

Empirical Investigation of a Sufficient Statistic for Monetary Shocks^{*}

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Abstract

In a broad class of sticky price models the non-neutrality of nominal shocks is captured by a simple sufficient statistic: the ratio of the kurtosis of the size-distribution of price changes over the frequency of price changes. We test this theoretical prediction using data for a large number of firms representative of the French economy. We use the micro data to measure the cross sectional moments, including kurtosis and frequency, for about 120 PPI industries and 220 CPI categories. We use a Factor Augmented VAR to measure the sectoral responses to a monetary shock, as summarized by the cumulative impulse response of sectoral prices (CIR^P), under three alternative identification schemes. The estimated CIR^P correlates with the kurtosis and the frequency consistently with the prediction of the theory (equal coefficients with opposite signs). The analysis also shows that other moments not suggested by the theory, such as the mean, standard deviation and skewness of the size-distribution of price changes, are not correlated with the CIR^P . Several robustness checks of these findings are analyzed.

Keywords: Impulse response functions, Monetary Shocks, Generalized Hazard Function, Sufficient statistic

JEL Classification Numbers: E3, E5

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1 Introduction

Many recent contributions in economics are characterized by the quest for a sufficient-statistic, a theoretical metric that summarizes the effect of a policy using only a few (high-level) parameters leaving aside a large number of modeling details. Chetty (2009) coined the phrase, tracing the origin of the idea to Koopmans (1953), Marschak (1953) and Harberger (1964), and discussed advantages and limitations of using sufficient statistics in public finance models. Summarizing the workings of complex models with a few measurable elasticities is convenient, useful for model selection and holds promise for policy analysis. Several papers in public finance and international trade have successfully followed the approach.¹

In the area of monetary economics recent results have identified a sufficient statistic for monetary shocks for a broad class of sticky-price models under low inflation. The key proposition is that the cumulative response of *output* to a once-and-for-all *small* monetary shock, essentially the *area under the output impulse response*, is proportional to the ratio of the *kurtosis* of the steady-state distribution of price changes over the *frequency* of price changes. A version of this theoretical result was first established in Alvarez, Le Bihan, and Lippi (2016) for the sticky price model of Nakamura and Steinsson (2010), that nests as special cases two workhorse of macroeconomics: Calvo (1983) and Golosov and Lucas (2007). The result was extended by Alvarez, Lippi, and Oskolkov (2020) to a broader class of models using the generalized hazard function setup of Caballero and Engel (1993, 1999).² Additional sufficient statistics have been discovered by Baley and Blanco (forthcoming) for a setup with non-negligible inflation and by Alexandrov (2020) for the case of large nominal shocks.

This paper presents an empirical test of the predictions of the sufficient-statistic proposition using the restrictions implied by the theory for an economy with low inflation. The theory prediction

¹Examples include Schmieder and von Wachter (2016); Badel and Huggett (2017); Kleven (2020) for an extension of Chetty's formula in the field of public finance with empirical applications, see Arkolakis, Costinot, and Rodriguez-Clare (2012); Anderson and Neary (2016) for international trade applications, Costinot and Werning (2018) for an application to optimal technology regulation.

²This class also includes time-dependent models a la Calvo, canonical menu-cost models a la Golosov-Lucas, intermediate cases such as the Calvo-plus by Nakamura and Steinsson or inherently random-menu cost models such as those of Caballero and Engel. Moreover, the result holds in multi-product models, and also holds in a class of costly information models that give rise to time-dependent rules, spanning classic models such as Taylor (1980); Caballero (1989); Reis (2006).

is somewhat bold: the ratio of kurtosis to frequency should explain different degrees of monetary non neutrality, while other moments should not matter (e.g. the variance, or the skewness, of price changes). This paper tests the sufficient-statistic result using producer price indices (PPI) and consumer price indices (CPI) micro data for a large number of firms that are representative of the French economy. Our test is made of three steps. We first estimate the sectoral responses to a monetary shock for about 120 PPI industries and 220 CPI categories, using a Factor Augmented VAR in the vein of [Bernanke, Boivin, and Eliasz \(2005\)](#); [Boivin, Giannoni, and Mihov \(2009\)](#). We identify the monetary shocks using three alternative schemes (recursive ordering - both with and without long-run restrictions being imposed-, and high frequency identification) and summarize the extent of the non neutrality using the cumulative impulse response of the sectoral prices (CIR^P). Since the sufficient statistic proposition concerns the cumulated response of *output*, we use the theory to derive the implication for the cumulated response of prices. This has two advantages: to increase the number of cross-sectoral observations that can be used in the tests, and to map the theoretical prediction into a metric that is more robust.³ Next, we use the micro data underlying the sectoral data to measure the cross sectional moments of the distribution of price changes. Finally, we inspect the relationship between the CIR^P and the cross-sectoral moments under the restrictions implied by the theory.

The results consistently show that the data do not reject the predictions of the theory across a variety of tests, specifications, and robustness exercises. Both the frequency and the kurtosis appear as statistically significant factors in accounting for the cross-sectional heterogeneity of the estimated CIR^P for both the PPI data as well for the CPI data. The sign and magnitudes of the estimated coefficients are consistent with the predictions of the theory in the specification where the variables enter the regression in a ratio, as the theory prescribes, as well as in an unrestricted specification where both variables are entered as separate regressors. Moreover, “placebo” tests show that moments not suggested by the theory, such as the size, standard deviation and skewness of price changes, are not correlated with the CIR^P . In addition, results are robust to allowing in various ways for measurement errors, an important concern as far as micro price data are concerned. When we compare results for PPI and CPI products, we find that the results for PPI

³The output response depends on sector specific elasticities that require additional information for the test.

are more robust and consistent than the results for CPI products. In the robustness analysis, we find that when removing products with frequent sales and substitutions (in particular, food, clothing and furniture), the CPI results are more aligned with the ones obtained for PPI products. This is consistent with the fact that the model underlying the sufficient statistic result assumes no seasonal sales or, more generally, price plans.

Our analysis is the first attempt to test the sufficient-statistic proposition for monetary shocks using the restrictions that are implied by the theory. A related analysis for the United States is presented in [Hong et al. \(2020\)](#), where the authors inspect the correlation between the response of sectoral producers price indices (24 months after the monetary shock) and several cross-sectional moments of the distribution of price changes. In spite of the wording, that presents the empirical evidence as a rejection of the sufficient statistic proposition, such evidence is not a proper test of the theory for two reasons. First, the outcome variable in the regressions is the *level response of prices*, while the theory concerns the *cumulated response of output*, and thus the dependent variable in the regressions of [Hong et al. \(2020\)](#) is *not* the one that the theory focuses on. Second, several regressions, such as those where kurtosis or frequency is used as the *only* regressor, are inconsistent with the theory that prescribes *both* kurtosis and frequency to be part of the specification.⁴

The paper is organized as follows. [Section 2](#) presents the sufficient-statistic proposition for small monetary shocks and derives the theoretical restrictions to be tested on the data. [Section 3](#) uses micro and sectoral data to measure the key ingredients needed to test the theory: (i) the sectoral response of prices and output to monetary shocks (ii) candidate sufficient statistics, i.e. several cross-sectoral micro moments. [Section 4](#) presents the baseline results of the tests using cross sectional data. [Section 5](#) investigates the robustness of our findings using a number of alternative measures and specifications. [Section 6](#) concludes and discusses avenues for future research.

⁴The reason is that kurtosis and frequency are in general not orthogonal, for instance both respond to a change in the distribution of adjustment costs. In spite of the disconnect between the outcome variable and the theory, we note that the *level* of the impulse response might still be informative about the CIR if the responses across the sectors are rank preserving. Indeed, we find it interesting that in the specifications where kurtosis and frequency are both entered as regressors the signs of the estimated coefficients are consistent with the sufficient statistic proposition, see e.g. column 1 and column 6 of Table 1 in [Hong et al. \(2020\)](#), or the cross sectional regressions of Table 12 where both kurtosis and frequency are statistically significant and with the expected signs.

2 A Sufficient Statistic for Monetary Shocks

This section presents the sufficient-statistic proposition for monetary shocks and derives some empirically testable implications. We first illustrate the theory using Caballero and Engel’s (1999) model, because of the generality of the setup that encompasses a vast class of sticky-price models. We highlight the assumptions that are important for the result to hold, discuss other setups where the result applies and setups where the result does not hold. Finally we derive various empirical tests for the theory (Section 2.3).

2.1 Set-up: a general structural model of price stickiness

We describe the price setting problem for a firm in steady state using the random menu cost model of Caballero and Engel (1999) and Caballero and Engel (2007), which covers a vast class of sticky-price models, including several well known cases such as the canonical Golosov and Lucas (2007), the pure Calvo (1983) model and the hybrid Calvo-plus model by Nakamura and Steinsson (2010). The setup considers a firm whose marginal nominal cost follows a Brownian motion with variance σ^2 and drift μ , where the latter is due to inflation (the model is summarized in Appendix A). The state of the firm x is given by its “price gap”, defined as the price currently charged by the firm relative to the price that maximizes current profits, which is proportional to the firm cost (measured as the log of the ratio between these prices). In the absence of control the price gap evolves as $dx = \mu dt + \sigma dW$ where W is a standard Brownian motion. At any moment the firm can change its price, and thus control x , by paying the menu cost $\Psi > 0$. Moreover, with probability κ per unit of time, the firm receives an opportunity to pay a menu cost $\psi \in [0, \Psi)$ drawn from the distribution $G(\psi)$. The distribution is allowed to have countably many mass points. We also allow for Ψ to diverge. For instance a distribution with a mass point at $\psi = 0$ and $\Psi \rightarrow \infty$ can be used to generate the Calvo model. When the distribution G is not degenerate the adjustment costs are random, which is why these models are often referred to as to “random menu cost” models. The firm maximizes the expected discounted value of profits and chooses the optimal times and size of price adjustment as a function of its state x . The firm’s optimal choices are encoded in the minimized value function $v(x)$, described in the appendix, which defines the optimal return point

$x^* = \arg \min_z v(z)$, namely the optimal price gap chosen by a firm that adjusts. The value function also defines the optimal boundaries of the inaction region $\underline{X} < \bar{X}$ which satisfy the smooth-pasting and value matching conditions described in the appendix.

Policy rules. Following Caballero and Engel (1999) the optimal policy can be summarized by a *generalized hazard function*, $\Lambda : (\underline{X}, \bar{X}) \rightarrow \mathbb{R}_+$, which gives the probability (per unit of time) that a firm with $x \in (\underline{X}, \bar{X})$ will change its price. The generalized hazard function is defined by the optimal decision rule, or the value function, as well the Poisson arrival rate $\kappa > 0$ and the distribution of fixed cost G . Formally, the generalized hazard function satisfies⁵

$$\Lambda(x) = \kappa G (v(x) - v(x^*)) \text{ for all } x \in (\underline{X}, \bar{X}) . \quad (1)$$

Intuitively, the probability of adjustment at x is given by the fraction of firms that draw a menu cost that is smaller than the benefit of adjusting. The value function $v(\cdot)$ and the generalized hazard function $\Lambda(\cdot)$ have a minimum at x^* , are decreasing in x for $x \in (\underline{X}, x^*)$, and increasing in x for $x \in (x^*, \bar{X})$.

Compared to the workhorse Calvo (1983) model, where the adjustment probability is constant, a generalized hazard function $\Lambda(x)$ allows it to depend on the state x , the firm's desired adjustment. Such state dependence is appealing theoretically, see e.g. Barro (1972); Sheshinski and Weiss (1977); Dixit (1991); Golosov and Lucas (2007), and has been found to be relevant empirically, see e.g. Fougere, Le Bihan, and Sevestre (2007); Dias, Marques, and Santos Silva (2007); Eichenbaum, Jaimovich, and Rebelo (2011); Gautier and Saout (2015).⁶ A large number of models are nested by this framework, including the canonical Calvo model with a constant hazard $\Lambda(x) = \lambda$, the Golosov and Lucas (2007) model with x bounded by the adjustment thresholds where the hazard equals zero for $x \in (\underline{X}, \bar{X})$ and spikes at the adjustment thresholds. Intermediate cases cover the so called Calvo-plus model by Nakamura and Steinsson (2010), the random menu cost problem of

⁵This is the continuous time version of equation (8) in Caballero and Engel (1999) discrete time model where the draws from the distribution G occur in every period.

⁶Several authors have employed the generalized hazard function in applications and empirical work. For recent applications see e.g. Costain and Nakov (2011); Carvalho and Kryvtsov (2018); Sheremirov (2019); for empirical work see e.g. Berger and Vavra (2018); Petrella, Santoro, and de la Porte Simonsen (2018), and for related theoretical work Baley and Blanco (forthcoming).

Dotsey and Wolman (2020).

Mapping the model to observables. Absent aggregate shocks the model is characterized by an invariant distribution of price gaps with density $f(\cdot) : (\underline{X}, \bar{X}) \rightarrow \mathbb{R}_+$. As shown in [Appendix A](#), the distribution $f(x)$ is uniquely determined by the generalized hazard function $\Lambda(x)$.

The functions f and Λ are used to compute several steady-state objects that are observable in the data, such as the frequency of price adjustments $N(\mu)$, and the distribution of the size of price changes c , $Q(c; \mu)$, where the notation emphasises the dependence of these moments on the rate of inflation μ . In turn, the latter is used to compute moments such as the variance of price changes, $Var(\mu)$, and the Kurtosis, $Kurt(\mu)$.

Statistics for low inflation economies. The main theoretical result that we present below will be established for economies where inflation is zero, i.e. $\mu = 0$. In this case the decision rules are symmetric, in the sense that $\bar{X} = -\underline{X}$, and the optimal return point is $x^* = 0$, located in the middle of the inaction region. Hence price changes are such that, upon adjustment, a firm with a price gap x chooses to “close the gap” completely, i.e. it chooses a price change $c = -x$ to reset the state at $x^* = 0$. The hazard function Λ , the invariant density f , and the size distribution of price changes Q are also symmetric around zero.

We will argue that the result for zero inflation provides an accurate approximation for economies where inflation is small, but not zero. The reason for this claim is that in models with idiosyncratic shocks (formally where $\mu/\sigma^2 < \infty$) the variables of interest, such as the frequency, the kurtosis and the variance of price changes, all exhibit a zero elasticity with respect to inflation when evaluated at zero inflation. Formally, it can be shown that⁷

$$\left. \frac{\partial N(\mu)}{\partial \mu} \right|_{\mu=0} = \left. \frac{\partial Var(\mu)}{\partial \mu} \right|_{\mu=0} = \left. \frac{\partial Kurt(\mu)}{\partial \mu} \right|_{\mu=0} = 0 . \quad (2)$$

Intuitively, this result states that the values of the even moments, such as frequency of price adjustment, variance or kurtosis, change very little when we move from zero to small inflation

⁷The proof can be established by using the symmetry properties of the even moments of the distribution. Noting that, for example, $N(\mu) = N(-\mu)$ and taking the derivative with respect to μ gives $2N'(0) = 0$. See proposition 5 in [Alvarez and Lippi \(2019\)](#) for a rigorous proof.

rates. [Alvarez, Beraja, Gonzalez-Rozada, and Neumeyer \(2019\)](#) offer new empirical evidence to validate this prediction.

2.2 The Sufficient Statistic Result

Next, we discuss the propagation of a monetary shock in this economy. In particular, we consider an economy in steady state, ie, with an invariant cross-sectional distribution of price gaps $f(x)$, and analyze the effect of an unexpected once-and-for-all monetary shock of size $\delta > 0$ on output. We consider the impulse response of output to such a shock, and focus on the area below such impulse response, named CIR^Y as in Cumulated Impulse Response, as a summary measure of the propagation mechanism.

Analytical results on the computation of the CIR^Y have been developed in recent papers focusing on small shocks and zero inflation by [Alvarez, Le Bihan, and Lippi \(2016\)](#); [Alvarez, Lippi, and Paciello \(2016\)](#). More results for environment with non-zero inflation and small shocks have been developed by [Baley and Blanco \(forthcoming\)](#); [Alvarez, Lippi, and Oskolkov \(2020\)](#); analytical results for large shocks in the presence of non-negligible inflation are studied in [Alexandrov \(2020\)](#). We find the CIR^Y statistic convenient for two reasons. First, it combines in a single value the persistence and the size of the output response. Second, for small monetary shocks, like the ones typically considered in the literature, the area is completely encoded by frequency of price changes and the kurtosis of price changes.

Formally, let the cumulative impulse response (CIR^Y) of output for a monetary shock δ be:

$$CIR^Y(\delta) = \int_0^{\infty} Y(t; \delta) dt \quad (3)$$

where $Y(t; \delta)$ is the aggregate output t periods after the shock δ , measured in deviation from the steady state output. Using $f(x, t)$ to denote the cross-sectional distribution of gaps at time t , and considering an aggregate nominal shock δ which uniformly displaces the invariant distribution of the desired adjustments at time zero, so that $f(x, 0) = f(x + \delta)$ the output at time t (in deviation

from steady state) is given by

$$Y(t; \delta) = \frac{1}{\epsilon} (\delta - P(t)) = \frac{1}{\epsilon} \int_{-\bar{x}}^{\bar{x}} f(x, t) x dx \quad (4)$$

where $P(t)$ stands for the response of the aggregate price level at time t and $1/\epsilon$ is the industry's Marshallian wage elasticity, so that output is proportional to real wages (or real money balances). The second equality formulates the same relation in terms of the distribution of price gaps x .

Our approach to characterize [equation \(3\)](#) is to compute the cumulated output measure for each firm, as indexed by its price gap x , and then aggregate over firms using the displaced time zero distribution $f(x + \delta)$, see [Appendix A](#) for details.⁸

We highlight a homogeneity property of the output CIR^Y . Let $Std \equiv \sqrt{Var}$ be the cross-sectoral standard deviation of price changes in the economy with zero inflation. Also, with slight abuse of notation, let us write $CIR^Y(\delta; N, Std, Kurt)$ to emphasize the dependence of the CIR^Y on the steady-state moments. We have:

$$CIR^Y(\delta; N, Std, Kurt) = \frac{Std}{N} CIR^Y\left(\frac{\delta}{Std}; 1, 1, Kurt\right) \quad (5)$$

The equation shows that the CIR^Y is homogenous of degree -1 with respect to the frequency of price changes. This is intuitive as changing N amounts to a rescaling of the time units, so that a doubling of N is equivalent to everything happening twice as fast. The equation also shows that the standard deviation of price changes scales both the size of the CIR^Y as well as the shock size. In particular this implies that for small shocks δ , where a small shock must be interpreted as small relative to Std , the only moments that matter for the CIR^Y are the frequency and the kurtosis of

⁸We stress that in computing such a measure we keep the decision rule constant at their steady state level. In Proposition 7 of [Alvarez and Lippi \(2014\)](#) we showed that, given the general equilibrium set-up in [Golosov and Lucas \(2007\)](#) and the lack of the strategic complementarities, such an approximation gives an accurate first order approximation. We also use the fact that *after* the first price change the expected contribution to output of each firm is zero since positive and negative output contributions are equally likely. This result, which holds around zero inflation, is convenient since it allows us to characterize the propagation of the monetary shocks without having to keep track of the time evolution for the whole distribution of price gaps.

price changes, since a first order expansion gives

$$CIR^Y(\delta; N, Std, Kurt) \approx \delta \frac{1}{N} \frac{\partial}{\partial \delta} CIR^Y(0; 1, 1, Kurt)$$

Interestingly, for small values of δ/Std , the first order expansion of the *CIR* does not depend on the standard deviation of price changes.

We next present the sufficient-statistic property – the cumulated output response following a small nominal shock δ is (see [Appendix A](#) for the proof):

$$CIR^Y(\delta; N, Std, Kurt) = \frac{\delta Kurt}{\epsilon 6N} + o(\delta^2). \quad (6)$$

The result states that the cumulated output response to a marginal shock, in a world with small inflation, is accurately approximated by the ratio of the kurtosis to the frequency of price changes, scaled by some constants. The approximation is accurate up to second order terms, so the remainder is of order δ^3 .⁹

The result in [equation \(6\)](#) is, to us, striking. It holds in a large class of inherently different models, from time dependent models a la Calvo, to canonical menu-cost models a la Golosov-Lucas, intermediate cases such as the Calvo-plus by Nakamura and Steinsson or inherently random-menu cost models such as those of Caballero and Engel. Moreover, we have shown in [Alvarez, Le Bihan, and Lippi \(2016\)](#) that [equation \(6\)](#) holds in multi product models, and we have shown in [Alvarez, Lippi, and Paciello \(2016\)](#) that the same equation holds in a large class of costly information models that give rise to time-dependent rules, spanning classic models such as [Taylor \(1980\)](#); [Caballero \(1989\)](#); [Reis \(2006\)](#). The broad applicability of the same equation across such a different set of models is the hallmark of the “sufficient statistic” result, a theoretical notion coined by [Koopmans \(1953\)](#); [Marschak \(1953\)](#), and recently revived by [Chetty \(2009\)](#); [Badel and Huggett \(2017\)](#) in public finance models. The central idea is to derive formulas to describe the effect of a policy that are functions of a few high-level elasticities rather than all the deep primitives of the models. In our case, this means that a two steady state moments fully capture the CIR across a wide range

⁹This happens since $CIR''(0)$ is zero, which follows from twice differentiating the *CIR* and noting its antisymmetric nature, or $CIR(\delta) = -CIR(-\delta)$.

of models that differ in terms of the number of primitives and even in their fundamental micro structure (think, for concreteness, about the difference between a time-dependent model and a multi product model).

Key assumptions and limitations of the sufficient statistic result. Three assumptions are key for the proof of [equation \(6\)](#). The first one is that the model has no inflation, so that several model objects display symmetry properties. While the assumption of zero inflation might seem restrictive, we argue that it provides a good approximation to models where inflation is low. The reason is that, as was noted above, that the CIR^Y function has a zero cross partial derivative, $CIR_{\delta,\mu}^Y(0,0) = 0$, which implies that $CIR_{\delta}^Y(0,0)$ is insensitive to small changes in the value of steady state inflation.

The second key assumption for the result to hold is that upon adjustment the firm completely closes the price gap, i.e. that x is reset to zero. This assumption is violated in models with high inflation, in models with strategic complementarities, or in models with price plans (as in [Eichenbaum, Jaimovich, and Rebelo \(2011\)](#)). In such cases [equation \(6\)](#) is not a good summary of the impulse response and other methods can be used to approximate CIR^Y . See [Alvarez and Lippi \(2019\)](#) and [Alexandrov \(2020\)](#) for some results on, respectively, price plans and high inflation.

A third assumption is that x follows a Brownian motion. This assumption allows us to exploit the identity $N \cdot Var = \sigma^2$, and to use the Kolmogorov forward equation in the proof. In a model with leptokurtic shocks, such as [Midrigan \(2011\)](#), such equation fails to hold and kurtosis and frequency are not enough to summarize the CIR^Y . However, we note that for moderate deviations from the normal benchmark, that are consistent with the data on the distribution of firms' nominal shocks, the formula continues to provide a useful benchmark (see Section 5 in [Alvarez, Le Bihan, and Lippi \(2016\)](#) and the numerical results in [Gautier and Le Bihan \(2020\)](#)).

2.3 An Empirical Test for the Sufficient Statistic Result

This section uses the predictions developed above to derive an empirical test of the theory. We will consider an economy made of several sectors, indexed by j , assuming that firms within a sector are similar, i.e. that they have the same response to a common monetary shock. The thought

experiment is to hit this economy with an aggregate monetary shock, and to use the variation in the responses observed across the sectors to test the theory.

The theory in [equation \(6\)](#) predicts that the CIR is related to the observed ratio $\frac{Kurt_j}{Freq_j}$ (in level) according to

$$CIR_T^{Y_j} \approx \frac{\delta}{6\epsilon_j} \frac{Kurt_j}{Freq_j} \quad (7)$$

where the approximation is due to the fact that the theory is based on a second order approximation and that our measurement will use a finite horizon ($T < \infty$). [Equation \(7\)](#) suggests testing the theory using a linear empirical relation between the product-level CIR of output over a long horizon, and the observed product-level ratios of kurtosis to the frequency of price changes. However, highly disaggregated sectoral output or real consumption series (at a monthly frequency) that match exactly the level of disaggregation and high frequency of observations of categories available for prices are typically not available. In particular, in the case of France, there are no available monthly consumption volume data available at the same level of disaggregation as the CPI (we conjecture the same holds for other countries). We thus rely in the following on the cumulated impulse response of *prices* rather than output. One advantage of this strategy is also that both the micro and sectoral sets of variables derive from the same source of micro prices, ensuring consistency.

To obtain this alternative test, let us derive the relation between the cumulated response of output in sector j at horizon T , $CIR_T^{Y_j}$, and the one of the prices, $CIR_T^{P_j} \equiv \int_0^T P^j(t) dt$ at the horizon T , following a monetary shock of size δ . Using the definition in [equation \(3\)](#) and [equation \(4\)](#) we have

$$CIR_T^{Y_j} \equiv \int_0^T Y_j(t) dt = \frac{1}{\epsilon_j} \int_0^T (\delta - P^j(t)) dt = \frac{1}{\epsilon_j} (\delta T - CIR_T^{P_j}) \quad (8)$$

where $\frac{\delta}{\epsilon_j} T$ is the cumulated change in nominal output following a permanent increase in money.¹⁰ Replacing $CIR_T^{Y_j}$ by its value in [equation \(7\)](#) we have the following prediction relating the cumu-

¹⁰Note that when T tends to infinity, as the CIR of output is finite, the CIR of prices diverges. This is an expected property as the price level is permanently higher (or lower).

lated response of prices and the ratio of the price change distribution for a large T :

$$CIR_T^{P_j} \approx \delta T - \frac{\delta Kurt_j}{6 Freq_j} \quad (9)$$

From this equation, we derive an empirical linear specification linking the product-level CIRs of prices to a monetary shock and the observed product-level ratios of kurtosis over frequency of price changes (in levels). One advantage of this specification (using CIR of prices instead CIR of output) is that the predictions for prices are independent of the sectoral elasticity ϵ_j , which simplifies how the regression coefficient should be interpreted. This provides an additional motivation for focusing on the response of prices rather than output. We will thus estimate, as a baseline, the following linear regression:

$$CIR_T^{P_j} = \alpha + \beta \left(\frac{Kurt_j}{Freq_j} \right) + \nu_j \quad (10)$$

where $\alpha = \delta T$ and $\beta = -\delta/6$ are the theory-implied values of the regression coefficients and ν_j is the regression's error term. In our empirical exercises, we have normalized our measure of the monetary policy shock so that $\delta = -1\%$, leading, under a strict interpretation of the model, to the prediction that $\beta = 1/6$ and that $\alpha = -T$ where T is the time horizon (in months). In our empirical tests, T will be set to either 24 months or 36 months. We refer to this regression as the baseline regression, or as a “constrained regression”, since the specification imposes that kurtosis and frequency enter the regression with coefficients of the same magnitude but opposite signs.¹¹

We can further decompose [equation \(9\)](#) to investigate the restriction imposed by the theory on how kurtosis and frequency relate to the CIR. For that, we rely on a first-order Taylor expansion around the sample means \bar{F} , \bar{K} , and we get:

$$CIR_T^{P_j} \approx CIR_T^{\bar{P}_r} - \frac{\delta \bar{K}}{6 \bar{F}} \frac{Kurt_j}{\bar{K}} + \frac{\delta \bar{K}}{6 \bar{F}} \frac{Freq_j}{\bar{F}} \quad (11)$$

From this expression we derive an unconstrained version of the empirical test where we relate

¹¹An interesting property of the specification in [equation \(10\)](#) is that, for some type of measurement errors - namely a fraction of price change being spurious changes, of a small size - , the induced multiplicative bias on measured kurtosis and frequency is identical, so these biases do cancel. In other terms the specification is correct even though both kurtosis and frequency are measured with errors. See [Appendix F](#) for details.

the CIR of prices to the ratio of the product-level kurtosis over its average, and the ratio of the product-level frequency over its average:

$$CIR_T^{P_j} = \gamma + \beta_k \left(\frac{Kurt_j}{\bar{K}} \right) + \beta_f \left(\frac{Freq_j}{\bar{F}} \right) + \nu_j \quad (12)$$

The theory suggests that β_k and β_f (i.e. the slope coefficients of the regressors $\frac{Kurt_j}{\bar{K}}$ and $\frac{Freq_j}{\bar{F}}$) are expected to be equal in absolute value.

We emphasize that the theory gives no prediction on the extent to which kurtosis and frequency contribute to the “explained share of variance”, or to the fit, of the regression. This contribution could be arbitrarily low or large, depending on the cross-sectoral dispersion of kurtosis and frequency, without invalidating the theory.

3 Measuring Monetary shocks and Cross Sectoral Moments

This section discusses the data used in the analysis, and the construction of the empirical statistics needed to test the sufficient statistic result. We will use variations across products to test the theory. We rely on the fact that there is cross product variability in the price adjustment statistics, and that [equation \(10\)](#) is expected to hold across different sectors.¹² We need to estimate two types of statistics: (i) the cumulative impulse response of prices (CIR^P) computed at the sectoral level, and (ii) the moments of the distribution of price changes for the corresponding products.

Before detailing the construction of the objects underlying our test, we stress two important features of our empirical approach. First, we make use of a cross section of moments computed from two micro data sets of prices in France: a first one covering consumer prices and the other one producer prices. Both data sets are relevant for our purpose, and each has distinctive advantages. Consumer prices are observed directly and somewhat less prone to measurement issues (since they can be directly observed in outlets), offer a broader coverage of the economy (goods and services

¹²In the paper we use indifferently the terms “sectors” and “products”. For PPI, product and sector classifications fully overlap, whereas for CPI, we will use product specific price indices but no monthly consumption or production statistics are available at this level of classification.

vs. only goods for PPI products) and consumer inflation is used for the definition of the monetary policy target. Producer price data are conceptually closer to the productive firms' pricing problem studied in standard macro models, and are not concerned by sales and temporary promotions.

The second feature is that we identify the monetary shocks by imposing that they have the properties highlighted by the theory (in the spirit of the “sign restriction” approach). In particular, we want a (contractionary) shock to decrease output in the short run, to have a permanent negative effect on the price level, and to have no long-run effect on output. These characteristics are consistent with the theoretical model described above, and are thus desirable to perform a test of the sufficient statistics result. Note that in principle any common shock to the marginal cost of firms could be used to test the theory. Oil price shocks would for instance qualify, but empirically the sectoral dynamics following such a shock is strongly heterogeneous making it hardly useable for a test in a finite sample. On the contrary, an aggregate monetary shock has the desirable features that it will eventually move all nominal prices by the same amount, leaving relative prices unaltered. We exploit this homogeneity property in our long-run identification of the monetary shock. Finally, we stress that another feature of our approach is that the construction of the CIR^P variables does not use the micro data nor the sectoral moments, so there is no reason to expect any bias in favor (or against) the sufficient statistics result.

3.1 Measuring the sectoral response to a monetary shock

To estimate the CIR^P for a large number of sectors of the French economy we employ a Factor Augmented VAR (FAVAR). The method was developed by [Bernanke, Boivin, and Eliasch \(2005\)](#) and [Boivin, Giannoni, and Mihov \(2009\)](#). We closely follow the approach of [Boivin, Giannoni, and Mihov \(2009\)](#) as they focus on the response of sectoral inflation rates to monetary policy shocks. A brief description is as follows:¹³ the FAVAR is a model in which the dynamics of a large number of time series is governed by the evolution of a small number of time series, the factors, that are typically – but not necessarily – unobserved and follow a VAR process.

Formally the vector of a large number n of time series X_t , called informational time series, are

¹³[Appendix B](#) describes the FAVAR model and discusses its framework.

related to the factors F_t by the following equation:

$$X_t = \Lambda F_t + e_t \quad (13)$$

where F_t is a vector of dimensions $K+M$ of respectively unobserved and observed factors, and e_t is a vector $n \times 1$ of error terms with zero mean. Following [Boivin, Giannoni, and Mihov \(2009\)](#) we allow one factor, the interest rate Y_t , to be observed, so $F_t \equiv [\tilde{F}_t \ Y_t]'$ where the unobservable factors \tilde{F}_t are to be estimated. Notice that the observable factors and the informative time series are two distinct objects that do not have any time series in common. The factors follow a VAR process:

$$F_t = \Phi(L)F_{t-1} + v_t \quad (14)$$

where $\Phi(L)$ is a lag polynomial of finite order and v_t is an error term with zero mean and covariance matrix Q .

The unobserved factors can be estimated, typically by using a principal component analysis on a large number of “informative” time series. After this step is performed, it is possible to estimate a VAR in the estimated factors (along with the unobserved ones, if any). Once the VAR is estimated, it is possible to retrieve the impulse response function (IRF) of any informative time series to a monetary policy shock. This last point is crucial for our purpose to estimate the cumulative impulse response of prices for all sectors.

Of primary interest for our purpose is the response of sectoral prices to an aggregate shock. The dynamics of inflation in sector j will, in our FAVAR set-up, governed by:

$$\pi_{jt} = \lambda_j F_t + e_{jt} \quad (15)$$

where λ_j is a vector of loadings, recovered as the relevant row of matrix Λ . [Equation \(15\)](#) makes it clear that one can easily recover the IRF of sectoral inflation (and hence, prices) to a monetary shock from the IRF of the factors F_t to the shocks.

We include three types of “informative time series” in vector X_t : (i) macroeconomic data for France including aggregate industrial production, aggregate producer price index (PPI), the

aggregate harmonized index of consumer prices (HICP), unemployment rate, (ii) financial and monetary variables relevant for the euro area including the monetary aggregate M3 in the euro area, the value of banknotes in circulation in the euro area, the euro exchange rate with respect to US dollar, yen, UK pound sterling, Swiss franc, Chinese Yuan Renminbi (iii) highly disaggregated series of industrial production, producer prices (PPI) and consumer prices (CPI), as well as some available disaggregated series for monthly consumption (16 broad categories of consumptions at an intermediate aggregation level, including, for instance, durables consumption, manufacturing goods consumption). As regards product-specific monthly price series, CPI price indices are available at the 5-digit level of the ECOICOP classification (e.g. ‘01.1.1.1’ ‘Rice’) whereas PPI price indices in the manufacturing sector are available at the 4-digit level of the NACE rev2 classification of sectors (e.g. ‘08.11’ ‘Quarrying of ornamental and building stone, limestone, gypsum, chalk and slate’). Overall, we use 223 product-specific consumer price indices covering both goods and services and 118 producer price indices covering the manufacturing sector. In addition, our analysis uses the 3-month Euribor as a measure of the monetary policy variable. This variable will be treated as an observable factor, and we filter it following motivations and a procedure that are detailed below. All the data are monthly and the sample period is Jan. 2005 to Dec. 2019. We are interested in estimating the response of the disaggregated time series of prices (PPI and CPI) after a monetary shock; in our analysis an exogenous shock to the 3-month Euribor. In a first step, factors are computed from a Principal Component Analysis using the informative time series (in log difference). We extract the five principal factors (those with the largest contribution to the overall variance). We subsequently estimate a VAR with 12 lags for the 5 factors and the interest rate.

Identifying monetary policy shocks and the price responses. To identify a contractionary monetary shock, and estimate the associated IRFs, based on our FAVAR results, we use a Cholesky decomposition of the variance-covariance matrix of the VAR innovation. Following a standard timing restriction, the Euribor is ordered as a last variable in the VAR. Notice that, imposing a Cholesky decomposition in this setup does not imply that the IRFs of informative time series cannot respond simultaneously to the monetary shock. The CIR^P is computed cumulat-

ing the responses of sectoral price levels over a large number of periods (see next section for a discussion).

In our baseline approach we impose a “long run neutrality” restriction. Specifically, we impose (i) that output comes back to its original level in the long run after a monetary shock and (ii) that all sectoral prices have identical responses -equal to that of the aggregate price index- in the long run. Both of these restrictions are consistent with the money neutrality hypothesis. To implement the latter restriction in the baseline FAVAR specification, we proceed following [Boivin, Giannoni, and Mihov \(2009\)](#). In an alternative FAVAR specification, we relax the long-run neutrality restriction and let the relative prices be unconstrained.

We also normalize the shock, so that the monetary policy shock produces a 1% long-run decrease in the aggregate price level. This normalization assumption (which has no bearings in terms of inference) departs from the usual approach to normalizations imposing that the shock produce a, say, 25 basis points impact effect on impact on the nominal interest rate. The normalization allows an easier comparison with our theoretical model (where the size of the shock is proportional to the long run response of the price level) and facilitates the interpretation of results relating the CIR^P to the sufficient statistic.

For robustness we consider the case when no long-run restriction is imposed (a case considered by [Boivin, Giannoni, and Mihov \(2009\)](#) along with the one with long-run restrictions). For robustness purposes, we also explore an alternative identification procedure, following [Gertler and Karadi \(2015\)](#), and use a High Frequency Identification in the VAR set-up. This allows us to handle simultaneity concern without resorting to a timing assumption as in the Cholesky approach. For the HFI approach, we use the data for monetary surprises in the euro area from [Altavilla et al. \(2019\)](#), who rely on market interest rate changes around the times of ECB Governing Council meetings.¹⁴

Filtering the Euribor. Given the marked downward trend in the nominal interest rate over the sample period (itself partly related to the decline in the natural rate of interest, see [Figure A](#))

¹⁴In robustness analysis, we also report results using the 2-year German Bond rate as the policy rate and using the same HFI approach.

the VAR estimates based on unfiltered interest rate data produce impulse response functions that are not consistent with the response of output and inflation to a monetary policy shock suggested by the model. The theory suggests that a (contractionary) monetary policy shock triggers a transient, and negative, impact on inflation and output.¹⁵ Our approach is to use an HP filter with parameter λ^{HP} delivering the expected properties. As is documented in [Appendix C](#), we select a value of $\lambda^{HP} = 1000$, a smaller value than the one traditionally used with monthly data. Using this parameter value, we are able to recover IRFs that (both for CPI and PPI) feature a negative response of prices and output for the aggregate price index, as well as the largest number of sectors for which the individual price response is negative after 24 or 36 months. We stress that our procedure for selecting the filter parameter makes no use of the microeconomic data or the sectoral moments, so it is not biasing toward finding a relevance of the sufficient statistics results. Our procedure is designed to produce a shock that has a common effect on all sectors and can be interpreted as a monetary policy shock.

VAR Results: IRFs and CIR^P 's. Our estimated FAVAR provides theory-consistent results for the responses of aggregate variables to a monetary shock. As presented in [Appendix Figure D](#), after a contractionary policy shock the interest rate increases and subsequently decreases, going back to its steady state level after two years. Industrial production immediately reduces after a contractionary monetary policy shock, then gradually recovers. The production price index declines following the shock, then recovers towards the new steady-state value. The aggregate consumption price index reacts similarly.

We focus our analysis on the objects used to test the sufficient statistic result, namely the responses of sectoral producer and consumer prices, as derived from the FAVAR. [Figure 1](#) reports the estimated IRFs of production and consumer price series. In each panel, the blue line (Aggr PPI/HICP) represents the IRF of the aggregate PPI/HICP series. Dashed red lines are the IRFs of different sectors disaggregated at the 2-digit level for PPI and 1-digit for CPI.¹⁶ The thick red

¹⁵Identifying well-behaved monetary policy shocks for the euro area is particularly challenging over the sample period, in particular due to the proximity of the effective lower bound on interest rates – see [Andrade and Ferroni \(2021\)](#) and [Jarocinski and Karadi \(2020\)](#) for investigations in the context of information shocks.

¹⁶Our PPI/CPI series are available at the 4-digit and 5-digit levels, and the dashed red lines are constructed as the arithmetic average of estimated IRFs.

line is the average of all the dashed red lines. In both panels, red and blue curves have a very similar shape, an expected property, as the average of sectoral responses should closely approach the response of the aggregate price index.¹⁷ In both figures, we impose that the long run price response is -1 percent at a long horizon. The transitory dynamics is however heterogeneous across sectors. Most of them display a through in prices after 1 to 2 years after the shock.¹⁸

Finally, using the estimated IRFs of the PPI and CPI, we construct the CIR^P s for each sector/product category, as the sum of the respective IRF from time zero up to a time horizon T . The sectoral CIR^P s are the most important object of interest in this section, since the sufficient statistic result relates these measures to the cross sectional moments of the price change distribution. We consider two different values for T , respectively 24 and 36 months (see [Table A](#) in Appendix for descriptive statistics on product-specific CIR^P s).

3.2 Measuring micro moments

Consumer Price (CPI) Micro Data For consumer price micro data, we rely on longitudinal data sets of monthly price quotes collected by the Institut National de la Statistique et des Études Économiques (INSEE) to compute the monthly French CPI (Consumer Price Index). Stacking data sets used in [Baudry et al. \(2007\)](#), [Berardi, Gautier, and Le Bihan \(2015\)](#) and [Berardi and Gautier \(2016\)](#) and extending the data set to September 2019, we obtain a long sample covering a period of about 25 years between August 1994 and September 2019.

The data set contains about 30 million of price quotes, and covers about 60% of the CPI weights.¹⁹ Price changes are computed as log-differences of prices, and we exclude price changes due to sales. To compute price adjustment moments, we have first dropped data collected around VAT changes (i.e. in Aug.-Sept. 1995, Sept.-Oct. 1999, April-May 2000, July-Sept. 2009, Jan.-Feb. 2012 and Jan.-Feb. 2014) and before and after the euro cash changeover (between Aug.

¹⁷The small discrepancy between these two curves is due to the fact that the aggregate price index is a weighted price index whereas the average of sectoral IRF is unweighted.

¹⁸In [Appendix D](#), we report similar results for all the different specifications of the FAVAR model we have estimated.

¹⁹Some categories of goods and services are not available in our sample: centrally collected prices, among which car prices and administered prices (e.g. tobacco) or public utility prices (e.g. electricity), as well as other types of products such as fresh food or rents.

2001 and June 2002). We have also dropped price changes smaller than 0.1% in absolute values, in both data sets, in order to control for possible small price changes due to measurement errors (Eichenbaum et al. (2014)).

We compute price adjustment statistics excluding sales, as the model is not able to reproduce price changes due to sales. For identifying sales we rely on an INSEE flag variable that identifies whether a price corresponds to a sale price, either in the form of seasonal sales or temporary promotional discounts. Sales are mostly concentrated in some sectors (i.e. clothing and shoes, and furnishings).

We identify products at the 5-digit level of the ECOICOP product classification, which is the most disaggregated level for which sectoral price indices are available. For each product, we compute the frequency of price changes as the ratio between the number of price changes (excluding price changes due to sales) and the total number of prices for this product. We also compute the kurtosis of price changes, as well as other moments of the price change distribution (such as average price changes, the standard deviation of price changes and the skewness of price change distribution), at the product level. Overall, for CPI products, our baseline data set contains price adjustment moments for 223 different “ECOICOP-5” products.

Measurement of kurtosis is notoriously a challenging issue, as large values of price changes, and outliers, can have an important impact on kurtosis. Very large kurtosis values tend to be obtained when not correcting for measurement errors.²⁰ In our baseline, we drop from the calculations price changes larger than 25% in absolute values, which corresponds to about 5% of all price changes. As robustness, we provide results with alternative values for the thresholds used to defining for outliers and address measurement errors concern (for very large or very small price changes in absolute values). Drawing on Alvarez, Lippi, and Oskolkov (2020), we also provide results using a measure of kurtosis including a correction for unobserved heterogeneity (see Appendix E for details). Alternative kurtosis measures are highly correlated across products.

²⁰Note however that excluding sales by itself does not decrease the degree of kurtosis, see for instance Gautier and Le Bihan (2020).

Producer price (PPI) Micro Data. We rely on micro price data collected by INSEE to construct the French Producer Price Index (the same data set as the one used in [Gautier \(2008\)](#)). Reported prices must be observed at the “factory gate”, excluding transport and commercialization costs, or invoiced VAT. Our sample contains more than 1.5 million price reports between January 1994 and June 2005. Overall, more than 90% of the price quotes used to compute the French PPI are available. The PPI covers all products manufactured and sold in France by industrial firms, which includes sections C (Mining and quarrying), D (Manufacturing) and E (Electricity, gas and water supply) of NACE Rev 2 classification.²¹ The data set has been investigated in [Gautier \(2008\)](#) where further details are available. Contrary to CPI prices, there is no flag for temporary promotions or sales. We assume, consistent with [Nakamura and Steinsson \(2008\)](#), that there are no sales in producer prices. Like for CPI, price changes are computed as log-differences of prices.

For each NACE 4-digit sector, we compute both the frequency of price changes and the kurtosis of non-zero price changes, as well as other moments of the price change distribution. Unlike with CPI, large price changes are much less frequent (reflecting, and confirming, that sales or temporary promotions are not a usual practice in the mainly business-to-business context of producer prices) and only 2% of all price changes are larger than 22% in absolute value. To measure kurtosis, we drop price changes larger than 15% in absolute values (which correspond to less than 5% of all price changes) and we test the robustness of our results to this definition of price change outliers. We restrict to the subsample of sectors for which an aggregate sectoral price index is available from the statistical office, so as to match micro moments with time-series macro evidence in our subsequent analysis. This result in a baseline sample containing 118 sectors.

Basic statistics for the micro data underlying both the CPI and the PPI, are presented in [Table 1](#) and [Figure 2a](#) and [Figure 2b](#). Consumer prices are more rigid than producer prices, with average frequencies of price changes of 10.6 percent and 19 percent respectively. The distribution of price change has fat-tails for both data sets, with a virtually identical value of the average unweighted kurtosis of 5.0 in both data sets. One main important takeaway is there is some cross-sectoral dispersion in frequency and kurtosis of price changes, for both consumer prices and

²¹NACE is the general “classification of economic activities within the European Community”. Some sectors are excluded from collection: mining of uranium and thorium, ores, publication, processing of nuclear fuel, weapons and ammunition, building and repairing of ships and boats, manufacturing of aircraft and spacecraft, and recycling.

producer prices - as apparent from the interquartile ranges or standard deviations, and from the full distribution of moments plotted on [Figure 2a](#) and [Figure 2b](#). The frequency of price changes however seems to show relatively more cross-sectoral variability than the kurtosis of price changes. While alternative corrections for measurement error and unobserved heterogeneity do change the average value of kurtosis, they do not substantially affect the degree of cross-sector heterogeneity however.

Cross-sectoral characteristics of both our CPI and PPI data sets are consistent with available international evidence. As regards consumer price data, [Berardi, Gautier, and Le Bihan \(2015\)](#) using the same data, provide a detailed comparison of CPI data moments in France with those in the United States, based on detailed moments reported by [Nakamura and Steinsson \(2008\)](#). They conclude that patterns are quite similar, whenever sales-related price changes are disregarded (as the pattern of sales is however much more prevalent in the United States). Regarding producer prices, [Vermeulen et al. \(2012\)](#) provide a comparison of the patterns of price setting in the United States and 6 euro-area countries, including France - relying for that particular country on the same data set as we use. They conclude patterns of producer price rigidities are very similar - albeit the size of price changes is typically larger in the United States than in Europe. The above-mentioned international evidence mainly focuses on the frequency of price changes, as well as on the first two moments of the distribution of price changes. Evidence is scarcer on kurtosis. For US PPI data, [Hong et al. \(2020\)](#) report an average kurtosis of 4.9. With consumer price data, [Cavallo \(2018\)](#) report a median kurtosis of 4.8 in a large sample of countries based on “scraped” data. These values, all obtained after correcting for measurement errors in the same spirit as we do, are thus much in line with our baseline values.

4 Testing the Theory: Results

This section presents the results of the empirical tests developed in [Section 2.3](#) using as inputs the variations across products in the real effects of monetary policy, as measured in [Section 3.1](#), and in the microeconomic price adjustment moments, as measured in [Section 3.2](#).

4.1 Estimates of the baseline empirical specification

This section presents our baseline estimation results. A visual summary of these results is provided by [Figure 3](#), that is the scatter plots of CIR^P at horizon 36 months against the ratio $Kurt/Freq$, for the different FAVAR specifications and for both PPI products (top panel) and CPI products (bottom panel). For most of the specifications, we find a positive relationship in the cross section of products between the value of CIR^P and the value of the ratio $Kurt/Freq$.

[Table 2](#) reports results for [equation \(10\)](#), the baseline “constrained” regressions for an horizon T equal to 24 or to 36 months. We consider separately the CIR^P of producer prices (Panel A) and the CIR^P of consumer prices (Panel B). In each panel we consider three specifications for the identification of the monetary policy shock: the baseline one, with Cholesky identification and long run restriction on relative prices; a first alternative with Cholesky identification and not imposing any restriction on the long-run effect on relative prices; and a second alternative where identification relies on High Frequency surprises and external instruments.

For producer prices (Panel A), the estimated slope coefficient associated with the $Kurt/Freq$ ratio turns out to be positive and statistically significant in all cases, whereas the constant term is negative and statistically different from zero. These results are consistent with the theoretical framework. A positive sign for the coefficient associated with $Kurt/Freq$ ratio is expected since a contractionary monetary policy shock has a negative effect on output, and the products with higher $Kurt/Freq$ ratios are expected to experience, in absolute terms, larger output effects. Consistently, they will experience a less negative effect on prices, resulting in the cross-section regression in a positive coefficient associated with the $Kurt/Freq$ ratio. A higher $Kurt/Freq$ ratio can reflect either less frequent price adjustments, less price selection or both, implying larger (absolute) real effects of monetary policy shock. The last columns of [Table 2](#) report results without the long-run restriction, and those using the HFI approach. Coefficients are significant and with expected signs, as in the baseline. In the former case however, coefficients are larger than in the baseline, presumably reflecting a larger degree of variability of the CIR^P 's in that case (see [Table A](#) in [Appendix D](#)). For consumer prices the results are mixed (Panel B of [Table 2](#)). In the baseline specification the $Kurt/Freq$ ratio is not significant, but it is significant for the two

alternative specifications (significance is at the 5% level for the 36 months horizon).²² Finally, in all specifications for both PPI and CPI, we find the intercept to be negative and statistically different from zero: on average, a contractionary monetary policy has a negative effect on prices (consistent with predictions of equation (10)).²³

To further investigate the relevance of both the kurtosis and the frequency of price adjustments in explaining the propagation of monetary shocks, we report in [Table 3](#) the estimate for [equation \(11\)](#), an “unconstrained” version of the regression that allows for a potentially different effect of frequency and kurtosis.²⁴

For PPI products (Panel A), we find that the estimates are consistent with the theoretical predictions. First, a larger product-level frequency is associated with a relatively more negative effect on the CIR^P for this product: if prices are more flexible, prices will decline faster. The real effects of monetary policy will also be smaller. Second, a higher kurtosis is associated to a smaller reaction of prices in that sector, resulting in a positive coefficient in the cross-section regression - since the CIR^P is negative following a contractionary shock. This effect is significant: a higher kurtosis is associated with less selection effect leading to a lower CIR^P of prices in absolute value and increasing the importance of the real effects of monetary policy. Moreover, we cannot reject that slope coefficients associated with frequency and kurtosis are equal in absolute value, as predicted by the theory.²⁵

For CPI products (Panel B of [Table 3](#)) we also find, in all cases, a negative and significant relationship across sectors between frequency and the CIR^P , and that the slope coefficient associated with kurtosis is positive. As for PPI products, we find that a positive relationship between kurtosis and CIR^P . In the Cholesky case with no long-run restriction the estimate is however not significantly different from zero. Moreover, in the Cholesky baseline case, we cannot reject

²²In [Table C](#) of [Appendix G](#) we also provide results for the specification using a log ratio as the right-hand-side variable, and results are consistent: we find a positive and statistically significant effect for PPI, and more mixed for CPI products at both horizons.

²³Note our identification assumptions for the FAVAR might contribute to this result, however, the restrictions are imposed on the long-run values of the IRF and not on the CIR^P per se (i.e. how the IRF converges to its long run value).

²⁴In [Figure I](#), [Figure J](#) and [Figure K](#) in [Appendix D](#) we provide scatter plots of the CIR^P and $Kurt/Freq$ log ratios but also scatter plots of the CIR^P and log of frequency and kurtosis for the different FAVAR specifications. They show a negative relationship between frequency and CIR^P and positive between CIR^P and kurtosis for all specifications (in particular PPI products) and these relations do not seem to be driven by any particular product.

²⁵[Table B](#) in [Appendix G](#) reports p-values of formal Fisher tests from the estimated parameters.

that slope coefficients associated with frequency and kurtosis are equal, as predicted by the theory (Appendix Table B). Finally, in almost all specifications, we find the intercept γ to be negative and statistically different from zero as predicted by theory (equation (11)).

Besides, the model provides not only predictions on the size of the coefficients of the regressions, but also on the amplitude of the coefficients for both constrained and unconstrained versions of the model. In the constrained version of the model, β is predicted to be equal to $\frac{-\delta}{6}$ which is equal to 0.16 in our case since we have normalized the shock to 1% whereas the intercept α should be equal to δT , hence in our cases to -24 or -36 .²⁶ In the unconstrained model, β_f and β_k are predicted to be both equal to $\frac{\delta \bar{K}}{6F}$ in absolute values, coefficients in absolute values should be equal to 4.4 for PPI and 7.9 for CPI whereas the constant of the model should be equal to $-\delta T + \frac{\bar{K}}{6F}$ which is equal to -20 for PPI and -16 for CPI at the horizon of 24 months and -16 for PPI and -28 for CPI at the horizon of 36 months. Note that testing these predictions is much more demanding for the empirical exercise and depends a lot on the degree of precision of our estimates. However, looking at the order of magnitudes of our results in Table 2 for our baseline case (Cholesky with long run restriction), estimates are broadly in line with the predictions for PPI whereas it less the case for CPI. Table B in Appendix G reports the results of more formal tests. For the unconstrained version of the model, we find that baseline results are fully in line with predictions on the amplitude of the coefficients for both PPI and CPI products and we cannot formally reject that the size of coefficients are consistent with model’s predictions.

4.2 “Placebo” tests

While the above results are consistent with the “sufficient statistic” property, a sufficient statistic property predicts something broader: it implies that the effect of a monetary shock should be related to the ratio “kurtosis over frequency” but it also implies that other moments of the price distribution should not matter in this relationship. To test this prediction, we run a regression in which we add to our baseline regressions three additional moments of the price change distribution computed at the product-level: the average size of (non-zero) price changes, the standard deviation

²⁶Besides, the ratio α/β is predict to be equal to $-6T$, i.e. -144 at horizon 24 months and -216 at the horizon of 36 months.

and the skewness of price adjustments. This exercise can be considered as a “placebo” test of our baseline regressions, testing that our main result is not driven by correlations between frequency or kurtosis and other moments of the price change distribution.

Table 4 shows the results for this specification using moments in levels. For the PPI samples (Panel A) we find that the ratio of kurtosis over frequency has a significant positive effect at 5% level across all specifications (except the Cholesky 24 month case where significance is at 10%) and the coefficients are highly similar to the ones obtained in Table 2, i.e. in the baseline case. We also find that neither the average size of price changes, nor the standard deviation of prices changes, nor the skewness of price changes, do have statistically significant effects (one single exception being the skewness variable in the HFI identification for the 36-month horizon). These two results are fully consistent with the theoretical predictions.

We have estimated an unconstrained version of this regression (see Table D in Appendix G). Results for PPI products are broadly robust, although the degree of significance decreases a bit, presumably owing to multicollinearity. Coefficients associated with placebo moments are never significant at 5% and only significant twice at 10% (note that given we consider results for 6 specifications and 3 placebo moments, the fraction of significant coefficients is in line with what one would expect under the null of no effect).

In the case of CPI products (Panel B of Table 4), support for the theoretical predictions is -as with the baseline specification- somewhat more mixed. The coefficient on $Kurt/Freq$ is positive and significant in only half of the cases. The coefficient associated with the “placebo” moments are in some cases significant, mainly at the 10% level (in 6 cases out of 18).

5 Robustness analysis

This section explores the robustness of our findings with respect to several dimensions: i) we test whether our main results are driven by products with extreme values of CIR^P , as the distribution of CIR^P values shows some very large positive and negative values - i.e. possible outliers; similarly, we present results removing products with extreme values of frequency of price changes, kurtosis or the ratio $Kurt/Freq$; ii) we investigate whether our results are robust to kurtosis’ measurement

issues related to product heterogeneity, or very large or very small values of price changes; iii) for CPI products, we investigate the extent to which sales and promotions affect the results; iv) we run regressions where we include sectoral effects to investigate which sources of sectoral variations are important to explain the relation between CIR^P and $Kurt/Freq$ ratio (across or within broad sectors); v) we report results using the 2-year German bond instead of the 3-month Euribor when identifying the model using a high-frequency identification with external instruments strategy; vi) finally, we report results for a subsample which excludes products with a large drift in prices.

5.1 Removing extreme values of CIR^P , $Freq$, $Kurt$ or $Kurt/Freq$

Our first robustness exercise consists of checking whether our main results are driven by some products for which the cumulative response of prices, frequency or kurtosis of price changes, is either extremely low or extremely high. For that we define 4 sub-samples, considering separately CPI and PPI products, in which we remove 5% of products corresponding to the 2.5% largest or the 2.5% smallest values for: (i) the CIR^P , (ii) ratio kurtosis over frequency, (iii) kurtosis of non-zero price changes or (iv) frequency of price changes.²⁷ We run our baseline regression (as well as unconstrained specifications) on each of these subsamples. Results of robustness regressions are all contained in tables in [Appendix G](#) (see [Table E](#) for the constrained specification with PPI products, [Table F](#) for CPI products, and [Table G](#) and [Table H](#) for results of unconstrained specifications).

For PPI products, removing products with extreme values of CIR^P (Panel A of [Table E](#)), ratio $Kurt/Freq$ (Panel B), kurtosis (Panel C), or frequency (Panel D), does not alter our baseline conclusions: the slope coefficient associated with the ratio $Kurt/Freq$ is positive and significantly different from 0, and estimated coefficients are very close to the ones estimated in our baseline exercise. Similarly, in unconstrained regressions, results are in line with the ones using the full sample of products: all the results are consistent with the theoretical predictions including the equality of coefficients in absolute values for slope coefficients associated with frequency and kurtosis.

For CPI products, in the constrained model, results obtained when removing 'extreme' prod-

²⁷For CPI, in each subsample, 10 different products are excluded whereas for PPI 5 different products are excluded.

ucts are in line with baseline results: in most specifications, the ratio $Kurt/Freq$ is positively related with CIR^P but weakly statistically different from 0. The strongest relationship is obtained when we exclude extreme values of kurtosis. For the unconstrained specification, in all cases, the coefficient associated with kurtosis is positive and significantly different from 0 (in most cases at 1% level) whereas the coefficient associated with frequency is negative and statistically significant in a majority of cases. However, the estimated parameter associated with frequency is found positive or non-significantly different from 0 in several cases for the model using Cholesky identification and a long-run restriction. Overall, for CPI products, results are more mixed and the negative relationship between frequency and CIR^P is less clear.

5.2 Issues with the measurement of kurtosis

The measurement of Kurtosis is known to be severely affected by unobserved heterogeneity. We run robustness regressions in which we use a measure of kurtosis, based on [Alvarez, Lippi, and Oskolkov \(2020\)](#), that takes into account product-level unobserved heterogeneity ([Appendix E](#) provides details on how we compute this robust measure of kurtosis). Results (reported in [Table I](#) of [Appendix G](#)) are very much in line with the ones in our baseline regressions. For PPI, the coefficient associated with the $Kurt/Freq$ ratio is positive, and significant in all specifications, whereas for CPI this is the case in two of the three specifications. In the unconstrained regression, results are very consistent with theoretical predictions for both PPI and CPI products in the specification using a Cholesky decomposition identification and imposing the long run restriction. In the two other specifications, most results of unconstrained specifications are in line with theoretical predictions.

Another possible measurement issue is the high sensitivity of kurtosis to the definition of price change outliers, namely here to either very large, or very small price changes, in absolute values. In the baseline regressions, we have used kurtosis measures calculated on the distribution of price changes smaller than 15% for PPI price changes and 25% for CPI price changes (i.e. 5% of all price changes in both cases) and we have excluded price changes below 0.1% in both cases. We here test the robustness of our results to modifying the thresholds defining extreme price changes.

In a first exercise, we investigate the role of large price changes and we set the thresholds defining extreme values to 25% for PPI price changes and 35% for CPI price changes (which corresponds to excluding about 2% of all price changes). In a second exercise, we set the threshold for small price changes to 0.5% (which corresponds to about 5% of all price changes).²⁸ The results overall remain in line with the baseline results (see [Table K](#) and [Table J](#) in [Appendix G](#)). Standard errors of coefficients associated with kurtosis are however much higher, lowering the significance of the estimated coefficients, in particular for large producer price changes.

5.3 Role of sales for CPI products

For CPI products we further investigate robustness by excluding products for which price changes are mainly due to sales. The extent of sales could indeed affect price adjustment moments even if we have removed price changes *observations* due to sales in the calculation of these moments. In particular, if a very large majority of price adjustments are due to sales or promotions in one sector, the pricing moments excluding these changes might be not very representative of the typical price changes. We thus run robustness exercises removing all food, clothing/footwear and furnishings goods, as within these broad sectors, most products are largely affected by seasonal sales and replacements.²⁹ In a second exercise, we exclude CPI products for which more than 10% of all price changes due to sales (which corresponds to the median value among all CPI products).

When removing the three broad sectors that are mostly affected by seasonal sales (Panels A and B of [Table L](#) in [Appendix G](#)), we find a positive and significant effect of the ratio kurtosis over frequency in all specifications. In unconstrained specifications, we also find that estimated coefficients associated with both frequency and kurtosis have the predicted sign and are significant. In the specification using the Cholesky identification and imposing a long-run restriction, we also cannot reject that the equality of coefficients in absolute values for slope coefficients associated with frequency and kurtosis (as predicted by the theory).

When removing products for which the share of sales in price changes is higher than that for

²⁸We have also run similar exercises with other definitions of small and large price changes and conclusions are very similar.

²⁹These products correspond to COICOP 01.1, 03 and 05 in the product classifications.

the median product, the coefficient associated with the $Kurt/Freq$ ratio is larger than in the baseline case, but still not significant in the specification Cholesky imposing a long run restriction. In the unconstrained specification of the regression, frequency has a negative and significant effect, whereas kurtosis has a positive effect but only significant in the “HFI IV” specification. In this exercise, however, the number of products, hence the sample size, is much more limited than in other regressions.

5.4 Including Product-level “fixed effects”

We run regressions in which higher-level sectoral “fixed-effects” are included, to investigate whether the relation between CIR^P and the pricing-moments still holds within a more disaggregated level of sectoral breakdown. This exercise informs us on the sources of product variability that help identify the relation between CIR^P and the cross sectional moments: broad sector differences versus within-sector variability. For that, we add sectoral fixed effects at the 2-digit level for both CPI and PPI products (there are 38 such “intermediate aggregation level” sectors for the CPI, and 24 in the case of the PPI). Results are reported in [Table M](#) in [Appendix G](#). For PPI products, adding sectoral dummy variables weakens the significance of the estimated parameters, but the results are qualitatively and -for most of coefficients- quantitatively the same as in our baseline regressions. The results are again consistent with the theoretical predictions: for CPI the $Kurt/Freq$ ratio is positive and significant in all specifications. In the specifications ‘Cholesky with long run restriction’ and ‘HFI with long run restriction’, both kurtosis and frequency have a significant effect with the expected sign. We note however that the addition of sectoral fixed effects significantly reduces the sources of cross-sectional variations, lowering the precision of the estimates.

5.5 2-year German Bond Rate

In this robustness, we alter the policy rate used in the FAVAR estimation where the shock is identified using an external instrument approach. The main motivation is that over the last part of our sample the short-run policy rate was arguably constrained by the proximity of the lower

bound for interested rates, and the ECB engaged in unconventional monetary policies intended to influence long term interest rates.³⁰ We use the 2-year German bond rate, a relevant risk-free long term interest rate, instead of the 3-month Euribor rate.³¹ Results using this specification of the FAVAR model are reported in [Appendix D](#).³²

Results relating sectoral CIR from this FAVAR model and the sufficient statistic, for PPI products, are in line with the baseline (see [Table N](#) in [Appendix G](#)). The coefficient associated with the *Kurt/Freq* ratio is positive and significantly different from 0 (and we cannot even reject the coefficient being equal to the predicted value 0.16 and the intercept being equal to $-T$ for both horizons). In the unconstrained specification, the frequency and kurtosis have significant effect and we cannot reject the equality of the absolute values of these coefficients. For CPI products, only the frequency has a significant coefficient in the case with long-un restriction, whereas the ratio kurtosis over frequency has no significant effect. In the case without imposing the long run restriction, the ratio has the positive expected sign.

5.6 Removing Products with Sizeable Drifts in Price Levels

The theoretical predictions of the model are derived under the assumption of small inflation. While this assumption is clearly fulfilled for the aggregate inflation rate in France on our sample period, a concern is that for some specific sectors it could not be the case. [Table 1](#) provides some statistics on the average product-specific inflation rates in absolute values. Product-level inflation rates (taken in absolute value) are typically small as well: average and median inflation rates are about 1.5% per year whereas the third quartiles of inflation distribution are around 2%. In this last robustness exercise, we remove all products for which we observe a non-small average inflation rate (in absolute values). In practice, we define small inflation rates as products with an average annual inflation lower than 5% in absolute values.³³ For PPI products, only two products are removed, whereas

³⁰Note however that the policy rate was negative from 2014, and statements by the ECB indicate that the lower bound was not actually reached afterward.

³¹[Jarocinski and Karadi \(2020\)](#) use the 1 year and 2 year German bond as a policy variable in their analysis of ECB monetary policy.

³²See in particular [Figure L](#) and [Figure M](#) in [Appendix D](#) for aggregate and sector IRF and [Figure N](#) for scatter plots relating CIR and price change moments.

³³[Gagnon \(2009\)](#), [Nakamura et al. \(2018\)](#) or [Alvarez et al. \(2019\)](#) for evidence on price rigidity in higher inflation rates, in Mexico, US and Argentina, they tend to show that when inflation is below 5%, patterns of price rigidity

for CPI 9 products are removed. For both PPI and CPI, results are very consistent with the ones obtained in the baseline regressions even if coefficients are less significant in some specifications.³⁴

6 Conclusion

This paper has presented results of an empirical test of the predictions of the sufficient-statistic proposition in a low-inflation economy, which relates the real effects of monetary policy shock to the ratio of kurtosis over the frequency of price changes.

To accomplish this goal we first estimated sectoral responses to a monetary shock for about 120 manufacturing goods and 220 consumer products in France, using a Factor Augmented VAR. Our monetary shock was identified using several identification schemes and long-term restrictions to test the robustness of our findings. From this estimation, we have calculated for each product the cumulative impulse response of prices over long horizons. Then, using micro data underlying French CPI and PPI, we measured cross sectional moments of price changes corresponding to these sectors. Finally, we estimated regressions relating CIR of prices to the ratio $Kurt/Freq$ to investigate empirically the predictions of the sufficient-statistic proposition.

For PPI products, the empirical results are fully in line with theory: the sign of the regression coefficients, and even the amplitude of the coefficients, correspond to the ones predicted by the theory. This result holds for a variety of FAVAR specifications and robustness tests (taking into account for measurement issues for instance). Moreover, the coefficients associated with both the frequency and the kurtosis of price changes are statistically significant, have the expected sign, and we cannot reject that the size of the coefficient associated with frequency is the same in absolute value as the one associated with kurtosis, as theory predicts. Moreover, “placebo” tests show that moments not suggested by the theory are not correlated with the CIR of prices. For CPI products the results are mixed and are less robust than for PPI: the ratio $Kurt/Freq$ has a positive and significant sign in several specifications, but this is less systematic than for PPI products. Similarly, we find that both the frequency and the kurtosis, when entering separately in the regressions, have

(in particular, frequency of price changes) are rather unchanged.

³⁴Using a threshold at 4% for defining ‘small’ vs ‘large’ inflation rates leads to similar results.

the expected sign and size in most, but not all, specifications. One candidate explanation for the difference between the CPI and the PPI results might come from the fact that (even if we have attempted to remove observations affected by sales) the sufficient statistic result holds in a setting where the pricing strategy of firms features no seasonal sales or price plans, both of which are empirically more prevalent for CPI than for PPI.

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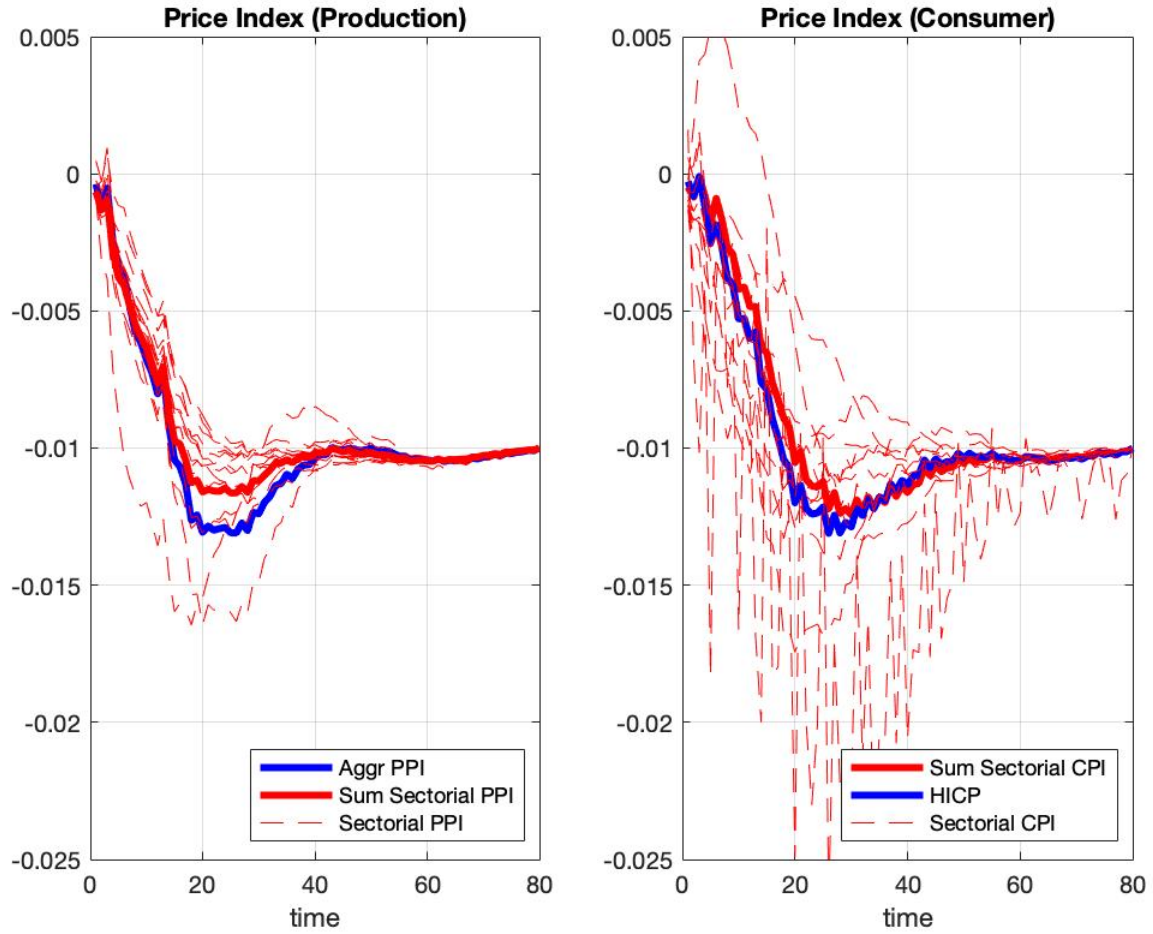
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Figures and Tables

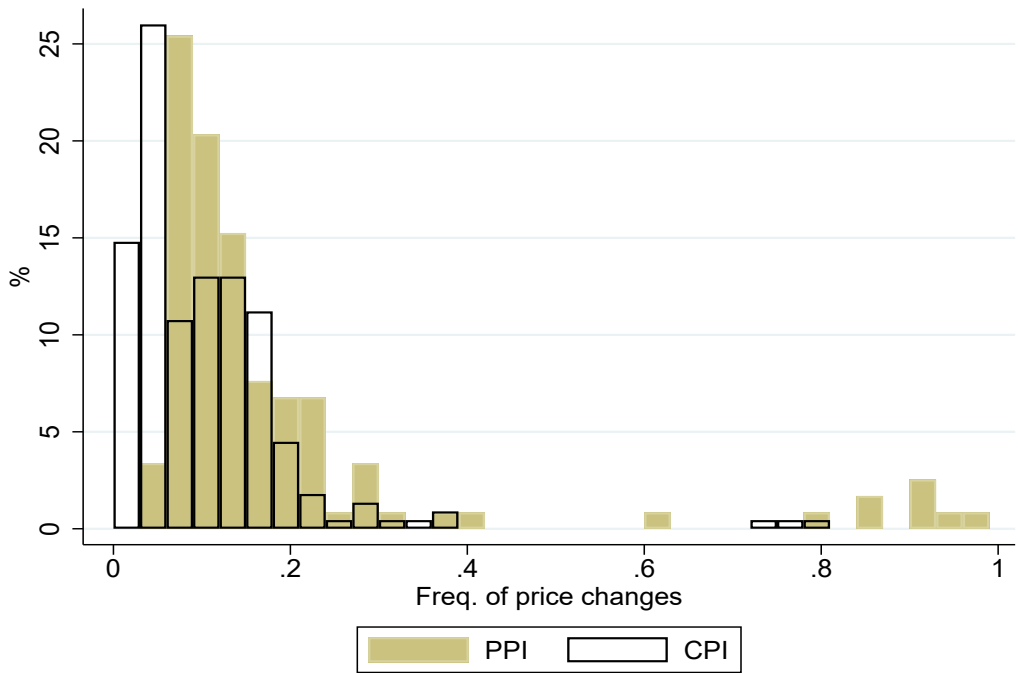
Figure 1: Sectoral Responses of PPI and CPI to a Contractionary Monetary Shock



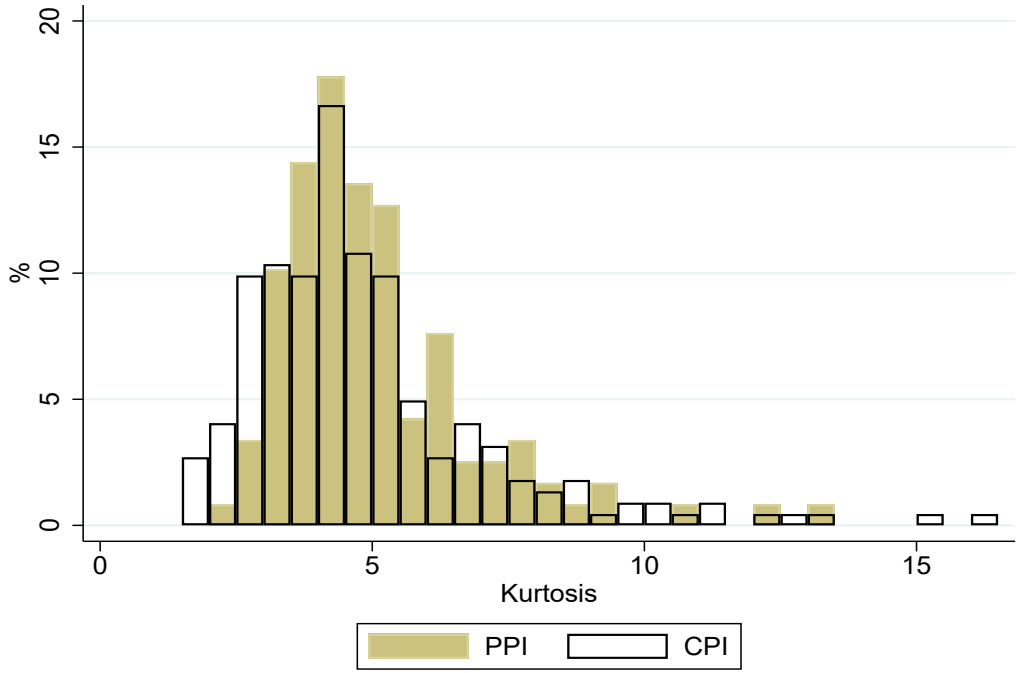
Note: y-axis: log points in deviation from the "steady state". Left panel sectoral IRFs of PPI, right panel sectoral IRFs of CPI. In both panel: blue line IRF of aggregate time series, dashed red lines sectoral IRFs, thick red line arithmetic average of sectoral IRFs.

Figure 2: Cross-sector Distribution of Frequency and Kurtosis of Price Changes (CPI-PPI)

(a) Frequency

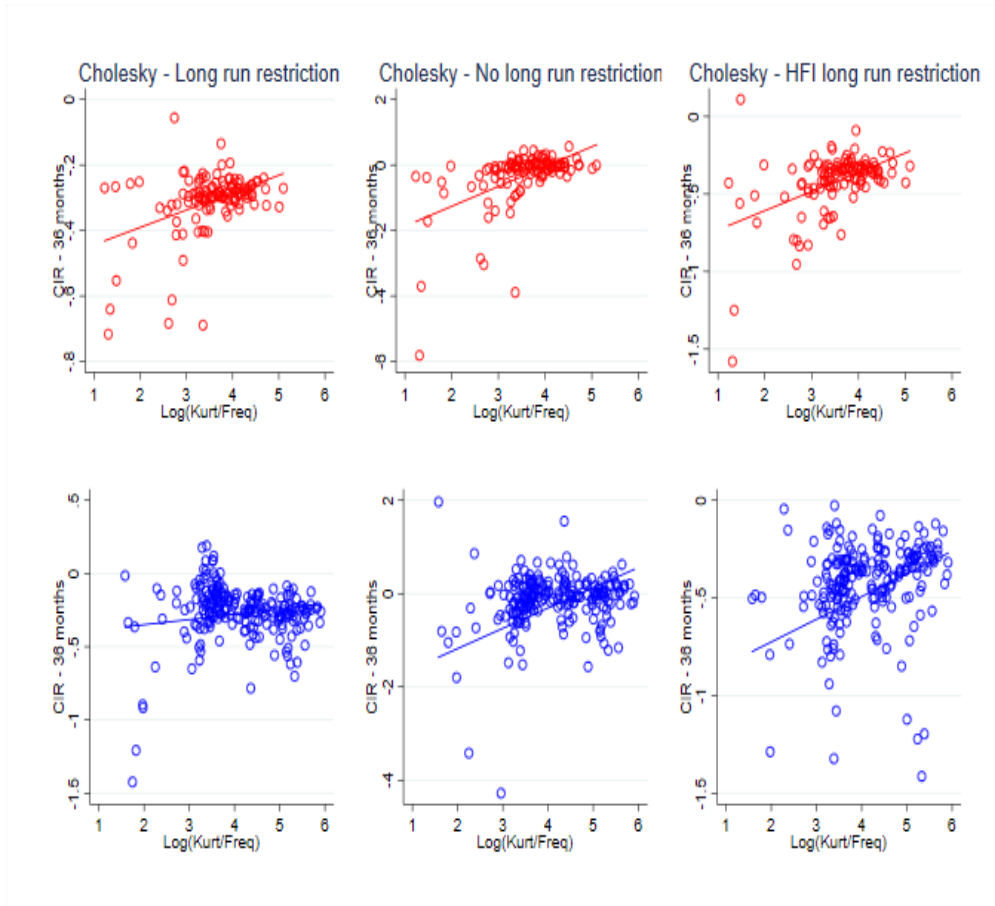


(b) Kurtosis



Note: histograms report the distribution of frequency and kurtosis separately for 118 PPI products and 227 CPI products.

Figure 3: Correlation CIR^P - Log ratio $\frac{Kurt}{Freq}$



Note: the figure plots for each of the three FAVAR specifications the product-specific CIR (at the horizon 36 months) and the log of the ratio kurtosis over frequency of price changes. The top panel (red dots) reports results for PPI products whereas the bottom panel (blue dots) reports results for CPI products.

Table 1: Micro Moments of Price Adjustments: Descriptive Statistics

	Nb products	Mean	Q1	Q2	Q3	SD
Panel A: Frequency of price changes						
CPI	223	0.106	0.039	0.088	0.143	0.104
PPI	118	0.190	0.086	0.123	0.185	0.208
Panel B: Kurtosis of non-zero price changes - with robustness						
CPI - baseline	223	5.039	3.355	4.434	5.652	2.952
PPI - baseline	118	5.068	3.927	4.615	5.857	1.851
CPI - outlier $ \Delta p < 0.5\%$	223	4.616	3.559	4.281	5.166	1.738
PPI - outlier $ \Delta p < 0.5\%$	118	4.777	3.183	4.220	5.411	2.821
CPI - outlier $ \Delta p > 35\%$	223	6.273	3.880	5.471	7.207	4.316
PPI - outlier $ \Delta p > 25\%$	118	7.805	5.532	6.956	9.042	3.952
CPI - hetero (S=5)	223	3.424	2.227	3.194	3.834	2.013
PPI - hetero (S=5)	118	3.917	2.638	3.435	4.497	2.036
Panel C: Mean of non-zero price changes (percent)						
CPI	223	1.219	0.294	0.947	2.074	2.124
PPI	118	0.793	0.204	0.722	1.405	0.906
Panel D: Standard deviation of non-zero price changes (percent)						
CPI	223	7.587	6.018	7.298	9.251	2.307
PPI	118	4.149	3.606	4.134	4.674	0.872
Panel E: Skewness of non-zero price changes						
CPI	223	-0.261	-0.419	-0.250	-0.098	0.367
PPI	118	-0.274	-0.559	-0.275	0.028	0.444
Panel F: Average inflation (in percent, absolute values)						
CPI	223	1.883	0.663	1.531	2.368	2.123
PPI	118	1.556	0.903	1.327	1.984	1.111

Note: Calculations on CPI micro data are made over the period 1994-2019 (30 million of monthly price quotes). Prices of rents, cars, fresh food products, electricity and clothing goods are non-available or excluded. Price changes due to sales and promotions are excluded (using the INSEE flag). VAT change and euro-cash changeover periods are excluded as well. Calculation on PPI data are made over the period 1994-2005. We here report some descriptive statistics of the distribution of product-specific moments of price rigidity for PPI and CPI products (statistics are unweighted). 'Frequency' reports the ratio between the number of price changes and the total number of prices. 'Mean', 'Standard deviation', 'Skewness' and 'Kurtosis' are calculated on the distribution of non-zero log price changes, expressed in percentages. In our baseline calculations, we have excluded all price changes below than 0.1% in absolute values and larger than 25% in absolute values for CPI price changes and 15% for PPI price changes. Panel F provides statistics on the average product-specific inflation in absolute values over the period 2005-2019.

Table 2: Baseline OLS Regression Results : “Constrained” Specification

Identification Long-run Restriction	Cholesky Yes		Cholesky No		High-Frequency IV Yes	
	24 months	36 months	24 months	36 months	24 months	36 months
<i>PANEL A: PRODUCER PRICES</i>						
Kurt/Freq	0.0669** (0.0326)	0.0974*** (0.0355)	0.690*** (0.220)	1.124*** (0.341)	0.192*** (0.0614)	0.242*** (0.0801)
Constant	-20.57*** (2.130)	-35.16*** (2.199)	-48.02*** (13.43)	-81.88*** (20.51)	-34.27*** (3.638)	-52.21*** (4.799)
Observations	118	118	118	118	118	118
R^2	0.041	0.082	0.117	0.135	0.131	0.118
<i>PANEL B: CONSUMER PRICES</i>						
Kurt/Freq	-0.0170 (0.0165)	-0.00245 (0.0199)	0.115* (0.0658)	0.233** (0.105)	0.0495** (0.0242)	0.0720** (0.0315)
Constant	-11.64*** (2.809)	-27.36*** (3.285)	-21.20* (10.81)	-47.72*** (17.13)	-34.43*** (3.434)	-54.70*** (4.419)
Observations	223	223	223	223	223	223
R^2	0.004	0.000	0.014	0.023	0.019	0.024

Note: this table reports results of OLS regressions (equation 10) where the endogenous variable is the product-specific CIR $CIR_T^{P_j}$ (expressed in %) and RHS variable is the ratio $Kurt/freq$. Each observation corresponds to a disaggregate CPI or PPI product. For CPI, the level of disaggregation is 5 digit-level of the ECOICOP classification (ie. ‘01.1.1.1’) whereas for PPI, the product level is the 4-digit level of the NACE rev2 classification of sectors. PPI covers the manufacturing sectors whereas CPI covers about 60% of the whole French CPI (main products excluded are rents, cars, utilities like electricity). Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 3: Regression Results - “Unconstrained” Specification

Identification Long-run Restriction	Cholesky Yes		Cholesky No		High-Frequency IV Yes	
	24 months	36 months	24 months	36 months	24 months	36 months
<i>PANEL A: PRODUCER PRICES</i>						
$Freq/\bar{F}$	-2.501* (1.279)	-3.153** (1.314)	-23.65*** (8.897)	-37.41*** (13.96)	-6.004** (2.776)	-7.239* (3.761)
$Kurt/\bar{K}$	3.663* (1.897)	4.665** (1.995)	28.83** (11.66)	45.17** (17.59)	6.662** (3.100)	7.922* (4.010)
Constant	-18.82*** (2.208)	-32.42*** (2.166)	-23.13* (12.73)	-40.64** (18.61)	-26.56*** (3.011)	-42.36*** (3.960)
Observations	118	118	118	118	118	118
R^2	0.106	0.161	0.240	0.259	0.217	0.179
<i>PANEL B: CONSUMER PRICES</i>						
$Freq/\bar{F}$	-4.920* (2.809)	-8.540** (3.331)	-54.17*** (13.77)	-91.14*** (21.32)	-16.36*** (3.894)	-21.30*** (4.812)
$Kurt/\bar{K}$	4.359* (2.328)	5.657** (2.594)	6.648 (4.523)	8.581 (7.023)	7.132*** (2.201)	9.175*** (2.806)
Constant	-12.61*** (3.684)	-24.70*** (4.267)	36.70*** (13.17)	55.84*** (20.40)	-20.74*** (4.907)	-36.08*** (6.274)
Observations	223	223	223	223	223	223
R^2	0.065	0.132	0.477	0.529	0.350	0.342

Note: this table reports results of OLS regressions (equation 11) where the endogenous variable is the product-specific CIR $CIR_T^{P_j}$ (expressed in %) and RHS variables are the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$. Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table 4: Regression Results - Placebo Specification

Identification Long-run Restriction	Cholesky Yes		Cholesky No		High-Frequency IV Yes	
	24 months	36 months	24 months	36 months	24 months	36 months
<i>PANEL A: PRODUCER PRICES</i>						
Kurt/Freq	0.0849* (0.0477)	0.110** (0.0488)	0.715** (0.284)	1.130** (0.432)	0.168** (0.0750)	0.202** (0.0989)
Mean	-0.418 (0.905)	-0.479 (1.000)	-6.211 (5.311)	-9.930 (8.251)	-1.212 (1.432)	-1.408 (1.857)
Skewness	1.759 (3.434)	1.006 (3.100)	-5.014 (15.18)	-12.47 (21.06)	-4.613 (2.783)	-6.889* (4.109)
Standard dev.	-0.940 (1.016)	-0.749 (1.083)	-4.317 (7.037)	-5.494 (10.86)	0.219 (1.964)	0.726 (2.535)
Constant	-16.65*** (4.669)	-31.95*** (4.791)	-27.65 (26.68)	-54.88 (40.24)	-34.44*** (7.221)	-54.26*** (9.552)
Observations	118	118	118	118	118	118
R^2	0.054	0.089	0.125	0.142	0.140	0.130
<i>PANEL B: CONSUMER PRICES</i>						
Kurt/Freq	-0.0529** (0.0206)	-0.0308 (0.0253)	0.143 (0.0902)	0.330** (0.144)	0.0585* (0.0348)	0.0923** (0.0455)
Mean	1.636** (0.755)	1.498* (0.826)	1.416 (1.902)	0.441 (3.081)	0.550 (0.864)	0.380 (1.159)
Skewness	2.109 (3.699)	5.033 (4.102)	16.84 (12.65)	30.51 (20.65)	11.30*** (4.227)	15.52*** (5.603)
Standard dev.	-1.992** (0.860)	-1.732* (1.044)	3.426 (3.666)	8.226 (5.727)	0.332 (1.166)	0.795 (1.486)
Constant	5.271 (7.708)	-12.17 (9.376)	-47.09 (37.03)	-111.4* (58.47)	-35.49*** (11.33)	-58.97*** (14.45)
Observations	223	223	223	223	223	223
R^2	0.067	0.038	0.027	0.050	0.036	0.045

Note: this table reports results of OLS regressions (equation 10) where the endogenous variable is the product-specific CIR $CIR_T^{P_j}$ (expressed in %) and RHS variables include the product-specific ratio $Kurt/freq$ but also three other moments of the product-specific price change distribution: the average price change $Mean$, the skewness of price changes $Skewness$, and the standard deviation of price changes $StandardDev.$. Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Empirical Investigation of a Sufficient Statistic for Monetary Policy Shocks

Fernando Alvarez, Herve le Bihan, Andrea Ferrara, Erwan Gautier, Francesco Lippi

A Analytics of the generalized random menu cost model

We describe the price setting problem for a firm in steady state using the random menu cost model of Caballero and Engel (1999, 2007), which covers a vast class of sticky-price models.

The firm's problem. The firm maximizes the expected discounted value of profits and chooses the optimal times and size of price adjustment as a function of its state x , as encoded in the value function $v(x)$ defined in the appendix. A second order approximation of the profit function at the optimal price gives a quadratic period return Bx^2 , where the constant term B relates to the curvature of the profit function.³⁵ The firm's value function solves the following HJB equation

$$r v(x) = \min \left\{ Bx^2 + v'(x)\mu + \frac{\sigma^2}{2}v''(x) + \kappa \int_0^\Psi \min \left\{ \psi + \min_z v(z) - v(x), 0 \right\} dG(\psi), r \left(\Psi + \min_z v(z) \right) \right\}$$

The first argument in the curly bracket represents the continuation value with a flow cost Bx^2 and the usual expected change in the value function, which includes the possibility to adjust if the firm draws a sufficiently small menu cost. For a firm with a gap x that draws a menu cost ψ the net cost effect of adjusting is $\psi + \min_z v(z) - v(x)$, which is optimally chosen by the firm only when it is smaller than zero. The second argument in the curly bracket represents the firms' option to reset the gap at any moment by paying the fixed cost Ψ . The value function features the smooth pasting conditions: $v'(\underline{X}) = v'(\bar{X}) = v'(x^*) = 0$ where $x^* = \arg \min_{\tilde{x}} v(\tilde{x})$ is the optimal price gap chosen by a firm that adjusts and \underline{X} and \bar{X} delimit the state space so that $x \in [\underline{X}, \bar{X}]$, and value matching conditions $v(\underline{X}) = v(\bar{X}) = v(x^*) + \Psi$.

Mapping the model to observables. The density f solves the following Kolmogorov forward equation:

$$f(x)\Lambda(x) = -\mu f'(x) + \frac{\sigma^2}{2} f''(x) \text{ for all } x \in (\underline{X}, \bar{X}), x \neq x^* \quad (16)$$

with boundary conditions: $\lim_{x \downarrow x^*} f(x) = \lim_{x \uparrow x^*} f(x)$; $1 = \int_{\underline{X}}^{\bar{X}} f(x) dx$ and $\lim_{x \rightarrow \bar{X}} f(x) = \lim_{x \rightarrow \underline{X}} f(x) = 0$.

Notice how the cross sectional distribution of price gaps $f(x)$ is fully determined by the generalized hazard function: Also, note that density is zero at the boundaries of the domain.³⁶

³⁵See Appendix B in Alvarez and Lippi (2014) for a detailed derivation of the approximation.

³⁶This is an implication of \underline{X} and \bar{X} being exit points, for every model where μ/σ^2 is finite. In the case where the domain of x is unbounded, then the zero density is a requirement for integrability.

The distribution of price changes has density q and two mass points at the boundaries of the inaction region:

$$\begin{aligned} q(x^* - x) &= \frac{f(x)\Lambda(x)}{N} \text{ for all } x \in [\underline{X}, \bar{X}] \\ dQ(x^* - \bar{X}) &= -\frac{\sigma^2 f'(\bar{X})}{2} \frac{1}{N} \quad , \quad dQ(x^* - \underline{X}) = +\frac{\sigma^2 f'(\underline{X})}{2} \frac{1}{N} \end{aligned} \quad (17)$$

where the number of price changes satisfies:

$$N(\mu) = \int_{\underline{X}}^{\bar{X}} \Lambda(x)f(x)dx + \frac{\sigma^2}{2} f'(\underline{X}) - \frac{\sigma^2}{2} f'(\bar{X}) \quad (18)$$

and the notation emphasises the dependence of the frequency on the rate of steady state inflation μ .

Computation of CIR for the zero drift case ($\mu = 0$). The contribution to the cumulative impulse response of a firm with price gap x is

$$m(x) = -\mathbb{E} \left[\int_0^\tau e^{-\Lambda(x)t} x(t) dt \mid x(0) = x \right] \quad (19)$$

where τ is the stopping time defined as the first time the price gap hits the barriers $\pm\bar{X}$. In words, $m(x)$ is the expected (cumulative) price gap of a firm that starts with a gap x .³⁷

The expectation in the right hand side of [equation \(19\)](#) is with respect to the process for x , a jump-diffusion with jump intensity $\Lambda(x)$, diffusion variance σ^2 , and zero drift. The function $m : [-X, X] \rightarrow \mathbb{R}$ is once continuously differentiable, antisymmetric around $x = 0$, and satisfies:

$$m(x)\Lambda(x) = -x + \frac{\sigma^2}{2} m''(x) \text{ for all } x \text{ at which } \Lambda \text{ is continuous} \quad (20)$$

$$0 = m(X) \text{ if } X < \infty \text{ and } \lim_{x \rightarrow \infty} \frac{|m(x)|}{x} \leq \frac{1}{\inf_y \Lambda(y)} \text{ if } X = \infty \quad . \quad (21)$$

Now we can define the cumulative impulse response to a monetary shock of size δ as

$$CIR(\delta) = \int_{-\bar{X}}^{\bar{X}} m(x)f(x + \delta)dx \quad . \quad (22)$$

Aggregation across heterogenous firms. We briefly discuss how the above results can be applied to economies composed of heterogenous firms. Assume that there are S groups of firms with different parameters, each with an expenditure weight $e_s > 0$, N_s price changes per unit of time, and a distribution of price changes with kurtosis Kur_s . In this case, after repeating the arguments above for each group and aggregating, we obtain that the area under the IRF of *aggregate* output for a small monetary shock δ is

$$CIR(\delta) = \frac{\delta}{6\epsilon} \sum_{s \in S} e_s \frac{Kur_s}{N_s} + o(\delta^2) = \frac{\delta}{6\epsilon} D \sum_{s \in S} d_s Kur_s + o(\delta^2) \quad (23)$$

³⁷The definition above uses the steady state decision rule $\Lambda(x)$, thus ignoring the general equilibrium feedback effect of the shock on the firm's decision. In Proposition 7 of [Alvarez and Lippi \(2014\)](#) it is shown that, given a combination of the general equilibrium setup in [Golosov and Lucas \(2007\)](#) and the lack of the strategic complementarities, these general equilibrium effects are of second order. In addition, we use the fact that after the first price change the expected contribution to output of each firm is zero, since positive and negative output contributions are equally likely, so $m(0) = 0$. This allows us to characterize the propagation of the monetary shocks without tracking the time evolution of the whole price gap distribution.

where D is the expenditure-weighted average duration of prices $D \equiv \sum_{s \in \mathcal{S}} \frac{e_s}{N_s}$, and $d_s \equiv \frac{e_s}{N_s D}$ are weights that take into account both relative expenditures and durations. When all groups have the same durations, then $d_s = e_s$ and the *CIR* is proportional to the average of the kurtosis of the sectors.³⁸ However, if groups are heterogenous in duration (or expenditures), then the kurtoses of the groups with longer duration (or higher expenditures) receive a higher weight in the computation of the *CIR*.³⁹

The proof of sufficient statistic result, equation (6). First note that the identity

$$N \cdot Var = \sigma^2 \tag{24}$$

holds in the model. Let $x(0) = 0$. Consider the process $z(t) \equiv x(t)^2 - \sigma^2 t$ for $t \geq 0$. Using Ito's lemma we can verify that the drift of x^2 is σ^2 , and hence $z(t)$ is a Martingale. Let τ be a stopping time, i.e. an instant where a price adjustment occurs (anywhere in the state space, including the boundaries), so that x is reset at $x(0) = 0$. By the optional sampling theorem $z(\tau)$, the process stopped at τ , is also a martingale. Then $\mathbb{E} [z(\tau) \mid x(0)] = \mathbb{E} [x(\tau)^2 \mid x(0)] - \sigma^2 \mathbb{E} [\tau \mid x(0)] = x(0) = 0$. Since $N = 1/\mathbb{E} [\tau \mid x(0)]$ and $Var = \mathbb{E} [x(\tau)^2 \mid x(0)]$ we get the identity in equation (24).

For simplicity, we focus next on the case with unbounded support $\bar{X} \rightarrow \infty$ (the logic for the case with bounded support is identical but the equations are slightly more cumbersome). Using the definition of the density of price changes in equation (17) we can rewrite the identity as

$$\int_{-\infty}^{\infty} x^2 \Lambda(x) f(x) dx = \sigma^2 \tag{25}$$

it is then straightforward to write the formula for kurtosis over $6N$ as:

$$\frac{Kurt}{6N} = \frac{\int_{-\infty}^{\infty} x^4 \Lambda(x) f(x) dx}{6 \left(\int_{-\infty}^{\infty} x^2 \Lambda(x) f(x) dx \right)^2} = \frac{\int_{-\infty}^{\infty} x^4 \Lambda(x) f(x) dx}{6\sigma^4}$$

where the last passage uses equation (25). Using the Kolmogorov forward equation,

$$\int_{-\infty}^{\infty} x^4 \Lambda(x) f(x) dx = \frac{\sigma^2}{2} \int_{-\infty}^{\infty} x^4 f''(x) dx$$

Integrating by parts twice gives

$$\int_{-\infty}^{\infty} x^4 \Lambda(x) f(x) dx = 6\sigma^2 \int_{-\infty}^{\infty} x^2 f(x) dx$$

This allows us to write

$$\frac{Kurt}{6N} = \frac{\int_{-\infty}^{\infty} x^2 f(x) dx}{\sigma^2} \tag{26}$$

³⁸The effect of heterogeneous N is well known for the Calvo model: due to Jensen's inequality D differs from the (reciprocal of) the average of N 's, see for example Carvalho (2006) and Nakamura and Steinsson (2010).

³⁹Suppose for instance that a fraction of firms have flexible prices (zero duration in our model, or infinitely many price changes per unit of time), as in Dotsey and Wolman (2020). The above formula implies that the group of the flexible price firms are excluded (zero duration yields a zero weight), and that the cumulative impulse response (CIR) is computed on the mass of firms with sticky prices. Notice that this is different from computing the CIR as the ratio of the cross-sectional average kurtosis and the average frequency. Since the latter is diverging because of the firms with flexible prices, the CIR computed this way would be zero, while obviously it is not.

Recall that we have a system of two equations:

$$\Lambda(x)f(x) = \frac{\sigma^2}{2}f''(x) \quad , \quad \Lambda(x)m(x) = \frac{\sigma^2}{2}m''(x) - x$$

Eliminate Λ to get:

$$\frac{\sigma^2}{2} \frac{m(x)f''(x)}{f(x)} = -x + \frac{\sigma^2}{2}m''(x)$$

Multiply both sides by $f(x)x$ and rearrange:

$$\frac{\sigma^2}{2}[m(x)f''(x) - m''(x)f(x)]x = -x^2f(x)$$

Integrate both sides from 0 to ∞ :

$$\frac{\sigma^2}{2} \int_0^{\infty} [m(x)f''(x) - m''(x)f(x)]x dx = - \int_0^{\infty} x^2f(x) dx$$

Perform integration by parts in the left-hand side using the fact that $[m(x)f'(x) - m'(x)f(x)]' = m(x)f''(x) - m''(x)f(x)$:

$$\begin{aligned} \frac{\sigma^2}{2} \int_0^{\infty} [m(x)f''(x) - m''(x)f(x)]x dx &= \frac{\sigma^2}{2} \left([m(x)f'(x) - m'(x)f(x)]x \Big|_0^{\infty} - \int_0^{\infty} [m(x)f'(x) - m'(x)f(x)] dx \right) \\ &= -\sigma^2 \int_0^{\infty} m(x)f'(x) dx \end{aligned}$$

where the last equality uses integration by parts again. We used $\mathbb{E}[m(x)] < \infty$ and $m(\cdot)$ being almost linear at infinity to justify setting $f'(x)m(x)x$ and $f(x)m'(x)x$ at infinity to 0. Hence, we have

$$\sigma^2 \int_0^{\infty} m(x)f'(x) dx = \int_0^{\infty} x^2f(x) dx$$

Plugging this result in [equation \(26\)](#) we have

$$\frac{Kurt}{6N} = \int_{-\infty}^{\infty} m(x)f'(x) dx$$

It appears from the CIR definition in [equation \(22\)](#) that the right hand side is just the first derivative of the CIR with respect to δ , evaluated at $\delta = 0$, or $CIR'(0)$. This completes the proof. \square

B FAVAR estimation

The Factor Augmented Vector Autoregression (FAVAR) was originally developed by [Bernanke, Boivin, and Elias](#) (2005) and by [Boivin, Giannoni, and Mihov](#) (2009). [Stock and Watson](#) (2016) provide also a clear explanation of the model.

Let Y_t be a vector of observable economic variables with dimension $M \times 1$, $M \geq 1$, and let \tilde{F}_t be a vector of unobserved factors with dimension $K \times 1$, $K \geq 1$. Assume that the dynamics of the economy is driven by (Y_t', \tilde{F}_t') which follows the transition equation:

$$\begin{bmatrix} \tilde{F}_t \\ Y_t \end{bmatrix} = \Phi(L) \begin{bmatrix} \tilde{F}_{t-1} \\ Y_{t-1} \end{bmatrix} + v_t \quad (27)$$

where $\Phi(L)$ is a lag polynomial of finite order and v_t is an error term with zero mean and covariance matrix Q . While [equation \(27\)](#) looks like a VAR, recall that F_t is unobserved and, thus, we cannot directly estimate [equation \(27\)](#). However, the factors \tilde{F}_t are interpreted as representing forces that potentially affect many economic variables from which we can estimate the factors. Indeed, assume that a large number of time series X_t , called informational time series, are related to the observed variables Y_t and to the unobservable factors \tilde{F}_t by the following equation:

$$X_t = \Lambda F_t + e_t \quad (28)$$

where $F_t \equiv [\tilde{F}_t' Y_t']'$ and e_t is a vector $N \times 1$ of error terms with zero mean⁴⁰. Notice that the number of informational time series, N , must be large which means N is much greater respect to the number of variables that drives the economy (F_t and Y_t), i.e. $N > K + M$, and potentially N can be bigger than the time period under consideration, T . Moreover, notice that F_t can always capture arbitrary lags of fundamental factors, thus it is not restrictive to assume that X_t depends only on the current values of the factors⁴¹.

Under the above assumptions, it is possible to estimate the model, using a two-step approach⁴²: in the first step, the common factors are estimated extracting the first K principal components, $\hat{C}^{(0)}$, from the information variables, X_t . Indeed, as shown by [Stock and Watson](#) (2002), for N large enough and if the number of principal components used are at least as the true number of factors, the principal components of X_t span the space generated by the factors \tilde{F} and the observable variables Y_t ; thus, the principal components represent independent but arbitrary linear combinations of \tilde{F}_t and Y_t . However, we want that these combinations do not depend on Y_t and that they are only independent combinations of the factors. For this reason, the factors are estimated as follow. Regress X_t on $\hat{C}^{(0)}$ and Y_t to obtain $\hat{B}_r^{(0)}$, the coefficient of Y_t . After compute $\tilde{X}_t^{(0)} = X_t - \hat{B}_r^{(0)} Y_t$ and estimate $\hat{C}^{(1)}$ as the first K principal components of $\tilde{X}_t^{(0)}$. Iterate until convergence of $\hat{B}_r^{(i)}$ to obtain the desired estimated factors, \hat{F}_t . The second step consists in estimating [equation \(27\)](#) as a structural VAR⁴³, replacing F_t with $\hat{F}_t \equiv [\hat{F}_t' Y_t']'$. Indeed, we can rewrite [equation \(27\)](#) as

$$\hat{F}_t = \Phi(L) \hat{F}_t - 1 + v_t \quad (29)$$

where $\hat{F}_t^+ \equiv [\hat{F}_t' Y_t']'$. Assuming $v_t = H\epsilon_t$, it is clear that [equation \(29\)](#) can be treated as a structural VAR.

We are left with only one open question: how is it possible to estimate the IRFs of X_t ? Consider again

⁴⁰If factors are estimated using a principal components analysis, errors can display a small amount of cross-correlation that must vanish as N goes to infinity. See [Stock and Watson](#) (2002) for a detailed discussion.

⁴¹For this reason [Stock and Watson](#) (1999) refer to [equation \(28\)](#) as a dynamic factor model.

⁴²The model can be estimated also using a single-step Bayesian likelihood approach.

⁴³In our application, we estimate the structural VAR using a Cholesky decomposition. However, any other approach can be used.

equation (29) and assume that the MA representation exists. Denoting the MA coefficient with $\Psi(L)$, we obtain

$$\hat{F}_t = \Psi(L)H\epsilon_t \quad (30)$$

Moreover, using \hat{F}_t instead of F_t in equation (28) and replacing in this equation equation (30), we get

$$X_t = \Lambda\Psi(L)^{-1}H\epsilon_t + e_t \quad (31)$$

Equation (31) links the information variables, X_t , to the shocks and provides the theoretical framework to retrieve the IRFs of X_t . However, in practice, the IRFs of X_t are not estimated using the MA representation and, thus, equation (31). Indeed, let $\widehat{IRF}(A)$ be the estimated IRFs of the time series A_t to a given shock. The IRFs of X_t is calculated as

$$\widehat{IRF}(X) = \hat{\beta} * \widehat{IRF}(\hat{F}) \quad (32)$$

where $\widehat{IRF}(\hat{F})$ is the VAR estimated IRF of \hat{F}_t and $\hat{\beta}$ is the estimated coefficient of the regression of X_t on \hat{F}_t .

C A Filter for the Euribor

For the purpose of our empirical test, we want the empirical monetary shock to capture a monetary policy shocks, characterized by a transient impact on inflation and output. We filter the 3-month Euribor so as to ensure this property, as with unfiltered data it is not fulfilled. One possible reason why it is not fulfilled (unlike in typical VAR) is because in our sample period, on the euro area, this variable is not stationary, as depicted in Figure A. We exploit two criteria to choose the value of the value of the HP filter, λ^{HP} . Both criterion are based on the behavior of the IRFs of PPI and CPI time series as λ^{HP} varies. We estimated the FAVAR model, for alternative values of filtered Euribor rates letting λ^{HP} vary from 6 to 10^5 .

One first criterion consists in considering the number of negative IRFs of PPI after two or three years, since our strong prior is that after a contractionary monetary shock, prices should decline as compared to the no-shock baseline. Thus, we are interested in estimating a FAVAR that is in line with this prediction. The top panel of Figure B shows our finding: the number of negative IRFs is maximized around the value of λ^{HP} of 500. The number of PPI sectors included in the analysis are 118 and for $\lambda^{HP} = 500$ we have around 100 sectors with negative IRF. Moreover, this curve is very flat around 500, as for any value of λ^{HP} in between 200 and 3000, more than the 60% of the sectors have a negative IRF after two years or three years.

As a second criterion to guide our choice of λ^{HP} , we consider the value of the *aggregate* IRF of PPI (or CPI) to the contractionary monetary shock as a function of λ^{HP} . For PPI, these responses are reported in the bottom panel of Figure B. This panel shows four different lines: we consider the response of an aggregate time series of PPI after 24 or 36 months; and that of the arithmetic average of the sectoral response of PPIs in addition to the response of the aggregate price index. In all cases, the minimum response is found λ^{HP} equal to a value around 1000. Developing the same criteria for CPI, we obtain very similar results, as depicted by Figure C. The only difference is that in the bottom panel for value of λ^{HP} bigger than 10^4 , the aggregate responses after two or three years become positive, implying that large values of λ^{HP} would defeat our purpose. Overall, based on the above results, we select as our benchmark to filter the 3-month Euribor an HP filter with $\lambda = 1000$.

Some additional figures: Filtering of interest rate, Identification of monetary policy shock

Figure A: 3-month Euribor: period 2005-2019

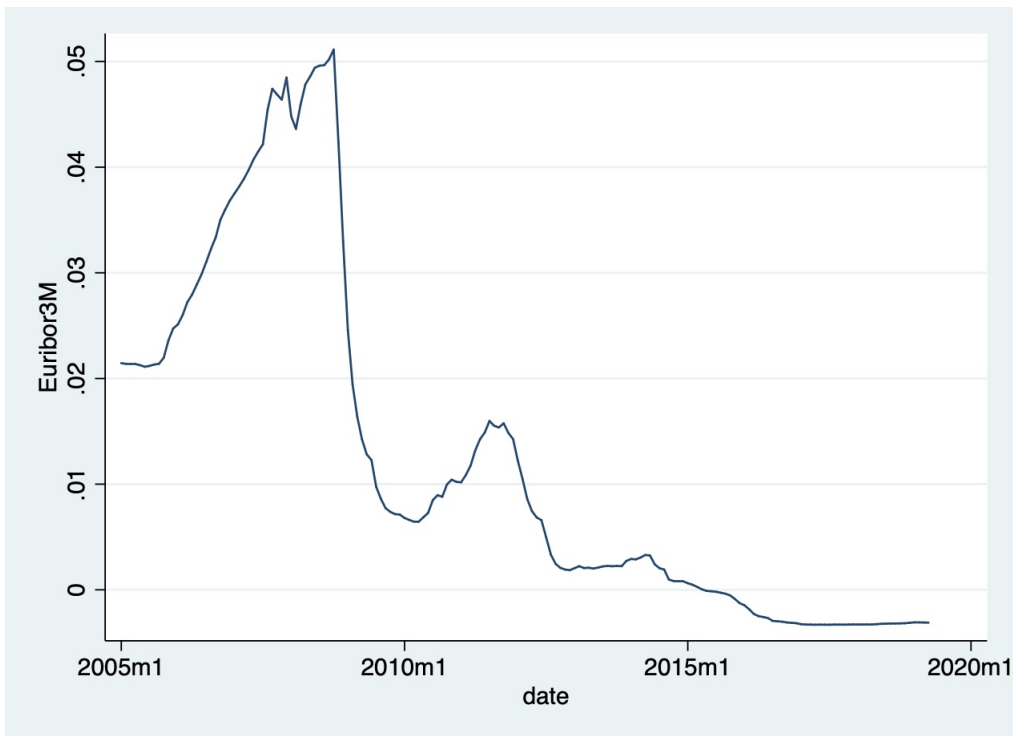
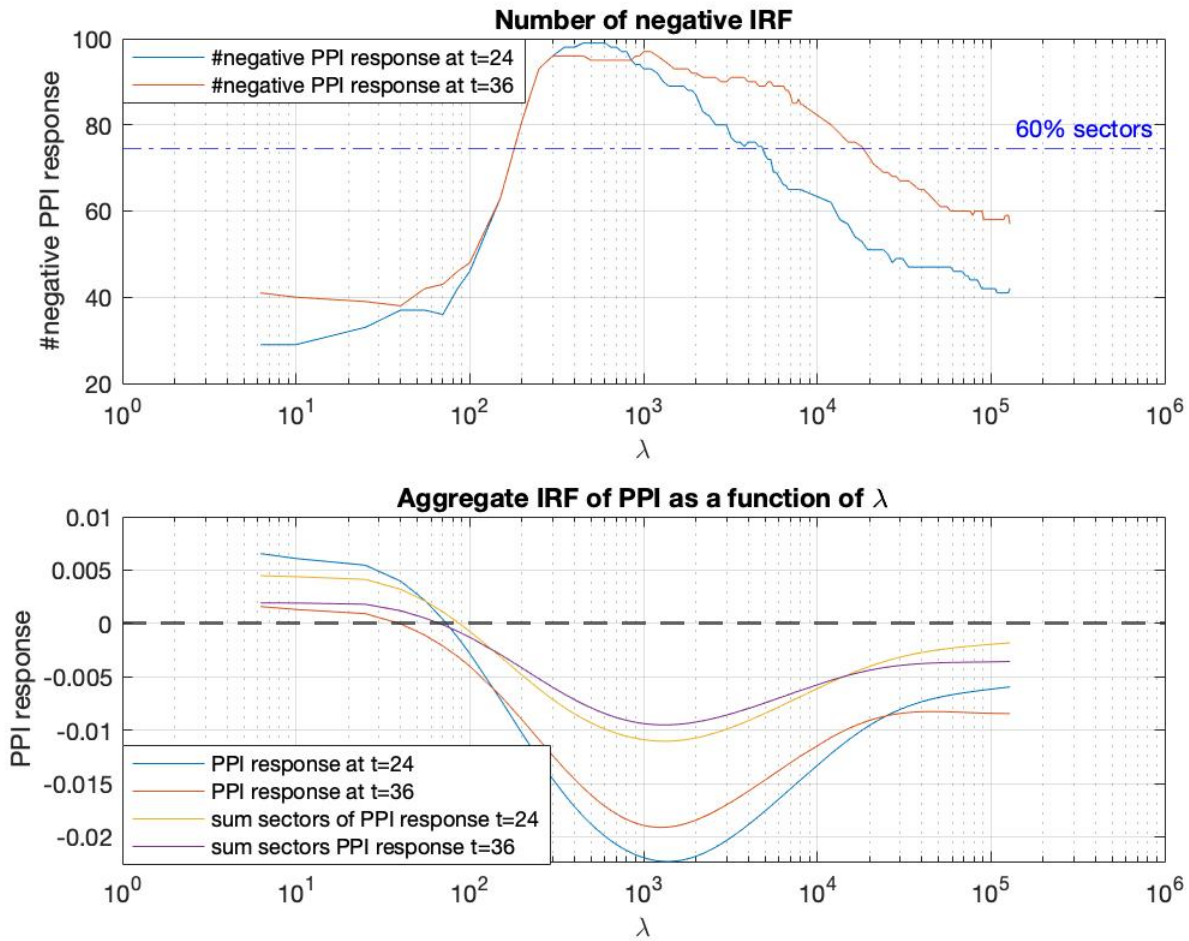
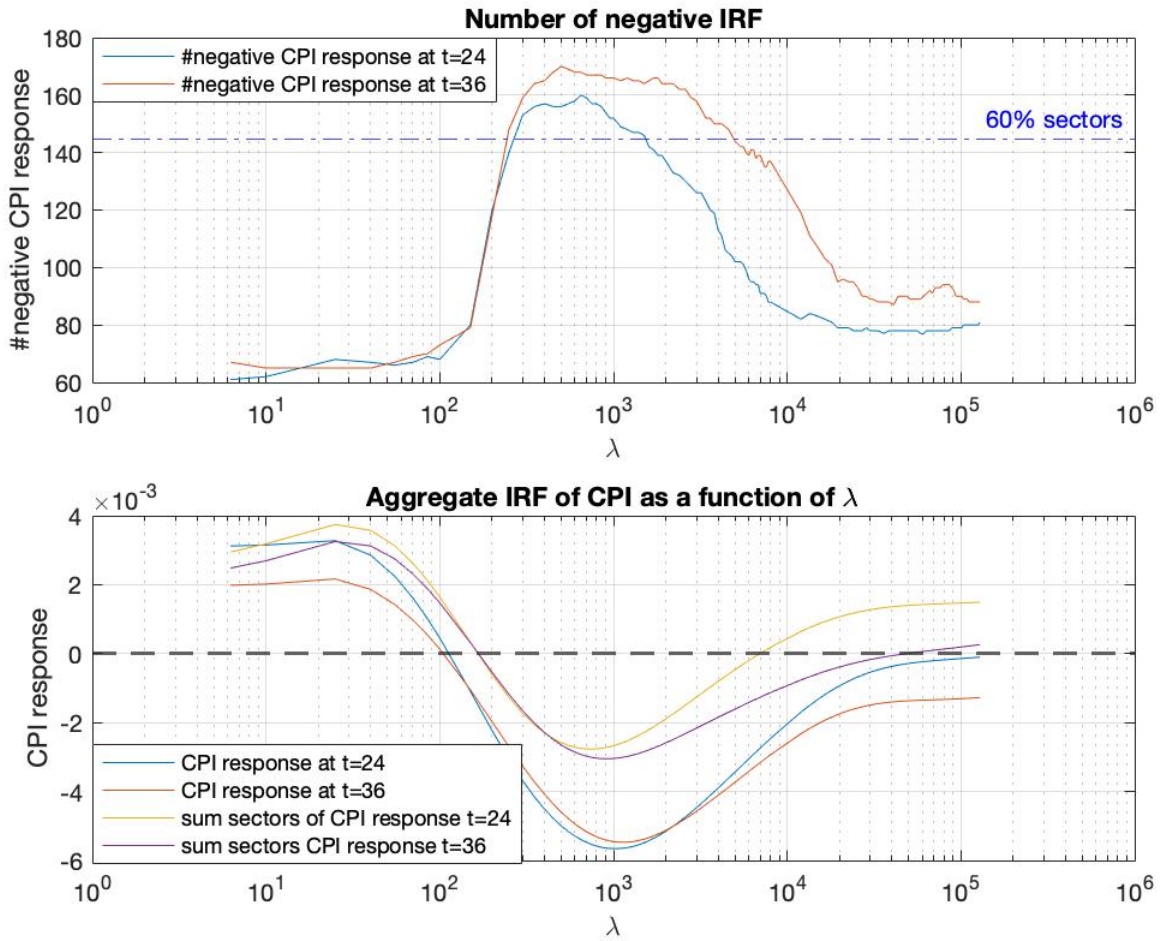


Figure B: Response of sectoral PPI as a function of λ



Top panel: number of PPI sectors with a negative IRF after two or three years to a contractionary monetary shock of 25 bp as a function of the HP filter parameter, λ^{HP} . Bottom panel: sectoral IRF of production prices to a contractionary monetary shock of 25 bp as a function of the HP filter parameter, λ^{HP} ; blue and red lines represent the IRF of the aggregate production price index after two and three years, respectively. Yellow and purple lines show the IRF of the arithmetic average of all the production price sectors after two and three years, respectively.

Figure C: Response of sectoral CPI as a function of λ

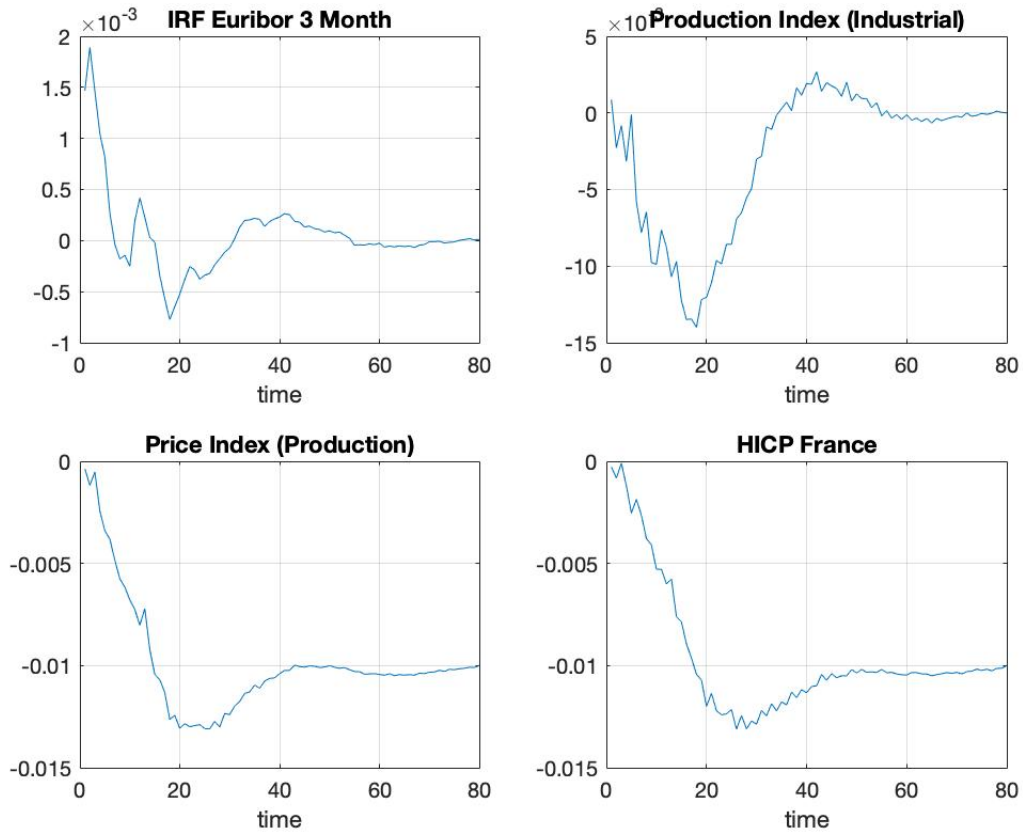


Top panel: number of CPI sectors with a negative IRF after two or three years to a contractionary monetary shock of 25 bp as a function of the HP filter parameter, λ^{HP} . Bottom panel: sectoral IRF of consumer prices to a contractionary monetary shock of 25 bp as a function of the HP filter parameter, λ^{HP} ; blue and red lines represent the IRF of the harmonized index of consumer prices after two and three years, respectively. Yellow and purple lines show the IRF of the arithmetic average of all the consumer price sectors after two and three years, respectively.

D Additional FAVAR Results

Cholesky - Long-run restriction

Figure D: Aggregate response to a contractionary monetary policy shock

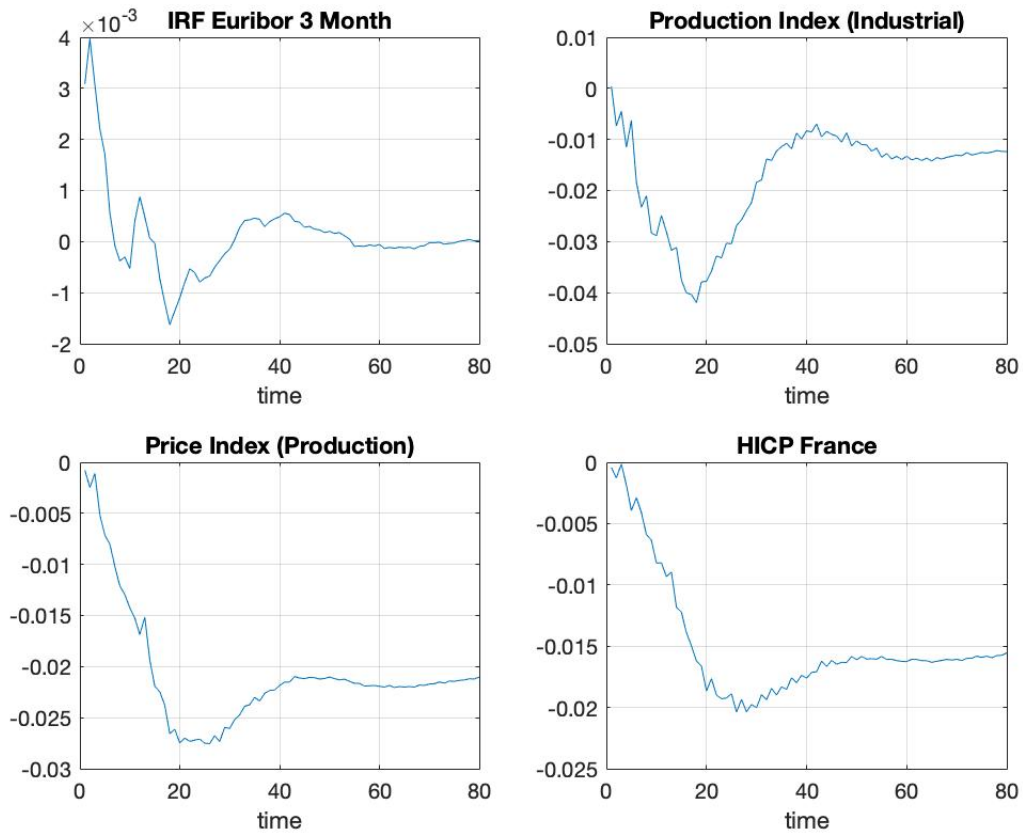


y-axis: log points in deviation from the "steady state".

Top panel: 3-month Euribor IRF. Top right panel: production index IRF. Bottom left panel: production price index IRF. Bottom right panel: IRF of the harmonized index of consumer prices

Cholesky - No Long-run restriction

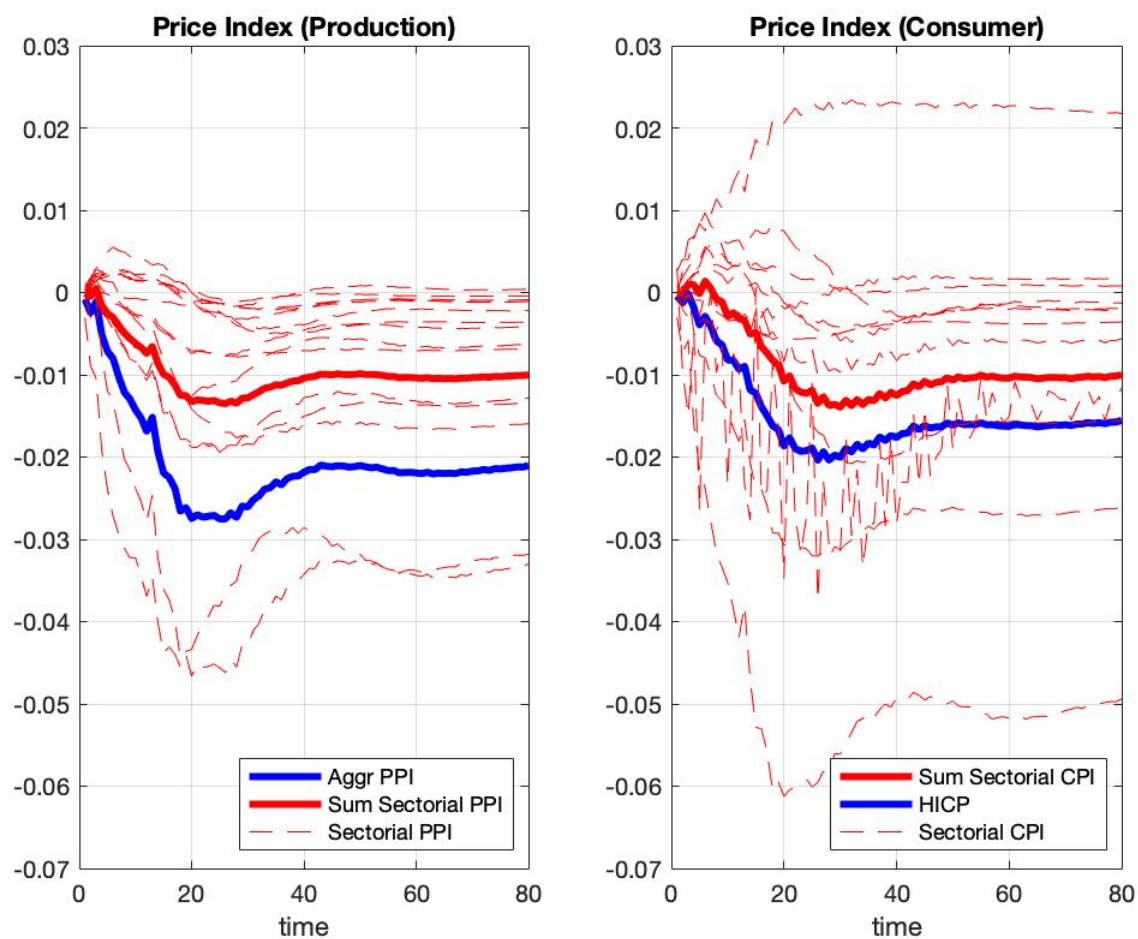
Figure E: Aggregate response to a contractionary monetary policy shock



y-axis: log points in deviation from the "steady state".

Top panel: 3-month Euribor IRF. Top right panel: production index IRF. Bottom left panel: production price index IRF. Bottom right panel: IRF of the harmonized index of consumer prices

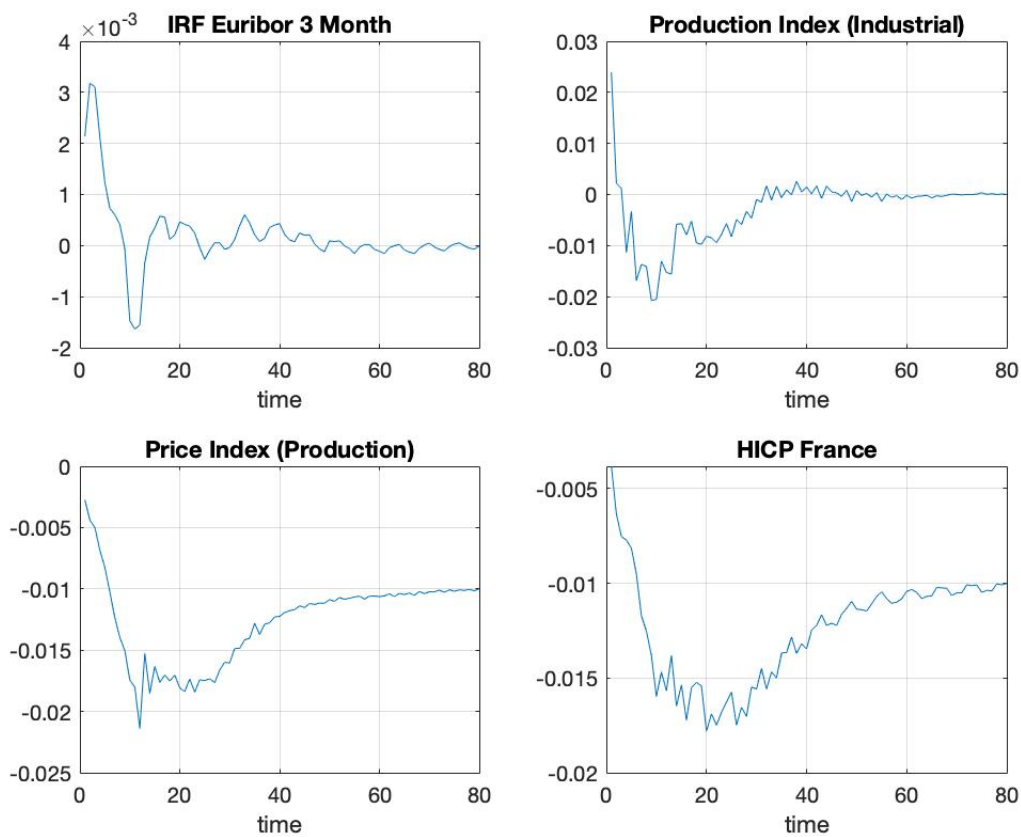
Figure F: Sectoral Responses of PPI and CPI to a Contractionary Monetary Shock



y-axis: log points in deviation from the "steady state". Left panel sectoral IRFs of PPI, right panel sectoral IRFs of CPI. In both panel: blue line IRF of aggregate time series, dashed red lines sectoral IRFs, thick red line arithmetic average of sectoral IRFs.

High-Freq. IV - Long-run restriction

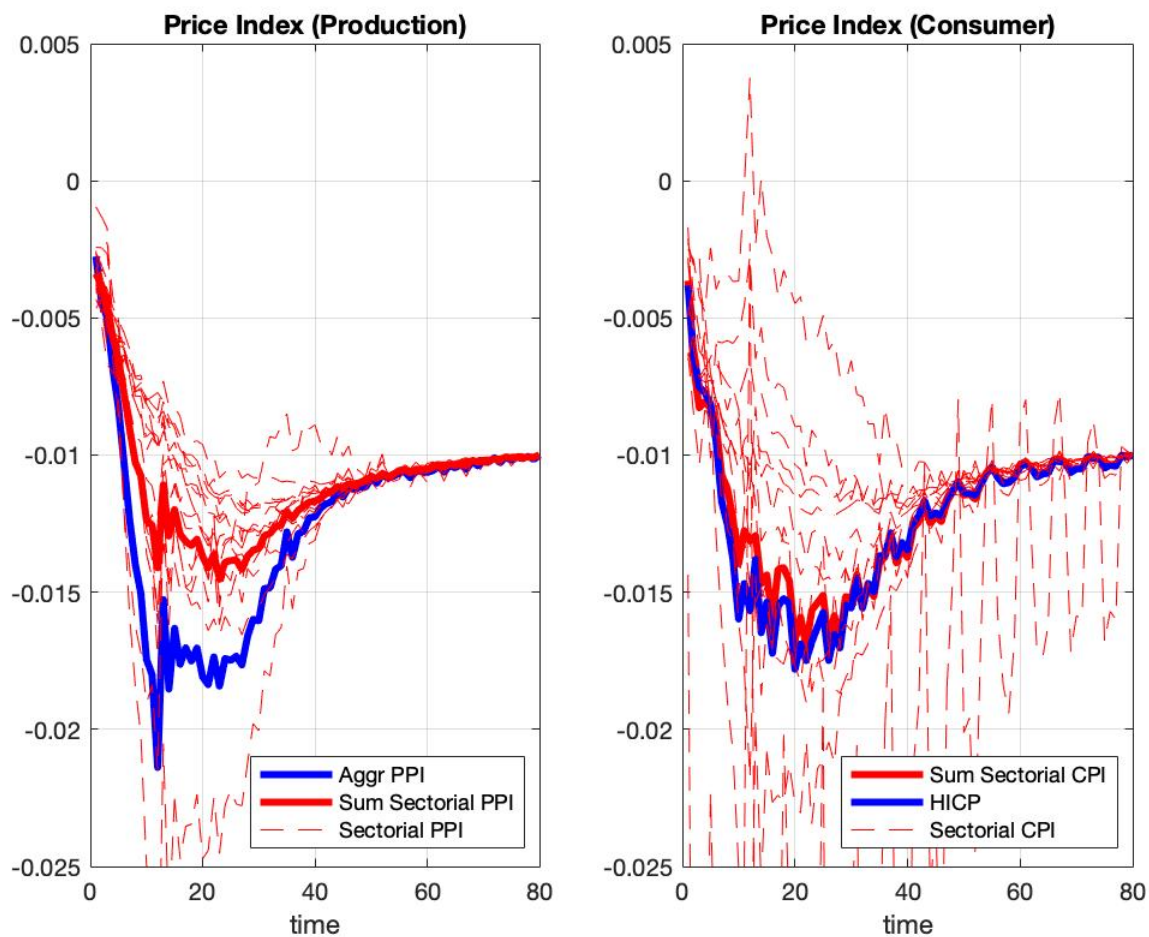
Figure G: Aggregate response to a contractionary monetary policy shock



y-axis: log points in deviation from the "steady state".

Top panel: 3-month Euribor IRF. Top right panel: production index IRF. Bottom left panel: production price index IRF. Bottom right panel: IRF of the harmonized index of consumer prices

Figure H: Sectoral Responses of PPI and CPI to a Contractionary Monetary Shock



y-axis: log points in deviation from the "steady state". Left panel sectoral IRFs of PPI, right panel sectoral IRFs of CPI. In both panel: blue line IRF of aggregate time series, dashed red lines sectoral IRFs, thick red line arithmetic average of sectoral IRFs.

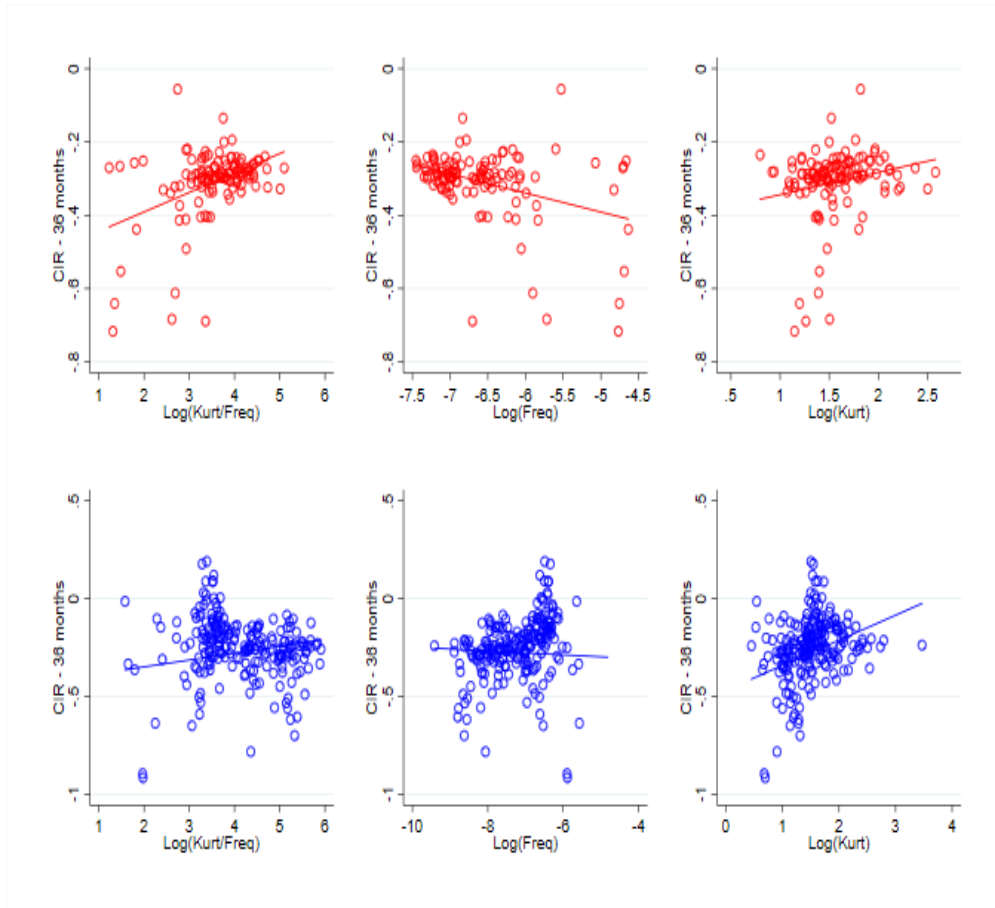
Table A: Product-specific CIR^P : Descriptive Statistics

	<i>Moments of the CIR distribution</i>								
	Mean	Std. Dev.	min	5%	25%	50%	75%	95%	max
<i>PANEL A: PRODUCER PRICES</i>									
<i>Cholesky - Long-run restriction</i>									
24 months	-0.18	0.09	-0.64	-0.39	-0.18	-0.16	-0.14	-0.09	0.12
36 months	-0.31	0.10	-0.72	-0.55	-0.32	-0.29	-0.27	-0.22	-0.06
<i>Cholesky - No long-run restriction</i>									
24 months	-0.18	0.57	-3.63	-1.50	-0.15	-0.02	0.04	0.22	0.35
36 months	-0.33	0.87	-5.81	-1.72	-0.34	-0.06	0.02	0.28	0.57
<i>High Frequency Instrument - Long-run restriction</i>									
24 months	-0.26	0.15	-1.21	-0.50	-0.27	-0.21	-0.19	-0.11	-0.01
36 months	-0.42	0.20	-1.58	-0.80	-0.44	-0.36	-0.32	-0.23	0.11
<i>PANEL B: CONSUMER PRICES</i>									
<i>Cholesky - Long-run restriction</i>									
24 months	-0.13	0.22	-1.92	-0.38	-0.20	-0.12	-0.04	0.15	0.49
36 months	-0.28	0.25	-2.44	-0.61	-0.32	-0.24	-0.16	-0.01	0.19
<i>Cholesky - No long-run restriction</i>									
24 months	-0.11	0.78	-7.40	-0.66	-0.14	0.01	0.14	0.38	1.41
36 months	-0.27	1.24	-12.17	-1.16	-0.34	-0.07	0.16	0.49	1.97
<i>High Frequency Instrument - Long-run restriction</i>									
24 months	-0.30	0.29	-2.50	-0.84	-0.32	-0.23	-0.17	-0.07	0.02
36 months	-0.48	0.38	-3.28	-1.22	-0.51	-0.39	-0.30	-0.16	-0.03

Note: this table reports descriptive statistics on the distribution of the product-specific CIR for the different specifications and at two horizons (24 and 36 months). These statistics are computed over 118 products for PPI and 223 products for CPI.

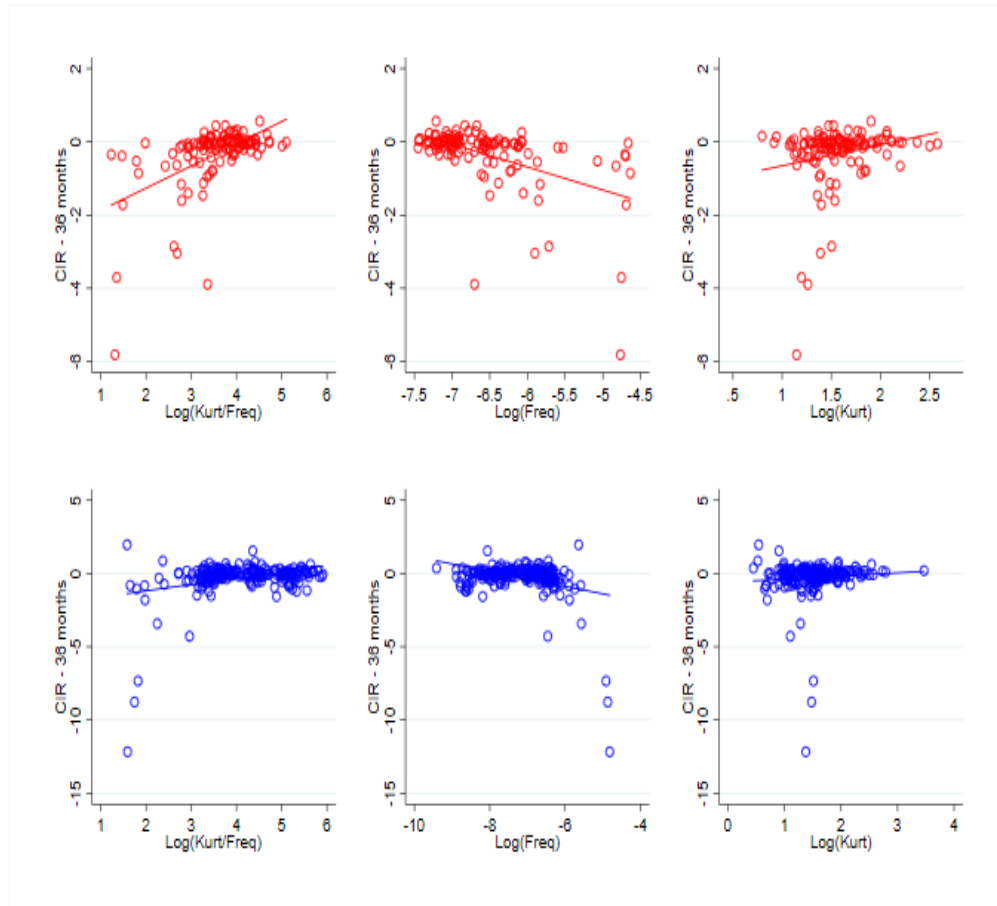
Additional Scatter Plots CIR^P - moments

Figure I: Correlation CIR^P - Log ratio $\frac{Kurt}{Freq}$ - Cholesky Long-run restriction



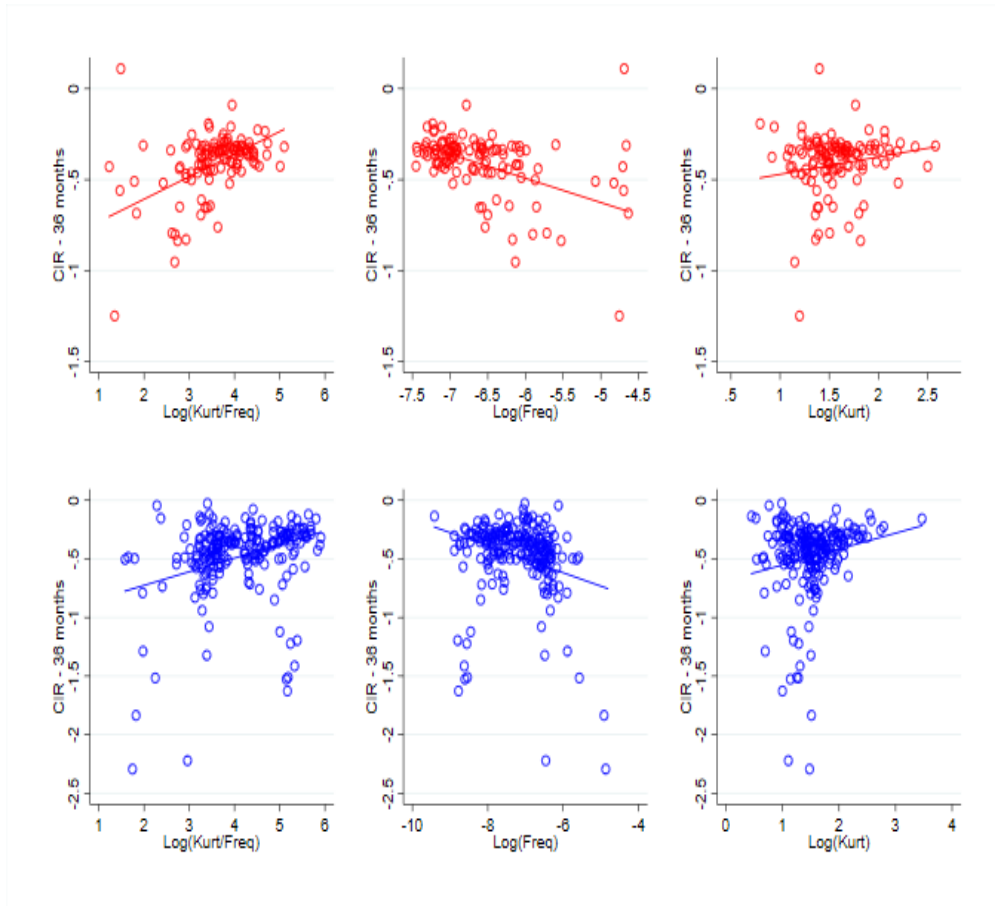
Note: the figure plots the product-specific CIR (at the horizon 36 months) obtained in the FAVAR specification using a Cholesky decomposition and imposing a long-run restriction and the log of the ratio kurtosis over frequency of price changes (left panel), the log of frequency of price changes (center panel), the log of kurtosis of price changes (right panel). The top panel reports results for PPI products whereas the bottom panel reports results for CPI products.

Figure J: Correlation CIR^P - Log ratio $\frac{Kurt}{Freq}$ - Cholesky No Long-run restriction



Note: the figure plots the product-specific CIR (at the horizon 36 months) obtained in the FAVAR specification using a Cholesky decomposition without imposing any long-run restriction and the log of the ratio kurtosis over frequency of price changes (left panel), the log of frequency of price changes (center panel), the log of kurtosis of price changes (right panel). The top panel reports results for PPI products whereas the bottom panel reports results for CPI products.

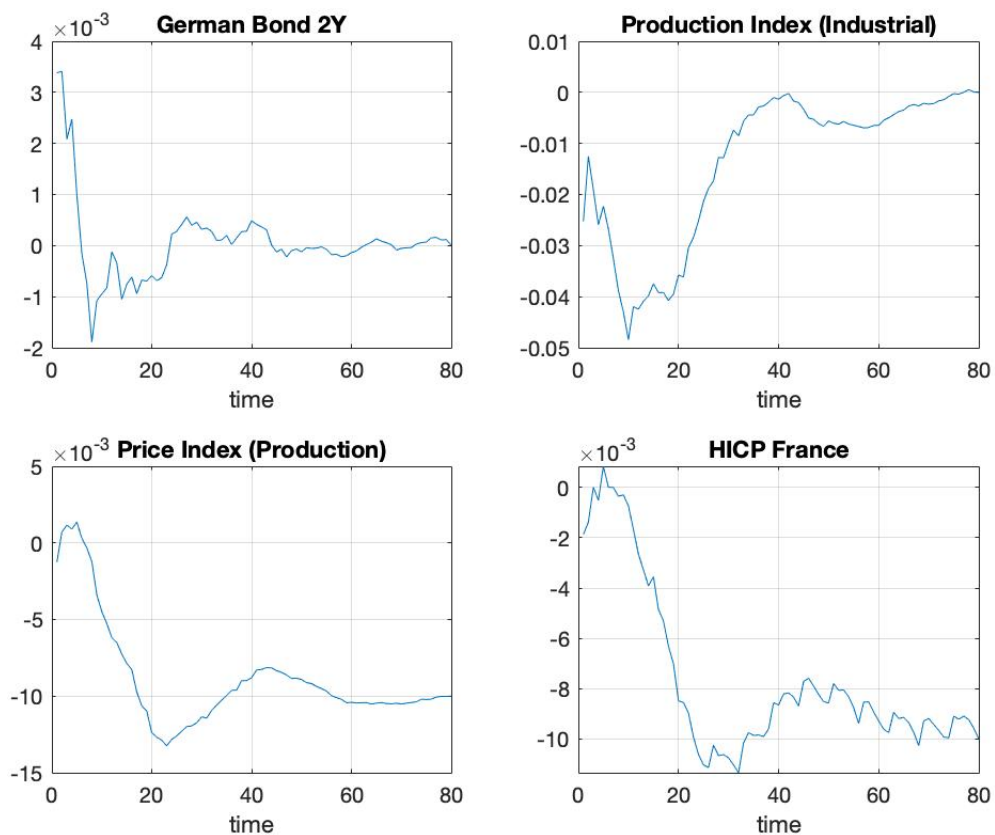
Figure K: Correlation CIR^P - Log ratio $\frac{Kurt}{Freq}$ - HFI Long-run restriction



Note: the figure plots the product-specific CIR (at the horizon 36 months) obtained in the FAVAR specification using a high-frequency instrument variable and imposing a long-run restriction and the log of the ratio kurtosis over frequency of price changes (left panel), the log of frequency of price changes (center panel), the log of kurtosis of price changes (right panel). The top panel reports results for PPI products whereas the bottom panel reports results for CPI products.

FAVAR - High-Freq. IV - German bond rate - Long-run restriction

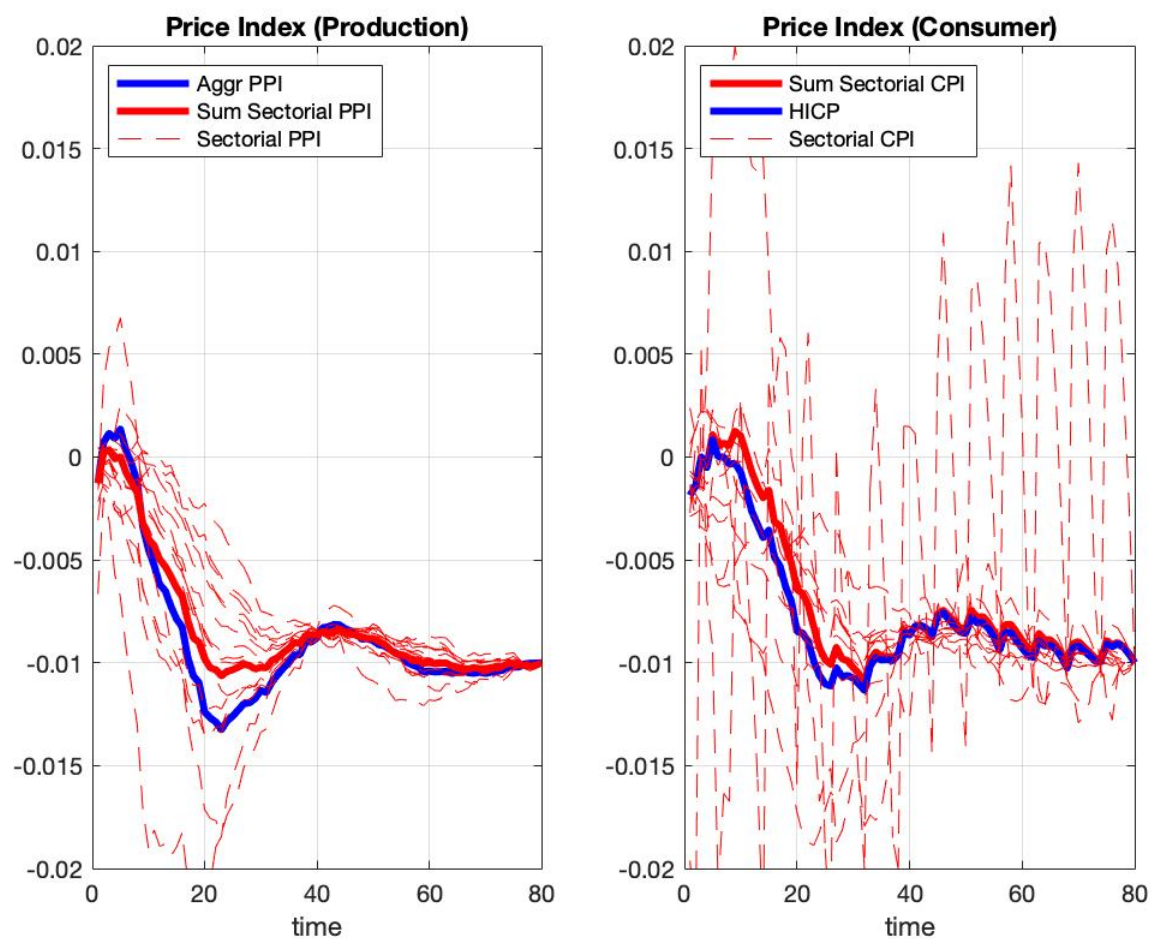
Figure L: Aggregate response to a contractionary monetary policy shock



y-axis: log points in deviation from the "steady state".

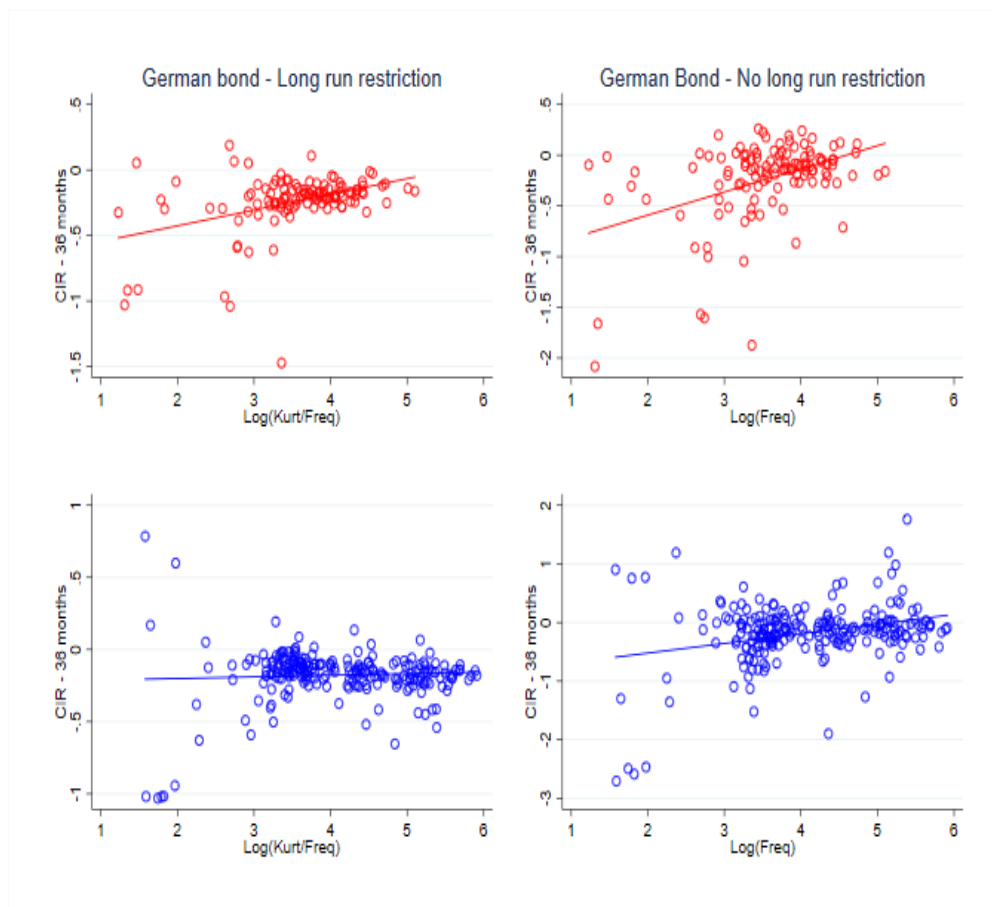
Top panel: 3-month Euribor IRF. Top right panel: production index IRF. Bottom left panel: production price index IRF. Bottom right panel: IRF of the harmonized index of consumer prices

Figure M: Sectoral Responses of PPI and CPI to a Contractionary Monetary Shock



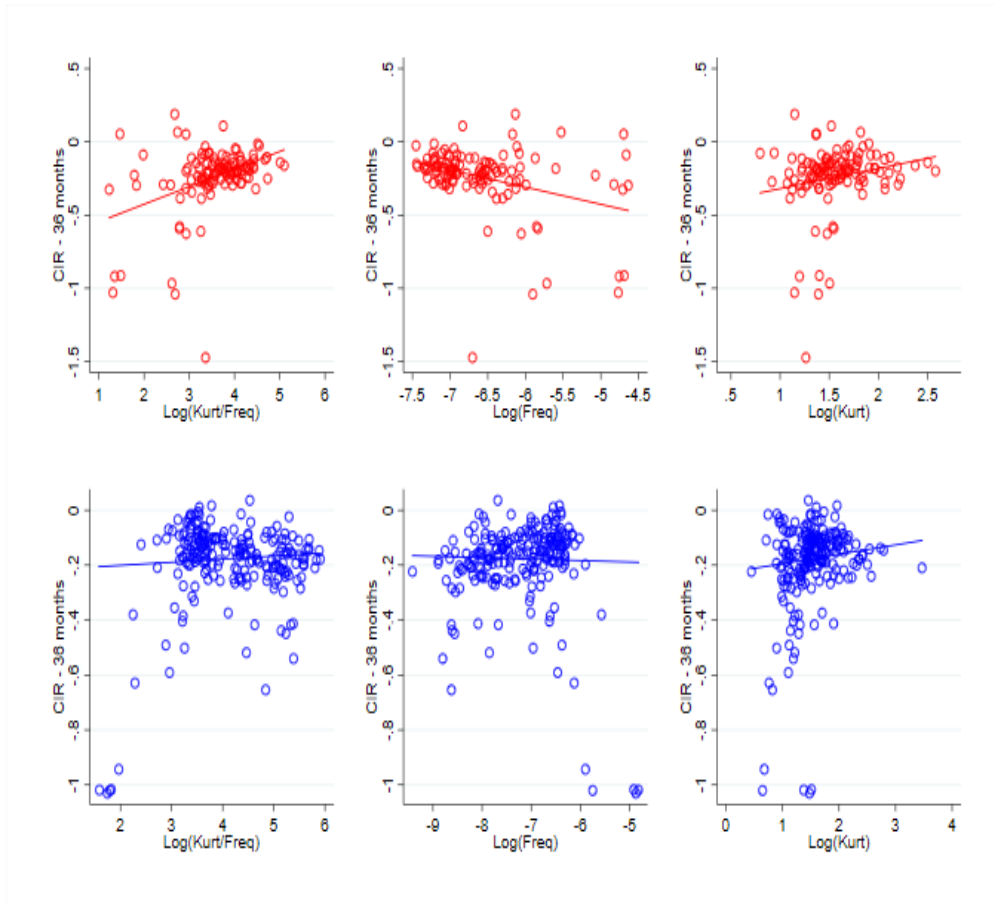
y-axis: log points in deviation from the "steady state". Left panel sectoral IRFs of PPI, right panel sectoral IRFs of CPI. In both panel: blue line IRF of aggregate time series, dashed red lines sectoral IRFs, thick red line arithmetic average of sectoral IRFs.

Figure N: Correlation CIR^P - Log ratio $\frac{Kurt}{Freq}$ - HFI 2-year German Bond Rate



Note: the figure plots for each FAVAR specification the product-specific CIR (at the horizon 36 months) and the log of the ratio kurtosis over frequency of price changes. The top panel reports result for PPI products whereas the bottom panel reports results for CPI products.

Figure O: Correlation CIR^P - Log ratio $\frac{Kurt}{Freq}$, $\log(Kurt)$ and $\log(Freq)$ - HFI 2-year German Bond Rate



Note: the figure plots the product-specific CIR (at the horizon 36 months) obtained in the FAVAR specification using a high-frequency instrument variable and imposing a long-run restriction and the log of the ratio kurtosis over frequency of price changes (left panel), the log of frequency of price changes (center panel), the log of kurtosis of price changes (right panel). The top panel reports results for PPI products whereas the bottom panel reports results for CPI products.

E Kurtosis Measurement with Unobserved Heterogeneity

The measure of Kurtosis is particularly sensitive to unobserved heterogeneity. Measured kurtosis is in particular known to suffer from an upward bias when a sample is composed of two (or more) sub-populations with different variances. To investigate the robustness of our results with respect to such a concern, we use an alternative measure of kurtosis derived along the lines of [Alvarez, Lippi, and Oskolkov \(2020\)](#). The assumption underlying this correction, is that within a given product category, there are several varieties (indexed by $i = 1, \dots, I$) that are pooled. For instance, one could have various brands of soda, in the case the brand of soda is not collected by the statistical office, or not disclosed to the researcher. At a given date t , the price change for all varieties is driven by a common factor $\Delta\tilde{p}_t$, but the variance differs across varieties, according to a scaling factor b_i .

$$\Delta p_{it} = b_i \Delta\tilde{p}_t \text{ for } i \in I \text{ and } t \in T(i)$$

where $T(i)$ is the set of adjustment instances for variety i . Under the assumption that $\Delta\tilde{p}_t$ is serially uncorrelated, and some other general assumptions, [Alvarez, Lippi, and Oskolkov \(2020\)](#) show that the following property then holds:

$$Kurt(\Delta\tilde{p}_t) = Kurt(\Delta p_{it}) \frac{E[(\Delta p_{it}^2)]^2}{E[(\Delta p_{it}^2)(\Delta p_{is}^2)]} \text{ for } t \neq s$$

or equivalently

$$Kurt(\Delta\tilde{p}_t) = \frac{Kurt(\Delta p_{it})}{1 + corr(\Delta p_{it}^2, \Delta p_{is}^2) CV(\Delta p_{it}^2) CV(\Delta p_{is}^2)} \text{ for } t \neq s$$

where $CV(\cdot)$ denotes the coefficient of variation and $corr(\cdot, \cdot)$ the correlation coefficient.

We use these equations to compute a measure of kurtosis robust to unobserved heterogeneity. In practice, we want to use information from several possible lags (the s 's as different from t), rather than picking up a single particular lag s .

To compute the covariance terms in the expression above we as use an estimator of $E = E[(\Delta p_{it}^2)(\Delta p_{is}^2)]$ the following expression:

$$E = (1/\#Terms) \sum_{t,s \in T(i), t \neq s} (\Delta p_{it})^2 (\Delta p_{is})^2 \quad (33)$$

In practice, we consider the first K lags of squared price changes. So, the numerator of the formula (33) above is computed as:

$$S = 2 * \left[\sum_{t=2}^T (\Delta p_t)^2 (\Delta p_{t-1})^2 + \sum_{t=3}^T (\Delta p_t)^2 (\Delta p_{t-2})^2 + \dots + \sum_{t=K+1}^T (\Delta p_t)^2 (\Delta p_{t-K})^2 \right] \quad (34)$$

Denotig by NN the number of terms in equation (34), then $\#Terms = 2 * NN$, where:

$$NN = (T - 1) + (T - 2) + \dots + (T - k) = T(T - 1)/2 - (T - K - 1) * (T - K)/2 \quad (35)$$

So when $K = T - 1$, $\#Terms = 2 * T(T - 1)/2 = T(T - 1)$ Then we recover

$$E = \frac{S}{T(T - 1)}$$

F Measurement Error

This appendix assesses the impact of (one form of) measurement errors on the micro moments of price adjustment and their ratio $Kurt/Freq$.

Assume measurement errors are of the following type: for a given store, measurement errors materialize at some points by extra spurious price changes, and these spurious price changes are small. Such patterns of error is plausible (as discussed in [Alvarez, Le Bihan, and Lippi \(2016\)](#)), both for CPI data because small coding error can stay undetected by the error checking procedures of the statistical institute, and for scanner data as the price is typically computed as the ratio of value purchased to quantity sold (and the numerator can vary reflecting e.g. coupons). These spurious price changes will increase both the measured Kurtosis, as well as the measured Frequency of price changes - with the size of the bias being a function of the fraction of spurious price changes. However, as is formally shown below, such measurement errors will leave ratio Kurtosis/Frequency unchanged. As a result, not only theory indicates that the ratio Kurtosis/Frequency is the relevant covariate, but it is also the case that this ratio should be more robust to measurement errors than each of the moments taken separately.

Formally, let $N_{\Delta p}$ be the number of “true” price changes per period (i.e. the frequency of price changes). Assume Δp , the price changes, have mean zero, variance $Var(\Delta p) = \sigma_{\Delta p}^2$ and Kurtosis $Kurt(\Delta p) = m_{4,\Delta p}/\sigma_{\Delta p}^4$, where $m_{4,\Delta p}$ is the fourth moment of variable Δp . Let N_e denote the number of spurious price changes per unit of time. Assume that spurious price changes, denoted e , have mean zero and variance $Var(e) = \sigma_e^2$, and kurtosis $Kurt(e) = m_{4,e}/\sigma_e^4$. Assume spurious and true price changes to be statistically independent. Then the observed (measured) frequency of price changes will be $\tilde{N} = N_{\Delta p} + N_e$. The distribution of the observed price changes, denoted $\tilde{\Delta p}$'s, will have mean zero and its Kurtosis will be

$$Kurt(\tilde{\Delta p}) = \frac{\theta Kurt(\Delta p)\sigma_{\Delta p}^4 + (1 - \theta)Kurt(e)\sigma_e^4}{\left(\theta\sigma_{\Delta p}^2 + (1 - \theta)\sigma_e^2\right)^2}$$

with $\theta \equiv \frac{N_{\Delta p}}{\tilde{N}}$ the fraction of “true” price changes. We consider the case of arbitrarily small measurement errors. From the above it results that $\lim_{\sigma_e^2 \rightarrow 0} Kurt(\tilde{\Delta p}) = \frac{Kurt(\Delta p)}{\theta}$. Then we have $\lim_{\sigma_e^2 \rightarrow 0} \frac{Kurt(\tilde{\Delta p})}{\tilde{N}} = \frac{Kurt(\Delta p)}{N_{\Delta p}}$. Thus, the ratio Kurtosis over Frequency is unaffected by these presence of small measurement error.

G OLS regressions - Additional results and robustness

Table B: Testing Model's Predictions

Identification Long-run Restriction	Cholesky Yes		Cholesky No		High-Frequency IV Yes	
	24 months	36 months	24 months	36 months	24 months	36 months
	<i>PANEL A: PRODUCER PRICES</i>					
<i>Constrained model</i>						
P-val $\beta = -1/6$	0.003	0.053	0.019	0.006	0.681	0.351
P-val $\alpha = -T$	0.111	0.702	0.076	0.027	0.006	0.000
Ratio α/β	-307.6	-360.9	-69.58	-72.82	-178.5	-216.0
<i>Unconstrained model</i>						
P-val $\beta_f = -\beta_k$	0.566	0.457	0.648	0.643	0.819	0.857
P-val $\beta_f = -\frac{\bar{K}}{6\bar{F}}$	0.130	0.325	0.033	0.020	0.577	0.460
P-val $\beta_k = \frac{\bar{K}}{6\bar{F}}$	0.679	0.915	0.039	0.022	0.477	0.389
P-val $\gamma = -T + \frac{\bar{K}}{6\bar{F}}$	0.743	0.687	0.779	0.626	0.022	0.000
<i>PANEL B: CONSUMER PRICES</i>						
<i>Constrained model</i>						
P-val $\beta = -1/6$	0.000	0.000	0.433	0.529	0.000	0.003
P-val $\alpha = -T$	0.000	0.009	0.796	0.495	0.003	0.000
Ratio α/β	685.5	11,160	-184.4	-205.0	-696.0	-759.4
<i>Unconstrained model</i>						
P-val $\beta_f = -\beta_k$	0.877	0.492	0.001	0.000	0.049	0.039
P-val $\beta_f = -\frac{\bar{K}}{6\bar{F}}$	0.281	0.860	0.001	0.000	0.032	0.006
P-val $\beta_k = \frac{\bar{K}}{6\bar{F}}$	0.124	0.377	0.773	0.929	0.710	0.664
P-val $\gamma = -T + \frac{\bar{K}}{6\bar{F}}$	0.352	0.433	0.000	0.000	0.340	0.202

Note: we report p-values of Wald tests performed on the parameters of our baseline OLS regressions presented in Table 2 and Table 2. These tests correspond to model's predictions presented in equation (10) and equation (11). We perform four different tests: (i) in the constrained version of the model we test whether β (parameter associated with the ratio $Kurt/Freq$ is equal to $-\delta/6$ (where δ is the MP shock here normalised to 1%); (ii) we test whether the constant of the constrained model (α) is equal to $-T$ and in the unconstrained model, whether γ is equal to $-T + \frac{\bar{K}}{6\bar{F}}$ and (iii) in the unconstrained model, we test whether the parameter associated with frequency (β_f) is equal to minus the parameter associated with kurtosis ($-\beta_k$); (iv) in the unconstrained version, we also perform tests on the parameter associated with frequency and kurtosis, they are predicted to be equal to $\frac{\bar{K}}{6\bar{F}}$ where \bar{K} and \bar{F} are sample averages of kurtosis and frequency. We also report the ratio of the estimated coefficients in OLS regressions.

Table C: Regression Results: Alternative Specifications - Log

Identification Long-run Restriction	Cholesky		Cholesky		High-Frequency IV	
	Yes		No		Yes	
	24 months	36 months	24 months	36 months	24 months	36 months
<i>PANEL A: Producer prices - Constrained model</i>						
Log(ratio)	3.948** (1.649)	5.301*** (1.723)	37.86*** (11.30)	60.62*** (17.62)	10.07*** (3.398)	12.39*** (4.541)
Constant	-31.63*** (6.440)	-49.67*** (6.681)	-152.0*** (43.44)	-247.5*** (67.54)	-61.54*** (12.97)	-85.53*** (17.35)
R^2	0.103	0.175	0.255	0.282	0.259	0.223
<i>PANEL B: Producer prices - Unconstrained model</i>						
Log(Freq)	-3.748** (1.833)	-5.155*** (1.873)	-38.32*** (11.75)	-61.78*** (18.24)	-10.56*** (3.582)	-13.11*** (4.845)
Log(Kurt)	4.834** (2.291)	5.948** (2.414)	35.84** (14.78)	55.48** (22.53)	7.895* (4.061)	9.189* (5.274)
Constant	-15.37*** (4.982)	-26.66*** (4.836)	26.79 (25.41)	42.85 (37.37)	-10.47 (7.124)	-21.55** (9.852)
R^2	0.104	0.176	0.255	0.282	0.262	0.226
Observations	118	118	118	118	118	118
<i>PANEL C: Consumer prices - Constrained model</i>						
Log(Ratio)	1.354 (2.289)	3.509 (2.877)	24.83** (11.76)	43.49** (18.74)	8.643** (3.580)	11.63** (4.539)
Constant	-18.72* (10.50)	-41.94*** (13.07)	-112.5** (52.51)	-204.8** (83.59)	-65.36*** (15.72)	-95.81*** (19.89)
R^2	0.003	0.017	0.092	0.111	0.082	0.086
<i>PANEL D: Consumer prices - Unconstrained model</i>						
Log(Freq)	1.460 (2.715)	-0.824 (3.442)	-27.27* (14.41)	-50.72** (22.87)	-8.005* (4.559)	-11.23* (5.792)
Log(Kurt)	11.07*** (3.020)	12.77*** (3.338)	16.43** (7.462)	18.56 (11.99)	10.84*** (3.289)	12.99*** (4.426)
Constant	-32.83*** (5.901)	-45.19*** (7.093)	19.46 (25.53)	47.73 (41.36)	-30.17*** (10.52)	-45.12*** (13.88)
R^2	0.051	0.050	0.095	0.121	0.084	0.087
Observations	223	223	223	223	223	223

Note: this table reports results of OLS regressions (equation 10) where the endogenous variable is the product-specific CIR $CIR_T^{P_j}$ (expressed in %) and the RHS variables include the log of the product-specific ratio $Kurt/freq$ in the constrained model and the log of product specific frequency and the log of product-specific kurtosis. Robust standard errors are reported in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table D: Regression Results - Placebo Unconstrained Specification

Identification Long-run Restriction	Cholesky Yes		Cholesky No		High-Frequency IV Yes	
	24 months	36 months	24 months	36 months	24 months	36 months
<i>PANEL A: PRODUCER PRICES</i>						
<i>Freq/\bar{F}</i>	-2.865*	-3.337**	-24.17**	-37.39**	-5.488*	-6.347
	(1.454)	(1.491)	(10.35)	(16.28)	(3.258)	(4.406)
<i>Kurt/\bar{K}</i>	3.026	4.066	21.29*	33.52*	5.823*	7.153
	(3.048)	(2.796)	(12.09)	(17.14)	(3.177)	(4.673)
Mean	-0.254	-0.186	-4.946	-7.775	-0.855	-0.927
	(0.792)	(0.851)	(4.713)	(7.327)	(1.285)	(1.699)
Skewness	1.798	0.793	-5.560	-13.96	-4.708*	-7.159*
	(3.359)	(2.989)	(14.85)	(20.32)	(2.589)	(3.855)
Standard dev.	-0.916	-0.625	-5.324	-7.003	0.245	0.837
	(1.297)	(1.306)	(8.740)	(13.51)	(2.460)	(3.229)
Constant	-13.33*	-28.68***	9.428	2.375	-27.87*	-47.18**
	(7.379)	(7.539)	(48.08)	(75.03)	(14.08)	(18.59)
Observations	118	118	118	118	118	118
R^2	0.118	0.164	0.246	0.264	0.228	0.195
<i>PANEL B: CONSUMER PRICES</i>						
<i>Freq/\bar{F}</i>	-6.170*	-10.12***	-58.12***	-96.96***	-17.81***	-23.07***
	(3.170)	(3.640)	(14.21)	(21.71)	(3.969)	(4.877)
<i>Kurt/\bar{K}</i>	-4.732*	-2.640	10.03	25.00	7.629	11.55*
	(2.733)	(3.216)	(10.82)	(18.26)	(4.688)	(6.434)
Mean	0.0898	-0.366	-5.173**	-9.461***	-1.505**	-2.092**
	(0.783)	(0.822)	(2.257)	(3.511)	(0.718)	(0.956)
Skewness	5.111	7.162	15.02*	22.97	8.489**	10.98**
	(4.335)	(4.393)	(8.898)	(14.50)	(3.586)	(4.974)
Standard dev.	-2.972***	-2.767**	0.0684	3.546	-0.00409	0.554
	(1.078)	(1.248)	(3.924)	(6.076)	(1.211)	(1.571)
Constant	21.50*	8.500	46.97	35.88	-15.71	-35.47*
	(11.82)	(13.60)	(45.35)	(70.99)	(14.13)	(18.48)
Observations	223	223	223	223	223	223
R^2	0.108	0.165	0.509	0.572	0.383	0.380

Note: this table reports results of OLS regressions (equation 11) where the endogenous variable is the product-specific CIR $CIR_T^{P_j}$ (expressed in %) and the RHS variables include the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$, but also three other moments of the product-specific price change distribution: the average price change $Mean$, the skewness of price changes $Skewness$, and the standard deviation of price changes $StandardDev.$. Robust standard errors are reported in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table E: Regression Results: Outliers - Constrained - Producer Prices

Identification Long-run Restriction	Cholesky Yes		Cholesky No		High-Frequency IV Yes	
	24 months	36 months	24 months	36 months	24 months	36 months
<i>PANEL A: CIR</i>						
Kurt/Freq	0.0591** (0.0271)	0.0841*** (0.0304)	0.505*** (0.160)	0.696*** (0.193)	0.139*** (0.0362)	0.189*** (0.0489)
Constant	-20.08*** (1.708)	-34.32*** (1.857)	-36.03*** (9.769)	-54.65*** (11.87)	-31.11*** (2.162)	-49.30*** (2.925)
R^2	0.059	0.095	0.125	0.135	0.143	0.145
<i>PANEL B: Ratio</i>						
Kurt/Freq	0.0762* (0.0392)	0.108*** (0.0352)	0.673*** (0.173)	1.086*** (0.243)	0.181*** (0.0444)	0.226*** (0.0668)
Constant	-20.39*** (2.388)	-34.85*** (2.146)	-41.71*** (10.61)	-71.31*** (14.78)	-32.18*** (2.580)	-49.54*** (3.861)
R^2	0.039	0.086	0.121	0.149	0.146	0.116
<i>PANEL C: Kurtosis</i>						
Kurt/Freq	0.0932** (0.0402)	0.134*** (0.0417)	0.918*** (0.258)	1.489*** (0.397)	0.258*** (0.0713)	0.324*** (0.0941)
Constant	-21.62*** (2.434)	-36.60*** (2.463)	-57.02*** (15.04)	-96.31*** (22.88)	-36.96*** (4.060)	-55.57*** (5.389)
R^2	0.057	0.111	0.149	0.169	0.171	0.154
<i>PANEL D: Frequency</i>						
Kurt/Freq	0.0653* (0.0348)	0.0961** (0.0379)	0.723*** (0.241)	1.182*** (0.374)	0.200*** (0.0668)	0.252*** (0.0871)
Constant	-20.55*** (2.241)	-35.12*** (2.303)	-49.50*** (14.25)	-84.36*** (21.78)	-34.52*** (3.844)	-52.51*** (5.068)
R^2	0.036	0.075	0.119	0.137	0.132	0.120
Observations	113	113	113	113	113	113

Note: this table reports results of OLS regressions (equation 10) for PPI products relating the product-specific CIR $CIR_T^{P_j}$ (expressed in %) to the ratio $Kurt/freq$. For each of the 4 regressions, we remove products with "extreme" values of CIR (Panel A); ratio $Kurt/Freq$ (Panel B), kurtosis (Panel C), frequency of price changes (Panel D). "Extreme values" are defined as products below the 2.5th percentile or above the 97.5th percentile of the distribution of each statistic. Robust standard errors are reported in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table F: Regression Results: Outliers - Constrained - Consumer Prices

Identification Long-run Restriction	Cholesky		Cholesky		High-Frequency IV	
	Yes		No		Yes	
	24 months	36 months	24 months	36 months	24 months	36 months
<i>PANEL A: CIR</i>						
Kurt/Freq	-0.0304*** (0.0107)	-0.0302** (0.0117)	-0.00224 (0.0206)	0.0469 (0.0332)	0.0260* (0.0153)	0.0384* (0.0220)
Constant	-9.375*** (1.637)	-22.80*** (1.616)	-2.007 (2.683)	-17.46*** (4.236)	-29.30*** (1.625)	-48.51*** (2.361)
R^2	0.033	0.029	0.000	0.009	0.015	0.015
<i>PANEL B: Ratio</i>						
Kurt/Freq	-0.0411** (0.0177)	-0.0315 (0.0205)	0.0158 (0.0357)	0.0842 (0.0548)	0.0164 (0.0237)	0.0320 (0.0318)
Constant	-8.503*** (2.706)	-23.39*** (2.984)	-4.693 (4.914)	-22.09*** (7.321)	-29.42*** (2.398)	-48.55*** (3.232)
R^2	0.021	0.011	0.001	0.011	0.003	0.006
<i>PANEL C: Kurtosis</i>						
Kurt/Freq	-0.0170 (0.0187)	-0.00270 (0.0226)	0.122* (0.0738)	0.245** (0.118)	0.0429 (0.0276)	0.0630* (0.0359)
Constant	-11.68*** (2.988)	-27.53*** (3.505)	-22.73** (11.44)	-50.45*** (18.14)	-34.42*** (3.681)	-54.70*** (4.734)
R^2	0.003	0.000	0.014	0.022	0.013	0.016
<i>PANEL D: Frequency</i>						
Kurt/Freq	-0.0326** (0.0153)	-0.0230 (0.0172)	0.00516 (0.0300)	0.0577 (0.0443)	0.0270 (0.0172)	0.0450* (0.0230)
Constant	-8.829*** (2.553)	-23.48*** (2.774)	-2.275 (4.547)	-17.34*** (6.499)	-29.32*** (2.046)	-48.33*** (2.747)
R^2	0.016	0.007	0.000	0.007	0.012	0.017
Observations	213	213	213	213	213	213

Note: this table reports results of OLS regressions (equation 10) for CPI products relating the product-specific CIR $CIR_T^{P_j}$ (expressed in %) to the ratio $Kurt/freq$. For each of the 4 regressions, we remove products with "extreme" values of CIR (Panel A), ratio $Kurt/Freq$ (Panel B), kurtosis (Panel C), frequency of price changes (Panel D). "Extreme values" are defined as products below the 2.5th percentile or above the 97.5th percentile of the distribution of each statistic. Robust standard errors are reported in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table G: Regression Results: Outliers - Unconstrained - Producer Prices

Identification Long-run Restriction	Cholesky Yes		Cholesky No		High-Frequency IV Yes	
	24 months	36 months	24 months	36 months	24 months	36 months
	<i>PANEL A: CIR</i>					
<i>Freq/\bar{F}</i>	-2.024* (1.115)	-2.662** (1.189)	-16.96*** (6.208)	-19.00*** (5.712)	-3.727*** (1.202)	-5.053*** (1.614)
<i>Kurto/\bar{K}</i>	2.310* (1.392)	2.921* (1.495)	17.05** (7.088)	20.76** (8.278)	3.941* (2.051)	5.685** (2.817)
Constant	-17.82*** (1.387)	-30.96*** (1.575)	-14.39* (7.629)	-26.79** (10.66)	-25.51*** (2.633)	-42.00*** (3.578)
R^2	0.109	0.149	0.220	0.146	0.127	0.131
<i>PANEL B: Ratio</i>						
<i>Freq/\bar{F}</i>	-1.984 (1.547)	-2.240* (1.164)	-14.86*** (5.390)	-22.60*** (6.645)	-2.874 (1.772)	-3.162 (2.990)
<i>Kurto/\bar{K}</i>	4.770* (2.431)	5.526** (2.116)	26.71** (10.80)	39.54*** (14.95)	4.710* (2.702)	4.962 (4.070)
Constant	-20.04*** (2.445)	-33.65*** (2.274)	-25.96** (12.70)	-43.67** (18.13)	-26.57*** (2.887)	-42.02*** (3.999)
R^2	0.062	0.088	0.118	0.127	0.071	0.043
<i>PANEL C: Kurtosis</i>						
<i>Freq/\bar{F}</i>	-2.520** (1.265)	-3.167** (1.297)	-23.76*** (8.788)	-37.55*** (13.82)	-5.994** (2.771)	-7.216* (3.766)
<i>Kurto/\bar{K}</i>	6.732** (2.787)	8.189*** (2.830)	47.35*** (17.03)	72.77*** (25.81)	11.03** (4.551)	12.85** (5.979)
Constant	-21.71*** (2.906)	-35.78*** (2.796)	-40.62** (16.81)	-66.79*** (24.73)	-30.83*** (3.969)	-47.22*** (5.258)
R^2	0.125	0.184	0.258	0.275	0.231	0.188
<i>PANEL D: Frequency</i>						
<i>Freq/\bar{F}</i>	-3.023* (1.626)	-3.710** (1.636)	-29.98*** (10.93)	-47.33*** (17.24)	-7.298** (3.572)	-8.713* (4.893)
<i>Kurto/\bar{K}</i>	3.267* (1.813)	4.218** (1.851)	24.65** (10.47)	38.62** (15.51)	5.751** (2.675)	6.870* (3.469)
Constant	-18.13*** (2.319)	-31.67*** (2.152)	-14.49 (12.60)	-27.03 (18.17)	-24.72*** (3.027)	-40.24*** (4.121)
R^2	0.116	0.171	0.285	0.307	0.245	0.198
Observations	113	113	113	113	113	113

Note: This table reports results of OLS regressions (equation 11) relating the product-specific CIR $CIR_T^{P_j}$ (expressed in %) to the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurto/\bar{K}$. For each of the 4 regressions, we remove products with "extreme" values of CIR (Panel A); ratio $Kurto/Freq$ (Panel B), kurtosis (Panel C), frequency of price changes (Panel D). "Extreme values" are defined as products below the 2.5th percentile or above the 97.5th percentile of the distribution of each statistic. Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table H: Regression Results: Outliers - Unconstrained - Consumer Prices

Identification Long-run Restriction	Cholesky Yes		Cholesky No		High-Frequency IV Yes	
	24 months	36 months	24 months	36 months	24 months	36 months
	<i>PANEL A: CIR</i>					
<i>Freq/\bar{F}</i>	4.340** (2.090)	4.832** (2.450)	-3.195 (3.550)	-13.78*** (4.902)	-5.913** (2.623)	-8.578** (3.670)
<i>Kurto/\bar{K}</i>	3.609* (1.869)	4.757** (2.031)	8.541*** (2.929)	12.46*** (3.718)	6.138*** (1.537)	8.495*** (2.170)
Constant	-19.75*** (2.281)	-34.72*** (2.855)	-8.113* (4.711)	-13.60* (7.168)	-27.70*** (3.460)	-45.70*** (5.075)
R^2	0.065	0.082	0.049	0.078	0.106	0.097
<i>PANEL B: Ratio</i>						
<i>Freq/\bar{F}</i>	6.264** (2.990)	3.664 (3.520)	-10.86 (6.827)	-28.78*** (10.85)	-6.202* (3.390)	-10.09** (4.530)
<i>Kurto/\bar{K}</i>	7.988*** (2.858)	9.923*** (3.078)	14.40*** (4.646)	18.75*** (6.909)	10.62*** (2.610)	13.32*** (3.483)
Constant	-25.47*** (2.925)	-39.09*** (3.249)	-7.688 (6.487)	-7.381 (11.06)	-32.82*** (4.749)	-49.76*** (6.495)
R^2	0.067	0.052	0.063	0.114	0.081	0.086
<i>PANEL C: Kurtosis</i>						
<i>Freq/\bar{F}</i>	-5.606** (2.758)	-9.607*** (3.162)	-59.80*** (12.61)	-100.4*** (19.37)	-17.86*** (3.666)	-23.21*** (4.536)
<i>Kurto/\bar{K}</i>	12.83*** (3.347)	15.14*** (3.757)	22.07** (8.766)	27.10* (14.30)	13.23*** (3.855)	16.00*** (5.120)
Constant	-19.94*** (4.172)	-32.85*** (4.782)	25.23* (13.36)	43.05** (21.25)	-25.88*** (5.801)	-41.83*** (7.637)
R^2	0.108	0.181	0.554	0.607	0.388	0.376
<i>PANEL D: Frequency</i>						
<i>Freq/\bar{F}</i>	6.723** (2.839)	5.051 (3.094)	-1.593 (6.755)	-12.30 (10.12)	-4.369* (2.587)	-7.549** (3.423)
<i>Kurto/\bar{K}</i>	4.587* (2.507)	5.874** (2.731)	8.233** (3.596)	10.98** (4.734)	6.992*** (1.937)	8.912*** (2.428)
Constant	-22.50*** (2.988)	-36.09*** (3.240)	-8.734 (5.719)	-12.18 (9.049)	-30.05*** (3.737)	-46.52*** (5.023)
R^2	0.055	0.042	0.019	0.035	0.067	0.073
Observations	213	213	213	213	213	213

Note: This table reports results of OLS regressions (equation 11) relating product-specific CIR $CIR_T^{P_j}$ (expressed in %) to the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurto/\bar{K}$. For each of the 4 regressions, we remove products with "extreme" values of CIR (Panel A); ratio $Kurto/Freq$ (Panel B), kurtosis (Panel C), frequency of price changes (Panel D). "Extreme values" are defined as products below the 2.5th percentile or above the 97.5th percentile of the distribution of each statistic. Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table I: Regression Results: Kurtosis Measurement - Heterogeneity

Identification Long-run Restriction	Cholesky Yes		Cholesky No		High-Frequency IV Yes	
	24 months	36 months	24 months	36 months	24 months	36 months
	<i>PANEL A: Producer Prices - Constrained model</i>					
Kurt/Freq	0.0837** (0.0345)	0.115*** (0.0384)	0.731*** (0.236)	1.170*** (0.367)	0.190*** (0.0662)	0.234*** (0.0862)
Constant	-20.49*** (1.807)	-34.79*** (1.917)	-42.68*** (11.98)	-72.47*** (18.44)	-32.33*** (3.278)	-49.60*** (4.293)
R^2	0.045	0.080	0.093	0.103	0.090	0.078
<i>PANEL B: Producer Prices - Unconstrained model</i>						
Freq/ \bar{F}	-2.408* (1.280)	-3.066** (1.327)	-23.46** (9.125)	-37.23** (14.34)	-6.048** (2.822)	-7.328* (3.803)
Kurt/ \bar{K}	4.458** (2.027)	4.376** (1.746)	12.96*** (4.757)	15.20** (6.591)	-0.641 (2.236)	-2.262 (3.571)
Constant	-19.71*** (2.272)	-32.22*** (1.985)	-7.442 (6.972)	-10.84 (9.815)	-19.21*** (2.556)	-32.08*** (3.995)
R^2	0.146	0.185	0.220	0.231	0.192	0.162
<i>PANEL C: Consumer Prices - Constrained model</i>						
Kurt/Freq	-0.0152 (0.0148)	-0.00274 (0.0175)	0.105* (0.0622)	0.211** (0.104)	0.0446* (0.0228)	0.0646** (0.0303)
Constant	-12.21*** (2.264)	-27.40*** (2.634)	-17.50** (8.703)	-40.13*** (13.88)	-32.81*** (2.792)	-52.32*** (3.616)
R^2	0.003	0.000	0.010	0.015	0.013	0.016
<i>PANEL D: Consumer Prices - Unconstrained model</i>						
Freq/ \bar{F}	-4.959* (2.820)	-8.601** (3.344)	-54.24*** (13.80)	-91.25*** (21.35)	-16.44*** (3.911)	-21.39*** (4.832)
Kurt/ \bar{K}	3.637* (2.084)	4.424** (2.225)	5.274 (4.155)	6.242 (6.457)	5.771*** (1.827)	7.303*** (2.337)
Constant	-11.85*** (3.538)	-23.40*** (4.079)	38.14*** (13.40)	58.29*** (20.75)	-19.31*** (4.801)	-34.12*** (6.128)
R^2	0.061	0.126	0.476	0.528	0.342	0.335
Observations	223	223	223	223	223	223

Note: This table reports results of OLS results of the constrained model (equation 10) relating product-specific CIR $CIR_T^{P_j}$ (expressed in %) to the ratio $Kurt/freq$ and OLS results of the unconstrained model (equation 11) relating product-specific CIR $CIR_T^{P_j}$ (expressed in %) to the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$. The measure of kurtosis takes into account for possible product heterogeneity following the methodology presented in Appendix E and using $S = 5$. Robust standard errors are reported in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table J: Regression Results: Kurtosis Measurement - Producer Prices

Identification Long-run Restriction	Cholesky Yes		Cholesky No		High-Frequency IV Yes	
	24 months	36 months	24 months	36 months	24 months	36 months
	<i>PANEL A: Outlier threshold - small price changes - Constrained model</i>					
Kurt/Freq	0.0554* (0.0283)	0.0806** (0.0323)	0.569*** (0.196)	0.928*** (0.306)	0.158*** (0.0561)	0.199*** (0.0728)
Constant	-19.78*** (1.801)	-33.99*** (1.878)	-39.71*** (11.36)	-68.34*** (17.37)	-31.95*** (3.075)	-49.28*** (4.048)
R^2	0.031	0.062	0.088	0.101	0.098	0.088
<i>PANEL B: Outlier threshold - small price changes - Unconstrained model</i>						
Freq/ \bar{F}	-2.417* (1.298)	-3.037** (1.327)	-22.91** (8.968)	-36.22** (14.03)	-5.819** (2.767)	-7.011* (3.743)
Kurt/ \bar{K}	2.490 (1.760)	3.403* (1.954)	21.70** (10.61)	34.71** (16.01)	5.349* (2.852)	6.565* (3.670)
Constant	-17.73*** (2.321)	-31.28*** (2.331)	-16.74 (13.35)	-31.37 (19.66)	-25.43*** (3.310)	-41.23*** (4.402)
R^2	0.096	0.147	0.227	0.245	0.209	0.173
<i>PANEL C: Outlier threshold - large price changes - Constrained model</i>						
Kurt/Freq	0.0273 (0.0193)	0.0415* (0.0245)	0.311** (0.147)	0.512** (0.236)	0.0881* (0.0453)	0.112* (0.0587)
Constant	-19.49*** (1.906)	-33.69*** (2.116)	-38.80*** (12.77)	-67.19*** (19.86)	-31.81*** (3.652)	-49.18*** (4.786)
R^2	0.023	0.050	0.081	0.094	0.093	0.085
<i>PANEL D: Outlier threshold - large price changes - Unconstrained model</i>						
Freq/ \bar{F}	-2.521* (1.306)	-3.181** (1.345)	-23.84*** (9.056)	-37.72*** (14.19)	-6.053** (2.806)	-7.298* (3.794)
Kurt/ \bar{K}	1.062 (1.267)	1.653 (1.513)	14.60 (8.948)	24.18* (14.06)	4.095 (2.812)	5.210 (3.647)
Constant	-16.20*** (1.623)	-29.38*** (1.675)	-8.704 (10.04)	-19.34 (15.14)	-23.94*** (2.774)	-39.59*** (3.682)
R^2	0.089	0.137	0.223	0.243	0.210	0.176
Observations	118	118	118	118	118	118

Note: This table reports results of OLS results of the constrained model (equation 10) for PPI products relating product-specific CIR $CIR_T^{P_j}$ (expressed in %) to the ratio $Kurt/freq$ and OLS results of the unconstrained model (equation 11) relating product-specific CIR $CIR_T^{P_j}$ (expressed in %) to the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$. In Panels A and B, we have modified the thresholds defining very small price changes for the calculation of kurtosis: we have removed all price changes below 0.5% in absolute values (instead 0.1% in our baseline). In Panels C and D, we have modified thresholds defining very large price changes for the calculation of kurtosis (25% for instead of 15% in the baseline). Robust standard errors are reported in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table K: Regression Results: Kurtosis Measurement - Consumer Prices

Identification Long-run Restriction	Cholesky Yes		Cholesky No		High-Frequency IV Yes	
	24 months	36 months	24 months	36 months	24 months	36 months
	<i>PANEL A: Outlier threshold - small price changes - Constrained model</i>					
Kurt/Freq	-0.0144 (0.0166)	0.00129 (0.0199)	0.126* (0.0657)	0.250** (0.105)	0.0542** (0.0240)	0.0781** (0.0312)
Constant	-11.86*** (2.758)	-27.59*** (3.226)	-21.40** (10.60)	-47.87*** (16.79)	-34.59*** (3.369)	-54.89*** (4.337)
R^2	0.003	0.000	0.016	0.025	0.022	0.027
<i>PANEL B: Outlier threshold - small price changes - Unconstrained model</i>						
Freq/ \bar{F}	-4.956* (2.845)	-8.610** (3.372)	-54.91*** (13.80)	-92.39*** (21.35)	-16.46*** (3.953)	-21.42*** (4.886)
Kurt/ \bar{K}	4.087* (2.381)	5.031* (2.557)	2.605 (3.971)	1.546 (6.585)	5.693*** (1.950)	7.245*** (2.490)
Constant	-12.22*** (3.881)	-23.90*** (4.449)	41.75*** (13.79)	64.52*** (21.43)	-19.16*** (5.045)	-34.00*** (6.433)
R^2	0.065	0.132	0.482	0.534	0.345	0.337
<i>PANEL C: Outlier threshold - large price changes - Constrained model</i>						
Kurt/Freq	-0.00911 (0.0128)	0.00346 (0.0154)	0.0994* (0.0512)	0.195** (0.0817)	0.0441** (0.0183)	0.0632*** (0.0238)
Constant	-12.17*** (2.744)	-27.96*** (3.206)	-21.76** (10.55)	-48.22*** (16.74)	-34.83*** (3.340)	-55.16*** (4.299)
R^2	0.002	0.000	0.017	0.025	0.024	0.029
<i>PANEL D: Outlier threshold - large price changes - Unconstrained model</i>						
Freq/ \bar{F}	-4.922* (2.783)	-8.546** (3.301)	-54.12*** (13.73)	-91.05*** (21.26)	-16.39*** (3.867)	-21.34*** (4.780)
Kurt/ \bar{K}	4.246* (2.540)	5.404* (2.854)	7.798* (4.218)	10.40* (5.795)	6.289*** (2.198)	7.988*** (2.696)
Constant	-12.50*** (3.682)	-24.44*** (4.268)	35.49*** (12.60)	53.93*** (19.33)	-19.86*** (4.708)	-34.85*** (5.972)
R^2	0.069	0.137	0.479	0.530	0.351	0.343
Observations	223	223	223	223	223	223

Note: This table reports results of OLS results of the constrained model (equation 10) for CPI products relating product-specific CIR $CIR_T^{P_j}$ (expressed in %) to the ratio $Kurt/freq$ and OLS results of the unconstrained model (equation 11) relating product-specific CIR $CIR_T^{P_j}$ (expressed in %) to the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$. In Panels A and B, we have modified the thresholds defining very small price changes for the calculation of kurtosis: we have removed all price changes below 0.5% in absolute values (instead 0.1% in our baseline). In Panels C and D, we have modified thresholds defining very large price changes for the calculation of kurtosis (35% for instead of 25% in the baseline). Robust standard errors are reported in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table L: Regression Results: Role of sales - Consumer Prices

Identification Long-run Restriction	Cholesky Yes		Cholesky No		High-Frequency IV Yes	
	24 months	36 months	24 months	36 months	24 months	36 months
	<i>PANEL A: Excluding food, clothing/footwear, furnishings - Constrained model</i>					
Kurt/Freq	0.0467** (0.0229)	0.0692** (0.0290)	0.247** (0.107)	0.400** (0.172)	0.0993*** (0.0342)	0.129*** (0.0437)
Constant	-23.04*** (4.330)	-39.33*** (5.454)	-44.61** (19.70)	-75.84** (31.64)	-38.04*** (6.312)	-57.39*** (8.042)
R^2	0.034	0.048	0.053	0.054	0.078	0.079
<i>PANEL B: Excluding food, clothing/footwear, furnishings - Unconstrained model</i>						
$Freq/\bar{F}$	-8.726*** (1.218)	-12.88*** (1.514)	-64.71*** (9.067)	-105.6*** (14.89)	-19.92*** (2.792)	-25.41*** (3.633)
$Kurt/\bar{K}$	5.318** (2.683)	6.718** (3.062)	11.18** (5.568)	15.32* (8.428)	6.500*** (2.173)	8.122*** (2.724)
Constant	-14.39*** (3.494)	-25.38*** (3.996)	36.71*** (9.837)	59.41*** (15.72)	-13.44*** (3.537)	-25.64*** (4.559)
R^2	0.260	0.361	0.725	0.745	0.644	0.636
Observations	134	134	134	134	134	134
<i>PANEL C: % of sales prices below the median - Constrained model</i>						
Kurt/Freq	-0.000929 (0.0276)	0.0316 (0.0342)	0.245** (0.122)	0.461** (0.193)	0.133*** (0.0374)	0.185*** (0.0473)
Constant	-12.32** (5.440)	-30.73*** (6.705)	-45.06* (23.19)	-91.14** (36.55)	-45.07*** (7.003)	-69.35*** (8.771)
R^2	0.000	0.009	0.046	0.064	0.130	0.153
<i>PANEL D: % of sales prices below the median - Unconstrained model</i>						
$Freq/\bar{F}$	-8.872*** (2.363)	-13.71*** (2.662)	-72.13*** (12.97)	-118.9*** (20.28)	-23.27*** (3.285)	-29.96*** (4.144)
$Kurt/\bar{K}$	0.410 (2.749)	2.409 (3.355)	3.005 (7.785)	7.709 (11.84)	7.194** (2.802)	10.26*** (3.579)
Constant	-3.958 (4.625)	-15.85*** (5.405)	51.75*** (14.10)	72.27*** (21.18)	-13.92*** (4.492)	-28.66*** (5.665)
R^2	0.166	0.273	0.645	0.693	0.676	0.690
Observations	111	111	111	111	111	111

Note: This table reports results of OLS results of the constrained model (equation 10) for CPI products relating product-specific CIR $CIR_T^{P_j}$ (expressed in %) to the ratio $Kurt/freq$ and OLS results of the unconstrained model (equation 11) relating product-specific CIR $CIR_T^{P_j}$ (expressed in %) to the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$. In Panels A and B, we have removed goods of three broad sectors where sales concentrate (COICOP01.1 Food, COICOP03 Clothing/Footwear, and COICOP05 Furnishing goods). In Panels C and D, we have removed products for which the share of sales and promotions represent more than 11% of all price changes (this threshold corresponds to the median of this ratio over all CPI products). Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table M: Regression Results: Alternative Specifications - Sector Fixed Effects

Identification Long-run Restriction	Cholesky Yes		Cholesky No		High-Frequency IV Yes	
	24 months	36 months	24 months	36 months	24 months	36 months
<i>PANEL A: Producer prices - Constrained model</i>						
Kurt/Freq	0.0366 (0.0301)	0.0565* (0.0321)	0.393** (0.157)	0.647*** (0.239)	0.119** (0.0483)	0.153** (0.0662)
Constant	-14.93*** (2.701)	-27.83*** (3.153)	-23.02 (14.04)	-44.51** (20.87)	-27.53*** (3.258)	-44.18*** (3.918)
R^2	0.371	0.440	0.521	0.549	0.527	0.476
<i>PANEL B: Producer prices- Unconstrained model</i>						
$Freq/\bar{F}$	-1.705 (1.310)	-1.951 (1.274)	-11.87 (7.540)	-17.96 (11.49)	-2.621 (2.336)	-2.957 (3.375)
$Kurt/\bar{K}$	2.562 (1.964)	2.722 (1.984)	21.48** (10.41)	32.66** (15.38)	3.630 (2.902)	3.823 (3.963)
Constant	-14.62*** (2.986)	-26.72*** (3.177)	-18.73 (13.94)	-36.08* (19.64)	-24.30*** (2.908)	-39.57*** (3.588)
R^2	0.396	0.462	0.544	0.567	0.525	0.467
Observations	118	118	118	118	118	118
<i>PANEL C: Consumer prices - Constrained model</i>						
Kurt/Freq	0.0246 (0.0187)	0.0422* (0.0224)	0.135* (0.0786)	0.228* (0.127)	0.0821*** (0.0259)	0.110*** (0.0337)
Constant	1.697 (2.922)	-14.68*** (2.760)	-0.883 (5.940)	-28.71*** (9.072)	-34.83*** (2.039)	-58.09*** (3.063)
R^2	0.530	0.544	0.334	0.338	0.486	0.491
<i>PANEL D: Consumer prices - Unconstrained model</i>						
$Freq/\bar{F}$	-11.03*** (1.623)	-14.88*** (1.883)	-69.03*** (9.599)	-110.0*** (15.41)	-19.96*** (2.697)	-24.87*** (3.495)
$Kurt/\bar{K}$	3.499 (2.458)	4.357* (2.456)	-4.398 (4.934)	-9.689 (8.415)	4.020** (1.988)	5.231* (2.688)
Constant	16.46*** (4.419)	5.828 (4.571)	114.4*** (15.93)	157.7*** (25.50)	-4.886 (4.972)	-20.75*** (6.628)
R^2	0.678	0.743	0.765	0.766	0.747	0.723
Observations	223	223	223	223	223	223

Note: This table reports results of OLS results of the constrained model (equation 10) relating product-specific CIR $CIR_T^{P_j}$ (expressed in %) to the ratio $Kurt/freq$ and OLS results of the unconstrained model (equation 11) relating product-specific CIR $CIR_T^{P_j}$ (expressed in %) to the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$. Regressions include sectoral fixed effects at the 2-digit level for both CPI and PPI products (38 sectors for CPI and 24 sectors for PPI). Robust standard errors are reported in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table N: Regression Results: 2-year German Bond - High-Frequency IV

Identification Long-run Restriction	High-Frequency IV Yes		High-Frequency IV No	
	24 months	36 months	24 months	36 months
	<i>PANEL A: Producer Prices - Constrained model</i>			
Kurt/Freq	0.186*** (0.0669)	0.244*** (0.0775)	0.281*** (0.0923)	0.446*** (0.149)
Constant	-20.34*** (4.393)	-34.78*** (4.993)	-24.42*** (5.708)	-43.44*** (8.921)
R^2	0.069	0.091	0.092	0.095
<i>PANEL B: Producer Prices - Unconstrained model</i>				
$Freq/\bar{F}$	-5.148* (2.627)	-6.623** (2.973)	-8.765** (3.545)	-14.20** (5.673)
$Kurt/\bar{K}$	8.553** (3.931)	10.98** (4.451)	7.425 (4.794)	8.914 (7.257)
Constant	-15.64*** (5.071)	-28.50*** (5.651)	-10.82* (6.015)	-18.71** (8.594)
R^2	0.104	0.131	0.144	0.149
Observations	118	118	118	118
<i>PANEL C: Consumer Prices - Constrained model</i>				
Kurt/Freq	-0.0265 (0.0171)	-0.00828 (0.0158)	0.0427* (0.0224)	0.135*** (0.0447)
Constant	-3.369 (2.802)	-16.95*** (2.516)	-9.561*** (3.390)	-29.80*** (6.309)
R^2	0.010	0.001	0.016	0.041
<i>PANEL D: Consumer Prices - Unconstrained model</i>				
$Freq/\bar{F}$	-3.898 (2.999)	-5.798* (3.015)	-15.56*** (2.965)	-29.74*** (4.492)
$Kurt/\bar{K}$	-1.131 (1.960)	1.167 (1.774)	-6.750*** (2.408)	-10.33** (4.825)
Constant	-0.732 (3.180)	-13.06*** (3.008)	16.61*** (4.051)	22.42*** (7.358)
R^2	0.032	0.091	0.324	0.304
Observations	223	223	223	223

Note: This table reports results of OLS results of the constrained model (equation 10) relating product-specific CIR $CIR_T^{P_j}$ (expressed in %) to the ratio $Kurt/freq$ and OLS results of the unconstrained model (equation 11) relating product-specific CIR $CIR_T^{P_j}$ (expressed in %) to the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$. CIR are here calculated using the 2-year German bond rate as a policy rate and the model is identified using a high frequency instrument method. Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1

Table O: Regression Results: No drift - Sectoral Average Inflation < 5%

Identification Long-run Restriction	Cholesky Yes		Cholesky No		High-Frequency IV Yes	
	24 months	36 months	24 months	36 months	24 months	36 months
<i>PANEL A: Producer Prices - Constrained model</i>						
Kurt/Freq	0.0597* (0.0318)	0.0917** (0.0350)	0.650*** (0.217)	1.071*** (0.338)	0.190*** (0.0616)	0.242*** (0.0805)
Constant	-19.95*** (2.037)	-34.68*** (2.150)	-44.21*** (13.04)	-76.73*** (20.12)	-34.05*** (3.671)	-52.25*** (4.851)
R^2	0.042	0.085	0.127	0.142	0.129	0.118
<i>PANEL B: Producer Prices - Unconstrained model</i>						
$Freq/\bar{F}$	-2.615** (1.290)	-3.243** (1.324)	-24.50*** (8.995)	-38.62*** (14.11)	-6.092** (2.798)	-7.294* (3.787)
$Kurt/\bar{K}$	2.817* (1.604)	4.008** (1.808)	23.74** (10.17)	38.25** (15.81)	6.322** (3.095)	7.900* (4.056)
Constant	-17.54*** (1.637)	-31.44*** (1.790)	-15.03 (9.562)	-29.54** (14.74)	-26.00*** (2.985)	-42.29*** (4.029)
R^2	0.133	0.184	0.295	0.303	0.220	0.179
Observations	116	116	116	116	116	116
<i>PANEL C: Consumer Prices - Constrained model</i>						
Kurt/Freq	-0.0261 (0.0171)	-0.00991 (0.0207)	0.113 (0.0689)	0.243** (0.110)	0.0542** (0.0249)	0.0805** (0.0322)
Constant	-9.926*** (2.909)	-25.94*** (3.448)	-21.08* (11.46)	-49.77*** (18.13)	-35.30*** (3.613)	-56.26*** (4.624)
R^2	0.009	0.001	0.014	0.025	0.023	0.030
<i>PANEL D: Consumer Prices - Unconstrained model</i>						
$Freq/\bar{F}$	-4.783 (2.896)	-8.439** (3.398)	-54.07*** (13.80)	-91.18*** (21.26)	-16.45*** (3.852)	-21.44*** (4.740)
$Kurt/\bar{K}$	3.505 (2.169)	5.047** (2.524)	6.555 (4.698)	9.697 (7.353)	7.892*** (2.418)	10.43*** (3.105)
Constant	-11.07*** (3.655)	-23.47*** (4.291)	36.95*** (13.10)	54.15*** (20.17)	-21.73*** (4.977)	-37.79*** (6.361)
R^2	0.059	0.128	0.478	0.533	0.360	0.357
Observations	214	214	214	214	214	214

Note: This table reports results of OLS results of the constrained model (equation 10) relating product-specific CIR $CIR_T^{P_j}$ (expressed in %) to the ratio $Kurt/freq$ and OLS results of the unconstrained model (equation 11) relating product-specific CIR $CIR_T^{P_j}$ (expressed in %) to the ratio of the product-level frequency over its average $Freq/\bar{F}$ and the ratio of the product-level kurtosis over its average $Kurt/\bar{K}$. We remove products for which the average annual inflation is above 5% (in absolute values) over the sample period. Robust standard errors are reported in parentheses. *** p<0.01, ** p<0.05, * p<0.1