Public Information and Survey Expectations

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Abstract

We improve on the standard tests for the FIRE hypothesis by allowing for both public and private information, and find new interesting results. First, we propose a new empirical strategy that can accommodate this richer information structure, and find that the true degree of information rigidity is about a third higher than previously estimated. Second, we find that individual forecasts over-react to private information but under-react to public information. We show that this is consistent with a theory of strategic diversification incentives in forecast reporting, where forecasters are rational but report a biased measure of their true expectations. This has two effects. First, it generates what looks like behavioral "over-reaction" in expectations, and second biases the information rigidity estimate further downward. Overall, our results caution against the use of survey of forecasts as a direct measure of expectations, and suggest that the true underlying beliefs are rational, but suffer from a much larger degree of imperfect information than previously thought. This has particularly profound implications for monetary policy, where inflation expectations play a key role.

1 Introduction

Expectations play a crucial role in macroeconomic models, and hence the process through which agents form their expectation has been a fundamental, and often debated, topic. An important new development in the literature has emphasized the use of survey data, which holds the promise of providing direct, micro-level measurement of agent expectations. Using such data, Coibion and Gorodnichenko (2012, 2015) find significant evidence of incomplete and imperfect information, while another set of studies documents extensive predictability in individual forecast errors, which calls into question the classic paradigm of rational expectations itself (e.g. Bordalo et al. (2020)). Both strands of the literature, however, rely on the strong assumption that the information set of agents are contaminated with purely idiosyncratic errors, excluding any correlation in the noise of agent beliefs.

We empirically investigate the importance of public information, which introduces a common error component, and provide new evidence on the full information and rational expectations (FIRE) hypotheses. Our key findings are two-fold. First, we indeed find significant evidence of a common noise component in expectations, which biases the standard estimates of informational rigidity downwards. In particular, our findings indicate that prior studies have under-estimated the degree of information rigidity by about a third on average across a variety of macroeconomic indicators, and by up to 50% in the case of long-term interest rates.

Second, we find that, while individual forecasters tend to *over-react* to new information on average (in-line with previous findings of Bordalo et al. (2020)), the forecasts actually *under-react* to new public (i.e. common) information. We show that this finding is in-line with models where strategic diversification incentives lead forecasters to provide a biased measure of their actual beliefs when responding to surveys (e.g. Ottaviani and Sørensen (2006)). To quantify this effect and recover the true underlying expectations, we estimate a dynamic model of strategic incentives in reporting forecasts.

Our findings indicate that strategic incentives indeed play an important role, and hence caution against the use of survey forecasts as a direct measure of agent expectations. Specifically, the estimated model can fully account for the "over-reaction" puzzle in surveys that has received a lot of recent attention, suggesting that Rational Expectations is in fact a good model for the underlying true beliefs of agents. Moreover, the model estimates also show that the strategic incentives themselves bias the estimated information rigidity downward by a further 20% on average. Hence, our results indicate that expectations are rational after all, but the degree of imperfect information is significantly greater than previously thought.

In our empirical work we use data from the Survey of Professional Forecasters (SPF), which by now has become the common dataset for survey of macro forecasts, and we proceed in two steps. First, we show that the seminal estimate of informational rigidity of Coibion and Gorodnichenko (2015) is biased downward, due to common noise in the forecast errors. Such noise could be due to the incorporation of public signals in the forecasts, for example central bank's communications (e.g. Morse and Vissing-Jorgensen (2020)). We then provide a new empirical strategy robust to the presence of common noise by exploiting the panel dimension of the survey data. After correcting for this bias, the resulting Kalman gain estimate we find is on average 30% lower than that estimated by Coibion and Gorodnichenko (2015), revealing a significantly higher degree of information rigidity than previously found. Moreover, in the particularly interesting case of inflation expectations, we find that information rigidity is actually 40% higher than previous estimates, suggesting an even more important role for imperfect information in the transmission of monetary policy.

Second, we refine tests of rational expectations in survey data by also incorporating public signals and information in the benchmark regression specifications. In particular, we find that the *lagged* consensus forecast has an outsize importance, as it is both publicly available to all forecasters when they make their current forecasts and is also highly informative about the future realization of the variable being forecasted. We show that, while the individual forecasts in the SPF appear to over-weight new information on average (as already documented by Bordalo et al. (2020), hereafter BGMS), they significantly *under-weight* the information in the previous quarter's consensus forecast.

To rationalize our empirical findings, we build a global game model à la Morris and Shin (2002) with strategic substitutability, where the forecaster is balancing the desire to be right with the desire to stand-out. Intuitively, the forecaster would most like to both be right and also be the only person that gave a correct forecast, introducing strategic diversification incentives in forecast reporting as in Ottaviani and Sørensen (2006).[1] We assume

[1] This setting can be interpreted as a general version of a winner-take-all game, in which being accurate is

agents have access to two types of noisy signals – a private signal with idiosyncratic noise, and a noisy public signal that is the same for everyone. We then show that, because of strategic substitutability incentives in responding to the survey, agents optimally decide to bias their response towards private information, leading to overreaction to private information and underreaction to public information, as we also find in the data.

Moreover, we prove that in this setting it is always the case that individual forecasts appear as if they are *over-reacting* to new information on average, which can explain the recent findings in BGMS. Intuitively, because of agents' desire to stand out, when revising their expectations they put too high of a weight on their private signals which then results in forecastable errors that look like "over-reaction".

While BGMS ascribe this predictability of forecast errors to departures from rational expectation, we preserve rational expectation and depart instead from the assumption of honest reporting. Furthermore, in models of behavioral extrapolation agents over-react to all new information, both public and private, but this is inconsistent with our key finding that forecasts in fact significantly under-react to public information. Moreover, we further refine our test by considering variation in the cross-section of plausible public signals. In particular, in addition to the past release of the consensus forecasts, we consider another type of public information – the past realization of the macroeconomic variable being forecasted (e.g. lagged inflation). It turns out that this second type of public signal is under-weighted to a much smaller degree, which is again qualitatively consistent with the hypothesis of strategic incentives. Because the past consensus is not only a signal that ev- everyone has access to, but is also a direct estimate of everyone else's recent beliefs, strategic diversification incentives imply that it will be doubly under-weighted. Thus, we provide an alternative, rational explanation of the over-reaction evidence, that is also consistent with additional, nuanced facts we uncover.

Finally, we estimate a dynamic, quantitative version of our model which allows us to back-out and measure the actual expectations of the forecasters, after removing the estimated bias due to strategic incentives. Our key results in this section are two-fold. First, we find that the reported consensus estimate is significantly more accurate than the true average belief – with the mean-squared error of the true average belief being roughly 30% to 100% higher, depending on the variable. This result is intuitive – the simultaneous

rewarded but the prize is shared among correct forecasters (Ottaviani and Sørensen, 2006) over-weighting of private information and under-weighting of public information acts as a positive externality in terms of the consensus estimate, as it limits the effects of common errors. Second, the true beliefs are also significantly less dispersed in the cross-section, with the cross-sectional standard deviation of beliefs being roughly 80% lower than the dispersion of the forecasts reported to SPF. This is also intuitive, and is a hallmark of the forecasters' attempts to "stand-out". It also speaks to the fact that the true disagreement and dispersion of beliefs is much lower than otherwise thought, and thus also consistent with an even higher degree of information stickiness.

Related literature This paper relates to three strands of the literature. First, papers using survey of professional forecasters to test the full information hypothesis. A common finding in this literature is consensus underreaction, meaning a positive relation between consensus forecast errors and consensus forecast revisions (Crowe, 2010; Coibion and Gorodnichenko, 2012, 2015). We contribute by (i) proposing an empirical strategy to consistently estimate information rigidity in presence of public information, and (ii) using the structural model to estimate the actual information rigidity of honest beliefs in presence of strategic incentives in forecast reporting.

Another strand of the literature uses surveys to test the rational expectation hypothesis. In particular, Bordalo et al. (2020) documents individual overreaction, meaning a negative relation between individual forecast errors and individual forecast revisions. As individual forecast errors should not be predictable using current information, the authors interpret this predictability as evidence of behavioral biases in belief formation. We show that this evidence can be explained by a departure from truthful revelation while preserving rational expectations. Moreover, we document underreaction to public information, which is consistent with a strategic incentive model but not with models of extrapolative beliefs. In a contemporaneous paper, Broer and Kohlhas (2018) also use public information to improve on the test of RE, and find mixed results in terms of under and over-reaction. The key difference is that in our empirical approach we isolate the surprise component of any given public signal, which leads to higher estimation precision, while they use the raw value of the public signal itself (which is correlated with other variables on the right-hand side of the main regressions).

A third group of papers investigate the role of strategic incentives in forecasters behavior (for a review, Marinovic et al. 2013). The most related is Ottaviani and Sørensen

(2006), that propose two models of strategic substitutability and complementarity that leads forecasters to over or underweight private information in their reported forecast. While the spirit of the analysis is the same, we employ a more general Morris and Shin (2002) game and focus only on strategic substitutability. Moreover, we (i) introduce a dynamic model which allows use to focus on the time-series dimension of survey data, (ii) introduce public signals to distinguish between strategic incentive and behavioral theories and (iii) estimate a structural model to recover the underlying true expectations.

Overall, our results also speak to the fact that imperfect and noisy information is the dominant paradigm in the data, supporting earlier results on the importance of information rigidities in the expectation formation process, such as Kiley (2007), Klenow and Willis (2007), Korenok (2008), Dupor et al. (2010), Knotek II (2010), Coibion and Gorodnichenko (2012), and Coibion and Gorodnichenko (2015). In contrast to this literature, however, we also specifically identify and quantify the contribution of common noise components in the (imperfect) information sets of agents, and of the biasing effects of strategic incentives survey responders face when reporting expectations.

Structure of the paper The remaining sections of the paper are organized as follows. In Section 2 we describe the data and replicate the empirical findings of CG and BGMS, i.e. respectively underreaction of consensus forecast and overreaction of individual forecasts. Then, we propose a novel empirical methodology to estimate information stickiness in the presence of common noise, and also document a novel fact: forecast underreact to new public information and overreact to new private information. We show that noisy information and diagnostic expectations are not enough to explain all three facts, and therefore we turn to departure from truthful reporting. In section 3 we develop a static model of strategic substitutability in forecast reporting which can rationalize the empirical evidence and provide the additional empirical implication,. i.e. contemporaneous underreaction to new public information and overreaction to new private information. In section 4 we extend the model to a dynamic setting and estimate it, allowing us to recover honest forecast and correctly measure information rigidity.

2 Empirical Analysis

Data on forecast We collect data on forecasts from the Survey of Professional Forecasters (SPF), currently run by the Federal Bank Reserve of Philadelphia. In each quarter around

40 professional forecasters contribute to the SPF with forecasts for outcomes in the current quarter and the next four quarters. Individual forecasts are collected at the end of the second month of the quarter, and the forecasters are anonymous but identified by forecasters IDs.

The SPF covers macroeconomic and financial outcomes, providing both consensus forecast and an unbalanced panel of individual forecasts. These variables include GDP, price indices, consumption, investment, unemployment, government consumption, yields on government bonds and corporate bonds.

While most macroeconomic variables are provided in SPF in level, we follow BGMS and CG in transforming them into implied growth rate. Because of the timing of the survey, the actual variable realization in $t-1$ is known to the forecasters at the time of the forecasts. Therefore, we compute the forecasted growth rate of the variable from $t-1$ to $t+3$. We apply this method for GDP, price indices, consumption, investment and government consumption, while we keep the forecast in level for unemployment and financial variables.

We winsorize outliers by removing forecasts that are more than 5 interquartile ranges away from the median of each horizon in each quarter. We keep forecasters with at least 10 observations in all analyses. Consensus forecast are computed as the average of the individual forecasts available in each quarter. Appendix A provides a description of variable construction.

Data on actual outcomes The values of macroeconomics variables are released quarterly but subsequently revised. At the time of the survey, the forecasters can observe the first release of the values of the variables in $t-1$. To match as closely as possible the information set of the forecasters, we follow BGMS and use the initial releases of macroeconomics variables from Philadelphia Fed's Real-Time Data Set for Macroeconomics. Financial variables are not revised, so we use historical data from the Federal Reserve Bank of St. Louis.

We use actual realization to compute forecast errors, defined as actual realization minus forecast, and forecast revisions, defined as forecast in t on some horizon $t+h$ minus forecast in $t-1$ about the same horizon $t+h$.

Summary Statistics Table 1 presents the summary statistics for each series. Columns 1-5 reports the statistics for the consensus errors and revisions, including mean, standard

Table 1: Summary Statistics

	Consensus					Individual		
	Errors			Revisions		Forecast dispersion	Nonrev share	Pr(< 80% revise same direction)
	Mean	SD	SE	Mean	SD			
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Nominal GDP	-0.26	1.69	0.19	-0.14	0.68	1.00	0.02	0.80
GDP price index inflation	-0.28	0.58	0.08	-0.08	0.25	0.49	0.07	0.85
Real GDP	-0.30	1.78	0.19	-0.16	0.58	0.78	0.02	0.74
Consumer Price Index	-0.08	1.04	0.15	-0.11	0.68	0.54	0.06	0.66
Industrial production	-0.83	3.94	0.46	-0.49	1.19	1.57	0.01	0.72
Housing Start	-3.36	17.79	2.20	-2.31	5.93	8.34	0.00	0.68
Real Consumption	0.32	1.10	0.15	-0.06	0.41	0.61	0.03	0.78
Real residential investment	-0.46	8.32	1.19	-0.61	2.33	4.37	0.04	0.87
Real nonresidential investment	0.20	5.60	0.79	-0.22	1.71	2.31	0.03	0.74
Real state and local government consumption	0.04	2.96	0.38	0.14	1.10	2.09	0.07	0.91
Real federal government consumption	0.02	1.10	0.15	-0.05	0.33	0.98	0.11	0.93
Unemployment rate	0.01	0.68	0.08	0.05	0.32	0.30	0.18	0.66
Three-month Treasury rate	-0.51	1.14	0.16	-0.19	0.51	0.43	0.15	0.59
Ten-year Treasury rate	-0.48	0.73	0.11	-0.12	0.36	0.37	0.11	0.55
AAA Corporate Rate Bond	-0.46	0.82	0.11	-0.11	0.38	0.49	0.09	0.66

Notes: Columns 1 to 5 show statistics for consensus forecast errors and revisions. Errors are defined as actuals minus forecasts, where actuals are the realized outcome corresponding to the variable forecasted. Revisions are forecast provided in t minus forecasts provided in $t-1$ about the same horizon. Columns 6 to 8 show statistics for individual forecasts, with Newey West (1994) standard errors. Forecast dispersion is the average standard deviation of individual forecasts at each quarter. The share of nonrevisions is the average quarterly share of instances in which forecast revision is less than 0.01 percentage points. The final column shows the fraction of quarters where less than 80 percent of the forecasters revise in the same direction.

deviation and standard errors.. Forecast errors are statistically indistinguishable from zero for most of the series except for the interest rates, for which the forecasts are systematically above realizations. As argued by BGMS, this is likely due to the downward trend of the interest rates during the sample period, to which the forecast adjust only partially.

Columns 6-8 reports the summary statistics of the individual forecasts, including forecasts dispersion, share forecast with no meaningful revisions[2] and the probability that less than 80 percent of forecasters revise in the same direction. The large dispersion of forecasts and revisions at each point in time suggest a role for dispersed information among forecasts, which we embed in our model. The share of non revisions is often small, contrary to a sticky-information model a la Mankiw and Reis (2002), and revisions go in different direction, suggestion a noisy information setting instead.

Theoretical framework We consider a general framework of belief updating with dispersed information. In particular, consider a variable following an autoregressive process

$$x_t = \rho x_{t-1} + u_t \tag{1}$$

where u_t is an i.i.d. normally distributed innovation to x_t with variance ξ^{-1}. Agents cannot directly observe x_t, but instead receive a private signal and a public signal

$$\begin{aligned} g_t &= x_t + e_t \\ s_t^i &= x_t + \eta_t^i \end{aligned} \tag{2}$$

where η_t^i represents a normally distributed mean-zero noise with variance τ^{-1} which is i.i.d. across time and across agents, while e_t represents a normally distributed mean-zero noise with variance ν^{-1} which is i.i.d. only across time, but common across agents. Each agent generates forecast $\tilde{E}_t^i[x_{t+h}]$ at time t about the variable at h periods ahead according to

$$\begin{aligned} \tilde{E}_t^i[x_t] &= \tilde{E}_{t-1}^i[x_t] + G_1(g_t - \tilde{E}_{t-1}^i[x_t]) + G_2(s_t^i - \tilde{E}_{t-1}^i[x_t]) \\ \tilde{E}_t^i[x_{t+h}] &= \rho^h \tilde{E}_t^i[x_t] \end{aligned} \tag{3}$$

[2] We follow BGMS in categorizing a non-missing forecast as a non meaningful revision if the forecasts change by less than 0.01 percentage points.

where G_1 is the weight agent put on the public signal, G_2 the weight agent put on the private signal and $\tilde{E}$ is a potentially non-optimal expectation operator. This general format embeds the rational Bayesian case, where $\tilde{E}_t[x_t] = E_t[x_t]$, $G_1 = \frac{\nu}{\tau+\nu+\Sigma^{-1}}$ and $G_2 = \frac{\tau}{\tau+\nu+\Sigma^{-1}}$, with $\Sigma \equiv var(x_t - E^i_t[x_t])$.

2.1 Fact 1: under-reaction in consensus forecasts

Denote with $\bar{\tilde{E}}_t[x_{t+h}]$ the mean forecast across forecasters. Consensus forecast error is defined as the actual realization minus the average forecast: $\bar{fe}_{t+h,t} = x_{t+h} - \bar{\tilde{E}}_t[x_{t+h}]$. Similarly, consensus forecast revision is defined as the average forecast provided today minus the forecast provided in the previous period about the same horizon: $\bar{fr}_{t+h,t} = \bar{\tilde{E}}_t[x_{t+h}] - \bar{\tilde{E}}_{t-1}[x_{t+h}]$.

Coibion and Gorodnichenko (2015), hereafter CG, test the full information rational expectation hypothesis by regressing consensus forecast errors on consensus forecast revisions. Intuitively if information was complete and forecasters fully rational, it should not be possible to predict future errors using today's revisions, which would be in the forecasters' information sets. They run the following regression

$$\bar{fe}_{t+h,t} = \alpha + \beta_{CG}\bar{fr}_{t+h,t} + err_t \tag{4}$$

A positive $\beta_{CG} > 0$ would instead imply that a upward revisions today are associated on average with forecast not optimistic enough, and therefore a systematic undershooting of the actual realization of x. On the other hand, a negative $\beta_{CG} < 0$ would imply that a upward revisions today are associated on average with forecast too optimistic, and therefore a systematic overshooting of the actual realization of x. CG document that $\beta_{CG} > 0$ is a robust finding for inflation expectation, while BGMS replicate their analysis for a wide range of macroeconomic and financial series and confirm the same result. We replicate their results in Table 2, for both three and two quarters horizons, in our data set.

In order to interpret the result, derive the structural equivalent from 3:

$$\bar{fe}_{t+h,t} = \frac{1-G}{G}\bar{fr}_{t+h,t} - \frac{G_1}{G}\rho^h e_t + \varepsilon_{t+h,t} \tag{5}$$

where $\varepsilon_{t+h,t} = \sum_{i=1}^{h} \rho^{h-i} u_{t+i}$.

First, consider a setting without public information: $G_1 = 0$. In this case, $\beta_{CG} = \frac{1-G}{G}$.

A $\beta = 0$ would imply $G = 1$, meaning forecast adjust completely to new information, as implied by the FIRE hypothesis. On the other hand, $\beta > 0$ would imply $G < 1$, meaning stickiness in forecast updating, as in the noisy information setting. Therefore, in absence of public information, $\beta > 0$ rejects full information model, but not necessarily rational expectation.

Intuitively, in a dispersed information setting individual forecasters do not observe the information of the others, and therefore the average forecast revisions can predict average forecast errors. Because of private noise the individual signal is more noisy and less accurate that the average signal, even if each individual update their forecast optimally given their signal the average forecast is suboptimally sticky with respect to the average signal.

While this intuition is accurate in absence of public information, it is not if $G_1 > 0$. Because of the bias introduce by the public noise in the regression error, β_{CG} does not identify the information gain G.

Proposition 1 *If agents forecasts follow 3, the coefficient from regression 4 is given by:*

$$\beta_{CG} = \frac{\bar{\bar{\Sigma}} - [G\bar{\bar{\Sigma}} + \frac{G_1^2}{G}\nu^{-1}]}{G\bar{\bar{\Sigma}} + \frac{G_1^2}{G}\nu^{-1}} \tag{6}$$

with $\bar{\bar{\Sigma}} \equiv var(x_t - \bar{\bar{E}}_{t-1}[x_t]) = \frac{\rho^2[G_1^2\nu^{-1}]+\xi^{-1}}{1-\rho^2(1-G)^2}$. *Ig* $G_1 = 0$, $\beta_{CG} = \frac{1-G}{G}$

Corollary 1 *If agents forecasts follow 3, under rational expectation the coefficient* β_{CG} *of regression 4 is equal to zero in either of these two cases:*

1. *Public information very imprecise and private information infinitely precise:* $\nu = 0$, $\tau \to \infty$
2. *Private information very imprecise:* $\tau = 0$

while it is positive in any other case.

It follows that $\beta_{CG} = \frac{1-G}{G}$ only if $G_1 = 0$. The CG regressions doesn't provide an estimate of the consensus stickiness (or gain) in the presence of public noise. A β close to zero, as in the case of Ten-year Treasury rate and AAA Corporate Rate Bond, does not imply absence of stickiness in adjustment, but either no public information with perfectly informative private information (as in CG), or no private information. We next provide an accurate measure of forecast update stickiness, which generalized CG method to public information.

Table 2: Consensus errors on revisions

	3 quarters horizon			2 quarters horizon		
	β	SE	p-value	β	SE	p-value
Variable	(1)	(2)	(3)	(4)	(5)	(6)
Nominal GDP	0.52	0.31	0.09	0.20	0.11	0.07
GDP price index inflation	0.29	0.22	0.18	-0.03	0.11	0.77
Real GDP	0.65	0.20	0.00	0.33	0.16	0.05
Consumer Price Index	0.22	0.25	0.39	-0.07	0.09	0.44
Industrial production	0.21	0.55	0.70	0.33	0.23	0.15
Housing Start	0.38	0.25	0.13	0.91	0.14	0.00
Real Consumption	0.31	0.33	0.35	0.03	0.13	0.81
Real residential investment	1.22	0.32	0.00	0.56	0.14	0.00
Real nonresidential investment	1.21	0.21	0.00	0.52	0.09	0.00
Real state and local government consumption	-0.23	0.19	0.22	-0.28	0.17	0.10
Real federal government consumption	0.63	0.33	0.06	-0.11	0.08	0.20
Unemployment rate	0.74	0.16	0.00	0.58	0.11	0.00
Three-month Treasury rate	0.62	0.17	0.00	0.41	0.12	0.00
Ten-year Treasury rate	-0.01	0.09	0.91	-0.09	0.15	0.57
AAA Corporate Rate Bond	-0.03	0.17	0.88	0.05	0.11	0.64

Notes: The table shows the coefficient of the CG regression (consensus forecast errors on consensus revisions) with standard errors and corresponding p-values. Columns (1)-(3) consider a forecast horizon of 3 quarters, while columns (4)-(6) consider a forecast horizon of 2 quarter. Standard errors are robust to heteroskedasticity and Newey-West with the automatic bandwidth selection procedure of Newey and West (1994).

2.2 Fact 2: stickiness with public information

We propose a different empirical strategy to recover the weight on new information G in presence of public information. Define individual forecast revision as $fr^i_{t+h,t} = \tilde{E}^i_t[x_{t+h}] - \tilde{E}^i_{t-1}[x_{t+h}]$ and forecast surprise as $fs^i_{t+h,t} = x_{t+h} - \tilde{E}^i_{t-1}[x_{t+h}]$. Rewrite 3 in terms of forecast revision on forecast errors at some horizon h

$$fr^i_{t+h,t} = G(fs^i_{t+h,t}) + G_1\rho^h e_t + G_2\rho^h\eta^i_t - G\sum_{i=1}^{h}\rho^{h-i}u_{t+i} \tag{7}$$

The estimated coefficient from regressing individual forecast revisions on individual surprise does not converge to G, as it is biased by the correlation between x_{t+h} and the sum of fundamental disturbances from $t+1$ to $t+h$. However, by demeaning 7 at every t one gets

$$(fr^i_{t+h,t}) - (\bar{fr}_{t+h,t}) = G(\bar{\tilde{E}}_{t-1}[x_{t+h}] - \tilde{E}^i_{t-1}[x_{t+h}]) - G_2\rho^h\eta^i_t \tag{8}$$

Equation 8 provide an unbiased strategy to measure information stickiness. The coefficient estimated by regressing the difference between individual and consensus forecast revision on the difference between individual and consensus prior converge to the posted weight on new information G. In particular, consider the regression

$$fr^i_{t+h,t} - \bar{fr}_{t+h,t} = \beta(\bar{\hat{x}}_{t+h,t-1} - \hat{x}^i_{t+h,t-1}) + err^i_t \tag{9}$$

the OLS coefficient $\hat{\beta}$ is an efficient estimator of gain G. This approach is more general than the CG regression as it doesn't rely on the assumption of no public information.

We run regression 9 in a panel data with time fixed effect to demean forecast revisions and priors at each quarter. Table 3 reports the estimated coefficient, standard errors and p-value of the panel data regression, and the median coefficient from demeaned individual regressions. We estimate the gains for both 3 quarters and 2 quarters horizons. There are two important observations. First, our estimated gains are relatively stable across variables at the same horizon. Second, the gains are systematically larger at the shorter horizon, consistently with the idea of more accurate information about shorter horizons.

In absence of public information, the gain $G_{CG} = \frac{1}{1+\beta_{CG}}$ implied by the CG regression 4 should equal the gain estimated directly from regression 9. However, in presence of public information, the former is larger than the latter.

Proposition 2 *If agents forecasts follow 3, the difference between $G_{CG} \equiv \frac{1}{1+\beta_{CG}}$, where β_{CG} is the coefficient from regression 4, and G, where G is the coefficient from regression 9, is given by:*

$$G_{CG} - G = G\left(\frac{G_1^2\nu^{-1}}{G^2\bar{\tilde{\Sigma}}}\right) > 0 \tag{10}$$

with $\bar{\tilde{\Sigma}} \equiv var(x_t - \tilde{E}_{t-1}[x_t]) = \frac{\rho^2[G_1^2\nu^{-1}]+\xi^{-1}}{1-\rho^2(1-G)^2}$.

The two estimated gains are the same if $G_1^2 = 0$, meaning no public information in agents forecast updating. However, if agents have access to public information, $G_1^2 > 0$ and the

Table 3: Stickiness estimation

Variable	3 quarters horizon				2 quarters horizon			
	β (1)	SE (2)	p-value (3)	Median (4)	β (5)	SE (6)	p-value (7)	Median (8)
Nominal GDP	0.53	0.02	0.00	0.49	0.61	0.01	0.00	0.62
GDP price index inflation	0.49	0.03	0.00	0.52	0.63	0.02	0.00	0.68
Real GDP	0.56	0.03	0.00	0.52	0.63	0.02	0.00	0.62
Consumer Price Index	0.49	0.02	0.00	0.53	0.70	0.02	0.00	0.71
Industrial production	0.50	0.03	0.00	0.52	0.59	0.02	0.00	0.63
Housing Start	0.49	0.03	0.00	0.55	0.53	0.02	0.00	0.56
Real Consumption	0.49	0.03	0.00	0.48	0.63	0.03	0.00	0.62
Real residential investment	0.41	0.03	0.00	0.44	0.56	0.02	0.00	0.64
Real nonresidential investment	0.48	0.02	0.00	0.49	0.61	0.03	0.00	0.61
Real state and local government consumption	0.43	0.04	0.00	0.40	0.60	0.05	0.00	0.56
Real federal government consumption	0.47	0.04	0.00	0.48	0.62	0.03	0.00	0.62
Unemployment rate	0.49	0.02	0.00	0.54	0.56	0.02	0.00	0.62
Three-month Treasury rate	0.55	0.02	0.00	0.59	0.63	0.03	0.00	0.67
Ten-year Treasury rate	0.51	0.02	0.00	0.54	0.60	0.02	0.00	0.63
AAA Corporate Rate Bond	0.54	0.02	0.00	0.56	0.61	0.02	0.00	0.62

Notes: The table shows the result from regression 9. Columns (1)-(3) report coefficients, standard errors and p-values from the panel data regression with time and individual fixed effect. Column (4) reports the median coefficient from individual regressions. Columns (5)-(8) reports the same statistics for the 2 quarters horizon. Standard errors are robust to heteroskedasticity and clustered at both forecaster and time level.

difference is positive: the gain implied by the CG regression overestimate the actual gain in presence of public information (or underestimate the stickiness).

Table 4 reports the estimated gain G from regression 9 in columns (1) -(2) and the gain G_{CG} implied by CG estimate 4 in absence of public information in columns (3)-(4). Figure 1 show the comparison graphically. Our estimate gain is less volatile across variables and consistently lower than the one implied by CG. We report the difference $G_{CG} - G$ in table 4, columns (5)-(6), and plot it graphically in figure 2. The difference is consistently positive as implied by proposition 2. We test whether the difference is statistically larger than zero and report the p-value in column (7). The null hypothesis is rejected at the 10% confidence level for 7 variables out of 15.

The evidence indicate that public information is in fact an important part of the information set of forecasters. While the CG estimate implies very different gains across variables, with some series with no apparent stickiness (Ten-year Treasury rate, AAA Corporate Bond and Real federal government consumption), our novel approach suggests that the overall gain on new information is instead similar across variables but with differences in the role of public information. In particular, public exceed private information in impor-

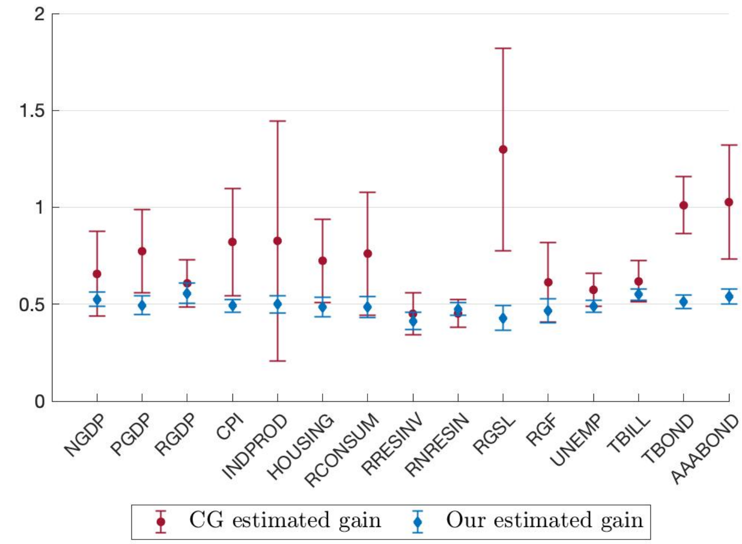

Figure 1: Estimated gains with the two methods

Notes: this figure plots the coefficient from panel data regression 13 with individual fixed effect. The blue diamonds represent the coefficient β_1 while the red circles represent the coefficient β_2. Standard errors are robust and clustered at the individual forecaster level.

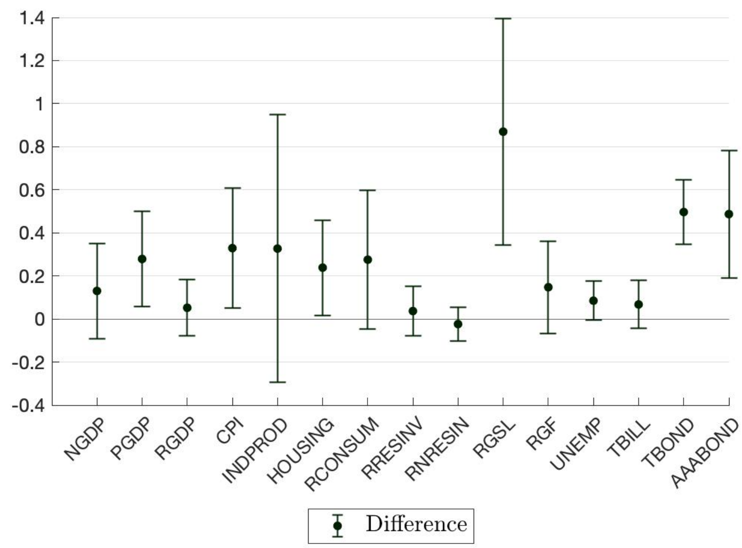

Figure 2: Difference between the two estimates

Notes: this figure plots the coefficient from panel data regression 13 with individual fixed effect. The blue diamonds represent the coefficient β_1 while the red circles represent the coefficient β_2. Standard errors are robust and clustered at the individual forecaster level.

Table 4: Difference between estimated gains

Variable	G_{CG} (1)	SE (2)	G (3)	SE (4)	Difference (5)	SE (6)	p-value (7)
Nominal GDP	0.66	0.13	0.53	0.02	0.13	0.13	0.17
GDP price index inflation	0.77	0.13	0.49	0.03	0.28	0.13	0.02
Real GDP	0.61	0.07	0.56	0.03	0.05	0.08	0.26
Consumer Price Index	0.82	0.17	0.49	0.02	0.33	0.17	0.03
Industrial production	0.83	0.38	0.50	0.03	0.33	0.38	0.19
Housing Start	0.72	0.13	0.49	0.03	0.24	0.13	0.04
Real Consumption	0.76	0.19	0.49	0.03	0.28	0.20	0.08
Real residential investment	0.45	0.07	0.41	0.03	0.04	0.07	0.30
Real nonresidential investment	0.45	0.04	0.48	0.02	-0.02	0.05	0.69
Real state and local government consumption	1.30	0.32	0.43	0.04	0.87	0.32	0.00
Real federal government consumption	0.61	0.12	0.47	0.04	0.15	0.13	0.13
Unemployment rate	0.57	0.05	0.49	0.02	0.08	0.05	0.06
Three-month Treasury rate	0.62	0.07	0.55	0.02	0.07	0.07	0.16
Ten-year Treasury rate	1.01	0.09	0.51	0.02	0.50	0.09	0.00
AAA Corporate Rate Bond	1.03	0.18	0.54	0.02	0.49	0.18	0.00

Notes: Columns (1)-(2) reports the implied gain from CG regressions of table 2. Columns (3)-(4) replicate the gain estimate in table 3. Columns (5)-(8) reports the difference between column (1) and (3), its standard error and the probability of rejecting the null of column (5) lower or equal to zero.

tance for financial variable, consistently with the idea that most of private information is priced in an efficient market.

2.3 Fact 3: over-reaction in individual forecast

Individual forecast error is defined as the actual realization minus the individual forecast: $fe^i_{t+h,t} = x_{t+h} - \tilde{E}^i_t[x_{t+h}]$. Similarly, individual forecast revision is defined as the individual forecast provided today minus the forecast provided in the previous period about the same horizon: $fr^i_{t+h,t} = \tilde{E}^i_t[x_{t+h}] - \tilde{E}^i_{t-1}[x_{t+h}]$.

Bordalo et al. (2018), hereafter BGMS, test the rational expectation hypothesis by regressing individual forecast errors on consensus forecast revisions. Intuitively, if forecasters were fully rational, it should not be possible to predict individual future errors using today individual revisions, which would be part of the forecasters' information sets. They run the following regression

$$fe^i_{t+h,t} = \alpha + \beta_{BMGS} fr^i_{t+h,t} + err^i_t \tag{11}$$

Proposition 3 *If agents forecasts follow 3, the coefficient of regression 11 is given by:*

$$\beta_{BGMS} = \frac{1-G}{G} - \frac{\frac{G_1^2}{K}\nu^{-1} + \frac{G_2^2}{G}\tau^{-1}}{G^2\tilde{\Sigma} + G_1^2\nu^{-1} + G_2^2\tau^{-1}} \tag{12}$$

with $\tilde{\Sigma} =\equiv var(x_t - \tilde{E}^i_{t-1}[x_t]) = \frac{\rho^2[G_1^2\nu^{-1}+G_2^2\tau^{-1}]+\xi^{-1}}{1-\rho^2(1-G)^2}$.

Corollary 2 *If agents forecasts follow 3, under rational expectation the coefficient* β_{BGMS} *of regression 11 is equal to zero.*

According to RE, individual forecast errors should not be predictable using individual forecast revisions. A positive $\beta_{BMGS} > 0$ would imply that after an positive surprise today agents don't update their forecast enough and they consistently underestimate the future value of x. On the opposite, a negative $\beta_{BMGS} < 0$ would imply that after an positive surprise today agents become too optimistic and they consistently overestimate the future value of x.

BMGS documents a robust $\beta_{BMGS} < 0$ for a wide range of macroeconomic and financial series. We replicate their panel data econometric specification with individual fixed effects in columns 1-3 of Table 6. However, the panel specification could introduce a bias in the coefficient.[3] Therefore we present also the median coefficients from the individual level regressions in the last column of Table 6, which confirms the panel results.

BGMS documents that overreaction holds also under the assumption that the fundamental process follows an AR(2). We replicate their finding in table 11 in appendix D.

2.4 Fact 4: under-reaction to public information in individual forecast

Motivated by the importance of public information in forecasters information set, we differentiate individual forecast reaction to private and public information. We document that while individual forecasts overreact to new information in general, they underreact to new public information.

In order to measure public information, we use the lagged consensus forecast, namely the average of the individual forecasts provided in the previous quarter about the same horizon. The consensus forecast is available to the forecasters at the time of the survey.

[3] RE implies that it is not possible for agents to predict their own forecast errors, $\beta_{BGMS} = 0$. However since the panel regression exploits the cross sectional variance in addition to the time series one, this specification effectively uses the average information set to pin down β_{BMGS} and not only the individual one.

Table 5: Individual errors on revisions

Variable	3 quarters horizon β_{BGMS} (1)	SE (2)	p-value (3)	Median (4)	2 quarters horizon β_{BGMS} (5)	SE (6)	p-value (7)	Median (8)
Nominal GDP	-0.25	0.08	0.00	-0.19	-0.11	0.06	0.10	-0.08
GDP price index inflation	-0.35	0.04	0.00	-0.35	-0.25	0.04	0.00	-0.26
Real GDP	-0.10	0.08	0.22	0.07	-0.07	0.10	0.47	0.04
Consumer Price Index	-0.30	0.09	0.00	-0.29	-0.24	0.07	0.00	-0.24
Industrial production	-0.30	0.14	0.04	-0.31	-0.01	0.10	0.94	0.03
Housing Start	-0.28	0.09	0.00	-0.28	0.12	0.05	0.03	0.07
Real Consumption	-0.26	0.12	0.04	-0.24	-0.16	0.08	0.07	-0.16
Real residential investment	-0.08	0.10	0.44	-0.07	0.07	0.08	0.41	0.02
Real nonresidential investment	0.08	0.13	0.56	0.15	0.10	0.07	0.18	0.10
Real state and local government consumption	-0.56	0.05	0.00	-0.52	-0.30	0.05	0.00	-0.26
Real federal government consumption	-0.48	0.04	0.00	-0.40	-0.28	0.04	0.00	-0.27
Unemployment rate	0.24	0.16	0.13	0.19	0.20	0.11	0.09	0.20
Three-month Treasury rate	0.24	0.10	0.03	0.29	0.14	0.08	0.09	0.21
Ten-year Treasury rate	-0.22	0.07	0.01	-0.24	-0.24	0.09	0.01	-0.27
AAA Corporate Rate Bond	-0.27	0.07	0.00	-0.32	-0.22	0.06	0.00	-0.29

Notes: The table reports the coefficients from the BGMS regression (individual forecast errors on individual revisions). Columns (1) to (3) shows the panel data with fixed effect coefficient with standard errors and corresponding p-values. Standard errors are robust to heteroskedasticity and clustered at both forecaster and time level. Columns (7) shows the median coefficient of the BGMS regression at the individual level.

To capture the surprise component in the public information, we compute the difference between the public signal and individual prior about the signal: $pi_{t,t+h} = g_t - \tilde{E}^i_{t-1}[x_{t+h}]$. We run the following regression:

$$fe^i_{t+h,t} = \alpha + \beta_1 fr^i_{t+h,t} + \beta_2 pi_{t+h,T} + err^i_t \tag{13}$$

where g_t is a public signal providing information about the variable at horizon $t+h$.

Proposition 4 *If agents forecasts follow 3, the coefficients of regression 13 are given by:*

$$\begin{aligned}\hat{\beta}_1 &= \frac{1-G_2}{G_2} - \frac{(\tilde{\Sigma}+\nu^{-1})G_2\frac{1}{\tau}}{(\tilde{\Sigma}+\nu^{-1})(G^2\tilde{\Sigma}^{-1}+G_1^2\nu^{-1}+G_2^2\tau^{-1})-(G\tilde{\Sigma}+G_1\nu^{-1})^2} \\ \hat{\beta}_2 &= -\frac{G_1}{G_2} + \frac{(G\tilde{\Sigma}+G_1\nu^{-1})G_2\frac{1}{\tau}}{(\tilde{\Sigma}+\nu^{-1})(G^2\tilde{\Sigma}+G_1^2\nu^{-1}+G_2^2\tau^{-1})-(G\tilde{\Sigma}+G_1\nu^{-1})^2}\end{aligned} \tag{14}$$

with $\tilde{\Sigma} \equiv var(x_t - \tilde{E}^i_{t-1}[x_t]) = \frac{\rho^2[G_1^2\nu^{-1}+G_2^2\tau^{-1}]+\xi^{-1}}{1-\rho^2(1-G)^2}$.

Corollary 3 *If agents forecasts follow 3, under rational expectation the coefficient β_1 and β_2 of regression 13 are equal to zero.*

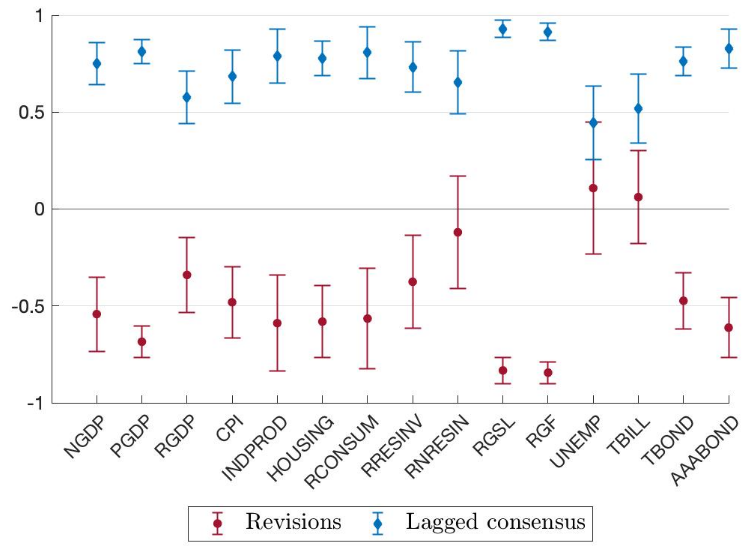

Figure 3: Forecast errors on forecast revisions and public information

Notes: this figure plots the coefficient from panel data regression 13 with individual fixed effect. The blue diamonds represent the coefficient β_1 while the red circles represent the coefficient β_2. Standard errors are robust and clustered at both time and individual forecaster level.

Under RE and truthful revelation, it would not be possible to predict individual errors using individual information sets. Panel A of Table **??** reports the panel data regressions with individual fixed effects and the median from individual regressions. Both specifications display a consistent $\beta_1 < 0$ and $\beta_2 > 0$ across variables, with few exceptions, meaning individual overreaction to private information and underreaction to public information. Figure 3 provide a graphical representation of the estimated coefficient from the panel data regression. In table 12 in appendix D we show that these results holds also under the assumption that the fundamental follows an AR(2) process.

Discussion We generalized two known fact in the information literature, fact 1 and fact 3, with the inclusion of public information and document two new facts, fact 2 and 4, which provides new insight on how to model agent's belief formation.

While our third fact, overreaction to new information, seems to indicates a departure for the rational expectation hypothesis, we distinguish between new private and public information and find that forecasters overreact to the first but underreact to the second, our fourth fact. This is not consistent with model of overreaction to all new information as Bordalo et al. (2020) and Broer and Kohlhas (2018), but it is consistent with other two

Table 6: Private and public information

Panel A: 3 quarters horizon

	Revision				Public signal			
	β_1	SE	p-value	Median	β_2	SE	p-value	Median
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Nominal GDP	-0.54	0.12	0.00	-0.44	0.75	0.07	0.00	0.76
GDP price index inflation	-0.68	0.05	0.00	-0.64	0.81	0.04	0.00	0.83
Real GDP	-0.34	0.12	0.01	-0.18	0.58	0.08	0.00	0.62
Consumer Price Index	-0.48	0.11	0.00	-0.46	0.68	0.08	0.00	0.69
Industrial production	-0.59	0.15	0.00	-0.60	0.79	0.08	0.00	0.78
Housing Start	-0.58	0.11	0.00	-0.53	0.78	0.05	0.00	0.71
Real Consumption	-0.57	0.16	0.00	-0.58	0.81	0.08	0.00	0.81
Real residential investment	-0.38	0.15	0.01	-0.39	0.73	0.08	0.00	0.66
Real nonresidential investment	-0.12	0.18	0.50	-0.10	0.65	0.10	0.00	0.51
Real state and local government consumption	-0.83	0.04	0.00	-0.81	0.93	0.03	0.00	0.89
Real federal government consumption	-0.84	0.03	0.00	-0.77	0.91	0.03	0.00	0.87
Unemployment rate	0.11	0.21	0.61	-0.02	0.44	0.11	0.00	0.42
Three-month Treasury rate	0.06	0.15	0.68	0.11	0.52	0.11	0.00	0.38
Ten-year Treasury rate	-0.47	0.09	0.00	-0.40	0.76	0.04	0.00	0.83
AAA Corporate Rate Bond	-0.61	0.09	0.00	-0.67	0.83	0.06	0.00	0.87

Panel B: 2 quarters horizon

	Revision				Public signal			
	β_1	SE	p-value	Median	β_2	SE	p-value	Median
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Nominal GDP	-0.35	0.09	0.00	-0.27	0.62	0.06	0.00	0.63
GDP price index inflation	-0.55	0.06	0.00	-0.50	0.70	0.04	0.00	0.66
Real GDP	-0.25	0.13	0.06	-0.14	0.54	0.08	0.00	0.54
Consumer Price Index	-0.38	0.09	0.00	-0.36	0.52	0.08	0.00	0.52
Industrial production	-0.16	0.12	0.19	-0.14	0.49	0.08	0.00	0.50
Housing Start	-0.15	0.08	0.08	-0.15	0.54	0.05	0.00	0.56
Real Consumption	-0.37	0.11	0.00	-0.29	0.64	0.07	0.00	0.69
Real residential investment	-0.13	0.11	0.23	-0.16	0.49	0.07	0.00	0.43
Real nonresidential investment	-0.02	0.10	0.81	-0.04	0.41	0.07	0.00	0.44
Real state and local government consumption	-0.63	0.08	0.00	-0.51	0.79	0.04	0.00	0.72
Real federal government consumption	-0.71	0.06	0.00	-0.64	0.80	0.04	0.00	0.74
Unemployment rate	0.09	0.15	0.57	0.03	0.39	0.10	0.00	0.37
Three-month Treasury rate	0.02	0.11	0.89	0.10	0.48	0.10	0.00	0.39
Ten-year Treasury rate	-0.46	0.11	0.00	-0.45	0.71	0.07	0.00	0.67
AAA Corporate Rate Bond	-0.49	0.08	0.00	-0.52	0.70	0.06	0.00	0.71

Notes: this table reports the coefficients of regression 13 (individual forecast errors on individual revisions and public information). Columns 1 to 3 show coefficient β_1 (forecast revision) from the panel regression with individual fixed effect, with standard errors and corresponding p-values. Standard errors are robust and clustered by forecaster. Column 4 shows the median coefficient of the same regression at the individual level. Columns 5 to 7 show coefficient β_2 (public information) from the panel regression with individual fixed effect, with standard errors and corresponding p-values. Standard errors are robust and clustered by forecaster. Column 8 shows the median coefficient of the same regression at the individual level. Panel A uses forecast at 3 quarters horizon and panel B uses forecast at 2 quarters horizon.

distinct frameworks. First, behavioral overconfidence, according to which agents overestimate the actual precision of their private signal. Second, strategic incentives, according to which agents are in fact rational in their beliefs formation, but provide to the public biased forecast in which their overweight private signals in order to stand out from the crowd.

While the behavioral overconfidence model departs from rational expectation, the strategic incentives model departs from truthful revelation. Both of them are consistent with facts 1-4, but with an important difference. Overconfident agents believe the forecast they post, and fact 2 correctly identify the stickiness of new information update. However, strategic agents beliefs are different from what they report to the survey, and fact 2 underestimate the actual stickiness (overestimate the gain) of their honest beliefs. Since posted forecasts overweight new private information, they appear to be less sticky than actual beliefs, with two consequences: first, the consensus forecast is more accurate, as it averages out private noise; second, the forecast dispersion is larger than the honest dispersion.

In the remaining part of the paper, we provide a general theoretical framework of strategic incentives consistent with our four empirical facts and estimate it structurally, in order to recover the actual stickiness of belief updating and forecast dispersion.

3 Strategic incentives in Forecast Reporting

3.1 Static setting

In this section we present a model of strategic interactions in forecast reporting, in which forecasters don't only want to provide accurate forecasts, but also to stand out with respect to the average forecast. We therefore depart from the assumption of truthful reporting by introducing strategic substitutability between forecasters.

The likelihood of strategic interactions in reporting is known in the forecasting literature. For example, Croushore (1997) suggests that "some [survey] participants might shade their forecasts more toward the consensus (to avoid unfavorable publicity when wrong), while others might make unusually bold forecasts, hoping to stand out from the crowd".[4] Ottaviani and Sørensen (2006) model the latter interpretation in a winner-take-all game. While we also focus on strategic substitutability, we keep a more general Morris

[4] While strategic considerations apply more intuitively to non-anonymous survey, in Appendix E we argue that they apply to anonymous survey as well, as forecasters are likely to provide the same forecast to both surveys.

and Shin (2002) global game setting.

In particular, the forecasters' problem is

$$min_{\hat{x}^i} \quad u^i = E^i\left[(\hat{x}^i - x)^2 - \lambda(\hat{x}^i - \bar{\hat{x}})^2\right] \tag{15}$$

where x is the true state and $\bar{\hat{x}} = \int \hat{x}^i di$ is the average of the reported forecast $\hat{x}^i$; $0 < \lambda < 1$ measures the degree of strategic substitutability in agent's reported forecasts.

The first order condition is:

$$\hat{x}^i = \frac{1}{1-\lambda}E^i[x] - \frac{\lambda}{1-\lambda}E^i[\bar{\hat{x}}] \tag{16}$$

If $\lambda = 0$, agents report their honest beliefs. If $\lambda > 0$, agents not only want to be accurate, but also to stand out with respect to the average forecast.

Information Suppose the actual x is unobserved. Forecasters have a common prior $x \sim N(\mu, \chi^{-1})$. Moreover, they received a private and a public signal, both unbiased and centered around the true x with some noise.

$$\begin{aligned} s^i &= x + \eta^i \\ g &= x + e \end{aligned} \tag{17}$$

with $\eta^i \sim N(0, \tau^{-1})$ and $e \sim N(0, \nu^{-1})$.

The resulting honest posterior is

$$E^i[x] = \mu + \gamma_1(g - \mu) + \gamma_2(s^i - \mu) \tag{18}$$

with $\gamma_1 = \frac{\nu}{\tau+\nu+\chi}$, $\gamma_2 = \frac{\tau}{\tau+\nu+\chi}$.

Introduce now strategic substitutability in expectation reporting as in equation 16. We guess a linear solution, solve for the fixed point problem and we get

$$\hat{x}^i = \mu + \delta_1(g - \mu) + \delta_2(s^i - \mu) \tag{19}$$

where $\delta_2 = \frac{\gamma_2}{(1-\lambda)+\lambda\gamma_2}$, $\delta_1 = \frac{(1-\lambda)\gamma_1}{(1-\lambda)+\lambda\gamma_2}$, $1 - \delta_1 - \delta_2 = \frac{(1-\lambda)(1-\gamma_1-\gamma_2)}{(1-\lambda)+\lambda\gamma_2}$. In order to stand out from the crowd, the forecasters overweight new private information in his posted forecast with respect to his actual beliefs ($\delta_2 > \gamma_2$) and underweight new public information ($\delta_1 < \gamma_1$).

At the same time, since the prior is common and new information partly private, the agent overweight new information as a whole ($1-\delta_1-\delta_2<1-\gamma_1-\gamma_2$).

Proposition 5 *In a strategic substitutability game as in 16, with* $0<\lambda<1$*, the coefficient of the individual regression 11 is given by:*

$$\beta_{BGMS}=\frac{-\lambda\tau\chi}{([(1-\lambda)\nu+\tau]^2+(1-\lambda)^2\nu\chi)} \tag{20}$$

Thus $\beta_{BGMS}<0$ *if* $\lambda>0$.

If $\lambda=0$, there is no strategic interaction between forecasters and they simply report their honest beliefs. In that case, $\beta_{BGMS}=0$, as forecast errors are not correlated with any information available in time t, and in particular her forecast revisions. This result follows directly from rational expectation. On the other hand, if $\lambda>0$, agents overweight private information to stand out from the crowd, which results in $\beta_{BGMS}<0$, meaning overreaction to new information. Ottaviani and Sørensen (2006) derive a similar result in a specific winner-take-all game only considering private information. The model reconciles the empirical result in section 2.

Proposition 6 *In a strategic substitutability game as in 16, with* $0<\lambda<1$*, the coefficient of the consensus regression 4 is given by:*

$$\beta_{CG}=\frac{(1-\lambda)\tau\chi}{([(1-\lambda)\nu+\tau]^2+[(1-\lambda)^2\nu+\tau]\chi)} \tag{21}$$

Thus $\beta_{CG}>0$ *if* $\lambda<1$.

If $\lambda=0$, there is no strategic interaction between forecasters and they simply report their honest beliefs. In that case, $\beta_{CG}>0$: the average forecast is sub-optimally sticky with respect to the average signal, which is less noisy than the individual one as shown by CG. On the other hand, if $\lambda>0$, agents overweight new information and the average forecast is less sticky. The higher is the strategic incentive λ, the lower is the rigidity of posted forecast. In the limit case of $\lambda\to 1$, individual forecasters adjust one-to-one their posteriors to new information, making the average forecast not sticky. It is not possible to have $\beta_{CG}<0$,consistently with the data.

Proposition 7 *In a strategic substitutability game as in 16, with $0 < \lambda < 1$, the coefficient of the individual regression 13 is given by:*

$$\begin{aligned} \beta_1 &= \frac{-\lambda(\nu + \chi)}{(\tau + \nu + \chi)} \\ \beta_2 &= \frac{\lambda\nu}{(\tau + \nu + \chi)} \end{aligned} \tag{22}$$

Thus $\beta_1 > 0$ and $\beta_2 < 0$ if $\lambda > 0$.

Proposition 7 represents our main theoretical result. If $\lambda = 0$, forecasters report their honest beliefs and both $\beta_1 = 0$ and $\beta_2 = 0$ as implied by rational expectation. However, with strategic incentives $\lambda > 0$, forecasters overweight private information and underweight public information, in order to stand from the crowd. This leads to an underreaction to public information, as measured by $\beta_2 > 0$, and overreaction to private information, as measured by $\beta_1 < 0$. This result reconciles our new empirical fact four documented in section 2.

3.2 Dynamic setting

We now extend the previous strategic incentives model to a dynamic setting. Assume the series follows a AR(1) process

$$x_t = \rho x_{t-1} + u_t \tag{23}$$

with $u \sim N(0, \xi^{-1})$.

Each agent receive a private signal s_t^i and a public signal g_t

$$\begin{aligned} s_t^i &= x_t + \eta_t^i \\ g_t &= x_t + e_t \end{aligned} \tag{24}$$

with $\eta_t^i \sim N(0, \tau^{-1})$, $e_t \sim N(0, \nu^{-1})$.

Honest beliefs Agents form beliefs about x at horizon h: $E_t^i[x_{t+h}] \equiv x_{t+h,t}^i$. The honest posterior belief about x is given by the Kalman filter

$$x_{t,t}^i = x_{t,t-1}^i + K_{1,1}(g_t - x_{t,t-1}^i) + K_{1,2}(s_t^i - x_{t,t-1}^i)$$

where the Kalman gains are

$$K_{1,1} = \frac{\nu}{\Sigma^{-1} + \nu + \tau}$$
$$K_{1,2} = \frac{\tau}{\Sigma^{-1} + \nu + \tau} \tag{25}$$

and the posterior forecast error variance

$$\Sigma \equiv E[(x_t - x^i_{t,t-1})(x_t - x^i_{t,t-1})'] \\ = \frac{-[(\rho^2-1)\xi + (\tau+\nu)] + \sqrt{[(\rho^2-1)\xi + (\tau+\nu)]^2 + 4(\tau+\nu)\xi}}{2} \tag{26}$$

Strategic interactions As in the previous section, the strategic substitutability in agents objective function leads them to report

$$\hat{x}^i_{t,t} = \frac{1}{1-\lambda} x^i_{t,t} - \frac{\lambda}{1-\lambda} E^i[\bar{\hat{x}}_{t,t}] \\ \hat{x}^i_{t+h,t} = \rho^h \hat{x}^i_{t,t} \tag{27}$$

where $\hat{x}^i_{t+h,t}$ is the forecast provided by individual i in t about realization in $t+h$, and $\bar{\hat{x}}_{t+h,t} = \int^i \hat{x}^i_{t+h,t} di$ is the average of forecasts provided in t about realization in $t+h$. If $\lambda = 0$, agents report their true beliefs. With $0 < \lambda < 1$, agents not only want to be accurate, but also to stand out with respect to the average forecast.

The model builds on Woodford (2001) and Coibion and Gorodnichenko (2012).[5] Following them, we average $\hat{x}^i_{t,t}$ across agents and use repeated substitution in 27 to express the reported average forecast as

$$F_t = -\frac{1}{1-\lambda} \sum_{k=0}^{\infty} \left(\frac{\lambda}{1-\lambda} \right)^k \bar{E}^{(k)}[x_t] = \frac{1}{1-\lambda} \bar{x}_{t+h,t} - \frac{\lambda}{1-\lambda} \bar{E}_t[\bar{\hat{x}}_{t+h,t}] \tag{28}$$

We guess and verify the law of motion for F_t and the other unobserved state variables.

[5] Our model depart from the latter in two dimensions. First, while they consider only strategic complementarity, we focus on strategic substitutability. Second, while they only consider consensus forecasts, we are interested in individual forecasts

In particular, we conjecture that the state vector evolves according to[6]

$$Z \equiv \begin{bmatrix} x_t \\ F_t \\ w_t \end{bmatrix} = MZ_{t-1} + m \begin{bmatrix} u_t \\ e_t \end{bmatrix} \tag{29}$$

Where

$$M = \begin{bmatrix} \rho & 0 & 0 \\ G & L & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \text{and} \qquad m = \begin{bmatrix} 1 & 0 \\ m_{2,1} & m_{2,2} \\ 0 & 1 \end{bmatrix} \tag{30}$$

the observable variables are the two signals about x_t

$$V_t^i \equiv \begin{bmatrix} g_t \\ s_t^i \end{bmatrix} = HZ_t + \begin{bmatrix} 0 \\ \eta_t^i \end{bmatrix} \tag{31}$$

where

$$H = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \tag{32}$$

Agents use their conjecture law of motion 29 and the observables 31 to infer the state using the individual Kalman filter. The posterior estimate of the state vector by agent i is

$$\begin{aligned} E_t^i[Z_t] &= ME_{t-1}^i[Z_{t-1}] + K(V_t^i - E_{t-1}^i[V_t]) \\ &= (I - KH)ME_{t-1}^i[Z_{t-1}] + KHMZ_{t-1} + KHm \begin{bmatrix} u_t \\ e_t \end{bmatrix} + K \begin{bmatrix} 0 \\ \eta_t^i \end{bmatrix} \end{aligned} \tag{33}$$

Where K is the Kalman gain. Average 33 to find the consensus believe on the state vector.

$$\bar{E}_t[Z_t] = (I - KH)M\bar{E}_{t-1}[Z_{t-1}] + KHMZ_{t-1} + KHm \begin{bmatrix} u_t \\ e_t \end{bmatrix} \tag{34}$$

[6] w_t takes care of the correlation between public signal and higher order beliefs F_t

From the definition on F_t in 28 it follows that

$$
\begin{aligned}
F_t &= \begin{bmatrix} \frac{1}{1-\lambda} & -\frac{\lambda}{1-\lambda} & 0 \end{bmatrix} \bar{E}_t[Z_t] \equiv \xi \bar{E}_t[Z_t] \\
&= \xi(I - KH)M\bar{E}_{t-1}[Z_{t-1}] + \xi KHMZ_{t-1} + \xi KHm \begin{bmatrix} u_t \\ e_t \end{bmatrix}
\end{aligned}
\tag{35}
$$

$$
\begin{aligned}
F_t =& ((1-\alpha)\rho + \alpha G)\bar{E}_{t-1}[x_{t-1}] + \alpha L \bar{E}_{t-1}[F_{t-1}] - C\rho \bar{E}_{t-1}[x_{t-1}] \\
& + C\rho x_{t-1} + C_1 e_t + C u_t \\
=& [\rho(1-\alpha) + \alpha G - (1-\alpha)L - C\rho]\bar{E}_{t-1}[x_{t-1}] + \\
& + LF_{t-1} + C\rho x_{t-1} + Cu_t + C_1 e_t
\end{aligned}
\tag{36}
$$

where we used 28 to substitute

$$
\alpha \bar{E}_t[F_{t-1}] = F_{t-1} - (1-\alpha)\bar{E}_{t-1}[x_{t-1}]
$$

and we defined

$$
C_1 \equiv \frac{K_{1,1} - \lambda(K_{2,1})}{1-\lambda}, \qquad C_2 \equiv \frac{K_{1,2} - \lambda K_{2,2}}{1-\lambda} \qquad \text{and} \qquad C = C_1 + C_2
$$

Equation 36 must equal the second line of the perceived law of motion 29. The solution to the fixed point is given by $G = C\rho$, $m_{2,1} = C$, $m_{2,2} = C_1$ and $L = \rho - G$.

Given the law of motion of unobserved state 29 and the observable 31, the posterior variance of the forecast solves the following Ricatti equation

$$
\begin{aligned}
&\Sigma \equiv E[(Z_t - Z^i_{t,t-1})(Z_t - Z^i_{t,t-1})'] \\
&\Sigma = M(\Sigma - \Sigma H' \left(H\Sigma H' + \begin{bmatrix} 0 & 0 \\ 0 & \tau^{-1} \end{bmatrix} \right)^{-1} H\Sigma)M' + m \begin{bmatrix} \xi^{-1} & 0 \\ 0 & \nu^{-1} \end{bmatrix} m'
\end{aligned}
\tag{37}
$$

and the Kalman filter is

$$
K = \Sigma H' \left(H\Sigma H' + \begin{bmatrix} 0 & 0 \\ 0 & \tau^{-1} \end{bmatrix} \right)^{-1}
\tag{38}
$$

Finally, the individual posted forecast is

$$\begin{aligned}\hat{x}^i_{t,t} &= \xi E^i_t[Z_t] \\ &= \hat{x}^i_{t,t-1} + C_1(g_t - \hat{x}^i_{t,t-1}) + C_2(s^i_t - \hat{x}^i_{t,t-1})\end{aligned} \tag{39}$$

Note that the individual posted forecast updating in 39 is similar to individual Kalman Filter in 33, with C_1 and C_2 as "modified" gains in place of K_1 and K_2. In particular, if $\lambda = 0$, $C_1 = K_1$ and $C_2 = K_2$: with no strategic incentives, agents simply report their honest beliefs. However, when $\lambda > 0$, one can show that $C_1 < K_1$ and $C_2 > K_2$: agents underweight public information and overweight private information in their posted forecast.[7]

The posted forecast updating 39 mirrors the general framework 3 in section 2 with $G_1 = C_1$, $G_2 = C_2$ and $\tilde{E}_t[x_t] = \hat{x}_{t,t}$. Therefore the coefficient from regressions 4, 9, 11 and 13 follows from propositions 1-4.

4 Structural estimation

We now proceed to estimate our model to test its ability to match the data recover the honest beliefs of forecasters to compute the actual belief stickiness. First, for each series we estimate the autoregressive coefficient ρ and the fundamental disturbance variance $\sigma^2_u \equiv \xi^{-1}$ directly from the data. Then we use the simulated method of moments to estimate the remaining parameters of the model: the public noise variance $\sigma^2_e \equiv \nu^{-1}$, the private noise variance $\sigma^2_\eta \equiv \tau^{-1}$ and the strategic incentive parameter λ. We prefer the simulated method of moments to maximum likelihood as the simplicity of our model come at the cost of likely misspecification which would be problematic when using maximum likelihood.

The data moments we target are the mean cross sectional dispersion of forecast errors, the coefficient β_1 from the public information regression 13 and the posted gain C from regression 9. We choose these three moments as they are differently affected by the three parameters to be estimated and therefore provide good identification.[8]

[7] To see this, note that $K_{1,1} < K_{2,1}$: intuitively, the public signal is more informative about the average forecast than about the actual state, because of the average belief depends also on the public noise. On the other hand, $K_{1,2} > K_{2,2}$: intuitively, the private signal is less informative about the average forecast than about the actual state, as the average forecast also depends on the public noise.

[8] While larger strategic incentive parameter λ increases posted gain C, an increase in either private or public noise decreases it. On the other hand, the coefficient β_1 decreases in private noise and strategic incentives but increase in public noise (see proposition 7). Finally, strategic incentives and public noise always increase mean dispersion, while private noise initially increases it and then decreases it.

Table 7: Estimasted parameters

Variable	ρ (1)	$\frac{\sigma_e}{\sigma_u}$ (2)	$\frac{\sigma_\eta}{\sigma_u}$ (3)	λ (4)
Nominal GDP	0.93	1.48	1.70	0.74
GDP price index inflation	0.93	1.60	2.13	0.88
Real GDP	0.80	1.30	1.36	0.47
Consumer Price Index	0.78	1.38	1.60	0.61
Industrial production	0.85	1.28	1.86	0.68
Housing Start	0.85	1.38	1.81	0.70
Real Consumption	0.87	1.33	1.84	0.67
Real residential investment	0.89	1.56	1.74	0.49
Real nonresidential investment	0.89	2.37	1.28	0.25
Real state and local government consumption	0.89	1.32	2.79	0.90
Real federal government consumption	0.80	1.29	2.90	0.87
Ten-year Treasury rate	0.83	1.81	1.56	0.72
AAA Corporate Rate Bond	0.85	1.76	1.82	0.87

Table 7 reports the estimated parameters for each series, while table 8 reports targeted and untargeted moments in the model and in the data. While the model is able to match the targeted model in columns 1-6, it also does a good job in matching the untargeted moments in columns 7-12 for most of the series. The only exceptions are the CG coefficient for the financial series in column 7-8 and a general underestimation of the public information coefficient in column 11-12.

We use the model to recover the honest beliefs of forecasters behind the biased posted forecast, and compute the related moments. We compare them with the moments calculated on the posted forecast in table 9. First, columns 1-3 reports the weight on new information in posted forecast, in honest beliefs and their ratio. Hones beliefs are stickier than posted forecasts, as the latter overweight new information. The magnitude of the difference can be appreciated by looking at the difference between posted and honest consensus mean forecast error. Because of the individual overweight of new information, the average posted forecast is less sticky and therefore more accurate than the honest one. For some variable (nominal GDP, CPI, Housing Start, Ten-year and AAA bond rate) the honest error is more than 1.5 times the posted one. Finally, columns 7-8 compare honest and posted cross sectional dispersion of forecast errors, which is used as a proxy for uncertainty in the literature (e.g. Kozeniauskas et al. 2018). The overweight of private information

Table 8: Moments in data and model

	Targeted moments						Untargeted moments					
	Mean Dispersion		C		β_1		β_{CG}		β_{BGMS}		β_2	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Nominal GDP	1.49	1.49	0.53	0.53	-0.54	-0.54	0.52	0.41	-0.25	-0.31	0.75	0.21
GDP price index inflation	0.33	0.33	0.49	0.49	-0.68	-0.68	0.29	0.50	-0.35	-0.44	0.81	0.31
Real GDP	0.92	0.92	0.56	0.56	-0.34	-0.34	0.65	0.33	-0.10	-0.15	0.57	0.13
Consumer Price Index	0.31	0.31	0.49	0.49	-0.48	-0.48	0.22	0.38	-0.30	-0.24	0.67	0.16
Industrial production	3.71	3.71	0.50	0.50	-0.59	-0.59	0.21	0.22	-0.30	-0.22	0.79	0.26
Housing Start	110.04	110.04	0.49	0.49	-0.58	-0.58	0.38	0.32	-0.28	-0.28	0.78	0.23
Real Consumption	0.51	0.51	0.49	0.49	-0.56	-0.56	0.31	0.25	-0.26	-0.23	0.80	0.23
Real residential investment	27.03	27.03	0.41	0.41	-0.37	-0.37	1.22	0.40	-0.08	-0.17	0.73	0.11
Real nonresidential investment	7.38	7.38	0.48	0.48	-0.12	-0.12	1.21	0.94	0.08	-0.10	0.65	0.01
Real state and local government consumption	1.41	1.41	0.47	0.47	-0.84	-0.84	0.63	0.17	-0.48	-0.41	0.91	0.45
Real federal government consumption	6.40	6.40	0.43	0.43	-0.83	-0.83	-0.23	0.12	-0.56	-0.35	0.93	0.37
Ten-year Treasury rate	0.17	0.17	0.51	0.51	-0.47	-0.47	-0.01	0.69	-0.22	-0.38	0.76	0.09
AAA Corporate Rate Bond	0.34	0.34	0.54	0.54	-0.61	-0.61	-0.03	0.62	-0.27	-0.48	0.83	0.18

increase substantially the dispersion of forecasts, as the honest beliefs dispersion is less than half the posted one for most of the series.

Table 9: Posted and honest moments

	Gain			Consensus MSE			Dispersion		
	Posted	Honest	Ratio	Posted	Honest	Ratio	Posted	Honest	Ratio
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Nominal GDP	0.53	0.40	0.76	0.49	1.07	2.19	1.49	0.29	0.19
GDP price index inflation	0.49	0.32	0.66	0.05	0.14	2.92	0.33	0.02	0.06
Real GDP	0.56	0.49	0.88	0.78	1.14	1.47	0.92	0.41	0.44
Consumer Price Index	0.49	0.40	0.82	0.23	0.36	1.58	0.31	0.08	0.27
Industrial production	0.50	0.44	0.87	3.51	5.11	1.46	3.71	0.60	0.16
Housing Start	0.49	0.40	0.82	69.95	115.75	1.65	110.04	18.10	0.16
Real Consumption	0.49	0.42	0.86	0.46	0.68	1.49	0.51	0.09	0.18
Real residential investment	0.41	0.36	0.87	29.60	40.95	1.38	27.03	10.76	0.40
Real nonresidential investment	0.48	0.43	0.90	4.12	5.30	1.29	7.38	6.01	0.82
Real state and local government consumption	0.47	0.40	0.86	0.54	0.81	1.51	1.41	0.02	0.02
Real federal government consumption	0.43	0.39	0.90	5.96	7.49	1.26	6.40	0.14	0.02
Ten-year Treasury rate	0.51	0.33	0.64	0.04	0.11	2.55	0.17	0.05	0.27
AAA Corporate Rate Bond	0.54	0.29	0.54	0.04	0.14	3.75	0.34	0.04	0.11

References

BORDALO, P., N. GENNAIOLI, Y. MA, AND A. SHLEIFER (2020): "Overreaction in macroeconomic expectations," *American Economic Review*, 110, 2748–82.

BORDALO, P., N. GENNAIOLI, AND A. SHLEIFER (2018): "Diagnostic expectations and credit cycles," *The Journal of Finance*, 73, 199–227.

BROER, T. AND A. KOHLHAS (2018): "Forecaster (mis-) behavior," .

COIBION, O. AND Y. GORODNICHENKO (2012): "What can survey forecasts tell us about information rigidities?" *Journal of Political Economy*, 120, 116–159.

——— (2015): "Information rigidity and the expectations formation process: A simple framework and new facts," *American Economic Review*, 105, 2644–78.

CROUSHORE, D. D. (1997): "The Livingston survey: Still useful after all these years," *Business Review-Federal Reserve Bank of Philadelphia*, 2, 1.

CROWE, C. (2010): "Consensus forecasts and inefficient information aggregation," *IMF Working Papers*, 1–43.

DUPOR, B., T. KITAMURA, AND T. TSURUGA (2010): "Integrating Sticky Information and Sticky Prices," *Review of Economics and Statistics*, 92, 657–669.

EUROPEAN CENTRAL BANK (2014): "Results of the Second Special Questionnaire for Participants in the ECB Survey of Professional Forecasters," .

KILEY, M. T. (2007): "A quantitative comparison of sticky-price and sticky-information models of price setting," *Journal of Money, Credit and Banking*, 39, 101–125.

KLENOW, P. J. AND J. L. WILLIS (2007): "Sticky information and sticky prices," *Journal of Monetary Economics*, 54, 79–99.

KNOTEK II, E. S. (2010): "A Tale of Two Rigidities: Sticky Prices in a Sticky-Information Environment," *Journal of Money, Credit and Banking*, 42, 1543–1564.

KORENOK, O. (2008): "Empirical comparison of sticky price and sticky information models," *Journal of Macroeconomics*, 30, 906–927.

KOZENIAUSKAS, N., A. ORLIK, AND L. VELDKAMP (2018): "What are uncertainty shocks?" *Journal of Monetary Economics*, 100, 1–15.

MANKIW, N. G. AND R. REIS (2002): "Sticky information versus sticky prices: a proposal to replace the New Keynesian Phillips curve," *The Quarterly Journal of Economics*, 117, 1295–1328.

MARINOVIC, I., M. OTTAVIANI, AND P. SORENSEN (2013): "Forecasters' objectives and strategies," in *Handbook of economic forecasting*, Elsevier, vol. 2, 690–720.

MORRIS, S. AND H. S. SHIN (2002): "Social value of public information," *american economic review*, 92, 1521–1534.

MORSE, A. AND A. VISSING-JORGENSEN (2020): "Information Transmission from the Federal Reserve to the Stock Market: Evidence from Governors' Calendars," Tech. rep., working paper.

OTTAVIANI, M. AND P. N. SØRENSEN (2006): "The strategy of professional forecasting," *Journal of Financial Economics*, 81, 441–466.

WOODFORD, M. (2001): "Imperfect common knowledge and the effects of monetary policy," Tech. rep., National Bureau of Economic Research.

A Variable definitions

All variables come from the Survey of Professional Forecasters, collected by the Federal Reserve Bank of Philadelphia. All surveys are collected around the 3rd week of the middle month in the quarter. In this section, x_t indicate the actual value and $F_t x_t + h$ the forecast provided in t about horizon h. All actual values of macroeconomic series (1-12) use the first release level, which are available to forecasters in the following quarter. We transform the macroeconomic level in growth. The series are constructed similarly as Bordalo et al. (2020)

1. NGDP

   - Variable: nominal GDP.
   - Question: The level of nominal GDP in the current quarter and the next 4 quarters.
   - Forecast: Nominal GDP growth from end of quarter $t-1$ to end of quarter $t+3$: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$
   - Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$
   - Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$

2. RGDP

   - Variable: real GDP.
   - Question: The level of real GDP in the current quarter and the next 4 quarters.
   - Forecast: real GDP growth from end of quarter $t-1$ to end of quarter $t+3$: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$
   - Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$
   - Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$

3. PGDP

   - Variable: GDP deflator.
   - Question: The level of GDP deflator in the current quarter and the next 4 quarters.

- Forecast: GDP price deflator inflation from end of quarter $t-1$ to end of quarter $t+3$: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$

4. CPI

- Variable: Consumer Price Index.
- Question: CPI growth rate in the current quarter and the next 4 quarters.
- Forecast: CPI inflation from end of quarter $t-1$ to end of quarter $t+3$: $F_t(z_t/4+1) * F_t(z_{t+1}/4+1) * F_t(z_{t+2}/4+1) * F_t(z_{t+3}/4+1)$, where z is the annualized quarterly CPI inflation in quarter t.
- Revision: $F_t(z_t/4+1) * F_t(z_{t+1}/4+1) * F_t(z_{t+2}/4+1) * F_t(z_{t+3}/4+1) - F_{t-1}(z_t/4+1) * F_{t-1}(z_{t+1}/4+1) * F_{t-1}(z_{t+2}/4+1) * F_{t-1}(z_{t+3}/4+1)$
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$. Real time data is not available before 1994Q3. For actual periods prior to this date, we use data published in 1994Q3 to measure the actual outcome.

5. RCONSUM

- Variable: Real consumption.
- Question: The level of real consumption in the current quarter and the next 4 quarters.
- Forecast: GDP price deflator inflation from end of quarter $t-1$ to end of quarter $t+3$: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$

6. INDPROD

- Variable: Industrial production index.
- Question: The average level of the industrial production index in the current quarter and the next 4 quarters.

- Forecast: Growth of the industrial production index from quarter $t-1$ to quarter $t+3$: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$

7. RNRESIN

- Variable: Real non-residential investment.
- Question: The level of real non-residential investment in the current quarter and the next 4 quarters.
- Forecast: Growth of real non-residential investment from quarter $t-1$ to quarter $t+3$: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$

8. RRESIN

- Variable: Real residential investment.
- Question: The level of real residential investment in the current quarter and the next 4 quarters.
- Forecast: Growth of real residential investment from quarter $t-1$ to quarter $t+3$: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$

9. RGF

- Variable: Real federal government consumption.
- Question: The level of real federal government consumption in the current quarter and the next 4 quarters.
- Forecast: Growth of real federal government consumption from quarter $t-1$ to quarter $t+3$: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$

- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$

10. RGSL

- Variable: Real state and local government consumption.
- Question: The level of real state and local government consumption in the current quarter and the next 4 quarters.
- Forecast: Growth of real state and local government consumption from quarter $t-1$ to quarter $t+3$: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$

11. HOUSING

- Variable: Housing starts.
- Question: The level of housing starts in the current quarter and the next 4 quarters.
- Forecast: Growth of housing starts from quarter $t-1$ to quarter $t+3$: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$

12. UNEMP

- Variable: Unemployment rate.
- Question: The level of average unemployment rate in the current quarter and the next 4 quarters.
- Forecast: Average quarterly unemployment rate in quarter $t+3$: $F_t x_{t+3}$
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$
- Actual: x_{t+3}

13. TB3M

- Variable: 3-month Treasury rate.
- Question: The level of average 3-month Treasury rate in the current quarter and next 4 quarters.
- Forecast: Average quarterly 3-month Treasury rate in quarter $t+3$: $F_t x_{t+3}$
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$
- Actual: x_{t+3}

14. TN10Y

- Variable: 10-year Treasury rate.
- Question: The level of average 10-year Treasury rate in the current quarter and next 4 quarters.
- Forecast: Average quarterly 10-year Treasury rate in quarter $t+3$: $F_t x_{t+3}$
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$
- Actual: x_{t+3}

15. AAA

- Variable: AAA corporate bond rate.
- Question: The level of average AAA corporate bond rate in the current quarter and next 4 quarters.
- Forecast: Average quarterly AAA corporate bond rate in quarter $t+3$: $F_t x_{t+3}$
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$
- Actual: x_{t+3}

B Proofs

Proposition 1. Let $\hat{x}_{t,t} \equiv \tilde{E}_t[x_t]$. From 3

$$\underbrace{(x_t - \bar{\hat{x}}_{t,t})}_{\bar{fe}_{t,t}} = \frac{1-G}{G}\underbrace{(\hat{\bar{x}}_{t,t} - \rho\hat{\bar{x}}_{t-1,t-1})}_{\bar{fr}_{t,t}} - \frac{G_1}{G}e_t \tag{40}$$

therefore by running CG regression 4, the regressor $\bar{fr}_{t,t}$ is correlated with the unobservable error. The resulting $\hat{\beta}_{CG}$ is equal to:

$$\begin{aligned}
\beta_{CG} &= \frac{1-G}{G} + \frac{cov(G[x_t - \rho\bar{\hat{x}}_{t-1,t-1}] + G_1 e_t, -\frac{G_1}{G}e_t)}{var(G[x_t - \rho\bar{\hat{x}}_{t-1,t-1}] + G_1 e_t)} \\
&= \frac{1-G}{G} - \frac{\frac{G_1^2}{C}\nu^{-1}}{G^2 var(x_t - \rho\bar{\hat{x}}_{t-1,t-1}) + G_1^2\nu^{-1}} \\
&= \frac{var(x_t - \rho\bar{\hat{x}}_{t-1,t-1}) - [G var(x_t - \rho\bar{\hat{x}}_{t-1,t-1}) + \frac{G_1^2}{G}\nu^{-1}]}{G var(x_t - \rho\bar{\hat{x}}_{t-1,t-1}) + \frac{G_1^2}{G}\nu^{-1}}
\end{aligned} \tag{41}$$

but $var(x_t - \rho\bar{\hat{x}}_{t-1,t-1}) \neq \bar{\Sigma} \equiv var(x_t - \rho\bar{x}_{t-1,t-1})$. In steady state

$$\begin{aligned}
x_{t+1} - \rho\bar{\hat{x}}_{t,t} =& \rho(x_t - \bar{\hat{x}}_{t,t}) + u_{t+1} \\
\hat{\Sigma} =& \rho^2\bar{\hat{\Phi}} + \xi^{-1}
\end{aligned} \tag{42}$$

where $\bar{\hat{\Phi}} = var(x_t - \hat{x}_{t,t})$.

$$\begin{aligned}
x_t - \bar{\hat{x}}_{t,t} =& (1-G)[x_t - \rho\bar{\hat{x}}_{t-1,t-1}] - G_1 e_t \\
\bar{\hat{\Phi}} =& (1-G)^2\bar{\hat{\Sigma}} + G_1^2\nu^{-1}
\end{aligned} \tag{43}$$

Substitute and solve for $\hat{\Sigma}$

$$\bar{\hat{\Sigma}} = \frac{\rho^2[C_1^2\nu^{-1}] + \xi^{-1}}{1 - \rho^2(1-G)^2} \tag{44}$$

■

Corollary 1. With rational expectation, $G_1 = \frac{\nu}{\tau+\nu+\Sigma^{-1}}$ and $G_2 = \frac{\tau}{\tau+\nu+\Sigma^{-1}}$, with $\Sigma \equiv var(x_t - E_t^i[x_t])$ and $\bar{\Sigma} \equiv var(x_t - \bar{E}_t[x_t])$.

- From 41, it follows that if $\nu = 0$, $G_1 = 0$ and $\beta_{CG} = \frac{1-G_2}{G_2}$. Moreover, if $\tau \to \infty$,

$G_2 = 1$ and $\beta_{CG} = 0$.

- From 41, it follows that If $\tau = 0$, $\Sigma = \bar{\Sigma}$, $G = G_1$ and $\frac{1-G}{G} = \frac{\Sigma^{-1}}{\nu^{-1}}$. Therefore $\beta_{CG} = 0$.

■

Proposition 2. From 41, $\frac{1}{1+\beta_{CG}}$ is given by

$$\begin{aligned} \frac{1}{1+\beta_{CG}} &= \frac{1}{1+\frac{1-G}{G} - \frac{\frac{G_1^2}{G}\nu^{-1}}{G^2\hat{\bar{\Sigma}}+G_1^2\nu^{-1}}} \\ &= G\left(\frac{G^2\hat{\bar{\Sigma}} + G_1^2\nu^{-1}}{G^2\hat{\bar{\Sigma}}}\right) > 0 \end{aligned} \tag{45}$$

which is equal to G if $G_1^2 = 0$. Subtracting the actual gain G

$$\begin{aligned} G\left(\frac{G^2\hat{\bar{\Sigma}} + G_1^2\nu^{-1}}{G^2\hat{\bar{\Sigma}}} - 1\right) \\ G\left(\frac{G_1^2\nu^{-1}}{G^2\hat{\bar{\Sigma}}}\right) > 0 \end{aligned} \tag{46}$$

■

Proposition 3. Let $\hat{x}_{t,t} \equiv \tilde{E}_t[x_t]$ and $x_{t,t} \equiv E_t[x_t]$. From 3

$$\begin{aligned} \hat{x}^i_{t,t} &= \rho\hat{x}^i_{t-1,t-1} + G(x_t - \rho\hat{x}^i_{t-1,t-1}) + G_1 e_t + G_2\eta^i_t \\ \hat{x}^i_{t,t} &= \rho\hat{x}^i_{t-1,t-1} + Gx_t - G\hat{x}^i_{t,t} + G(\hat{x}^i_{t,t} - \rho\hat{x}^i_{t-1,t-1}) + G_1 e_t + G_2\eta^i_t \\ (1-G)(\hat{x}^i_{t,t} - \rho\hat{x}^i_{t-1,t-1}) &= +G(x_t - \hat{x}^i_{t,t}) + G_1 e_t + G_2\eta^i_t \\ \underbrace{(x_t - \hat{x}^i_{t,t})}_{fe^i_{t,t}} &= \frac{1-G}{G}\underbrace{(\hat{x}^i_{t,t} - \rho\hat{x}^i_{t-1,t-1})}_{fr^i_{t,t}} - \frac{G_1}{G}e_t - \frac{G_2}{G}\eta^i_t \end{aligned} \tag{47}$$

therefore by running BGMS regression 11, the regressor $fr^i_{t,t}$ is correlated with the unobservable error. The resulting $\hat{\beta}_{BGMS}$ is equal to:

$$\begin{aligned}
\beta_{BGMS} &= \frac{1-G}{G} + \frac{cov(G[x_t - \rho\hat{x}^i_{t-1,t-1}] + G_1 e_t + G_2\eta^i_t, -\frac{G_1}{G}e_t - \frac{G_2}{G}\eta^i_t)}{var(G[x_t - \rho\hat{x}^i_{t-1,t-1}] + G_1 e_t + G_2\eta^i_t)} \\
&= \frac{1-G}{G} - \frac{\frac{G_1^2}{G}\nu^{-1} + \frac{G_2^2}{G}\tau^{-1}}{G^2 var(x_t - \rho\hat{x}^i_{t-1,t-1}) + G_1^2\nu^{-1} + G_2^2\tau^{-1}} \\
&= \frac{var(x_t - \rho\hat{x}^i_{t-1,t-1}) - [G var(x_t - \rho\hat{x}^i_{t-1,t-1}) + \frac{G_1^2}{G}\nu^{-1} + \frac{G_2^2}{G}\tau^{-1}]}{G var(x_t - \rho\hat{x}^i_{t-1,t-1}) + \frac{G_1^2}{G}\nu^{-1} + \frac{G_2^2}{G}\tau^{-1}}
\end{aligned} \tag{48}$$

but $var(x_t - \rho\hat{x}^i_{t-1,t-1}) \neq \Sigma \equiv var(x_t - \rho x^i_{t-1,t-1})$. In steady state

$$\begin{aligned}
x_{t+1} - \rho\hat{x}^i_{t,t} =& \rho(x_t - \hat{x}^i_{t,t}) + u_{t+1} \\
\hat{\Sigma} =& \rho^2\hat{\Phi} + \xi^{-1}
\end{aligned} \tag{49}$$

where $\hat{\Phi} = var(x_t - \hat{x}^i_{t,t})$.

$$\begin{aligned}
x_t - \hat{x}^i_{t,t} =& (1-G)[x_t - \rho\hat{x}^i_{t-1,t-1}] - G_1 e_t - G_2\eta^i_t \\
\hat{\Phi} =& (1-G)^2\hat{\Sigma} + G_1^2\nu^{-1} + G_2^2\tau^{-1}
\end{aligned} \tag{50}$$

Substitute and solve for $\hat{\Sigma}$

$$\hat{\Sigma} = \frac{\rho^2[G_1^2\nu^{-1} + G_2^2\tau^{-1}] + \xi^{-1}}{1 - \rho^2(1-G)^2} \tag{51}$$

■

Corollary 2. With rational expectation, $G_1 = \frac{\nu}{\tau+\nu+\Sigma^{-1}}$ and $G_2 = \frac{\tau}{\tau+\nu+\Sigma^{-1}}$, with $\Sigma \equiv var(x_t - E^i_t[x_t])$. Define $\chi = \Sigma^{-1}$. From 48

$$\begin{aligned}
\beta_{BGMS} &= \frac{\chi}{\nu+\tau} - \frac{\frac{1}{\chi+\nu+\tau}}{\frac{1}{(\chi+\nu+\tau)^2}[(\nu+\tau)^2\chi^{-1} + \nu + \tau]} \\
\beta_{BGMS} &= \frac{\chi}{\nu+\tau} - \frac{(\chi+\nu+\tau)\chi}{(\nu+\tau)^2 + \nu\chi + \tau\chi} \\
\beta_{BGMS} &= \frac{\chi}{\nu+\tau} - \frac{(\chi+\nu+\tau)\chi}{(BGMS+\nu+\tau)(\nu+\tau)} \\
\beta_{BGMS} &= 0
\end{aligned} \tag{52}$$

■

Proposition 4. Let $\hat{x}_{t,t} \equiv \tilde{E}_t[x_t]$. From 3

$$\begin{aligned}
\hat{x}^i_{t,t} - \rho\hat{x}^i_{t-1,t-1} &= G(x_t - \rho\hat{x}^i_{t-1,t-1}) + G_1 e_t + G_2\eta^i_t \\
&= G_2(x_t - \rho\hat{x}^i_{t-1,t-1}) + G_1(g_t - \rho\hat{x}^i_{t-1,t-1}) + G_2\eta^i_t \\
&= G_2(x_t - \hat{x}^i_{t,t}) + G_2(\hat{x}^i_{t,t} - \rho\hat{x}^i_{t-1,t-1}) + G_1(g_t - \rho\hat{x}^i_{t-1,t-1}) + G_2\eta^i_t \\
\underbrace{(x_t - \hat{x}^i_{t,t})}_{fe^i_{t,t}} &= \frac{1-G_2}{G_2}\underbrace{(\hat{x}^i_{t,t} - \rho\hat{x}^i_{t-1,t-1})}_{fr^i_{t,t}} - \frac{G_1}{G_2}\underbrace{(g_t - \rho\hat{x}^i_{t-1,t-1})}_{pi^i_{t,t}} - \eta^i_t
\end{aligned} \tag{53}$$

Write regression 13 as

$$fe^i_{t,t} = X\beta + err^i_t \tag{54}$$

where $X = [fr^i_{t,t} \quad pi^i_{t,t}]$ and $\beta = \begin{bmatrix}\beta_1 \\ \beta_2\end{bmatrix}$.

$$\hat{\beta} = \beta + \Sigma^{-1}_{XX}\Sigma_{Xu} \tag{55}$$

where

$$\begin{aligned}
\Sigma_{XX} &= \begin{bmatrix} var(fr^i_{t,t}) & cov(fr^i_{t,t}, pi^i_{t,t}) \\ cov(fr^i_{t,t}, pi^i_{t,t}) & var(pi^i_{t,t}) \end{bmatrix} \\
\Sigma^{-1}_{XX} &= \frac{1}{var(fr^i_{t,t})var(pi^i_{t,t}) - cov(fr^i_{t,t}, pi^i_{t,t})^2}\begin{bmatrix} var(pi^i_{t,t}) & -cov(fr^i_{t,t}, pi^i_{t,t}) \\ -cov(fr^i_{t,t}, pi^i_{t,t}) & var(fr^i_{t,t}) \end{bmatrix} \\
\Sigma_{Xu} &= \begin{bmatrix} cov(fr^i_{t,t}, err^i) \\ cov(pi^i_{t,t}, err^i) \end{bmatrix} \\
\hat{\beta} = \beta + \Sigma^{-1}_{XX}\Sigma_{Xu} &= \beta + \begin{bmatrix} \frac{var(pi^i_{t,t})cov(fr^i_{t,t}, err)}{var(fr^i_{t,t})var(pi^i_{t,t}) - cov(fr^i_{t,t}, pi^i_{t,t})^2} \\ \frac{-cov(fr^i_{t,t}, pi^i_{t,t})cov(fr^i_{t,t}, err)}{var(fr^i_{t,t})var(pi^i_{t,t}) - cov(fr^i_{t,t}, pi^i_{t,t})^2} \end{bmatrix}
\end{aligned} \tag{56}$$

and

$$
\begin{aligned}
var(fr^i_{t,t}) &= [G^2\hat{\Sigma} + G_1^2\nu^{-1} + G_2^2\tau^{-1}] \\
var(pi^i_{t,t}) &= \hat{\Sigma} + \nu^{-1} \\
cov(fr^i_{t,t}, pi^i_{t,t}) &= [G\hat{\Sigma} + G_1\nu^{-1}] \\
cov(fr^i_{t,t}, err^i) &= -G_2\tau^{-1} \\
cov(pi^i_{t,t}, err^i) &= 0
\end{aligned}
\tag{57}
$$

where $\hat{\chi} = \hat{\Sigma}^{-1}$ therefore

$$
\begin{aligned}
\hat{\beta}_1 &= \frac{1-G_2}{G_2} + \frac{var(pi^i_{t,t})cov(fr^i_{t,t}, err)}{var(fr^i_{t,t})var(pi^i_{t,t}) - cov(fr^i_{t,t}, pi^i_{t,t})^2} \\
&= \frac{1-G_2}{G_2} - \frac{(\hat{\Sigma}+\nu^{-1})G_2\frac{1}{\tau}}{(\hat{\Sigma}+\nu^{-1})(G^2\hat{\Sigma} + G_1^2\nu^{-1} + G_2^2\tau^{-1}) - (G\hat{\Sigma}+G_1\nu^{-1})^2}
\end{aligned}
\tag{58}
$$

and

$$
\begin{aligned}
\hat{\beta}_2 &= -\frac{G_1}{G_2} + \frac{-cov(fr^i_{t,t}, pi^i_{t,t})cov(fr^i_{t,t}, err)}{var(fr^i_{t,t})var(pi^i_{t,t}) - cov(fr^i_{t,t}, pi^i_{t,t})^2} \\
&= -\frac{G_1}{G_2} + \frac{(G\hat{\Sigma}+G_1\nu^{-1})G_2\frac{1}{\tau}}{(\hat{\Sigma}+\nu^{-1})(G^2\hat{\Sigma} + G_1^2\nu^{-1} + G_2^2\tau^{-1}) - (G\hat{\Sigma}+G_1\nu^{-1})^2}
\end{aligned}
\tag{59}
$$

■

Corollary 3. With rational expectation, $G_1 = \frac{\nu}{\tau+\nu+\Sigma^{-1}}$ and $G_2 = \frac{\tau}{\tau+\nu+\Sigma^{-1}}$, with $\Sigma \equiv var(x_t - E^i_t[x_t])$. Define $\chi = \Sigma^{-1}$.

From 58

$$
\begin{aligned}
\beta_1 &= \frac{\chi+\nu}{\tau} - \frac{(\chi^{-1}+\nu^{-1})G_2\frac{1}{\tau}}{(\chi^{-1}+\nu^{-1})(G^2\chi^{-1} + G_1^2\nu^{-1} + G_2^2\tau^{-1}) - (G\chi^{-1}+G_1\nu^{-1})^2} \\
&= \frac{\chi+\nu}{\tau} - \frac{(\frac{1}{\chi+\nu+\tau})(\frac{\nu+\chi}{\nu\chi})}{(\frac{1}{\chi+\nu+\tau})^2[(\frac{\nu+\chi}{\nu\chi})(\frac{(\nu+\tau)^2}{\chi}+\nu+\tau) - (\frac{(\nu+\tau)}{\chi})+1)^2]} \\
&= \frac{\chi+\nu}{\tau} - \frac{(\frac{\nu+\chi}{\nu\chi})(\chi+\nu+\tau)}{(\frac{\nu+\chi}{\nu\chi})(\nu+\tau)(\chi+\nu+\tau)\frac{1}{\chi} - (\chi+\nu+\tau)^2\frac{1}{\chi^2}} \\
&= \frac{\chi+\nu}{\tau} - \frac{(\nu+\chi)}{(\chi+\nu+\tau)-(\nu+\chi)} \\
&= 0
\end{aligned}
\tag{60}
$$

While from 59

$$
\begin{aligned}
\hat{\beta}_2 &= -\frac{\nu}{\tau} + \frac{(G\chi^{-1} + G_1\nu^{-1})G_2\frac{1}{\tau}}{(\chi^{-1}+\nu^{-1})(G^2\chi^{-1} + G_1^2\nu^{-1} + G_2^2\tau^{-1}) - (G\chi^{-1} + G_1\nu^{-1})^2} \\
&= -\frac{\nu}{\tau} + \frac{(\frac{1}{\chi+\nu+\tau})^2(\frac{\nu+\tau}{\chi}+1)}{(\frac{1}{\chi+\nu+\tau})^2[(\frac{\nu+\chi}{\nu\chi})(\frac{(\nu+\tau)^2}{\chi}+\nu+\tau) - (\frac{(\nu+\tau)}{\chi})+1)^2]} \\
&= -\frac{\nu}{\tau} + \frac{(\chi+\nu+\tau)\frac{1}{\chi}}{(\frac{\nu+\chi}{\nu\chi})(\nu+\tau)(\chi+\nu+\tau)\frac{1}{\chi} - (\chi+\nu+\tau)^2\frac{1}{\chi^2}} \\
&= -\frac{\nu}{\tau} + \frac{\nu\chi}{(\nu+\chi)(\nu+\tau) - (\nu+\tau+\chi)\nu} \\
&= 0
\end{aligned}
\tag{61}
$$

■

Proposition 5. From 19, using $\delta = \delta_1 + \delta_2$

$$
\begin{aligned}
\hat{x}^i &= \mu + \delta(x-\mu) + \delta_1 e + \delta_2\eta^i \\
\hat{x}^i &= \mu + \delta x - \delta\hat{x}^i + \delta(\hat{x}^i - \mu) + \delta_1 e + \delta_2\eta^i \\
(1-\delta)(\hat{x}^i - \mu) &= +\delta(x - \hat{x}^i) + \delta_1 e + \delta_2\eta^i \\
(\underbrace{x - \hat{x}^i}_{fe^i}) &= \frac{1-\delta}{\delta}(\underbrace{\hat{x}^i - \mu}_{fr^i}) - \frac{\delta_1}{\delta}e - \frac{\delta_2}{\delta}\eta^i
\end{aligned}
\tag{62}
$$

therefore by running BGMS regression 11, the regressor $fr^i = \hat{x}^i - \mu$ is correlated with the unobservable error. The resulting $\hat{\beta}_{BGMS}$ is equal to:

$$
\begin{aligned}
\hat{\beta}_{BGMS} &= \frac{1-\delta}{\delta} + \frac{cov(\hat{x}^i - \mu, -\frac{\delta_1}{\delta}e - \frac{\delta_2}{\delta}\eta^i)}{var(\hat{x}^i - \mu)} \\
&= \frac{1-\delta}{\delta} + \frac{cov(\delta(x_t-\mu) + \delta_1 e + \delta_2\eta^i, -\frac{\delta_1}{\delta}e - \frac{\delta_2}{\delta}\eta^i)}{var(\delta(x_t-\mu) + \delta_1 e + \delta_2\eta^i)} \\
&= \frac{1-\delta}{\delta} + \frac{-\frac{\delta_1^2}{\delta}\nu^{-1} - \frac{\delta_2^2}{\delta}\tau^{-1}}{\delta^2\chi^{-1} + \delta_1^2\nu^{-1} + \delta_2^2\tau^{-1}}
\end{aligned}
\tag{63}
$$

substitute for δ_1 and δ_2

$$\hat{\beta}_{BGMS} = \frac{(1-\lambda)(1-\gamma_1-\gamma_2)}{(1-\lambda)\gamma_1+\gamma_2} - \frac{\frac{(1-\lambda)^2}{(1-\lambda)+\lambda\gamma_2}\frac{\gamma_1^2}{(1-\lambda)\gamma_1+\gamma_2}\nu^{-1} + \frac{1}{(1-\lambda)+\lambda\gamma_2}\frac{\gamma_2^2}{(1-\lambda)\gamma_1+\gamma_2}\tau^{-1}}{\frac{1}{[(1-\lambda)+\lambda\gamma_2]^2}([(1-\lambda)\gamma_1+\gamma_2]^2\chi^{-1} + (1-\lambda)^2\gamma_1^2\nu^{-1} + \gamma_2^2\tau^{-1})} \tag{64}$$

use definition of γ_1 and γ_2

$$\begin{aligned}
\hat{\beta}_{BGMS} &= \frac{(1-\lambda)\chi}{(1-\lambda)\nu+\tau} - \frac{\frac{1}{(1-\lambda)\nu+\tau}[(1-\lambda)^2\nu+\tau]}{\frac{1}{(1-\lambda)(\nu+\chi)+\tau}([(1-\lambda)\nu+\tau]^2\chi^{-1} + (1-\lambda)^2\nu+\tau)} \\
&= \frac{(1-\lambda)\chi}{(1-\lambda)\nu+\tau} - \frac{[(1-\lambda)(\nu+\chi)+\tau][(1-\lambda)^2\nu+\tau]\chi}{[(1-\lambda)\nu+\tau]([(1-\lambda)\nu+\tau]^2 + [(1-\lambda)^2\nu+\tau]\chi)} \\
&= \frac{\chi\{(1-\lambda)[(1-\lambda)\nu+\tau]^2 - [(1-\lambda)\nu+\tau][(1-\lambda)^2\nu+\tau]\}}{[(1-\lambda)\nu+\tau]([(1-\lambda)\nu+\tau]^2 + [(1-\lambda)^2\nu+\tau]\chi)} \\
&= \frac{-\lambda\tau\chi[(1-\lambda)\nu+\tau]}{[(1-\lambda)\nu+\tau]([(1-\lambda)\nu+\tau]^2 + [(1-\lambda)^2\nu+\tau]\chi)} \\
&\frac{-\lambda\tau\chi}{([(1-\lambda)\nu+\tau]^2 + [(1-\lambda)^2\nu+\tau]\chi)} < 0
\end{aligned} \tag{65}$$

which is negative as long as $0 < \lambda < 1$. ■

Proposition 6. From 19

$$\begin{aligned}
\bar{\hat{x}} &= \mu + \delta(x-\mu) + \delta_1 e \\
\bar{\hat{x}} &= \mu + \delta x - \delta\bar{\hat{x}} + \delta(\bar{\hat{x}}-\mu) + \delta_1 e \\
(1-\delta)(\bar{\hat{x}}-\mu) &= +\delta(x-\bar{\hat{x}}) + \delta_1 e \\
(\underbrace{x-\bar{\hat{x}}}_{fe^i}) &= \frac{1-\delta}{\delta}(\underbrace{\bar{\hat{x}}-\mu}_{fr^i}) - \frac{\delta_1}{\delta}e
\end{aligned} \tag{66}$$

therefore by running CG regression 4, the regressor $\bar{fr} = \bar{\hat{x}} - \mu$ is correlated with the unobservable error. The resulting $\hat{\beta}_{CG}$ is equal to:

$$\begin{aligned}\hat{\beta}_{CG} &= \frac{1-\delta}{\delta} + \frac{cov(\bar{\hat{x}}-\mu, -\frac{\delta_1}{\delta}e)}{var(\bar{\hat{x}}-\mu)} \\ &= \frac{1-\delta}{\delta} + \frac{cov(\delta(x_t-\mu)+\delta_1 e, -\frac{\delta_1}{\delta}e)}{var(\delta(x_t-\mu)+\delta_1 e)} \\ &= \frac{1-\delta}{\delta} + \frac{-\frac{\delta_1^2}{\delta}\nu^{-1}}{\delta^2\chi^{-1}+\delta_1^2\nu^{-1}}\end{aligned} \tag{67}$$

substitute for δ_1 and δ_2

$$\hat{\beta}_{CG} = \frac{(1-\lambda)(1-\gamma_1-\gamma_2)}{(1-\lambda)\gamma_1+\gamma_2} - \frac{\frac{(1-\lambda)^2}{(1-\lambda)+\lambda\gamma_2}\frac{\gamma_1^2}{(1-\lambda)\gamma_1+\gamma_2}\nu^{-1}}{\frac{1}{[(1-\lambda)+\lambda\gamma_2]^2}([(1-\lambda)\gamma_1+\gamma_2]^2\chi^{-1}+(1-\lambda)^2\gamma_1^2\nu^{-1})} \tag{68}$$

use definition of γ_1 and γ_2

$$\begin{aligned}\hat{\beta}_{CG} &= \frac{(1-\lambda)\chi}{(1-\lambda)\nu+\tau} - \frac{\frac{1}{(1-\lambda)\nu+\tau}[(1-\lambda)^2\nu]}{\frac{1}{(1-\lambda)(\nu+\chi)+\tau}([(1-\lambda)\nu+\tau]^2\chi^{-1}+(1-\lambda)^2\nu)} \\ &= \frac{(1-\lambda)\chi}{(1-\lambda)\nu+\tau} - \frac{[(1-\lambda)(\nu+\chi)+\tau](1-\lambda)^2\nu\chi}{[(1-\lambda)\nu+\tau]([(1-\lambda)\nu+\tau]^2+(1-\lambda)^2\nu\chi)} \\ &= \frac{(1-\lambda)\tau\chi[(1-\lambda)\nu+\tau]}{[(1-\lambda)\nu+\tau]([(1-\lambda)\nu+\tau]^2+(1-\lambda)^2\nu\chi)} \\ & \frac{(1-\lambda)\tau\chi}{([(1-\lambda)\nu+\tau]^2+(1-\lambda)^2\nu\chi)} > 0\end{aligned} \tag{69}$$

which is positive as long as $0 < \lambda < 1$. If $\lambda = 1$, it is zero. ■

Proposition 7. From 19

$$\begin{aligned} x^i &= \mu + \delta_1(y-\mu) + \delta_2(x-\mu) + \delta_2\eta^i \\ x^i &= \mu + \delta_1(y-\mu) + \delta_2 x_t - \delta_2 x^i + \delta_2(\hat{x}^i-\mu) + \delta_2\eta^i \\ (1-\delta_2)(\hat{x}^i-\mu) &= \delta_1(y-\mu) + \delta_2(x-\hat{x}^i) + \delta_2\eta^i \\ (\underbrace{x-\hat{x}^i}_{fe^i}) &= \frac{1-\delta_2}{\delta_2}(\underbrace{\hat{x}^i-\mu}_{fr^i}) - \frac{\delta_1}{\delta_2}(\underbrace{g-\mu}_{pi}) - \eta^i\end{aligned} \tag{70}$$

write regression 13 as

$$fe^i = X\beta + err^i \tag{71}$$

where $X = [fr^i \quad pi]$ and $\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$.

$$\hat{\beta} = \beta + \Sigma_{XX}^{-1}\Sigma_{Xu} \tag{72}$$

where

$$\begin{aligned}
\Sigma_{XX} &= \begin{bmatrix} var(fr^i) & cov(fr^i, pi) \\ cov(fr^i, pi) & var(pi) \end{bmatrix} \\
\Sigma_{XX}^{-1} &= \frac{1}{var(fr^i)var(pi) - cov(fr^i, pi)^2} \begin{bmatrix} var(pi) & -cov(fr^i, pi) \\ -cov(fr^i, pi) & var(fr^i) \end{bmatrix} \\
\Sigma_{Xu} &= \begin{bmatrix} cov(fr^i, err^i) \\ cov(pi, err^i) \end{bmatrix} \\
\hat{\beta} = \beta + \Sigma_{XX}^{-1}\Sigma_{Xu} &= \beta + \begin{bmatrix} \frac{var(pi)cov(fr, err)}{var(fr)var(pi) - cov(fr, pi)^2} \\ \frac{-cov(fr, pi)cov(fr, err)}{var(fr)var(pi) - cov(fr, pi)^2} \end{bmatrix}
\end{aligned} \tag{73}$$

and

$$\begin{aligned}
var(fr^i) &= \delta^2\chi^{-1} + \delta_1^2\nu^{-1} + \delta_2^2\tau^{-1} \\
var(pi) &= \chi^{-1} + \nu^{-1} \\
cov(fr^i, pi) &= \delta\chi^{-1} + \delta_1\nu^{-1} \\
cov(fr^i, err^i) &= -\delta_2\tau^{-1} \\
cov(pi, err^i) &= 0
\end{aligned} \tag{74}$$

$$
\begin{aligned}
\hat{\beta}_1 &= \frac{1-\delta_2}{\delta_2} + \frac{var(pi)cov(fr,err)}{var(fr)var(pi) - cov(fr,pi)^2} \\
&= (1-\lambda)\frac{1-\gamma_2}{\gamma_2} - \frac{(\chi^{-1}+\nu^{-1})\delta_2\tau^{-1}}{(\chi^{-1}+\nu^{-1})(\delta^2\chi^{-1}+\delta_1^2\nu^{-1}+\delta_2^2\tau^{-1}) - (\delta\chi^{-1}+\delta_1\nu^{-1})^2} \\
&= (1-\lambda)\frac{1-\gamma_2}{\gamma_2} - \\
&\frac{\frac{\chi+\nu}{\chi\nu}\gamma_2\tau^{-1}}{\frac{\chi+\nu}{\chi\nu}\frac{1}{(1-\lambda)+\lambda\gamma_2}([(1-\lambda)\gamma_1+\gamma_2]^2\chi^{-1}+(1-\lambda)^2\gamma_1^2\nu^{-1}+\gamma_2^2\tau^{-1}) - \frac{1}{(1-\lambda)+\lambda\gamma_2}([(1-\lambda)\gamma_1+\gamma_2]\chi^{-1}+(1-\lambda)\gamma_1}
\end{aligned}
\tag{75}
$$

use definition of γ_1 and γ_2

$$
\begin{aligned}
\hat{\beta}_1 &= (1-\lambda)\frac{\nu+\chi}{\tau} - \frac{(\chi+\nu)[(1-\lambda)(\nu+\chi)+\tau]\chi}{(\chi+\nu)([(1-\lambda)\nu+\tau]^2+[(1-\lambda)^2\nu+\tau]\chi) - ([(1-\lambda)\nu+\tau]+(1-\lambda)^2\nu\chi)^2\nu} \\
&= (1-\lambda)\frac{\nu+\chi}{\tau} - \frac{(\chi+\nu)[(1-\lambda)(\nu+\chi)+\tau]\chi}{\chi\tau(\tau+\nu+\chi)} \\
&= \frac{-\lambda(\nu+\chi)\chi\tau}{\chi\tau(\tau+\nu+\chi)} \\
&= \frac{-\lambda(\nu+\chi)}{(\tau+\nu+\chi)}
\end{aligned}
\tag{76}
$$

negative as long as $0 < \lambda < 1$.

$$
\begin{aligned}
\hat{\beta}_2 &= -\frac{\delta_1}{\delta_2} + \frac{-cov(fr,pi)cov(fr,err)}{var(fr)var(pi) - cov(fr,pi)^2} \\
&= -\frac{\delta_1}{\delta_2} + \frac{(\delta\chi^{-1}+\delta_1\nu^{-1})\delta_2\frac{1}{\tau}}{(\chi^{-1}+\nu^{-1})(\delta^2\chi^{-1}+\delta_1^2\nu^{-1}+\delta_2^2\tau^{-1}) - (\delta\chi^{-1}+\delta_1\nu^{-1})^2} \\
&= -(1-\lambda)\frac{\gamma_1}{\gamma_2} - \\
&\frac{[(1-\lambda)\nu+\tau]\chi^{-1}+(1-\lambda)}{\frac{\chi+\nu}{\chi\nu}([(1-\lambda)\gamma_1+\gamma_2]^2\chi^{-1}+(1-\lambda)^2\gamma_1^2\nu^{-1}+\gamma_2^2\tau^{-1}) - ([(1-\lambda)\gamma_1+\gamma_2]\chi^{-1}+(1-\lambda)\gamma_1\nu^{-1})^2}
\end{aligned}
\tag{77}
$$

use definition of γ_1 and γ_2

$$\begin{aligned}\hat{\beta}_2 &= -(1-\lambda)\frac{\nu}{\tau} + \frac{([(1-\lambda)\nu+\tau]+(1-\lambda)\chi)\chi\nu}{(\chi+\nu)([(1-\lambda)\nu+\tau]^2+[(1-\lambda)^2\nu+\tau]\chi)-([(1-\lambda)\nu+\tau]+(1-\lambda)^2\nu\chi)^2\nu} \\ &= -(1-\lambda)\frac{\nu}{\tau} + \frac{([(1-\lambda)\nu+\tau]+(1-\lambda)\chi)\chi\nu}{\chi\tau(\tau+\nu+\chi)} \\ &= \frac{\lambda\nu\chi\tau}{\chi\tau(\tau+\nu+\chi)} \\ &= \frac{\lambda\nu}{(\tau+\nu+\chi)}\end{aligned} \tag{78}$$

positive as long as $0 < \lambda < 1$. ■

C Different public signal measure

In addition to our baseline measure in section 2, we use the current value of the forecasted series as an additional possible proxy for public signal . In particular, assume that the observable series y agents are asked to forecast depends on a latent unobservable factor x and some noise e. Moreover, agents receive some private noisy signal on it s_t^i.

$$\begin{aligned} y_t &= x_t + e_t \\ x_t &= \rho x_{t-1} + u_t \\ s_t^i &= x_t + \eta_t^i \end{aligned} \tag{79}$$

with u_t, e_t and η^i normally distributed with zero mean and $\rho < 1$. The observable contemporaneous y_t is a public noisy signal about the underlying fundamental x_t. This structure is consistent with CG and BGMS econometric specification as long as $\tilde{E}_t[y_{t+h}] = \tilde{E}_t[x_{t+h}]$.

To measure the contemporaneous public signal for financial series, we use the average value of the series in the same quarter up to the survey date, which is the second month of the quarter. On the other hand, macroeconomic series are released with some lag, therefore we use the first release of the previous period value, which is available at the time of the forecast. To capture the surprise component in the public information, we compute the difference between the public signal and individual prior about the signal. In this case $pi_{t,t+h} = y_t - E_{t-1}^i[y_t]$. For macroeconomic variables, we compare contemporaneous release of lagged value with lagged nowcasting. In thi case $pi_{t,t+h} = y_{t-1} - E_{t-1}^i[y_{t-1}]$.

We run regression 13 using this different measure of public information. Panel A of Table 10 reports the panel data regressions at 3 quarters horizon with individual fixed effects and the median from individual regressions. The tables displays consistent $\beta_{GV,1} < 0$ and $\beta_{GV,2} > 0$ across variables, though less consistently than in table 6 in section 2. The reason being that the measure of public signal considered here doesn't refer direction to horizon $h = 3$, but to horizon $h = 0$ or even ($h = -1$ for macro variable) and it is therefore less informative about longer horizons. Panel B of Table 10 reports the same regression using a shorter horizon $h = 2$ and shows that the result are much more consistent and significant.[9] Figure **??** shows the coefficients graphically.

[9] At both horizons forecasts about the Consumer Price Index seem to overreact to this measure of public information instead of underreacting. However, in unreported result we show that if we consider the actual

Table 10: Private and public information: alternative measure of public information

Panel A: 3 quarters horizon

	Revision				Public signal			
	β_1	SE	p-value	Median	β_2	SE	p-value	Median
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Nominal GDP	-0.25	0.08	0.00	-0.18	-0.05	0.12	0.69	-0.13
GDP price index inflation	-0.40	0.04	0.00	-0.40	0.39	0.15	0.01	0.30
Real GDP	-0.10	0.08	0.23	0.06	-0.07	0.08	0.39	-0.10
Consumer Price Index	-0.19	0.08	0.03	-0.14	-0.56	0.28	0.06	-0.52
Industrial production	-0.30	0.14	0.03	-0.35	0.08	0.14	0.57	0.11
Housing Start	-0.09	0.09	0.36	-0.13	0.57	0.13	0.00	0.37
Real Consumption	-0.30	0.12	0.01	-0.25	0.27	0.13	0.06	0.15
Real residential investment	-0.09	0.10	0.39	-0.07	0.57	0.18	0.00	0.48
Real nonresidential investment	0.06	0.14	0.65	0.18	0.20	0.22	0.38	0.14
Real state and local government consumption	-0.53	0.05	0.00	-0.53	0.12	0.10	0.24	0.17
Real federal government consumption	-0.47	0.04	0.00	-0.39	0.28	0.09	0.00	0.19
Unemployment rate	0.26	0.16	0.10	0.18	-0.39	0.25	0.12	-0.44
Three-month Treasury rate	-0.26	0.10	0.02	-0.31	0.93	0.26	0.00	1.30
Ten-year Treasury rate	-0.63	0.05	0.00	-0.64	0.61	0.11	0.00	0.62
AAA Corporate Rate Bond	-0.69	0.04	0.00	-0.78	0.80	0.10	0.00	0.75

Panel B: 2 quarters horizon

	Revision				Public signal			
	β_1	SE	p-value	Median	β_2	SE	p-value	Median
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Nominal GDP	-0.14	0.09	0.11	-0.10	0.12	0.12	0.34	0.04
GDP price index inflation	-0.41	0.04	0.00	-0.38	0.46	0.12	0.00	0.34
Real GDP	-0.09	0.10	0.41	0.07	0.06	0.09	0.51	-0.03
Consumer Price Index	-0.07	0.14	0.59	-0.12	-0.50	0.34	0.16	-0.54
Industrial production	-0.19	0.17	0.26	-0.15	0.35	0.22	0.12	0.32
Housing Start	0.03	0.06	0.67	-0.04	0.29	0.11	0.01	0.27
Real Consumption	-0.25	0.11	0.02	-0.21	0.21	0.13	0.11	0.14
Real residential investment	-0.09	0.09	0.32	-0.12	0.41	0.14	0.00	0.41
Real nonresidential investment	0.13	0.12	0.28	0.17	-0.02	0.20	0.94	-0.11
Real state and local government consumption	-0.40	0.04	0.00	-0.36	0.20	0.10	0.05	0.25
Real federal government consumption	-0.42	0.05	0.00	-0.33	0.29	0.10	0.00	0.08
Unemployment rate	0.22	0.12	0.06	0.20	-0.30	0.18	0.10	-0.28
Three-month Treasury rate	-0.33	0.14	0.02	-0.43	0.78	0.30	0.01	1.04
Ten-year Treasury rate	-0.80	0.06	0.00	-0.92	0.75	0.12	0.00	0.76
AAA Corporate Rate Bond	-0.77	0.05	0.00	-0.83	0.92	0.07	0.00	0.88

Notes: this table reports the coefficients of regression 13 (individual forecast errors on individual revisions and public information). Columns 1 to 3 show coefficient β_1 (forecast revision) from the panel regression with individual fixed effect, with standard errors and corresponding p-values. Standard errors are robust and clustered by forecaster. Column 4 shows the median coefficient of the same regression at the individual level. Columns 5 to 7 show coefficient β_2 (public information) from the panel regression with individual fixed effect, with standard errors and corresponding p-values. Standard errors are robust and clustered by forecaster. Column 8 shows the median coefficient of the same regression at the individual level. Panel A uses forecast at 3 quarters horizon and panel B uses forecast at 2 quarters horizon.

D Empirical evidence with AR2

Table 11: Motivating evidence: BGMS regressions with 2 lags

	$fr^i_{t+2,t}$				$fr^i_{t+1,t}$			
	$\beta_{BGMS,1}$	SE	p-value	Median	$\beta_{BGMS,2}$	SE	p-value	Median
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	
Nominal GDP	-0.24	0.14	0.10	-0.19	0.15	0.15	0.30	0.11
GDP price index inflation	-0.36	0.09	0.00	-0.38	0.27	0.13	0.04	0.36
Real GDP	-0.08	0.16	0.62	0.24	0.08	0.19	0.69	-0.15
Consumer Price Index	-0.89	0.18	0.00	-1.20	0.70	0.28	0.02	1.05
Industrial production	-0.30	0.19	0.12	-0.21	0.25	0.22	0.27	0.09
Housing Start	-0.26	0.15	0.09	-0.32	0.67	0.17	0.00	0.68
Real Consumption	-0.34	0.19	0.08	-0.35	0.34	0.20	0.10	0.29
Real residential investment	-0.54	0.19	0.01	-0.29	0.83	0.20	0.00	0.62
Real nonresidential investment	0.26	0.31	0.42	0.57	-0.05	0.38	0.91	-0.27
Real state and local government consumption	-0.10	0.13	0.46	-0.20	-0.06	0.16	0.70	0.01
Real federal government consumption	-0.46	0.13	0.00	-0.44	0.34	0.14	0.02	0.25
Unemployment rate	0.14	0.22	0.51	0.00	0.21	0.20	0.29	0.26
Three-month Treasury rate	-0.35	0.12	0.01	-0.58	0.75	0.21	0.00	1.25
Ten-year Treasury rate	-0.97	0.13	0.00	-0.96	0.85	0.16	0.00	0.76
AAA Corporate Rate Bond	-0.68	0.14	0.00	-1.08	0.54	0.20	0.01	0.84

value of GDP deflator as a public signal for consumer inflation (highly correlated with CPI), the forecasts underreact to it.

Table 12: Motivating evidence: public information regressions with 2 lags

	$fr^i_{t+2,t}$				$fr^i_{t+1,t}$				$pi^i_{t+2,t}$				$pi^i_{t+1,t}$			
	$\beta_{fr,1}$	SE	p-value	Median	$\beta_{fr,2}$	SE	p-value	Median	$\beta_{pi,1}$	SE	p-value	Median	$\beta_{pi,2}$	SE	p-value	Median
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
Nominal GDP	-0.71	0.21	0.00	-0.55	1.12	0.14	0.00	1.25	0.38	0.21	0.09	0.16	-0.48	0.17	0.01	-0.67
GDP price index inflation	-0.85	0.13	0.00	-0.82	1.17	0.11	0.00	1.21	0.48	0.18	0.01	0.46	-0.57	0.14	0.00	-0.74
Real GDP	-0.58	0.25	0.02	-0.32	1.25	0.22	0.00	1.12	0.42	0.32	0.19	0.14	-0.84	0.33	0.01	-0.67
Consumer Price Index	-1.55	0.27	0.00	-1.53	1.55	0.26	0.00	1.85	1.27	0.39	0.00	1.43	-1.14	0.34	0.00	-1.61
Industrial production	-0.72	0.25	0.01	-0.66	1.08	0.16	0.00	0.78	0.49	0.26	0.07	0.22	-0.52	0.19	0.01	-0.22
Housing Start	-0.75	0.20	0.00	-0.78	1.03	0.14	0.00	1.03	0.97	0.23	0.00	0.84	-0.69	0.18	0.00	-0.63
Real Consumption	-0.72	0.25	0.01	-0.89	1.02	0.21	0.00	0.89	0.51	0.24	0.04	0.57	-0.40	0.23	0.10	-0.29
Real residential investment	-1.03	0.24	0.00	-0.79	1.45	0.21	0.00	1.47	1.17	0.27	0.00	0.90	-1.18	0.24	0.00	-0.73
Real nonresidential investment	0.08	0.38	0.84	0.26	0.63	0.32	0.06	0.95	-0.01	0.45	0.99	-0.34	-0.18	0.43	0.69	-0.66
Real state and local government consumption	-0.46	0.15	0.00	-0.29	0.95	0.19	0.00	0.98	-0.01	0.18	0.98	-0.15	-0.26	0.21	0.23	-0.32
Real federal government consumption	-1.10	0.16	0.00	-0.90	1.35	0.12	0.00	1.07	0.55	0.17	0.00	0.36	-0.62	0.15	0.00	-0.45
Unemployment rate	-0.15	0.30	0.62	-0.33	0.83	0.21	0.00	0.93	0.43	0.27	0.13	0.65	-0.55	0.20	0.01	-0.74
Three-month Treasury rate	-1.13	0.20	0.00	-1.75	1.64	0.19	0.00	2.06	1.43	0.33	0.00	1.98	-1.47	0.35	0.00	-2.31
Ten-year Treasury rate	-1.72	0.19	0.00	-1.68	1.91	0.17	0.00	1.98	1.39	0.25	0.00	1.45	-1.42	0.23	0.00	-1.53
AAA Corporate Rate Bond	-1.50	0.13	0.00	-1.59	1.64	0.15	0.00	1.47	1.10	0.24	0.00	1.09	-1.03	0.25	0.00	-0.90

E Survey anonimity

The forecast data used in this paper are from the Survey of Professional Forecasters, compiled by the Federal Reserve of Philadelphia. Even if this particular survey is anonymous, we argue that it can nonetheless be affected by strategic incentives as well. In particular, we argue that the survey provided by forecasters to anonymous surveys appear to be the same as the one provided to other non-anonymous survey. This has been noted before in the forecasting literature: ”According to industry experts, forecasters often seem to submit to the anonymous surveys the same forecasts they have already prepared for public (i.e. non-anonymous) release. There are two reasons for this. First, it might not be convenient for the forecasters to change their report, unless they have a strict incentive to do so. Second, the forecasters might be concerned that their strategic behavior could be uncovered by the editor of the anonymous survey.” (Marinovic et al., 2013)

Two observations support this claim. First, Bordalo et al. (2020) establish fact 1 and 2 in section 2 by using both the SPF data and the Blue Chip data, which are not anonymous. They show that the two series provide very similar results, which is in line with the hypothesis of forecasters provided similar forecast to both surveys. Second, in a survey by the European Central Bank supplementary to their Survey of Professional Forecasters, respondents are asked explicitly ”When responding to the SPF, what forecast do you provide?”. In 2013, more than 80% of the panelists responded ”the last available, while in 2008 more than 90% gave the same answer (European Central Bank, 2014). It is also important to note that this is a conservative estimate of agents compiling a new forecast exclusively for the ECB survey, as the new forecast provided might be compiled to be used for other non-anonymous surveys as well.

F Dynamic model with AR(2)

We consider here a dynamic setting with a fundamental AR(2) process

$$
\begin{aligned}
x_t &= \rho_1 x_{t-1} + \rho_2 x_{t-2} + u_t \\
\begin{bmatrix} x_t \\ x_{t-1} \end{bmatrix} &= \begin{bmatrix} \rho_1 & \rho_2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ x_{t-2} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_t \\ e_t \end{bmatrix} \\
\bar{X}_t &= A\bar{X}_{t-1} + a \begin{bmatrix} u_t \\ e_t \end{bmatrix}
\end{aligned}
\tag{80}
$$

With $u \sim N(0, \nu^{-1})$.

Each agent receive a private signal s_t^i and a public signal g_t

$$
\begin{aligned}
s_t^i &= x_t + \eta_t^i \\
g_t &= x_t + e_t
\end{aligned}
\tag{81}
$$

with $\eta_t^i \sim N(0, \tau^{-1})$, $e_t \sim N(0, \nu^{-1})$. In matrix form

$$
V_t^i \equiv \begin{bmatrix} g_t \\ s_t^i \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \bar{X}_t + \begin{bmatrix} 0 \\ \eta_t^i \end{bmatrix}
\tag{82}
$$

Honest beliefs Agents form beliefs about x at horizon h: $E_t^i[\bar{X}_{t+h}]$. The honest posterior belief about $\bar{X}$ is given by the Kalman filter

$$
E_t^i[\bar{X}_t] = AE_{t-1}^i[\bar{X}_{t-1}] + K(V_t^i - E_{t-1}^i[V_t])
$$

With the first line yields the posterior $E_t^i[x_t] \equiv x_{t,t}^i$

$$
x_{t,t}^i = x_{t,t-1}^i + K_{1,1}(g_t - x_{t,t-1}^i) + K_{1,2}(s_t^i - x_{t,t-1}^i)
$$

where the Kalman gains are

$$
\begin{aligned}
K_{1,1} &= \frac{\nu}{\Sigma^{-1} + \nu + \tau} \\
K_{1,2} &= \frac{\tau}{\Sigma^{-1} + \nu + \tau}
\end{aligned}
\tag{83}
$$

and the posterior forecast error variance

$$
\Sigma \equiv E[(x_t - x^i_{t,t-1})(x_t - x^i_{t,t-1})'] \tag{84}
$$

Strategic interactions As in the previous section, the strategic substitutability in agents objective function leads them to report

$$
\begin{aligned}
\begin{bmatrix} \hat{x}^i_{t,t} \\ \hat{x}^i_{t-1,t} \end{bmatrix} &= \begin{bmatrix} \frac{1}{1-\lambda} E^i_t[x_t] - \frac{\lambda}{1-\lambda} E^i[\bar{\hat{x}}_{t,t}] \\ \frac{1}{1-\lambda} E^i_t[x_t - 1] - \frac{\lambda}{1-\lambda} E^i[\bar{\hat{x}}_{t-1,t}] \end{bmatrix} \\
F^i_t &= \begin{bmatrix} \frac{1}{1-\lambda} & 0 & -\frac{\lambda}{1-\lambda} & 0 \\ 0 & \frac{1}{1-\lambda} & 0 & -\frac{\lambda}{1-\lambda} \end{bmatrix} E^i_t \begin{bmatrix} \bar{X}_t \\ F_t \end{bmatrix}
\end{aligned}
\tag{85}
$$

where $\hat{x}^i_{t+h,t}$ is the forecast provided by individual i in t about realization in $t+h$, and $\bar{\hat{x}}_{t+h,t} = \int^i \hat{x}^i_{t+h,t} di$ is the average of forecasts provided in t about realization in $t+h$. Define $F_t \equiv \begin{bmatrix} \bar{\hat{x}}_{t,t} \\ \bar{\hat{x}}_{t-1,t} \end{bmatrix}$ and $F^i_t \equiv \begin{bmatrix} \hat{x}^i_{t,t} \\ \hat{x}^i_{t-1,t} \end{bmatrix}$. If $\lambda = 0$, agents report their true beliefs. With $1 > \lambda > 0$, agents not only want to be accurate, but also to stand out with respect to the average forecast.

We average $\hat{x}^i_{t,t}$ across agents and use repeated substitution in 85 to express the reported average forecast as

$$
F_t = -\frac{1}{1-\lambda} \sum_{k=0}^{\infty} \left(\frac{\lambda}{1-\lambda} \right)^k \bar{E}^{(k)}[\bar{X}_t] = \frac{1}{1-\lambda} \bar{E}_t \bar{X}_t - \frac{\lambda}{1-\lambda} F_t \tag{86}
$$

We guess and verify the law of motion for F_t and the other unobserved state variables. In particular, we conjecture that the state vector evolves according to[10]

$$Z \equiv \begin{bmatrix} \bar{X}_t \\ F_t \\ w_t \end{bmatrix} = MZ_{t-1} + m \begin{bmatrix} u_t \\ e_t \end{bmatrix} \tag{87}$$

Where

$$M = \begin{bmatrix} \underset{2x2}{A} & \underset{2x2}{0} & \underset{2x1}{0} \\ \underset{2x2}{G} & \underset{2x2}{L} & \underset{2x1}{0} \\ \underset{1x2}{0} & \underset{1x2}{0} & \underset{1x1}{0} \end{bmatrix} \quad \text{and} \quad m = \begin{bmatrix} \underset{2x2}{a} \\ \underset{2x2}{\mu} \\ 0 \quad 1 \end{bmatrix} \tag{88}$$

the observable variables are the two signals about x_t

$$V_t^i \equiv \begin{bmatrix} g_t \\ s_t^i \end{bmatrix} = HZ_t + \begin{bmatrix} 0 \\ \eta_t^i \end{bmatrix} \tag{89}$$

where

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{90}$$

Agents use their conjecture law of motion 87 and the observables 89 to infer the state using the individual Kalman filter. The posterior estimate of the state vector by agent i is

$$\begin{aligned} E_t^i[Z_t] &= ME_{t-1}^i[Z_{t-1}] + K(V_t^i - E_{t-1}^i[V_t]) \\ &= (I - KH)ME_{t-1}^i[Z_{t-1}] + KHMZ_{t-1} + KHm \begin{bmatrix} u_t \\ e_t \end{bmatrix} + K \begin{bmatrix} 0 \\ \eta_t^i \end{bmatrix} \end{aligned} \tag{91}$$

Where K is the Kalman gain. Average 91 to find the consensus believe on the state vector.

$$\bar{E}_t[Z_t] = (I - KH)M\bar{E}_{t-1}[Z_{t-1}] + KHMZ_{t-1} + KHm \begin{bmatrix} u_t \\ e_t \end{bmatrix} \tag{92}$$

[10] w_t takes care of the correlation between public signal and higher order beliefs F_t

From the definition on F_t in 28 it follows that

$$\begin{aligned} F_t &= \begin{bmatrix} \frac{1}{1-\lambda} & 0 & -\frac{\lambda}{1-\lambda} & 0 \\ 0 & \frac{1}{1-\lambda} & 0 & -\frac{\lambda}{1-\lambda} \end{bmatrix} \bar{E}_t[Z_t] \equiv \xi \bar{E}_t[Z_t] \\ &= \xi(I-KH)M\bar{E}_{t-1}[Z_{t-1}] + \xi KHMZ_{t-1} + \xi KHm \begin{bmatrix} u_t \\ e_t \end{bmatrix} \end{aligned} \tag{93}$$

Compute (i) $\xi M\bar{E}_{t-1}[Z_{t-1}]$, (ii) $HM\bar{E}_{t-1}[Z_{t-1}]$, (iii) HMZ_{t-1}, (iv) Hm.

1. write ξ as a vector of matrices

$$\xi \equiv \begin{bmatrix} \begin{bmatrix} \frac{1}{1-\lambda} & 0 \\ 0 & \frac{1}{1-\lambda} \end{bmatrix} & \begin{bmatrix} -\frac{\lambda}{1-\lambda} & 0 \\ 0 & -\frac{\lambda}{1-\lambda} \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{bmatrix} \equiv \begin{bmatrix} \frac{1}{1-\lambda}I & -\frac{\lambda}{1-\lambda}I & 0 \end{bmatrix} \tag{94}$$

Then

$$\begin{aligned} \xi M\bar{E}_{t-1}[Z_{t-1}] &= \begin{bmatrix} \frac{1}{1-\lambda}I & -\frac{\lambda}{1-\lambda}I & 0 \end{bmatrix} \begin{bmatrix} A & 0 & 0 \\ G & L & 0 \\ 0 & 0 & 0 \end{bmatrix} \bar{E}_{t-1}[Z_{t-1}] \\ &= \begin{bmatrix} \frac{1}{1-\lambda}A - \frac{\lambda}{1-\lambda}G & -\frac{\lambda}{1-\lambda}L & 0 \end{bmatrix} \bar{E}_{t-1} \begin{bmatrix} \bar{X}_{t-1} \\ F_{t-1} \\ w_{t-1} \end{bmatrix} \\ &= \left(\frac{1}{1-\lambda}A - \frac{\lambda}{1-\lambda}G\right) \bar{E}_{t-1}[\bar{X}_{t-1}] - \frac{\lambda}{1-\lambda}L\bar{E}_{t-1}[F_{t-1}] \end{aligned} \tag{95}$$

2. write H as a vector of matrices $H \equiv \begin{bmatrix} \begin{bmatrix} H_{1,1} & H_{1,2} \\ H_{1,1} & H_{1,2} \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{bmatrix} \equiv [H_1 \quad H_2 \quad H_3]$.

Then

$$\begin{aligned} HM\bar{E}_{t-1}[Z_{t-1}] &= [H_1 \quad H_2 \quad H_3] \begin{bmatrix} A & 0 & 0 \\ G & L & 0 \\ 0 & 0 & 0 \end{bmatrix} \bar{E}_{t-1}[Z_{t-1}] \\ &= [H_1 A \quad 0 \quad 0] \bar{E}_{t-1} \begin{bmatrix} \bar{X}_{t-1} \\ F_{t-1} \\ w_{t-1} \end{bmatrix} \\ &= H_1 A \bar{E}_{t-1}[\bar{X}_{t-1}] \end{aligned} \tag{96}$$

3. Similarly,

$$HMZ_{t-1} = H_1 A \bar{X}_{t-1} \tag{97}$$

4. Similarly

$$Hm = H_1 a + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{98}$$

Substitute back in the posted KF, using that $-\frac{\lambda}{1-\lambda}\bar{E}_{t-1}[F_{t-1}] = F_{t-1} - \frac{1}{1-\lambda}\bar{E}_{t-1}[X_{t-1}]$. After some algebra, one gets

$$\begin{aligned} F_t = \left(\frac{1}{1-\lambda} A + -\frac{\lambda}{1-\lambda} G - \frac{1}{1-\lambda} L - \xi K H_1 A \right) \bar{E}_{t-1}[\bar{X}_{t-1}] + \xi K H_1 A \bar{X}_{t-1} + L F_{t-1} \\ + \xi K \left(H_1 a + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right) \begin{bmatrix} u_t \\ e_t \end{bmatrix} \end{aligned} \tag{99}$$

Equation 99 must equal the second line (a 2x1 vector) of the perceived law of motion 87. The solution to the fixed point is given by $G = \xi K H_1 A$, $\mu = \xi K \left(H_1 a + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \right)$ and $L = A - G$.

In particular, define

$$C_1 \equiv \frac{K_{1,1} - \lambda(K_{3,1})}{1-\lambda}, \qquad C_2 \equiv \frac{K_{1,2} - \lambda K_{3,2}}{1-\lambda} \qquad \text{and} \qquad C = C_1 + C_2$$
$$D_1 \equiv \frac{K_{2,1} - \lambda(K_{4,1})}{1-\lambda}, \qquad D_2 \equiv \frac{K_{2,2} - \lambda K_{4,2}}{1-\lambda} \qquad \text{and} \qquad D = D_1 + D_2$$

Then $G = \begin{bmatrix} \rho_1 C & \rho_2 C \\ \rho_1 D & \rho_2 D \end{bmatrix}$, $\mu = \begin{bmatrix} C & C_1 \\ D & 0 \end{bmatrix}$ and $L = \begin{bmatrix} \rho_1(1-C) & \rho_2(1-C) \\ 1-\rho_1 D & -\rho_2 D \end{bmatrix}$.

Given the law of motion of unobserved state 29 and the observable 31, the posterior variance of the forecast solves the following Ricatti equation

$$\Sigma \equiv E[(Z_t - Z^i_{t,t-1})(Z_t - Z^i_{t,t-1})']$$
$$\Sigma = M(\Sigma - \Sigma H' \left(H\Sigma H' + \begin{bmatrix} 0 & 0 \\ 0 & \tau^{-1} \end{bmatrix} \right)^{-1} H\Sigma)M' + m \begin{bmatrix} \xi^{-1} & 0 \\ 0 & \nu^{-1} \end{bmatrix} m' \tag{100}$$

and the Kalman filter is

$$K = \Sigma H' \left(H\Sigma H' + \begin{bmatrix} 0 & 0 \\ 0 & \tau^{-1} \end{bmatrix} \right)^{-1} \tag{101}$$

Step 6: derive the action of individual With the model's solution, one can obtain the individual forecast as

$$
\begin{aligned}
F_t^i &= \xi \quad E_t^i[Z_t] \\
&= \xi(I-KH)ME_{t-1}^i[Z_{t-1}] + \xi KHMZ_{t-1} + \xi KHm \begin{bmatrix} u_t \\ e_t \end{bmatrix} \\
&= A \underbrace{\left(\frac{1}{1-\lambda}E_{t-1}^i[\bar{X}_{t-1}] - \frac{\lambda}{1-\lambda}E_{t-1}^i[F_{t-1}]\right)}_{F_{t-1}^i} + [-\frac{\lambda}{1-\lambda}G - G]E_{t-1}^i[x_{t-1}] + \frac{\lambda}{1-\lambda}GE_{t-1}^i[F_{t-1}] \\
&\quad - \xi KH_1AE_{t-1}^i[\bar{X}_{t-1}] + \xi KH_1A\bar{X}_{t-1} + \xi KH_1a \begin{bmatrix} u_t \\ e_t \end{bmatrix} + \xi K \begin{bmatrix} e_t \\ 0 \end{bmatrix} + \xi K \begin{bmatrix} 0 \\ \eta_t^i \end{bmatrix} \\
F_t^i - AF_{t-1}^i &= -\xi KH_1AF_{t-1}^i + \xi KH_1\bar{X}_t + \xi K \begin{bmatrix} e_t \\ 0 \end{bmatrix} + \xi K \begin{bmatrix} 0 \\ \eta_t^i \end{bmatrix} \\
F_t^i - AF_{t-1}^i &= \xi K \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} (\bar{X}_t - AF_{t-1}^i) + \xi K \begin{bmatrix} 1 \\ 0 \end{bmatrix} e_t + \xi K \begin{bmatrix} 0 \\ 1 \end{bmatrix} \eta_t^i \\
F_t^i - AF_{t-1}^i &= \begin{bmatrix} C \\ D \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} (\bar{X}_t - AF_{t-1}^i) + \begin{bmatrix} C_1 \\ D_1 \end{bmatrix} e_t + \begin{bmatrix} C_2 \\ D_2 \end{bmatrix} \eta_t^i
\end{aligned} \tag{102}
$$

consider the first line

$$\hat{x}_{t,t}^i - \hat{x}_{t,t-1}^i = C[x_t - \hat{x}_{t,t-1}^i] + C_1e_t + C_2\eta_t^i \tag{103}$$

Which is similar to the basic framework in section 2. Consider the second line

$$\hat{x}_{t-1,t}^i - \hat{x}_{t-1,t-1}^i = D[x_{t-1} - \hat{x}_{t-1,t-1}^i] + D_1e_t + D_2\eta_t^i \tag{104}$$

G Structural estimation at: 2 quarters horizon

Table 13: Estimasted parameters

Variable	ρ (1)	$\frac{\sigma_e}{\sigma_u}$ (2)	$\frac{\sigma_\eta}{\sigma_u}$ (3)	λ (4)
Nominal GDP	0.93	1.51	1.31	0.61
GDP price index inflation	0.90	1.10	1.08	0.32
Real GDP	0.80	1.20	1.19	0.38
Consumer Price Index	0.97	1.14	1.22	0.56
Industrial production	0.85	1.41	1.16	0.29
Housing Start	0.85	2.12	1.20	0.31
Real Consumption	0.73	1.05	1.32	0.39
Real residential investment	0.89	1.44	1.20	0.23
Real nonresidential investment	0.88	3.16	1.04	0.14
Real state and local government consumption	0.74	1.07	1.67	0.73
Real federal government consumption	0.77	1.11	1.61	0.69
Unemployment rate	0.97	3.15	1.03	-0.28
Three-month Treasury rate	0.94	3.16	1.03	0.06
Ten-year Treasury rate	0.83	1.48	1.39	0.69
AAA Corporate Rate Bond	0.85	2.13	1.53	0.83

Table 14: Moments in data and model

	Targeted moments						Untargeted moments					
	Mean Dispersion		C		β_1		β_{CG}		β_{BGMS}		β_2	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Nominal GDP	0.94	0.94	0.61	0.61	-0.35	-0.35	0.20	0.43	-0.11	-0.22	0.62	0.11
GDP price index inflation	0.34	0.34	0.70	0.70	-0.20	-0.20	0.48	0.20	-0.02	-0.06	0.44	0.10
Real GDP	0.69	0.69	0.63	0.63	-0.25	-0.25	0.33	0.28	-0.07	-0.10	0.54	0.11
Consumer Price Index	0.27	0.27	0.70	0.70	-0.38	-0.38	-0.05	0.21	-0.24	-0.14	0.51	0.20
Industrial production	2.49	2.49	0.59	0.59	-0.16	-0.16	0.33	0.41	-0.01	-0.09	0.49	0.05
Housing Start	75.76	75.76	0.53	0.53	-0.15	-0.15	0.91	0.75	0.12	-0.13	0.54	0.01
Real Consumption	0.34	0.34	0.63	0.63	-0.31	-0.31	0.12	0.16	-0.11	-0.07	0.61	0.16
Real residential investment	16.69	16.69	0.56	0.56	-0.13	-0.13	0.56	0.42	0.07	-0.07	0.49	0.04
Real nonresidential investment	5.02	5.02	0.61	0.59	-0.02	-0.05	0.53	0.67	0.10	-0.05	0.41	0.00
Real state and local government consumption	0.92	0.92	0.61	0.61	-0.65	-0.65	0.05	0.15	-0.24	-0.23	0.77	0.35
Real federal government consumption	4.40	4.40	0.60	0.60	-0.60	-0.60	-0.21	0.18	-0.27	-0.21	0.77	0.31
Unemployment rate	0.09	0.06	0.56	0.55	0.09	0.08	0.59	0.78	0.20	0.08	0.39	0.00
Three-month Treasury rate	0.21	0.12	0.63	0.60	0.02	-0.02	0.40	0.66	0.14	-0.02	0.48	0.00
Ten-year Treasury rate	0.12	0.12	0.60	0.60	-0.46	-0.46	-0.09	0.45	-0.24	-0.31	0.71	0.14
AAA Corporate Rate Bond	0.25	0.25	0.61	0.61	-0.49	-0.49	0.05	0.58	-0.22	-0.44	0.70	0.07

Table 15: Posted and honest moments

	Gain			Consensus MSE			Dispersion		
	Posted	Honest	Ratio	Posted	Honest	Ratio	Posted	Honest	Ratio
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Nominal GDP	0.61	0.49	0.80	0.27	0.60	2.22	0.94	0.41	0.44
GDP price index inflation	0.70	0.66	0.94	0.27	0.37	1.36	0.34	0.22	0.65
Real GDP	0.63	0.57	0.91	0.56	0.80	1.41	0.69	0.39	0.57
Consumer Price Index	0.70	0.62	0.89	0.13	0.23	1.86	0.27	0.10	0.39
Industrial production	0.59	0.54	0.92	1.71	2.29	1.34	2.49	1.79	0.72
Housing Start	0.53	0.47	0.87	35.99	50.84	1.41	75.76	57.87	0.76
Real Consumption	0.63	0.59	0.95	0.57	0.71	1.24	0.34	0.16	0.48
Real residential investment	0.56	0.53	0.94	13.81	17.09	1.24	16.69	12.95	0.78
Real nonresidential investment	0.59	0.57	0.95	2.08	2.43	1.17	5.02	4.65	0.93
Real state and local government consumption	0.61	0.55	0.89	0.73	1.10	1.51	0.92	0.11	0.12
Real federal government consumption	0.60	0.53	0.88	3.60	5.48	1.52	4.40	0.69	0.16
Unemployment rate	0.55	0.60	1.08	0.04	0.03	0.78	0.06	0.07	1.12
Three-month Treasury rate	0.60	0.59	0.98	0.05	0.05	1.07	0.12	0.11	0.97
Ten-year Treasury rate	0.60	0.44	0.73	0.03	0.08	2.48	0.12	0.04	0.29
AAA Corporate Rate Bond	0.61	0.31	0.51	0.02	0.10	4.58	0.25	0.06	0.24