The Long-Run Phillips Curve is... a Curve [1]

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[1] Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank

The question

An old debate: is there any trade-off between inflation and output/unemployment in the long run?

- Phelps (1967), Friedman (1968): Natural rate hypothesis
- "there is no permanent trade-off":

  =>*the long-run Phillips curve is vertical*
- Cornerstone role in macroeconomic theory and practice
- The working assumption of central banks in the implementation of monetary policy

The question

It is surprising to note that:

- **Empirically**: There is little econometric work devoted to test the absence of a long-run trade-off.

  Some literature: King and Watson (1994); Beyer and Farmer (2007); Berentsen et al. (2011); Haug and King (2014); Benati (2015)

- **Theoretically**: Modern macroeconomic sticky price frameworks generally do not imply the absence of a long- run relation
  - The Generalized NK model delivers a negative relationship between steady state inflation and output. See Ascari (2004); Ascari and Sbordone (2014)

Results

What is the long-run relation between inflation and output?

1. Time series model
   - The LRPC is not vertical, it is negatively sloped (higher inflation is related to lower output in the LR)
   - The key to get this result: model the LRPC as non linear
   - Methodological contribution: a "convenient" non-linear approach
2. Structural model
   - GNK model (Ascari and Ropele, 2009; Ascari and Sbordone, 2014): higher trend inflation causes lower GDP in the LR
   - The model has the two key features from the statistical analysis: non-linear and negatively sloped LRPC
   - The model is also able to capture the quantitative features of the time series analysis

The time series approach: A time-varying equilibrium VAR

Generalization of Steady State VAR (Villani, 2009; Del Negro et al., 2017; Johannes and Mertens, 2021):

$$A(L)\left(X_t - \bar{X}_t\right) = \varepsilon_t \quad \varepsilon_t \sim N\left(0, \Sigma_{\epsilon,t}\right) \tag{1}$$

- X_t is a $(n \times 1)$ vector with observed variables at time
- $\bar{X}_t$ is the vector with the long-run values of X_t

Trend-cycle decomposition:

$$X_t = \bar{X}_t + \hat{X}_t$$

- $\hat{X}_t$ described by (1): stable component with unconditional expectation equal to zero
- $\bar{X}_t = h\left(\theta_t\right)$
  $\theta_t = f\left(\theta_{t-1}, \eta_t\right) \quad \eta_t \sim N\left(0, \Sigma_\eta\right)$

The model

- Three observables: GDP per capita, inflation and interest rate
- The short-run component: VAR with 4 lags

THE MODEL FOR THE LONG RUN

$$\bar{y}_t = y^*_t + \delta(\bar{\pi}_t) \quad \text{the equilibrium level of output as function of inflation}$$

$$y^*_t = y^*_{t-1} + g_t + \eta^y_t$$

$$g_t = g_{t-1} + \eta^g_t$$

$$\delta(\bar{\pi}_t) : \delta(0) = 0$$

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \eta^\pi_t \quad \text{trend inflation is random walk}$$

$$\bar{i}_t = \bar{\pi}_t + cg_t + z_t \quad \text{long-run Fisher equation}$$

$$z_t = z_{t-1} + \eta^z_t$$

A non-linear long-run Phillips curve

Our choice of $\delta(\bar{\pi}_t)$ is a piecewise linear function:

$$\bar{y}_t = y_t^* + \delta(\bar{\pi}_t)$$
$$\delta(\bar{\pi}_t) = \begin{cases} k_1 \bar{\pi}_t & \text{if } \bar{\pi}_t \leq \tau \\ k_2 \bar{\pi}_t + c_k & \text{if } \bar{\pi}_t > \tau \end{cases}$$

- It is simpler to treat: methodological contribution
- It can approximate the kind of non-linearity we have in mind without imposing strong assumptions on a specific functional form
- It is easy to interpret

A piecewise linear approach

The model can be written in state space form:

$$Y_t = D(\theta_t) + F(\theta_t)\theta_t + \epsilon_t \tag{2}$$

$$\theta_t = M(\theta_t) + G(\theta_t)\theta_{t-1} + P(\theta_t)\eta_t \tag{3}$$

where, in particular

$$(D, F, M, G, P) = \begin{cases} (D_1, F_1, M_1, G_1, P_1) & \text{if } \bar{\pi}_t \leq \tau \\ (D_2, F_2, M_2, G_2, P_2) & \text{if } \bar{\pi}_t > \tau \end{cases} \tag{4}$$

- Methodological contribution: we find the likelihood and the posterior distribution of θ_t analytically
- Compromise between efficiency and misspecification

Estimation

- US data, sample from 1960Q1 to 2008Q2
- Bayesian approach

Two sources of non linearity: stochastic volatility and a piecewise linear LRPC => Particle filtering approach

1. "Rao-Blackwellization", thanks to the analytical results on the piecewise linear model
2. Particle filtering also to approximate the posterior distribution of the parameters
   - Particle learning by Carvalho Johannes Lopes and Polson (2010); see also Mertens and Nason (2020)
   - Mixture of Normal distributions as approximation of the posterior of τ (Liu and West, 2001)

Strategy for parameter learning follows Chen, Petralia and Lopes (2010) and Ascari, Bonomolo and Lopes (2019)

Filter

Estimation results - Linear model

A vertical (or flat) long-run Phillips curve

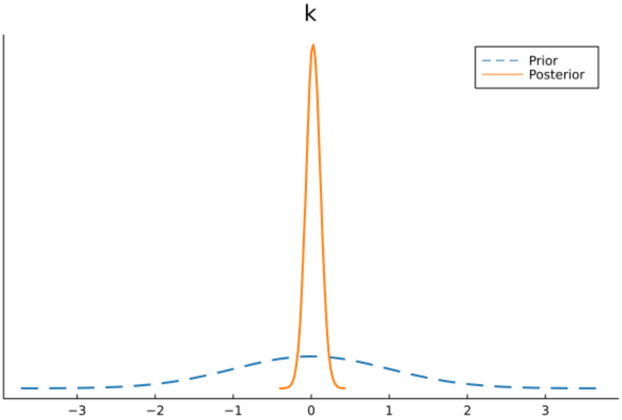

Figure: Posterior distributions of the slope of the LRPC - Linear model.

Estimation results - Non-linear model

Non linear and negatively sloped long-run Phillips curve

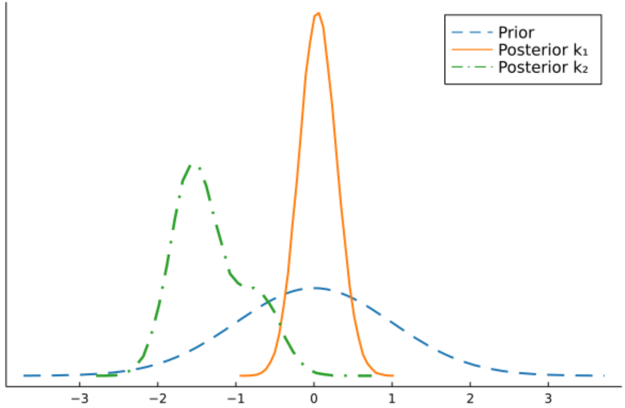

Figure: Posterior distributions of the slopes of the LRPC - Non-linear model.

Estimation results - Non-linear model

The threshold:

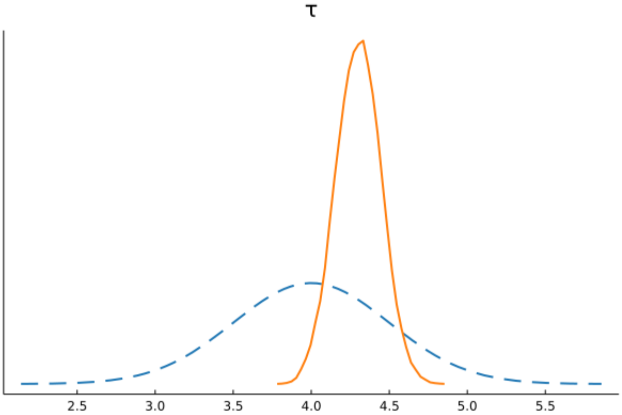

Figure: Posterior distributions of τ - Non-linear model.

Estimation results - non-linear model

A non-linear, negatively sloped long-run Phillips curve

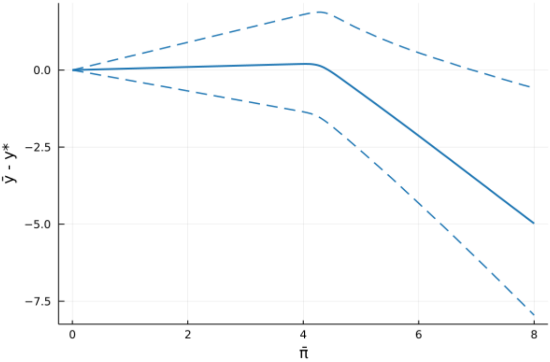

Figure: LRPC - Non-linear model. Median and 90% probability interval.

Estimation results - non-linear model

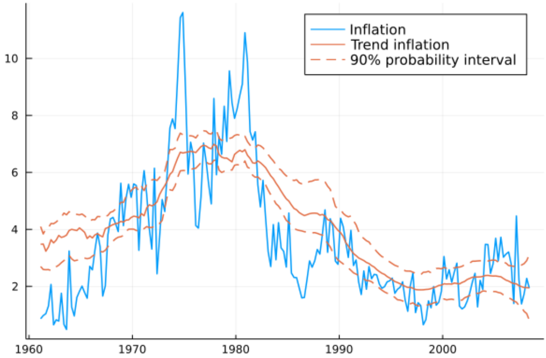

Figure: Inflation and trend inflation - Non linear model.

The cost of trend inflation: the long-run output gap

$$\hat{Y}_t = \frac{Y_t}{\bar{Y}_t} = \frac{Y_t}{Y_t^*}\frac{Y_t^*}{\bar{Y}_t} \tag{5}$$

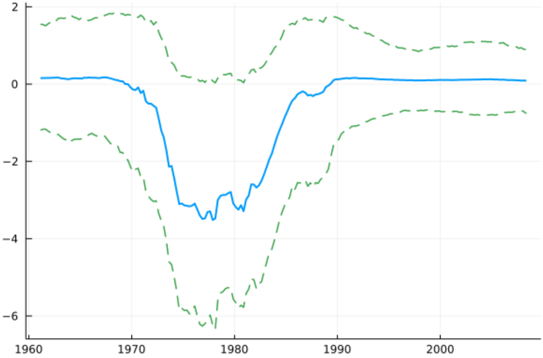

Figure: Long-run output gap estimated through the non-linear model.

The structural model

- A variant of Ascari and Ropele (2009), Ascari and Sbordone (2014) GNK model:
  - Inter-temporal Euler equation featuring (external) habit formation in consumption
  - Generalized New Keynesian Phillips curve featuring positive trend inflation
  - Taylor-type monetary policy rule
- Time varying trend inflation => LRPC is:
  - Non-linear
  - Negatively sloped
- When taking decisions the agents consider trend inflation as a constant parameter: anticipated-utility model (Kreps, 1998; Cogley and Sbordone, 2008)
- Stochastic volatility to the four shocks: discount factor, technology, monetary policy and trend inflation

The costs of trend inflation

- Price stickiness => price dispersion and inefficiency in the quantity produced
- Higher trend inflation leads to higher price dispersion and increases output inefficiency

Formally:

$$N_t = \int_0^1 N_{i,t} di = \int_0^1 \left(\frac{Y_{i,t}}{A_t}\right)^{\frac{1}{1-\alpha}} di = \underbrace{\int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{\frac{-\varepsilon}{1-\alpha}} di}_{s_t} \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}}$$

Aggregate output is:

$$Y_t = \frac{A_t}{s_t^{1-\alpha}} N_t^{1-\alpha}$$

with long-run price dispersion: $\bar{s}_t = g(\underset{+}{\bar{\pi}_t})$

▸ Model

Comparing long-run Phillips curves: VAR and GNK

The GNK model measures the costs of trend inflation consistently with the VAR

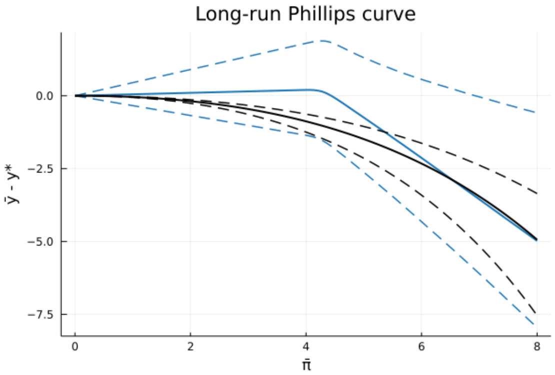

Figure: Long-run Phillips curve: median (continuous line) and 90% probability interval (dashed lines) - comparison between VAR (blue) and GNK (black) estimates.

Conclusions

- What is the long-run relation between inflation and output?
- A time series model suggests that the LRPC is:
  - Non linear
  - Negatively sloped
- We interpret these findings through the lens of a GNK model
- This model is able to measure the costs implied by the LRPC consistently with the time series model

EXTRA

Econometric strategy

We use a particle filtering strategy to approximate the joint posterior distribution of latent processes and parameters:

Latent processes: a "conditional piecewise linear model"

$$p\left(\theta_t, \Sigma_{\epsilon,t} | Y_t\right) = \underbrace{p\left(\theta_t | \Sigma_{\epsilon,t}, Y_t\right)}_{\text{"}optimal\ \ importance\ \ kernel\text{"}} \quad \underbrace{p\left(\Sigma_{\epsilon,t} | Y_t\right)}_{\text{"}blind\ \ proposal\text{"}}$$

Parameters:

- Particle learning by Carvalho Johannes Lopes and Polson (2010); see also Mertens and Nason (2020)
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A fully adapted particle filter

At $t-1$: $\left\{\theta_{t-1}^{(i)}\right\}_1^N$ approximate $p\left(\theta_{t-1}|\psi, X_{1:t-1}\right)$

1. Resample
   - Compute $\tilde{w}_t^{(i)} \propto p\left(X_t|\theta_{t-1}^{(i)}, \psi, X_{1:t-1}\right)$
   - Resample $\left\{\tilde{\theta}_{t-1}^{(i)}\right\}_1^N$ using $\left\{\tilde{w}_t^{(i)}\right\}_1^N$
2. Propagate
   - draw $\theta_t^{(i)} \sim p\left(\theta_t|\tilde{\theta}_{t-1}^{(i)}, \psi, X_{1:t-1}\right)$

where:

- $p\left(X_t|\theta_{t-1}^{(i)}, \psi, X_{1:t-1}\right)$ is a weighted sum of Unified Skew Normal distributions (Arellano-Valle and Azzalini, 2006)
- $p\left(\theta_t|\tilde{\theta}_{t-1}^{(i)}, \psi, X_{1:t-1}\right)$ is a weighted sum of multivariate truncated Normal distributions

Back

Household

The economy is populated by a representative agent with utility

$$E_0 \sum_{t=0}^{\infty} \beta^t d_t \left[\ln \left(C_t - h\mathbf{C}_{t-1} \right) - d_n \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$

Budget constraint is given by

$$P_t C_t + R_t^{-1} B_t = W_t N_t + D_t + B_{t-1}$$

d_t is a discount factor shock which follows an AR(1) process

$$\ln d_t = \rho_d \ln d_{t-1} + \epsilon_{d,t}$$

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Final good firm

Perfectly competitive final good firms combine intermediate inputs

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad \varepsilon > 1$$

Price index is a CES aggregate of intermediate input prices

$$P_t = \left[\int_0^1 {P_{i,t}}^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}$$

The demand schedule for intermediate input

$$Y_{i,t} = \left[\frac{P_{i,t}}{P_t} \right]^{-\varepsilon} Y_t$$

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Intermediate good firm

Each firm i produces according to the production function

$$Y_{i,t} = A_t N_{i,t}^{1-\alpha}$$

where A_t denotes the level of technology and its growth rate $g_t \equiv A_t / A_{t-1}$ follows

$$\ln g_t = \ln \overline{g} + \epsilon_{g,t}$$

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Price setting

Firms adjust prices $P^*_{i,t}$ to maximize expected discounted profits with probability $0 < 1 - \theta < 1$

$$E_t \sum_{j=0}^{\infty} \theta^j \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left[\frac{P^*_{i,t}}{P_{t+j}} Y_{i,t+j} - \frac{W_{t+j}}{P_{t+j}} \left[\frac{Y_{i,t+j}}{A_{t+j}} \right]^{\frac{1}{1-\alpha}} \right]$$

subject to the demand schedule

$$Y_{i,t+j} = \left[\frac{P^*_{i,t}}{P_{t+j}} \right]^{-\varepsilon} Y_{t+j},$$

where λ_t is the marginal utility of consumption.

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The Phillips curve

The first order condition for the optimized relative price $x_t (= \frac{P^*_{i,t}}{P_t})$ is given by

$$\left(x_t\right)^{1+\frac{\varepsilon\alpha}{1-\alpha}} = \frac{\varepsilon}{(\varepsilon-1)(1-\alpha)} \frac{E_t \sum_{j=0}^{\infty} (\theta\beta)^j \lambda_{t+j} \frac{W_{t+j}}{P_{+j}} \left[\frac{Y_{t+j}}{A_{t+j}}\right]^{\frac{1}{1-\alpha}} \pi_{t|t+j}^{\frac{\varepsilon}{(1-\alpha)}}}{E_t \sum_{j=0}^{\infty} (\theta\beta)^j \lambda_{t+j} \pi_{t|t+j}^{\varepsilon-1} Y_{t+j}}.$$

where $\pi_{t|t+j} = \frac{P_{t+1}}{P_t} \times ... \times \frac{P_{t+j}}{P_{t+j-1}}$ for $j \geqslant 1$ and $\pi_{t|t} = \pi_t$.

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Price setting contd.

Aggregate price level evolves according to

$$
\begin{aligned}
P_t &= \left[\int_0^1 {P_{i,t}}^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \Rightarrow \\
x_t &= \left[\frac{1-\theta\pi_t^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}} .
\end{aligned}
$$

Finally, price dispersion $s_t \equiv \int_0^1 (\frac{P_{i,t}}{P_t})^{-\varepsilon} di$ can be written recursively as:

$$
s_t = (1-\theta)x_t^{-\varepsilon} + \theta\pi_t^{\varepsilon} s_{t-1}
$$

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Monetary policy

$$\frac{R_t}{\overline{R}_t} = \left(\frac{R_{t-1}}{\overline{R}_t}\right)^{\rho} \left[\left(\frac{\pi_t}{\overline{\pi}_t}\right)^{\psi_\pi} \left(\frac{Y_t}{Y_t^n}\right)^{\psi_x} \left(\frac{g_t^y}{\overline{g}}\right)^{\psi_{\Delta y}}\right]^{1-\rho} e^{\varepsilon_{r,t}}$$

$$\ln \overline{\pi}_t = \ln \overline{\pi}_{t-1} + \epsilon_{\overline{\pi},t}$$

where $\overline{\pi}_t$ denotes trend inflation, Y_t^n is the flex-price output and g_t^y is growth rate of output.

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Estimates of the parameters

Table 1: Prior and Posterior Distributions

Parameter	Prior			Posterior
	Density	Mean	St Dev	
ψ_π	Gamma	1.5	0.5	2.05 [1.83 2.3]
ψ_x	Gamma	0.125	0.05	0.1 [0.06 0.16]
$\psi_{\Delta y}$	Gamma	0.125	0.05	0.42 [0.26 0.67]
ρ	Beta	0.7	0.1	0.74 [0.71 0.77]
h	Beta	0.5	0.1	0.41 [0.36 0.46]
r^*	Gamma	2	0.5	1.88 [1.68 2.1]
θ	Beta	0.5	0.1	0.51 [0.46 0.56]
ρ_d	Beta	0.7	0.1	0.79 [0.74 0.83]
	Density	Mean	Degree of freedom	
δ_d^2	Inverse Gamma	0.02^2	5	0.047^2 [0.042^2 0.054^2]
δ_g^2	Inverse Gamma	0.02^2	5	0.047^2 [0.042^2 0.053^2]
δ_r^2	Inverse Gamma	0.02^2	5	0.029^2 [0.026^2 0.033^2]
δ_π^2	Inverse Gamma	0.02^2	5	0.012^2 [0.011^2 0.014^2]

Posterior median and 90% credibility interval in brackets

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