'(i^e_t)$). In the monetary cost specification, we also need to consider how the costs of increasing interest rates have changed in utility terms. Specifically, households will only pay to increase i^e_t (and so $C'(i^e_t)$) if expected future consumption has fallen *relative to* current consumption. In that situation future income is worth more in utility terms relative to current income, so the household gives up some current income to increase their future asset income.

In the consumption Euler equation, we can see that a higher interest rate implies that the current marginal utility of consumption must be higher relative to future marginal utility. The household choice to increase interest rates therefore causes current consumption to fall relative to future expected consumption. After a persistent contractionary shock, expected future consumption will fall, so households will pay to increase their interest rate[48], which will cause current consumption to fall further, amplifying the shock. This is the mechanism explored in section 4: the rational inattention problem is a tractable way to motivate and model the cost $C(i^e_t)$ as a utility cost, and allows for the distribution of interest rates available to be endogenised as a bank pricing equilibrium. It is not, however, the only way to do this. I now show that a model with frictional search for banks also fits into this class of models.

Suppose that there is a large household made up of many individuals. Many banks offer savings products, with interest rates that are distributed according to some CDF $F(i)$. Individuals can only choose a bank for their saving if they have observed its interest rate, and these observations are subject to a search friction. The search friction is such that individuals all observe one bank drawn at random from F, then with probability ψ an individual observes a second bank (again drawn at random) before choosing where to place their savings. This meeting rate ψ is some increasing function of the search effort of the individual, denoted e, which is decided by the household.

[48]In the monetary cost specification households will only increase their interest rate if future consumption is expected to fall by more than current consumption. While this is not the case in the model presented in section 4, in richer business cycle models there is usually found to be large amounts of internal persistence which gives rise to 'hump-shaped' dynamics after shocks, which would imply that households would pay to increase rates after a contractionary shock in both cost specifications.

If an individual observes the interest rates of two banks, they will choose the bank offering the higher interest rate, so the interest rate chosen follows the distribution $(F(i))^2$. The expected interest rate for an individual before we know how many banks they will observe, that is the effective interest rate faced by the household overall, is therefore:

$$i_t^e = (1 - \psi(e_t)) \int if(i)di + 2\psi(e_t) \int if(i)F(i)di \tag{44}$$

This is increasing in the probability of seeing a second bank $\psi(e_t)$, as the expected maximum of two draws from a distribution must be (weakly) greater than the expectation of a single draw. We can rearrange this to express search effort in terms of the interest rate the household ends up facing:

$$e_t = \psi^{-1}\left(\frac{i_t^e - \int if(i)di}{2\int if(i)F(i)di - \int if(i)di}\right) \tag{45}$$

The fraction inside the inverse ψ function increases linearly in i_t^e. If there are diminishing returns to effort (ψ is concave) then effort will be a convex function of the desired interest rate. If we think of effort as being (psychologically) costly in its own right, or because it uses up valuable time, then the costs of increasing i_t^e will be a direct cost in the household utility function. As long as there are weakly diminishing returns to effort, and the cost of effort is weakly convex in effort, and at least one of those two curvatures is strict, then we obtain the first specification discussed above: there is a direct cost in utility which is convex in the desired (chosen) level of the interest rate. Formally, if the cost of effort in the utility function is $C_e(e)$, then we have:

$$C(i_t^e) = C_e\left(\psi^{-1}\left(\frac{i_t^e - \int if(i)di}{2\int if(i)F(i)di - \int if(i)di}\right)\right) \tag{46}$$

$$C''(i_t^e) > 0 \text{ if } C_e''(i_t^e) \geq 0 \text{ and } \psi''(e_t) \leq 0, \text{ one inequality strict} \tag{47}$$

As an example, consider the case where interest rates are exponentially distributed $F(i) = 1 - \exp(-\lambda i)$. The effective interest rate is then:

$$i_t^e = \frac{2 + \psi_t}{2\lambda} \tag{48}$$

Furthermore, suppose that the matching rate of individuals with a second draw from the bank distribution is given by $\psi(e_t) = \bar{\psi} e_t^{\alpha}$, and effort takes time, so the cost in the utility function is of the same form as labour supply in textbook business cycle models (e.g. in Gali, 2008): $C_e(e_t) = \frac{e_t^{1+\eta}}{1+\eta}$. We then have that the cost of increasing the household's interest rate is:

$$C(i_t^e) = \frac{1}{1+\eta}\left(\frac{2}{\bar{\psi}}(\lambda i_t^e - 1)\right)^{\frac{1+\eta}{\alpha}} \tag{49}$$

This cost is increasing and convex as long as $\eta \geq 0$, $\alpha \in (0,1]$, and at least one of $\alpha < 1$ and $\eta > 0$. The first order condition on i_t^e is then:

$$\frac{\lambda}{\alpha}\left(\frac{2}{\bar{\psi}}\right)^{\frac{1+\eta}{\alpha}}(\lambda i_t^e - 1)^{\frac{1+\eta-\alpha}{\alpha}} = \beta b_t \mathbb{E}_t u'(c_{t+1}) \tag{50}$$

The restrictions on α and γ ensure that $\frac{1+\eta-\alpha}{\alpha} > 0$, so when expected future consumption falls (so $\mathbb{E}_t u'(c_{t+1})$ rises) the household chooses to increase the interest rate they face i_t^e, which amplifies the contraction in consumption.

B Loans and misallocation

In this section I discuss two alternative channels through which attention to financial product choice could affect the business cycle: attention to loan choice and misallocation of credit. I argue that they are potentially less powerful than the consumption channel of attention to savings that I study in the main body of the paper.

At first glance, it may seem that variable attention to loans should counteract the effects of variable attention to savings. If attention to both choices rises in contractions, then savers will face higher interest rates and so reduce their consumption (the main channel studied in this paper), but borrowers will on average find out about lower interest rate loans, and so will have an incentive to increase their consumption. However, there are two reasons to expect that the two channels do not operate in the same way, and are not of equal importance.

Firstly, the most significant debt for the majority of indebted households is a mortgage, and the evidence from the FCA (2019) suggests that there is strong price competition leading to only limited interest rate dispersion in mortgages. The scope for attention to drive interest rate changes is therefore small, and indeed one reason why this might be the case is that the large sums of money involved lead almost all mortgagors to pay a large amount of attention to their choice of product whatever the state of the economy when they make their choice.

Secondly, it is not clear that attention to loan choice will rise in contractions as attention to savings does. For savings, I find that the marginal utility of income is very important in determining the extent of attention, and for savers the marginal utility of income is high in contractions for two reasons: labour income and asset income are both low, as wages and interest rates are low. In contrast, in a contraction a debtor sees their labour income fall, but the decline in interest rates (assuming the contraction is demand-led, so interest rates do indeed fall) leads to lower debt repayments, and so to a greater disposable income. It is not therefore clear that attention to loan choice will rise in contractions: for the most indebted households a fall in interest rates will increase disposable income so much that the marginal utility of income could even fall. Unfortunately, the Moneyfacts data on loans is not suitable for an empirical examination of attention to credit products, as the products tend to be very complicated. This means that the equivalent Bank of England data on quoted household interest rates averages over a set of products with substantially different characteristics, and so the comparison of this with the Moneyfacts panel does not accurately reflect search or attention behaviour.

The second alternative mechanism relates to what banks do with the deposits they receive. If higher interest rates reflect more productive investment opportunities for the bank, then as households pay more attention to their savings choices in recessions there will be a reduction in misallocation, dampening the output effects of the contraction. This is a very interesting potential mechanism, and I leave detailed study of it to future research.

In the specific case of retail savings, however, it is unlikely that this channel has much effect. Particularly in the UK (though this is the case in other countries as well), retail banks take in deposits in order to fund residential lending. We know from the arguments and evidence in the previous paragraphs that there is very little interest rate dispersion in mortgages, so it is unlikely that banks offering higher deposit rates are doing so because they have access to more profitable lending opportunities.

This argument does not apply to other forms of saving, particularly saving in equities. Greater attention to equity choices in recessions should indeed lead to lower misallocation, which would mitigate the amplification of the business cycle that I find through the consumption channel. It is useful to note, however, that less than 20% of equities in the UK are owned by UK individuals[49] (ONS 2020), and even that figure masks the fact that many of those individuals hold their equities through managed funds and other institutions. This means that an increase in attention by households can only have a small effect on misallocation, as the majority of equity investment decisions are controlled by professional investors, who should spend all of their time paying as much attention to their choices as possible.

[49]The ONS does not distinguish between foreign individual and foreign institutional investors, but even with these included it is clear that the majority of equities are not under the direct control of households.

C Monte Carlo simulations for φ

The link between the empirics in section 3 and the model in section 4 relies on attention being related to the position of average household interest rates within the distribution of rate offers, as measured by φ. In section 5.1 I discuss why this is the case in the model developed in this paper, but that makes a number of simplifying assumptions. In particular, the assumption that individuals belong to large households means that the individuals processing information about banks are risk neutral. I also assume that all individuals pay the same amount of attention as each other within a period, and that prior beliefs are not biased towards one bank or another. Moreover, outside of the simple case with two banks, I can only prove that there is a positive relationship between attention as defined in the Rational Inattention literature and φ, so it is possible that changes in the shape of the interest rate distribution could be shifting φ mechanically, without a corresponding change in attention.

To show that attention and φ are closely related, without large contamination from other aspects of the rate distribution, even in a model without the assumptions made in section 4, in this appendix I run simulations of a significantly richer model of household choice in partial equilibrium and examine how successful φ is at capturing attention.

There are a large number of households indexed $h \in H$. There are N banks, plus a 'default option' bank. This default bank offers an interest rate of i_d, and all households observe this without cost. They also observe the distribution of interest rates on offer at the other banks, $g(i)$, but have uninformative prior beliefs about which bank is offering which rate. They can process costly information to update their beliefs before choosing a bank. The marginal cost of information for household h is μ_h, and information is measured using the entropy measure for discrete distributions as in Matějka and McKay (2015). The total cost of information for household h is therefore:

$$C_h(\mathcal{I}_h) = \mu_h \Bigg(\log(N) + \mathbf{E}_s \sum_{n=1}^{N} \Pr_h(\text{choose } n|\text{state } s) \log \Pr_h(\text{choose } n|\text{state } s) \Bigg) \tag{51}$$

Households either choose the default bank and pay no information processing costs, or choose to pay attention to the other banks. They know the distribution of interest rates, so if they pay attention there is no chance of them deciding to choose the default option after all. They pay attention if the expected utility of entering the market for other banks is higher than that of choosing the default bank. I assume that households value higher interest rates according to:

$$v(i) = A \log(Bi + C) \tag{52}$$

From the results in Matějka and McKay (2015), a household that pays attention to other banks has:

$$\Pr_h(\text{choose } n|\text{state } s) = \frac{(Bi_{n,s} + C)^{A/\lambda_h}}{\sum_{m=1}^{N}(Bi_{m,s} + C)^{A/\lambda_h}} \tag{53}$$

For each simulation, I set N at 34, the average number of products per quarter qualifying for the Quoted Household Interest Rate data. I set the number of households H at 10000. In each simulation, I first draw parameters governing the mean and standard deviation of the 'other bank' interest rates and the information costs λ_h from given distributions. I then draw the interest rates and information costs from log-normal distributions with these means and variances. I also draw the no-attention interest rate i_d from a given distribution[50]. For simplicity I set the parameters in the preferences at $A = 1, B = 1, C = 0$.

I then calculate the average attention $\mathcal{I}_h$ across the population, and the measure φ using:

$$\varphi = \frac{\mathbf{E}_h i_h - i_d}{\sigma(i_n)} \tag{54}$$

Where i_h is the interest rate on the product chosen by household h.

The graph below plots the log of φ against the log of average household attention $\log(\mathbf{E}_h\mathcal{I}_{\langle})$ over 50000 simulations.

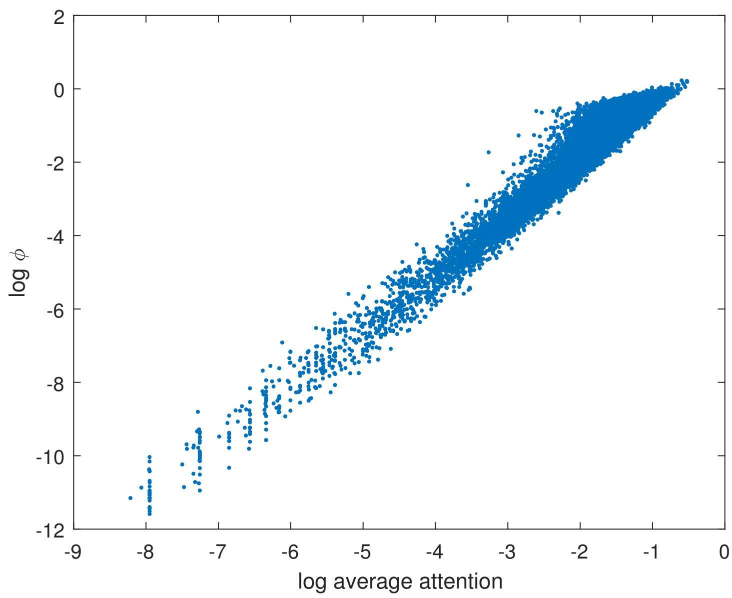

Figure 6: Simulation results

While there is some noise, there is a clear positive correlation between φ and average

[50]Equal to the distribution of other interest rates, with a slightly lower mean to reflect the observation that the no-attention benchmark interest rates in the Moneyfacts data are, on average, below the mean of the rest of the distribution. The standard deviation is often high, however, so in several simulations i_d is near the top of the distribution of other interest rates. In this situation only households with extremely low information costs λ_h choose to process any information.

attention even in this rich model of household choice.

In dividing the gap between $\mathbf{E}_h i_h$ and the benchmark no-attention interest rate by the standard deviation of interest rates, I attempted to remove contamination of φ by features of the interest rate distribution on top of the effects those features have on optimal attention choice. To examine how successful this is in this rich simulation, I run the following regression on the simulated data:

$$\log \varphi_s = \alpha + \beta \log(\mathbf{E}_{hs}\mathcal{I}_{hs}) + \gamma_1 \log \bar{i}_s + \gamma_2 \log \sigma(i)_s + \gamma_3 \log \text{skew}(i)_s + \varepsilon_s \tag{55}$$

The results are presented below.

Table 3: Regressions on simulation results.

	$\log(\varphi)$
$\log(\mathbf{E}_{hs}\mathcal{I}_{hs})$	1.676^{***}
	(0.0021)
$\log \bar{i}_s$	0.048
	(0.0359)
$\log \sigma(i)_s$	-0.049^{*}
	(0.0251)
$\log \text{skew}(i)_s$	-0.096^{***}
	(0.0074)
Constant	1.596^{***}
	(0.0367)
Observations	50000

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

As indicated in figure 5, there is a strong positive link between average attention in the model and the measure φ. Although the standard deviation has a marginally significant negative effect on φ and the skewness of the interest rate distribution has a significant negative effect, in excess of their effects on attention, the coefficients are very small. From this I conclude that φ is a reasonable measure of attention if the true data generating process is similar to that used in these simulations: households have diminishing marginal utility of interest rates, and face a discrete choice problem with costly information acquisition, with a 'default option' which they can choose and get a known return without paying any attention.

D Alternative measures of φ

Here I present two alternative summary statistics for household savings product choice, which corroborate the evidence in section 3.3 that households move up through the distribution of interest rates when the level of average rates is low.

First, I define a new variable φ_{best} in a similar way to φ, except that rather than comparing the average rate achieved by households each month with the 'blind rate', I compare it with the 'best buy': the highest interest rate available in the market. Intuitively, rather than comparing choices to a 'no attention' benchmark, this compares choices to a full information benchmark.

$$\varphi_{best} = \frac{\mathbf{E}_h i_t - i_t^{best}}{\sigma(i_t)} \tag{56}$$

Second, I consider the percentile of the interest rate distribution at which the average interest rate achieved by the household sits. This is even more model-free than φ and φ_{best}, taking no stance on what an appropriate benchmark for choices should be. As with the previous two statistics, it is homogeneous of degree 0 in rates. The downside is that it does not consider the shape of the rate distribution either side of the average rate achieved by households. I call this variable φ_{pct}:

$$\varphi_{pct} = \Pr(i_t^n < \mathbf{E}_h i_t) \tag{57}$$

Both of these measures, like the φ considered in the main text, are higher when households are more successful at choosing the higher interest rate products in the market. And like φ, they both rise when average interest rates are low. The three measures are also positively correlated with one another. The pairwise correlations between the three statistics on household choice and mean interest rates are shown in the table below.

Table 4: Pairwise contemporaneous correlations

	φ	φ_{best}	φ_{pct}	$\bar{i}$
φ	1			
φ_{best}	0.4146	1		
φ_{pct}	0.2709	0.8238	1	
$\bar{i}$	-0.2598	-0.1133	-0.1638	1

The qualitative implications explored in section 4.6.3 therefore hold with other measures of household choice. Changing to a different choice measure would, however, have quantitative implications for the estimated model in section 5.

E Market size cannot explain fluctuations in φ_t

In a recent paper Dreschler, Savov and Schnable (2017) (DSS) show that when the Federal Funds Rate rises in the US, retail banks increase their deposit spreads and deposits flow out of the retail market. Here I show that such switching out of the deposit market and into other asset types (such as government bonds) cannot explain the cyclicality of φ_t, because the proportion of households who hold fixed interest rate bonds does not vary significantly through the Great Recession.

In principle, the switching identified by DSS could drive my empirical findings. If households differ in their propensity to pay attention to their savings, then it could be that when the level of interest rates rises the high-attention households switch out of the retail deposit market. The savers that remain buying fixed-rate savings bonds from banks are the low-attention households, and so the average attention of households in the market falls without any individual household changing the amount of attention they are paying. We could observe a countercyclical φ_t without any variation in the attention paid by each household.

To explore if this switching is occurring, I study waves 1-3 (2006, 2008, 2010) of the Wealth and Assets Survey (WAS). This survey asks a large number of households several questions about their assets, including whether they hold fixed interest rate savings bonds, and if they do how large their deposits are in such products. As the three waves span the Great Recession, if the DSS switching behaviour is driving the cyclicality of φ_t we should find that the proportion of households who hold the products studied in section 3 increases over time.

The products considered in section 3 were those available with an investment of £5000. In the vast majority of cases, this means that the minimum investment was £5000, and the maximum was £9999, as above that level the banks usually offer a different interest rate. I therefore study the proportion of households who hold fixed-rate savings bonds with balances in this range in each wave of the WAS. Table 5 shows the results from regressing a dummy variable indicating whether the household owns a fixed-rate bond and has the given deposit size in those bonds on the wave of the WAS they are in, with wave 1 (2006) as the baseline. The probability of holding the bond is not (statistically or economically) significantly different in each wave, even though the Great Recession occurs during this period. This remains true if I widen the range of deposit sizes, which I do as a check since a minority of products in the sample have a lower minimum investment than £5000, or don't have a corresponding product for balances above £10000.

This is not inconsistent with the mechanism in DSS. Table 6 shows that the proportion of

Table 5: Proportion of households holding the relevant fixed-rate bonds does not change significantly over the Great Recession

	(1) Hold bond £5,000-£9,999	(2) Hold bond £2,500-£12,499
Wave=2	-0.000575	0.000431
	(-0.56)	(0.28)
Wave=3	-0.00112	-0.000229
	(-1.05)	(-0.15)
Constant	0.0129***	0.0275***
	(19.85)	(29.08)
Observations	72197	72197

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

households holding very large balances in fixed-rate bonds increased through the Great Recession (following the same method as table 5 for larger balances). While a minority of products studied in section 3 would allow these higher balances, in general households could get higher interest rates by depositing larger balances in banks offering specific large-balance bonds, so if these households are high-attention types they are unlikely to be buying products in the set studied above.

Table 6: Proportion of households holding larger fixed-rate bonds does change significantly over the Great Recession

	(1) Hold bond £25,000-£49,999	(2) Hold bond £50,000+
Wave=2	0.00290**	0.00169
	(2.91)	(1.84)
Wave=3	0.00262**	0.00428***
	(2.67)	(4.40)
Constant	0.0114***	0.0114***
	(19.14)	(20.23)
Observations	72197	72197

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Finally, if the proportion of savers in the fixed-rate market who are high-attention types increases in recessions, we should expect to see an increase through the Great Recession in the average education of those who hold fixed-rate bonds. Table 7 shows that the year in which they participated in the survey does not significantly correlate with the education of a holder of a fixed-rate bond of the size considered in section 3.

Table 7: Educational attainment conditional on holding a fixed-rate bond with balance between £5,000 and £9,999 does not change significantly over the Great Recession

	(1) Has degree level education or above	(2) Has some educational qualification
Wave=2	0.0466	0.00341
	(1.23)	(0.11)
Wave=3	0.0216	-0.0294
	(0.54)	(-0.80)
Constant	0.269***	0.809***
	(12.00)	(40.34)
Observations	1052	1052

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

F Proofs

F.1 The household FOCs are sufficient for utility maximisation

Here I prove that the household first order conditions (equations 6 - 8) are sufficient for utility maximisation. I will only consider a household away from the no-borrowing constraint, as at the constraint they fix attention at zero and attention cannot affect the problem.

For an unconstrained household, their utility maximisation problem can be written as an unconstrained maximisation by substituting out for consumption using the budget constraint:

$$\max_{n_t, b_t, i_t^e} U = \sum_t \beta^t \left(\frac{\left(\frac{b_{t-1}}{\Pi_t}(1+i_{t-1}^e) + w_t n_t + \mathcal{D}_t - T_t - b_t\right)^{1-\gamma}}{1-\gamma} - \frac{n_t^{1+\eta}}{1+\eta} - \mu \mathcal{I}(i_t^e) \right) \quad (58)$$

The first order conditions are:

$$\frac{\partial U}{\partial b_t} = -\left(\frac{b_{t-1}}{\Pi_t}(1+i_{t-1}^e) + w_t n_t + \mathcal{D}_t - T_t - b_t\right)^{-\gamma} + \mathbb{E}_t \frac{\beta}{\Pi_{t+1}}(1+i_t^e)\left(\frac{b_t}{\Pi_{t+1}}(1+i_t^e) + w_{t+1}n_{t+1} + \mathcal{D}_{t+1} - T_{t+1} - b_{t+1}\right)^{-\gamma} = 0 \quad (59)$$

$$\frac{\partial U}{\partial n_t} = w_t\big(\frac{b_{t-1}}{\Pi_t}(1+i^e_{t-1}) + w_t n_t + \mathcal{D}_t - T_t - b_t\big)^{-\gamma} - n_t^{\eta} = 0 \tag{60}$$

$$\frac{\partial U}{\partial i^e_t} = \mathbb{E}_t\frac{\beta b_t}{\Pi_{t+1}}\big(\frac{b_t}{\Pi_{t+1}}(1+i^e_t) + w_{t+1}n_{t+1} + \mathcal{D}_{t+1} - T_{t+1} - b_{t+1}\big)^{-\gamma} - \mu\mathcal{I}'(i^e_t) = 0 \tag{61}$$

The elements of the Hessian matrix are therefore (substituting consumption back in using the budget constraint where it simplifies the expressions):

$$\frac{\partial^2 U}{\partial b_t^2} = -\gamma c_t^{-\gamma-1} - \mathbb{E}_t\frac{\gamma\beta}{\Pi^2_{t+1}}c_{t+1}^{-\gamma-1}(1+i^e_t)^2 \tag{62}$$

$$\frac{\partial^2 U}{\partial n_t^2} = -w_t^2\gamma c_t^{-\gamma-1} - \eta n_t^{\eta-1} \tag{63}$$

$$\frac{\partial^2 U}{\partial i^{e2}_t} = -\mathbb{E}_t\frac{\gamma\beta b_t^2}{\Pi^2_{t+1}}c_{t+1}^{-\gamma-1} - \mu\mathcal{I}''(i^e_t) \tag{64}$$

$$\frac{\partial^2 U}{\partial b_t\partial n_t} = \gamma w_t c_t^{-\gamma-1} \tag{65}$$

$$\frac{\partial^2 U}{\partial b_t\partial i^e_t} = \mathbb{E}_t\frac{\beta}{\Pi_{t+1}}c_{t+1}^{-\gamma} - \mathbb{E}_t\frac{\beta\gamma b_t(1+i^e_t)}{\Pi^2_{t+1}}c_{t+1}^{-\gamma-1} \tag{66}$$

$$\frac{\partial^2 U}{\partial n_t\partial i^e_t} = 0 \tag{67}$$

The first order conditions are sufficient for utility maximisation of U is concave. This is the case if the Hessian matrix is negative semi-definite, i.e. if for any real-valued vector $[x \quad y \quad z]$:

$$[x \quad y \quad z]\begin{bmatrix} \frac{\partial^2 U}{\partial b_t^2} & \frac{\partial^2 U}{\partial b_t\partial n_t} & \frac{\partial^2 U}{\partial b_t\partial i^e_t} \\ \frac{\partial^2 U}{\partial b_t\partial n_t} & \frac{\partial^2 U}{\partial n_t^2} & \frac{\partial^2 U}{\partial n_t\partial i^e_t} \\ \frac{\partial^2 U}{\partial b_t\partial i^e_t} & \frac{\partial^2 U}{\partial n_t\partial i^e_t} & \frac{\partial^2 U}{\partial i^{e2}_t}\end{bmatrix}\begin{bmatrix} x \\ y \\ z\end{bmatrix} \leq 0 \tag{68}$$

Multiplying this out, we obtain:

$$\begin{aligned} -\gamma c_t^{-\gamma-1}(x - yw_t)^2 - \mathbb{E}_t\frac{\gamma\beta c_{t+1}^{-\gamma-1}}{\Pi^2_{t+1}}(x(1+i^e_t) - zb_t)^2 \\ + \mathbb{E}_t\frac{2xz\beta c_{t+1}^{-\gamma}}{\Pi_{t+1}} - \eta y^2 n_t^{\eta-1} - \mu z^2\mathcal{I}''(i^e_t) \leq 0 \end{aligned} \tag{69}$$

This expression would clearly be true without the term in xz, which comes from $\frac{\partial^2 U}{\partial b_t\partial i^e_t}$. This is to be expected: it is the feedback between saving and attention (so interest rates) that is the cause for our concern that U may not be concave. I proceed by showing a condition under which this feedback is sufficiently weak that U remains concave, and so the first order conditions remain sufficient for utility maximisation.

The terms containing c_{t+1} can be written as:

$$\begin{aligned}
&-\mathbb{E}_t\frac{\gamma\beta c_{t+1}^{-\gamma-1}}{\Pi_{t+1}^2}(x(1+i_t^e)-zb_t)^2+\mathbb{E}_t\frac{2xz\beta c_{t+1}^{-\gamma}}{\Pi_{t+1}}\\
&\quad=-\mathbb{E}_t\frac{x^2\gamma\beta(1+i_t^e)^2c_{t+1}^{-\gamma-1}}{\Pi_{t+1}^2}-\mathbb{E}_t\frac{\gamma\beta b_t^2c_{t+1}^{-\gamma-1}}{\Pi_{t+1}^2}\left(z^2-\frac{2xzc_{t+1}\Pi_{t+1}}{\gamma}+\frac{2xz(1+i_t^e)}{b_t}\right)\\
&=\mathbb{E}_t\frac{x^2\beta c_{t+1}^{-\gamma}b_t}{\Pi_{t+1}}\left(\frac{c_{t+1}\Pi_{t+1}b_t}{\gamma}-2(1+i_t^e)\right)-\mathbb{E}_t\frac{\gamma\beta b_t^2c_{t+1}^{-\gamma-1}}{\Pi_{t+1}^2}\left(z-\frac{x\left(c_{t+1}\Pi_{t+1}b_t-\gamma(1+i_t^e)\right)}{\gamma b_t}\right)^2
\end{aligned}\tag{70}$$

From this we get that the utility function is concave if:

$$\frac{b_t}{\gamma}\mathbb{E}_tc_{t+1}^{1-\gamma}\leq 2(1+i_t^e)\mathbb{E}_t\frac{c_{t+1}^{-\gamma}}{\Pi_{t+1}}\tag{71}$$

Assuming that the effective nominal interest rate is never below zero, this will be satisfied if:

$$b_t\mathbb{E}_tc_{t+1}^{1-\gamma}\leq 2\gamma\mathbb{E}_t\frac{c_{t+1}^{-\gamma}}{\Pi_{t+1}}\tag{72}$$

That is, as long as consumption, inflation and assets are sufficiently small relative to the coefficient of risk aversion. In the quantitative model I set $\gamma=1$, and $b_t=1$ for all t, which means the condition becomes:

$$\frac{1}{2}\leq\mathbb{E}_t\frac{1}{c_{t+1}\Pi_{t+1}}\tag{73}$$

The steady state for inflation is $\Pi=1$, so in the neighbourhood of steady state the condition holds as long as c_{t+1} is always sufficiently below 2. As the steady state for consumption is 0.86, we are extremely unlikely to approach the region where the utility function becomes non-concave, and indeed the estimation does not find that we approached that region at any point in the data sample.

To show this more precisely, we can use the fact that if the utility function is concave, the household is on their consumption Euler equation. Substituting this into the cocavity condition gives:

$$\beta(1+i_t^e)c_t\leq 2\tag{74}$$

The maximum value taken by the left hand side of this condition in the simulation is 0.87, so we remain a substantial distance from the region where the utility function would become non-concave.

F.2 Individual choice probabilities

The individual bank choice problem follows the structure of discrete choice rational inattention problems studied in Matějka and McKay (2015) (hereafter MM). Given a known aggregate state of the economy, the individuals know the distribution of interest rates in the bank market, but they do not know the realisations of the bank cost draws, so they do not know the ordering of banks within that distribution. I refer to each of the possible $N!$ rank orderings as a state of the world v. All interest rates, information choices and choice probabilities in the equations below are time-varying, but the time-variation does not matter for the derivation of the formulae so I drop the time subscripts to save on notation.

At the start of each period each individual must choose an information strategy and an action strategy in order to maximise their expected payoff subject to the information constraint imposed by the household. The information strategy is a joint distribution details what kinds of signals the individual will receive about banks. Formally, this is a joint probability distribution between signals s and states of the world v: $f(s, v)$. The action strategy then dictates how the individual translates a signal realisation into a choice of a particular bank. There is no communication between individuals: all receive idiosyncratic signals, which are only correlated because all individuals choose signals which are correlated with the same state of the world v. The information constraint set by the household will restrict the information strategy, so that the individual cannot choose signals which will perfectly reveal the state of the world unless the household has chosen to pay large amounts of attention.

Before setting this out formally, it is helpful to note that the action strategy is necessarily one-to-one (lemma 1 in MM). That is, it is always the case that each signal realisation will have a unique bank choice associated with it, and each bank choice is triggered by a unique signal realisation. An action strategy not satisfying this could not maximise the individual's expected payoff subject to the information constraint. If two signal realisations both implied the same bank choice, it would be a waste of information processing because information would have been processed (to distinguish between the two possible signal realisations) but not used in the action. We can therefore leave the signal structure chosen in the information strategy in the background and rewrite the individual decision rule as $f(n, v)$, the joint probability that the individual chooses bank n in state of the world v.

Remembering that the individual wants to maximise their expected interest rate because the intra-household redistribution makes them risk neutral we can write the individual choice problem as:

$$\max_f \mathbb{E}_v i^{n^*} = \sum_v \sum_n i_v^n f(n,v) \tag{75}$$

subject to

$$\sum_n f(n,v) = g(v) \quad \forall v \tag{76}$$

$$f(n,v) \geq 0 \quad \forall n, v \tag{77}$$

$$H(g(v)) - \mathbb{E}_n H(f(v|n^* = n)) = \kappa \tag{78}$$

Where n^* denotes the bank that is actually chosen, and $g(v)$ is the prior belief of the individual about the probability of being in each state v before receiving any signals. I assume uninformative priors throughout, so $g(v) = \frac{1}{N!}$ for all v. The first two constraints ensure that the decision rule $f(n,v)$ is consistent with those prior beliefs (so really the individual is only choosing the conditional probability of choosing each bank given the state of the world $f(n|v)$) and is always positive.

The final constraint is the information constraint imposed by the household. The function $H(\cdot)$ gives the entropy of a distribution, a measure of its dispersion:

$$H(h) = -\sum_v h(v) \log h(v) \tag{79}$$

The constraint implies that the expected reduction in entropy between prior beliefs g and the posterior beliefs about the states of the world implied by choices, which themselves embody the signals received, is capped at κ, which is set in the household problem. The more accurately signals chosen in the information strategy can identify the state of the world, the closer those posterior beliefs will become to 1 (for the true state) and 0 (all other states), which implies the expected posterior entropy shrink. Capping this entropy reduction with κ restricts the accuracy with which individuals can identify the state of the world, and so the bank offering the highest interest rate in the market.

MM show the solution to this kind of problem in their theorem 1. In this particular case with uninformative priors and risk-neutral preferences the probability that an individual chooses bank n in state v is given by:

$$\Pr(n^* = n|v) = \frac{\exp(\frac{i_v^n}{\lambda})}{\sum_m \exp(\frac{i_v^m}{\lambda})} \tag{80}$$

Where λ is the Lagrange multiplier on the information constraint (equation 78). As the household increases individual attention, κ rises and the information constraint relaxes, causing λ to fall. As λ approaches 0, the choice probability approaches 1 for the highest interest rate bank in the market, and 0 for all other banks. As attention approaches 0, λ gets arbitrarily large and the choice probabilities approach $\frac{1}{N}$ for all banks, irrespective of the state.

F.3 Proof that without cost heterogeneity there is no interest rate dispersion in the model

Rearranging the first order condition for bank n we have:

$$i_t^n = i_t^{CB} - \chi_t^n - \lambda_t \frac{\sum_{j=1}^N \exp(\frac{i_t^j}{\lambda_t})}{\sum_{k\neq n}^N \exp(\frac{i_t^k}{\lambda_t})} \tag{81}$$

The difference between the rates offered by two banks n and m is therefore:

$$i_t^n - i_t^m = (\chi_t^m - \chi_t^n) + \lambda_t \sum_{j=1}^N \exp(\frac{i_t^j}{\lambda_t}) \left(\frac{1}{\sum_{k\neq m}^N \exp(\frac{i_t^k}{\lambda_t})} - \frac{1}{\sum_{k\neq n}^N \exp(\frac{i_t^k}{\lambda_t})} \right) \tag{82}$$

Suppose that the banks have the same costs, $\chi_t^n = \chi_t^m$. Now suppose that bank n is offering a strictly greater interest rate than bank m, $i_t^n > i_t^m$. The left hand side of equation 82 is positive. Furthermore:

$$\exp(\frac{i_t^n}{\lambda_t}) > \exp(\frac{i_t^m}{\lambda_t}) \tag{83}$$

$$\Rightarrow \sum_{j=1}^N \exp(\frac{i_t^j}{\lambda_t}) - \exp(\frac{i_t^n}{\lambda_t}) < \sum_{j=1}^N \exp(\frac{i_t^j}{\lambda_t}) - \exp(\frac{i_t^m}{\lambda_t}) \tag{84}$$

$$\Rightarrow \sum_{k\neq n}^N \exp(\frac{i_t^k}{\lambda_t}) < \sum_{k\neq m}^N \exp(\frac{i_t^k}{\lambda_t}) \tag{85}$$

$$\Rightarrow \left(\frac{1}{\sum_{k\neq m}^N \exp(\frac{i_t^k}{\lambda_t})} - \frac{1}{\sum_{k\neq n}^N \exp(\frac{i_t^k}{\lambda_t})} \right) < 0 \tag{86}$$

The right hand side of equation 82 is therefore negative, so there is no solution to equation 82 for which banks n and m have the same costs, but bank n offers strictly higher rates. Since n and m are arbitrary banks, the only equilibrium when two banks have the same costs is for those banks to offer the same interest rates. If all banks share the same costs, there is no interest rate dispersion in equilibrium. Away from this case, the extent of rate heterogeneity will be determined by heterogeneity in costs, and by attention choices embodied in λ_t.

F.4 Derivation of equation 35

We start with the first order condition for bank n (equation 16), and differentiate it with respect to λ_t, denoting $\mathcal{S}_t^n = \frac{\exp(i_t^n/\lambda_t)}{\sum_k=1^N \exp(i_t^k/\lambda_t)}$ as the market share of bank n in period t, and $d_t^n = i_t^{CB} - i_t^n - \chi_t^n$ as the profit bank n makes per bond sold:

$$-d_t^n \frac{d\mathcal{S}_t^n}{d\lambda_t} - (1 - \mathcal{S}_t^n)\frac{di_t^n}{d\lambda_t} = 1 \tag{87}$$

Then we use the definition of $\mathcal{S}_t^n$ to find $\frac{d\mathcal{S}_t^n}{d\lambda_t}$:

$$\frac{d\mathcal{S}_t^n}{d\lambda_t} = \frac{\mathcal{S}_t^n(1-\mathcal{S}_t^n)}{\lambda_t}\frac{di_t^n}{d\lambda_t} - \frac{\mathcal{S}_t^n(1-\mathcal{S}_t^n)i_t^n}{\lambda_t^2} + \mathcal{S}_t^n\left(\sum_{j\neq n}\frac{\mathcal{S}_t^j}{\lambda_t^2}(i_t^j - \lambda_t\frac{di_t^j}{d\lambda_t})\right) \tag{88}$$

Substituting this in to equation 87 and rearranging we obtain:

$$\frac{di_t^n}{d\lambda_t} = \frac{1}{\lambda_t(1-\mathcal{S}_t^n)(\lambda_t + d_t^n\mathcal{S}_t^n)}\left[i_t^n d_t^n \mathcal{S}_t^n(1-\mathcal{S}_t^n) - \lambda_t^2 - d_t^n\mathcal{S}_t^n\big(\sum_{j\neq n}\mathcal{S}_t^j(i_t^j - \lambda_t\frac{di_t^j}{d\lambda_t})\big)\right] \tag{89}$$

Finally, note that from the bank first order condition (equation 16) we can write $\lambda_t = d_t^n(1-\mathcal{S}_t^n)$. Using this to substitute out for d_t^n we obtain equation 35.

G Persistence of interest rate rankings in the data

In the model, I assume that the ranking of a bank in the interest rate distribution has no persistence, which simplifies the individual information processing problem as information from one period has no value in future periods. In the data, however, there is some persistence. The tables below show the probabilities of a bank transitioning between quintiles of the interest rate distribution studied in section 3 over a month and a year. The length of a period in the model is one month, but the annual transition probabilities are also relevant since these products have a term of one year, so individual savers buying these products only return to the decision a year later.

Without persistence, every transition probability would equal 0.2. The values on the diagonal of the transition matrices are all greater than this, so there is some persistence. However, even in the top and bottom quintiles where persistence is greatest the persistence is limited. If a saver chose a bank in the top quintile of the interest rate distribution in a given period, then a year later when their product matures there is only a 33% probability of that bank still being in the top quintile. Information from one period does therefore have some future value in reality, but it is small.

	1	2	3	4	5
1	0.77	0.15	0.04	0.02	0.02
2	0.21	0.47	0.20	0.08	0.04
3	0.04	0.27	0.44	0.19	0.07
4	0.01	0.08	0.30	0.40	0.21
5	0.01	0.03	0.09	0.22	0.65

(a) Monthly

	1	2	3	4	5
1	0.59	0.21	0.11	0.06	0.04
2	0.28	0.25	0.22	0.14	0.11
3	0.17	0.24	0.26	0.19	0.15
4	0.08	0.21	0.26	0.23	0.22
5	0.07	0.15	0.21	0.24	0.33

(b) Annual

Table 8: Bank quintile transition matrices. In each table the cell (n, m) indicates the probability of transitioning from the nth quintile to the mth quintile in the following period.

We can test if these transition matrices are significantly different from a matrix where every element is 0.2 (the no-persistence case) using a likelihood ratio test:

$$2\ln\left(\frac{\prod_{n=1}^{5}\prod_{m=1}^{5} p_{n,m}}{\prod_{n=1}^{5}\prod_{m=1}^{5} 0.2}\right) \sim \chi^2_{19} \tag{90}$$

The critical value of the test statistic for 5% significance is 30.1. The monthly and annual transition matrices give test statistics of 31.2 and 7.2 respectively. We therefore cannot reject the hypothesis of no persistence at an annual frequency, and we only marginally reject that hypothesis at the monthly frequency.

H $\mathcal{I}$ φ link in the model

Here I show that φ and attention $\mathcal{I}$ are closely related in the model with N banks. There are no dynamics to the relationship, so for ease I drop the time subscripts on all variable. Denoting the unweighted mean interest rate (which is the blind rate in the model) as $\bar{i}$, and the standard deviation of interest rates as $\sigma(i)$, the model-implied φ is:

$$\varphi = \frac{\sum_n i^n \Pr(\text{choose } n) - \bar{i}}{\sigma(i)} = \frac{\frac{\sum_n i^n \exp(\frac{i^n}{\lambda})}{\sum_m \exp(\frac{i^m}{\lambda})} - \bar{i}}{\sigma(i)} \tag{91}$$

The first thing to note is that as $\mathcal{I}$ approaches 0, λ tends to infinity, and so when attention is 0, $\varphi = 0$:

$$\lim_{\lambda \to \infty} \varphi = \frac{\frac{1}{N}\sum_n i^n - \bar{i}}{\sigma(i)} = 0 \tag{92}$$

Also note that if attention $\mathcal{I}$ reaches $log(N)$, then each individual can perfectly identify the highest interest rate bank with probability 1, so if we denote this as bank 1 (without loss of generality) we have $\varphi > 0$:

$$\varphi(\mathcal{I} = \log(N)) = \frac{i^1 - \bar{i}}{\sigma(i)} = \frac{\frac{1}{N}\sum_n (i^1 - i^n)}{\sigma(i)} > 0 \tag{93}$$

The information constraint is continuous for $\in(0, \log(N))$, so the statements above guarantee that $\mathcal{I}$ and φ are positively related at least in some portions of this range.

To make further progress, I now consider how φ changes in the model assuming that interest rates are held fixed. In reality, the interest rates also respond (as discussed in section 4.6). The total response, including this equilibrium adjustment, is shown in the plots below. We use the chain rule to write:

$$\frac{d\varphi}{d\mathcal{I}} = \frac{d\varphi}{d\lambda}\frac{d\lambda}{d\mathcal{I}} \tag{94}$$

I start with $\frac{d\lambda}{d\mathcal{I}}$. Substituting the optimal choice probabilities into the information constraint 78 and rearranging gives:

$$\mathcal{I} = \log(N) + \frac{i^e}{\lambda} - \log(\sum_n \exp(\frac{i^n}{\lambda})) \tag{95}$$

Differentiating this with respect to λ:

$$1 = \frac{d\lambda}{d\mathcal{I}}\Big[-\frac{i^e}{\lambda^2} + \frac{1}{\lambda}\frac{di^e}{d\lambda} - \frac{d}{d\lambda}\log(\sum_n \exp(\frac{i^n}{\lambda}))\Big] \tag{96}$$

I begin with the final term in the square brackets:

$$\frac{d}{d\lambda}\log(\sum_n \exp(\frac{i^n}{\lambda})) = -\frac{\sum_n i^n \exp(\frac{i^n}{\lambda})}{\lambda^2 \sum_m \exp(\frac{i^m}{\lambda})} = -\frac{i^e}{\lambda^2} \tag{97}$$

Substituting this into equation 96 the first and third terms in the square brackets cancel, so we have:

$$1 = \frac{d\lambda}{d\mathcal{I}}\frac{1}{\lambda}\frac{di^e}{d\lambda} \tag{98}$$

The sign of $\frac{d\lambda}{d\mathcal{I}}$ is therefore the same as the sign of $\frac{di^e}{d\lambda}$, the derivative of the effective interest rate with respect to λ:

$$\frac{di^e}{d\lambda} = \frac{\big(\sum_n i^n \exp(\frac{i^n}{\lambda})\big)^2 - \big(\sum_n \exp(i^{n^2}\frac{i^n}{\lambda})\big)\big(\sum_m \exp(\frac{i^m}{\lambda})\big)}{\lambda^2\big(\sum_n \exp(\frac{i^n}{\lambda})\big)^2} \tag{99}$$

The denominator of this fraction is always positive, so the sign is determined by the sign of the numerator, which after expanding the terms in brackets gives:

$$\sum_n i^{n^2} \exp(\frac{2i^n}{\lambda}) + \sum_{m\neq n} i^n i^m \exp(\frac{i^n + i^m}{\lambda}) - \sum_n i^{n^2} \exp(\frac{2i^n}{\lambda}) - \sum_{m\neq n} i^{n^2} \exp(\frac{i^n + i^m}{\lambda})$$
$$= -\sum_{m\neq n} (i^{n^2} - i^n i^m) \exp(\frac{i^n + i^m}{\lambda}) \quad (100)$$

Inside the sum, notice that each pair of banks $\{j, k\}$ appear twice: once when $m = k, n = j$ and again when $m = j, n = k$. For each distinct *pair* of banks $\{j, k\}$, the terms inside the sum are equal to:

$$\exp(\frac{i^j + i^k}{\lambda})(i^{j^2} - i^j i^k + i^{k^2} - i^k i^j) = \exp(\frac{i^j + i^k}{\lambda})(i^j - i^k)^2 > 0 \quad (101)$$

Each pair of terms inside the sum in equation 100 is therefore positive, and so the numerator in equation 99 is negative, and $\frac{di^e}{d\lambda}$ is negative. That is, when the shadow cost of information in the individual problem falls, the effective interest rises, if we hold the distribution of interest rates constant.

This implies that $\frac{d\lambda}{d\mathcal{I}}$ is also negative. If attention rises, then holding the distribution of interest rates constant the shadow price of attention falls.

Now consider $\frac{d\varphi}{d\lambda}$:

$$\frac{d\varphi}{d\lambda} = \frac{1}{\sigma(i)} \frac{di^n}{d\lambda} < 0 \quad (102)$$

As we have already shown that $\frac{di^n}{d\lambda} < 0$ above.

Therefore holding the distribution of interest rates constant, there is a positive monotonic relationship between attention and φ.

To consider the role of equilibrium changes in interest rates, we can write the full derivative of φ with respect to $\mathcal{I}$ as:

$$\frac{d\varphi}{d\mathcal{I}} = \frac{\partial\varphi}{\partial\mathcal{I}}|_{i^n} + \sum_n \frac{d\varphi}{di^n} \frac{di^n}{d\mathcal{I}} \quad (103)$$

The derivation above refers to the first term here, and finds that it is positive. As discussed in section 4.6 and appendix F, $\frac{di^n}{d\mathcal{I}} > 0$ for the range of attention encountered in simulations of the model. However, the sign of $\frac{d\varphi}{di^n}$ is ambiguous. To see why, it is helpful to examine the numerator and denominator of φ separately.

The numerator is $\sum_n \Pr(\text{choose } n) i^n - \frac{1}{N}\sum_n i^n$. This will rise when a given bank's rate i^j rises if $\Pr(\text{choose } j) > \frac{1}{N}$. For all $\mathcal{I} > 0$ this is true for the lowest cost bank, and is not true for the highest cost bank. However for intermediate banks the sign of this term will depend on the rate distribution and the level of attention. The second-lowest cost bank, for example, will be chosen with a probability greater than $\frac{1}{N}$ at low and modest levels of attention. Initially, rises in attention will increase this probability further. However, as attention gets very high individuals will accurately distinguish even between the top two interest rates in the market, and so the probability of this bank being chosen will fall below $\frac{1}{N}$, eventually reaching 0 when $\mathcal{I} = \log(N)$.

Furthermore, it can be shown that the derivative of the denominator of φ (i.e. $\sigma(i)$) with respect to any individual rate i^j is:

$$\frac{d\sigma(i)}{di^j} = \frac{1}{N\sigma(i)}\Big(i^j - \frac{1}{N}\sum_n i^n\Big) \tag{104}$$

That is, the denominator of φ rises with i^j if i^j is above the mean interest rate in the market. Therefore at moderate levels of attention, both the numerator and denominator of φ rise with the interest rates of the banks offering the highest rates, and fall when the lower rates in the market rise. The overall effect on φ of a rise in any individual rate is therefore ambiguous without further specification of how much attention is being paid, and what costs are.

Simulations of the model with $N > 2$ banks suggest that these equilibrium interest rate response terms are not sufficient to outweigh the direct effect discussed above that $\frac{\partial\varphi}{\partial\mathcal{I}}|_{i^n} > 0$, so there is a positive relationship between attention and φ in the model even outside of the simple case with $N = 2$ banks.

Note, however, that the correspondence is not perfect outside of the two bank case. In general, the state of the economy will influence $\frac{d\varphi}{d\mathcal{I}}$ through the policy rate, which affects the standard deviation of interest rates if $\tau_2 \neq 0$. This means that an increase in attention when policy rates are low will have a different effect on φ than it would if the increase happened when policy rates are high. As there is not a one-to-one mapping from policy rates to attention (attention decisions co-move differently with the policy rate depending on the shock causing the dynamics, as discussed in section 4.6), the link from attention to φ is not one-to-one.

I Estimation Details

I.1 Data Sources and Treatment

All data is for the UK, and is monthly, with the exception of consumption which is quarterly. Consumption data is from the ONS (series ABJR+HAYO). It is divided by population (ONS series MGSL), then logged. Inflation is measured by the monthly CPI growth rate, expressed as a gross rate (i.e. $= 1 + \frac{\text{ONS series MM23}}{100}$). This series is not available seasonally adjusted, so I adjust it using the X13-SEATS procedure from the U.S. Census Bureau. All interest rate series are taken from Moneyfacts and the calculations discussed in section 3, and are left as annualised rates. The observation equations in the model are adjusted to annualise the monthly model interest rates. These data do not display a strong seasonal pattern, so I do not seasonally adjust these series. Mean interest rates are taken as the unweighted numeric mean of the rates on offer in Moneyfacts, expressed net, so the only data treatment is to divide the observed rates (in percentage points) by 100. The within-month standard deviation of interest rates is treated similarly. Finally φ does not need to by divided by 100, as the measure is homogeneous of degree 0 in interest rates. I do treat it by multiplying the φ data by the mean standard deviation of rates over the sample. This is because the current estimation is done with two banks, and no prior bias towards a particular bank, so the blind rate in the model is the mean rate. This means that the maximum possible φ in the model is 1, which poses a problem when comparing with the data in which the mean of φ is slightly above 1, and it does go substantially higher than that in certain periods. This is possible because there are always many more than two banks in the data, and the blind rate is below the unweighted mean interest rate. Multiplying by the steady state within-month rate standard deviation helps to make up for this discrepancy, as τ_1 is calibrated such that the steady state gap between the blind rate and the maximum rate in the market is equal to the average of this gap in the data over the sample. As this gap is from the mean rate to the best rate in the model, rather than between the maximum and a blind rate below the mean, this means that the steady state standard deviation of rates is higher in the model than in the data. When I move to estimating the model with more banks this will no longer be necessary, as it will be possible for φ in the model to exceed 1.

All data are then filtered using a one-sided Hodrick-Prescott filter, with smoothing parameter 129600 for the monthly data and 1600 for the quarterly consumption data. The observation equation in the model for consumption is specified as the log of the sum of the current month and previous two months. This object is observed only every three months. Leaving the intervening observations blank, the Kalman filter in the estimation imputes them using the estimated parameters and values of the state variables (the shocks).

I.2 Estimation Results

These parameters are estimated using classical maximum likelihood.

Table 9: Estimated parameters

	Estimate	**s.d.**	**t-stat**
μ	0.0379	0.0179	2.1160
τ_2	-0.1656	0.0565	2.9319
ρ_ζ	0.9504	0.0136	70.0628
σ_ζ	0.0026	0.0005	5.2257
ρ_m	0.9114	0.0017	529.7408
σ_m	0.0021	0.0002	9.2705
ρ_a	0.0380	0.0259	1.4642
σ_a	0.1668	0.0115	14.5584
$\sigma_{s.d.ME}$	0.0007	0.0000	21.1505
$\sigma_{\varphi ME}$	0.0013	0.0001	18.8526