Monetary Policy without Commitment

Hassan Afrouzi
Columbia

Marina Halac
Yale

Kenneth Rogoff
Harvard

Pierre Yared
Columbia GSB

ECB Annual Conference
September 20, 2024

How does Lack of Commitment Interact with Inflation Dynamics?

- Commitment to inflation targeting is a hallmark of modern central banking

- Limitations of previous literature on central bank credibility
  - e.g. Barro-Gordon 83 and Rogoff 85
  - No connection to underlying economic parameters
  - No transitional dynamics
  - No quantitative implications

- This paper: Lack of commitment in the New Keynesian model
  Requires dynamic, non-linearized framework

Preview of Model

- Deterministic non-linear NK model with Calvo pricing
  - Firms underproduce and underhire because of monopoly power w/o stimulus
  - Price dispersion with labor misallocated to low-price varieties with stimulus
- Monetary Non-Neutrality
  - $\uparrow$ inflation $\Longrightarrow \uparrow$ dispersion (misallocation), $\downarrow$ monopoly distortions
- Markov Perfect Competitive Equilibrium: CB optimizes at every date
  - CB undoes monopoly distortion, sets labor share to 1 (MRS = MPL)
  - Model reduces to three equations:
    - Forward-looking Phillips curve + pass-through of real wage to inflation
    - Backward-looking dispersion dynamics equation

Main Results

- Economic environment drives long-run inflation
  - $\uparrow$ labor wedge or $\downarrow$ elasticity of substitution $\implies \uparrow$ inflation
  - Driven by interaction of environment with lack of commitment
- Inflation overshoots in transition to high-inflation steady state
  - Driven by evolution of CB incentives as dispersion increases
- Quantitative magnitudes are large
  - Small shocks $\implies$ Large change in inflation, significant overshooting
  - Loss due to lack of commitment (versus targeting) is high

Related Literature

- Linearized models of central bank credibility
  - Barro-Gordon 83, Rogoff 85, Athey-Atkeson-Kehoe 05, Halac-Yared 20,22
  - **This paper**: Transition dynamics, quantitative implications
- Credibility in non-linear environments without dispersion
  - Alvarez-Kehoe-Neumeyer 04, Davila-Schaab 23
  - **This paper:** Focus on dispersion and inflation-output tradeoff
- Non-linear models of central bank credibility
  - Albanesi-Chari-Christiano 03, King-Wolman 04, Zandweghe-Wolman 19
  - **This paper**: Theoretical analysis of non-linear Calvo model
- Non-linear models of optimal commmitment policy
  - Benigno-Woodford 05, Yun 05
  - **This paper**: No commitment, recursive auxiliary variable

Model

Household Problem

$$\max_{C_t, L_t, B_t, (s_{j,t}, C_{j,t})_{j\in[0,1]}} \sum_{t=0}^{\infty} \beta^t \left(\log(C_t) - \frac{L_t^{1+\psi}}{1+\psi} \right)$$

subject to

$$\int_0^1 P_{j,t} C_{j,t} dj + B_t \leq W_t L_t + (1 + i_{t-1}) B_{t-1} + \int_0^1 s_{j,t} X_{j,t} dj + \int_0^1 (s_{j,t-1} - s_{j,t}) P^S_{j,t} dj - T_t,$$

$$\text{where } C_t = \left(\int_0^1 C_{j,t}^{1-\sigma^{-1}} dj \right)^{\frac{1}{1-\sigma^{-1}}}$$

Optimality Conditions

- Allocation across varieties:

$$C_{j,t} = C_t \left(\frac{P_{j,t}}{P_t} \right)^{-\sigma} \quad \text{where} \quad P_t = \left(\int_0^1 P_{j,t}^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}$$

Optimality Conditions

- Allocation across varieties:

$$C_{j,t} = C_t \left(\frac{P_{j,t}}{P_t} \right)^{-\sigma} \quad \text{where} \quad P_t = \left(\int_0^1 P_{j,t}^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}$$

- Intertemporal and intratemporal conditions:

$$\frac{W_t}{P_t} = C_t L_t^{\psi} \quad \text{and} \quad 1 = \beta(1 + i_t) \frac{P_t C_t}{P_{t+1} C_{t+1}}$$

Optimality Conditions

- Allocation across varieties:

$$C_{j,t} = C_t \left(\frac{P_{j,t}}{P_t} \right)^{-\sigma} \quad \text{where} \quad P_t = \left(\int_0^1 P_{j,t}^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}$$

- Intertemporal and intratemporal conditions:

$$\frac{W_t}{P_t} = C_t L_t^{\psi} \quad \text{and} \quad 1 = \beta(1 + i_t) \frac{P_t C_t}{P_{t+1} C_{t+1}}$$

- Firm pricing:

$$P_{j,t}^S = \sum_{h=0}^{\infty} \beta^h \frac{P_t C_t}{P_{t+h} C_{t+h}} \mathbb{E}_t^j [X_{j,t+h}] + \lim_{h \to \infty} \beta^h \frac{P_t C_t}{P_{t+h} C_{t+h}} \mathbb{E}_t^j [P_{j,t+h}^S]$$

- In the paper: A TVC as a sufficient condition for the **lim** term to vanish

Firm Problem

- Initial prices $P_{j,-1}$. Can change in every period with probability $1 - \theta$

Firm Problem

- Initial prices $P_{j,-1}$. Can change in every period with probability $1-\theta$
- Objective to maximize

$$P^S_{j,t} = \sum_{h=0}^{\infty} \beta^h \frac{P_t C_t}{P_{t+h} C_{t+h}} \left[P_{j,t} Y_{j,t} - (1+\tau) W_t L_{j,t}\right] \text{ where } L_{j,t} = Y_{j,t}$$

after substitution

$$\max_{P^*_t} \sum_{h=0}^{\infty} (\beta\theta)^h \frac{P_t C_t}{P_{t+h} C_{t+h}} [P^*_t - (1+\tau) W_{t+h}] C_{t+h} \left(\frac{P^*_t}{P_{t+h}}\right)^{-\sigma}$$

Firm Problem

- Initial prices $P_{j,-1}$. Can change in every period with probability $1-\theta$
- Objective to maximize

$$P^S_{j,t} = \sum_{h=0}^{\infty} \beta^h \frac{P_t C_t}{P_{t+h} C_{t+h}} \left[P_{j,t} Y_{j,t} - (1+\tau) W_t L_{j,t} \right] \text{ where } L_{j,t} = Y_{j,t}$$

after substitution

$$\max_{P_t^*} \sum_{h=0}^{\infty} (\beta\theta)^h \frac{P_t C_t}{P_{t+h} C_{t+h}} [P_t^* - (1+\tau) W_{t+h}] C_{t+h} \left(\frac{P_t^*}{P_{t+h}} \right)^{-\sigma}$$

Assumption

$\tau > -1/\sigma$

Government

- CB sets i_t to maximize social welfare
- Fiscal authority sets T_t and B_t
- Government budget constraint

$$(1 + i_{t-1})B_{t-1} = B_t + T_t + \tau W_t L_t$$

Order of Events

1. Flexible firms choose $P_{j,t} = P_t^*$. Sticky firms choose $P_{j,t} = P_{j,t-1}$
2. CB chooses i_t
3. Households choose $C_t, L_t, B_t, (s_{i,t}, C_{j,t})_{j\in[0,1]}$

Order of Events

1. Flexible firms choose $P_{j,t} = P_t^*$. Sticky firms choose $P_{j,t} = P_{j,t-1}$
2. CB chooses i_t
3. Households choose $C_t, L_t, B_t, (s_{i,t}, C_{j,t})_{j\in[0,1]}$
4. Fiscal authority chooses T_t and B_t

Markov Perfect Competitive Equilibrium

- All decisions are functions of minimal payoff relevant variables
    - Ricardian Equivalence $\implies$ wlog, set $B_t = 0$ at every t (not payoff relevant)

Markov Perfect Competitive Equilibrium

- All decisions are functions of minimal payoff relevant variables
  - Ricardian Equivalence $\implies$ wlog, set $B_t = 0$ at every t (not payoff relevant)
- Strategies:
  - Flex-price firms at t choose P_t^* as a function of price distribution Ω_{t-1}
    - This determines Ω_t
  - Central bank chooses i_t as a function of Ω_t
  - Households make decisions as a function of Ω_t and i_t

Markov Perfect Competitive Equilibrium

- All decisions are functions of minimal payoff relevant variables
  - Ricardian Equivalence $\implies$ wlog, set $B_t = 0$ at every t (not payoff relevant)
- Strategies:
  - Flex-price firms at t choose P_t^* as a function of price distribution Ω_{t-1}
    - This determines Ω_t
  - Central bank chooses i_t as a function of Ω_t
  - Households make decisions as a function of Ω_t and i_t
- An MPCE is a collection of all of these mappings

Competitive Equilibrium

Aggregate Production

- Aggregate production: labor market clearing and $y_{j,t} = l_{j,t} = Y_t(P_{j,t}/P_t)^{-\sigma}$

$$L_t = \int_0^1 l_{j,t} dj = \int_0^1 y_{j,t} dj = Y_t \int_0^1 (P_{j,t}/P_t)^{-\sigma} dj$$

Aggregate Production

- Aggregate production: labor market clearing and $y_{j,t} = l_{j,t} = Y_t (P_{j,t}/P_t)^{-\sigma}$

$$L_t = \int_0^1 l_{j,t} dj = \int_0^1 y_{j,t} dj = Y_t \int_0^1 (P_{j,t}/P_t)^{-\sigma} dj$$

- Define price (markup) dispersion as $D_t \equiv \int_0^1 (P_{j,t}/P_t)^{-\sigma} dj$:

$$D_t \geq 1 \implies Y_t = \frac{L_t}{D_t} \leq L_t \qquad \text{(misallocation)}$$

Aggregate Production

- Aggregate production: labor market clearing and $y_{j,t} = l_{j,t} = Y_t(P_{j,t}/P_t)^{-\sigma}$

$$L_t = \int_0^1 l_{j,t}dj = \int_0^1 y_{j,t}dj = Y_t \int_0^1 (P_{j,t}/P_t)^{-\sigma}dj$$

- Define price (markup) dispersion as $D_t \equiv \int_0^1 (P_{j,t}/P_t)^{-\sigma}dj$:

$$D_t \geq 1 \implies Y_t = \frac{L_t}{D_t} \leq L_t \qquad \text{(misallocation)}$$

- Using household's labor supply, real wage and labor share are

$$\text{labor share} \quad : \quad \mu_t \equiv \frac{W_t L_t}{P_t Y_t} = \frac{MRS_t}{MPL_t} = L_t^{1+\psi}$$

$$\text{real wage} \quad : \quad \frac{W_t}{P_t} = \mu_t Y_t / L_t = \mu_t / D_t$$

Dispersion Dynamics

- Letting $\Pi_t = P_t / P_{t-1}$, dispersion follows

$$D_t = (1-\theta)\left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{\frac{\sigma}{\sigma-1}} + \theta\Pi_t^{\sigma} D_{t-1} \tag{DD}$$

Dispersion Dynamics

- Letting $\Pi_t = P_t / P_{t-1}$, dispersion follows

$$D_t = (1-\theta)\left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{\frac{\sigma}{\sigma-1}} + \theta\Pi_t^{\sigma} D_{t-1} \tag{DD}$$

- Two forces:
    - Inflation leaves sticky price firms behind $\rightarrow$ More dispersion across firms
    - All flex price firms choose P_t^* $\rightarrow$ Less dispersion within flex price firms

Dispersion Dynamics

- Letting $\Pi_t = P_t / P_{t-1}$, dispersion follows

$$D_t = (1-\theta)\left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{\frac{\sigma}{\sigma-1}} + \theta\Pi_t^{\sigma} D_{t-1} \tag{DD}$$

- Two forces:
  - Inflation leaves sticky price firms behind $\rightarrow$ More dispersion across firms
  - All flex price firms choose P_t^* $\rightarrow$ Less dispersion within flex price firms
- For $\Pi_t > 1$, first force dominates

Non-Linear Phillips Curve

- Flex-price firm optimality yields non-linear Phillips Curve:

$$\left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}} = \frac{\sigma(1+\tau)}{\sigma-1}\delta_t\frac{\mu_t}{D_t} + (1-\delta_t)\Pi_{t+1}\left(\frac{1-\theta\Pi_{t+1}^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}} \tag{NLPC}$$

where δ_t is an aux. variable capturing wage pass-through dynamics (WPD):

$$\delta_t^{-1} = 1 + \beta\theta\Pi_{t+1}^{\sigma-1}\delta_{t+1}^{-1} \tag{WPD}$$

Non-Linear Phillips Curve

- Flex-price firm optimality yields non-linear Phillips Curve:

$$\left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}} = \frac{\sigma(1+\tau)}{\sigma-1}\delta_t\frac{\mu_t}{D_t} + (1-\delta_t)\Pi_{t+1}\left(\frac{1-\theta\Pi_{t+1}^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}} \tag{NLPC}$$

where δ_t is an aux. variable capturing wage pass-through dynamics (WPD):

$$\delta_t^{-1} = 1 + \beta\theta\Pi_{t+1}^{\sigma-1}\delta_{t+1}^{-1} \tag{WPD}$$

- Two forces:
  - Higher current real wages → Higher current inflation
  - Higher future inflation → Higher current inflation

Non-Linear Phillips Curve

- Flex-price firm optimality yields non-linear Phillips Curve:

$$\left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}}=\frac{\sigma(1+\tau)}{\sigma-1}\delta_t\frac{\mu_t}{D_t}+(1-\delta_t)\Pi_{t+1}\left(\frac{1-\theta\Pi_{t+1}^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}} \qquad \text{(NLPC)}$$

  where δ_t is an aux. variable capturing wage pass-through dynamics (WPD):

$$\delta_t^{-1}=1+\beta\theta\Pi_{t+1}^{\sigma-1}\delta_{t+1}^{-1} \qquad \text{(WPD)}$$

- Two forces:
  - Higher current real wages $\rightarrow$ Higher current inflation
  - Higher future inflation $\rightarrow$ Higher current inflation
- Nature of time inconsistency: Π_{t+1} and δ_{t+1} affect allocation at t.

Long-Run Monetary Non-Neutrality

Lemma

Consider hypothetical steady state $\{\Pi, D, \mu\}$ for $\Pi \in [1, \theta^{-1/\sigma})$. D and μ are unique and increasing in Π.

Long-Run Monetary Non-Neutrality

Lemma

Consider hypothetical steady state $\{\Pi, D, \mu\}$ for $\Pi \in [1, \theta^{-1/\sigma})$. D and μ are unique and increasing in Π.

- Mechanism: If $\uparrow \Pi$, then
  - $\uparrow D$ since more sticky price firms left behind
  - $\uparrow \mu$ since more overhiring by sticky price firms
    - Not fully counterbalanced by flex-firm price increases since $\beta < 1$

Long-Run Monetary Non-Neutrality

Lemma

Consider hypothetical steady state $\{\Pi, D, \mu\}$ for $\Pi \in [1, \theta^{-1/\sigma})$. D and μ are unique and increasing in Π.

- Mechanism: If $\uparrow \Pi$, then
  - $\uparrow D$ since more sticky price firms left behind
  - $\uparrow \mu$ since more overhiring by sticky price firms
    - Not fully counterbalanced by flex-firm price increases since $\beta < 1$
- Implications:
  - Steady state tradeoff between dispersion and monopoly distortions
    - Zero inflation/dispersion steady state is distorted ($\tau > -1/\sigma$)
  - Inflation dynamics not pinned down by model
    - Immediate transition from one steady state inflation to another possible

Equilibrium Policy

Central Bank Problem with Commitment

- Recall $L_t^{1+\psi} = \mu_t$ and $\ln(C_t) = -\ln(D_t) + \ln(\mu_t)/(1+\psi)$:

$$\ln(C_t) - \frac{L_t^{1+\psi}}{1+\psi} = -\ln(D_t) + \frac{\ln(\mu_t) - \mu_t}{1+\psi}$$

Central Bank Problem with Commitment

- Recall $L_t^{1+\psi} = \mu_t$ and $\ln(C_t) = -\ln(D_t) + \ln(\mu_t)/(1+\psi)$
- The central bank—with commitment to Π_t and δ_t—solves:

$$W\left(D_t, \Pi_t, \delta_t\right) = \max_{D_{t+1}, \Pi_{t+1}, \delta_{t+1}, \mu_t} \left\{ -\ln(D_t) + \frac{\ln(\mu_t) - \mu_t}{1+\psi} + \beta W\left(D_{t+1}, \Pi_{t+1}, \delta_{t+1}\right) \right\}$$

s.t.

NLPC, DD, WPD

Central Bank Problem with Commitment

- Recall $L_t^{1+\psi} = \mu_t$ and $\ln(C_t) = -\ln(D_t) + \ln(\mu_t)/(1+\psi)$
- The central bank—with commitment to Π_t and δ_t—solves:

$$W\left(D_t, \Pi_t, \delta_t\right) = \max_{D_{t+1}, \Pi_{t+1}, \delta_{t+1}, \mu_t} \left\{ -\ln(D_t) + \frac{\ln(\mu_t) - \mu_t}{1+\psi} + \beta W\left(D_{t+1}, \Pi_{t+1}, \delta_{t+1}\right) \right\}$$

s.t.

NLPC, DD, WPD

Lemma

In any steady state, $\Pi_t = 0$.

Central Bank Problem with Commitment

- Recall $L_t^{1+\psi} = \mu_t$ and $\ln(C_t) = -\ln(D_t) + \ln(\mu_t)/(1+\psi)$
- The central bank—with commitment to Π_t and δ_t—solves:

$$W\left(D_t, \Pi_t, \delta_t\right) = \max_{D_{t+1}, \Pi_{t+1}, \delta_{t+1}, \mu_t} \left\{ -\ln(D_t) + \frac{\ln(\mu_t) - \mu_t}{1+\psi} + \beta W\left(D_{t+1}, \Pi_{t+1}, \delta_{t+1}\right) \right\}$$

s.t.

NLPC, DD, WPD

Lemma

In any steady state, $\Pi_t = 0$.

- Intuition: No long-run intertemporal distortions
- Implication: Economic environment does not affect long-run inflation

Central Bank Problem without Commitment

- Central bank objective

$$V(D_t) = -\ln(D_t) + \frac{\ln(\mu_t) - \mu_t}{1+\psi} + \beta V(D_{t+1})$$

Central Bank Problem without Commitment

- Central bank objective

$$V(D_t) = -\ln(D_t) + \frac{\ln(\mu_t) - \mu_t}{1+\psi} + \beta V(D_{t+1})$$

- Since D_{t+1}, Π_{t+1}, and D_t are predetermined, FOC yields $\mu_t = 1$ ($MRS_t = MPL_t$)
  - Stimulate labor share to 1 from its inefficient level due to monopoly distortions

Central Bank Problem without Commitment

- Central bank objective

$$V(D_t) = -\ln(D_t) + \frac{\ln(\mu_t) - \mu_t}{1+\psi} + \beta V(D_{t+1})$$

- Since D_{t+1}, Π_{t+1}, and D_t are predetermined, FOC yields $\mu_t = 1$ ($MRS_t = MPL_t$)
    - Stimulate labor share to 1 from its inefficient level due to monopoly distortions
- Remarks
    - CB does not internalize policy's impact on D_t
    - CB reaction function: $1 + i_t = \frac{1}{\beta}\Pi_{t+1}Y_{t+1}D_t$. Stimulus declines in D_t
    - CB's policy is independent of underlying price-setting model

System of Equations

- Dispersion dynamics

$$D_t = (1-\theta)\left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{\frac{\sigma}{\sigma-1}} + \theta\Pi_t^{\sigma} D_{t-1}$$

System of Equations

- Dispersion dynamics

$$D_t = (1-\theta)\left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{\frac{\sigma}{\sigma-1}} + \theta\Pi_t^{\sigma} D_{t-1}$$

- Phillips curve (substituting CB reaction function)

$$\begin{aligned}\left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}} &= \frac{\sigma(1+\tau)}{\sigma-1}\delta_t D_t^{-1} + (1-\delta_t)\Pi_{t+1}\left(\frac{1-\theta\Pi_{t+1}^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}} \\ \text{for } \delta_t^{-1} &= 1+\beta\theta\Pi_{t+1}^{\sigma-1}\delta_{t+1}^{-1}\end{aligned}$$

System of Equations

- Dispersion dynamics

$$D_t = (1-\theta)\left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{\frac{\sigma}{\sigma-1}} + \theta\Pi_t^{\sigma} D_{t-1}$$

- Phillips curve (substituting CB reaction function)

$$\begin{aligned}\left(\frac{1-\theta\Pi_t^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}} &= \frac{\sigma(1+\tau)}{\sigma-1}\delta_t D_t^{-1} + (1-\delta_t)\Pi_{t+1}\left(\frac{1-\theta\Pi_{t+1}^{\sigma-1}}{1-\theta}\right)^{\frac{1}{1-\sigma}} \\ \text{for } \delta_t^{-1} &= 1+\beta\theta\Pi_{t+1}^{\sigma-1}\delta_{t+1}^{-1}\end{aligned}$$

- To facilitate analysis, consider continuous-time limit of model
    - Define $\pi_t \equiv d\log P_t/dt$ (rate of inflation) and $\lambda \equiv -\ln(\theta)$ (Poisson arrival rate)

System of Equations: Continuous Time Limit

- Dispersion dynamics and the Phillips curve:

$$\dot{D}_t = \lambda\left(1 - \frac{\sigma - 1}{\lambda}\pi_t\right)^{\frac{\sigma}{\sigma-1}} - (\lambda - \sigma\pi_t)D_t$$

$$\dot{\pi}_t = -\lambda\frac{\sigma(1+\tau)}{\sigma - 1}\left(1 - \frac{\sigma - 1}{\lambda}\pi_t\right)^{\frac{\sigma}{\sigma-1}}\frac{\delta_t}{D_t} + (\delta_t - \pi_t)(\lambda - (\sigma - 1)\pi_t)$$

$$\dot{\delta}_t = \delta_t^2 + [(\sigma - 1)\pi_t - (\rho + \lambda)]\delta_t$$

System of Equations: Continuous Time Limit

- Dispersion dynamics and the Phillips curve:

$$\dot{D}_t = \lambda \left(1 - \frac{\sigma - 1}{\lambda}\pi_t\right)^{\frac{\sigma}{\sigma-1}} - (\lambda - \sigma\pi_t)D_t$$

$$\dot{\pi}_t = -\lambda\frac{\sigma(1+\tau)}{\sigma - 1}\left(1 - \frac{\sigma - 1}{\lambda}\pi_t\right)^{\frac{\sigma}{\sigma-1}} \frac{\delta_t}{D_t} + (\delta_t - \pi_t)(\lambda - (\sigma - 1)\pi_t)$$

$$\dot{\delta}_t = \delta_t^2 + [(\sigma - 1)\pi_t - (\rho + \lambda)]\delta_t$$

- Consolidated dynamical system for $X_t = (D_t, \pi_t, \delta_t)$:

$$\dot{X}_t = f(X_t) \implies 0 = f(X_{ss})$$

- Prove X_{ss} is hyperbolic $\implies$ Hartman-Grobman and Stable Manifold Thms

Steady State

Proposition

There is a unique steady state $\{D, \pi\}$. Moreover,

- $\uparrow \tau$ (labor wedge) $\Longrightarrow \uparrow D$ and $\uparrow \pi$
- $\downarrow \sigma$ (elasticity of substitution) $\Longrightarrow \uparrow D$ (if τ low enough) and $\uparrow \pi$

Steady State

Proposition

There is a unique steady state $\{D, \pi\}$. Moreover,

- $\uparrow \tau$ (labor wedge) $\Longrightarrow \uparrow D$ and $\uparrow \pi$
- $\downarrow \sigma$ (elasticity of substitution) $\Longrightarrow \uparrow D$ (if τ low enough) and $\uparrow \pi$

- Intuition. Take economy with $\tau = -1/\sigma$ and increase τ
  - Under inflation targeting (IT), economy jumps to lower labor share

Steady State

Proposition

There is a unique steady state $\{D, \pi\}$. Moreover,

- $\uparrow \tau$ (labor wedge) $\implies \uparrow D$ and $\uparrow \pi$
- $\downarrow \sigma$ (elasticity of substitution) $\implies \uparrow D$ (if τ low enough) and $\uparrow \pi$

- Intuition. Take economy with $\tau = -1/\sigma$ and increase τ
  - Under inflation targeting (IT), economy jumps to lower labor share
  - IT not incentive compatible $\implies$ CB wants to stimulate

Steady State

Proposition

There is a unique steady state $\{D, \pi\}$. Moreover,

- $\uparrow \tau$ (labor wedge) $\Longrightarrow \uparrow D$ and $\uparrow \pi$
- $\downarrow \sigma$ (elasticity of substitution) $\Longrightarrow \uparrow D$ (if τ low enough) and $\uparrow \pi$

- Intuition. Take economy with $\tau = -1/\sigma$ and increase τ
  - Under inflation targeting (IT), economy jumps to lower labor share
  - IT not incentive compatible $\Longrightarrow$ CB wants to stimulate
  - Flex-price firms anticipate higher stimulus and raise prices

Steady State

Proposition

There is a unique steady state $\{D, \pi\}$. Moreover,

- $\uparrow \tau$ (labor wedge) $\Longrightarrow \uparrow D$ and $\uparrow \pi$
- $\downarrow \sigma$ (elasticity of substitution) $\Longrightarrow \uparrow D$ (if τ low enough) and $\uparrow \pi$

- Intuition. Take economy with $\tau = -1/\sigma$ and increase τ
  - Under inflation targeting (IT), economy jumps to lower labor share
  - IT not incentive compatible $\Longrightarrow$ CB wants to stimulate
  - Flex-price firms anticipate higher stimulus and raise prices
  - Sequential price increases by flex-price firms raise dispersion

Steady State

Proposition

There is a unique steady state $\{D, \pi\}$. Moreover,

- $\uparrow \tau$ (labor wedge) $\Longrightarrow \uparrow D$ and $\uparrow \pi$
- $\downarrow \sigma$ (elasticity of substitution) $\Longrightarrow \uparrow D$ (if τ low enough) and $\uparrow \pi$

- Intuition. Take economy with $\tau = -1/\sigma$ and increase τ
  - Under inflation targeting (IT), economy jumps to lower labor share
  - IT not incentive compatible $\Longrightarrow$ CB wants to stimulate
  - Flex-price firms anticipate higher stimulus and raise prices
  - Sequential price increases by flex-price firms raise dispersion
  - Benefit of stimulating the economy decreases as dispersion $\uparrow$ and MPL $\downarrow$

Steady State

Proposition

There is a unique steady state $\{D, \pi\}$. Moreover,

- $\uparrow \tau$ (labor wedge) $\Longrightarrow \uparrow D$ and $\uparrow \pi$
- $\downarrow \sigma$ (elasticity of substitution) $\Longrightarrow \uparrow D$ (if τ low enough) and $\uparrow \pi$

- Intuition. Take economy with $\tau = -1/\sigma$ and increase τ
  - Under inflation targeting (IT), economy jumps to lower labor share
  - IT not incentive compatible $\Longrightarrow$ CB wants to stimulate
  - Flex-price firms anticipate higher stimulus and raise prices
  - Sequential price increases by flex-price firms raise dispersion
  - Benefit of stimulating the economy decreases as dispersion $\uparrow$ and MPL $\downarrow$
  - Stimulus ends at higher inflation/dispersion steady state
- Analogous logic starting from other τ and for changes in σ

Transition Dynamics

Proposition

Take economy at steady state at t_0. In transition to new steady state $\{D', \pi'\}$ following unanticipated permanent increase in τ or decrease in σ (for low enough τ), inflation overshoots (i.e., there exists $t' \geq t_0$ with $\pi_t > \pi' \ \forall t > t'$)

Transition Dynamics

Proposition

Take economy at steady state at t_0. In transition to new steady state $\{D', \pi'\}$ following unanticipated permanent increase in τ or decrease in σ (for low enough τ), inflation overshoots (i.e., there exists $t' \geq t_0$ with $\pi_t > \pi' \; \forall t > t'$)

- Result proved by analyzing the three-dimensional non-linear system
- Saddle-path stability with a one-dimensional stable saddle-path

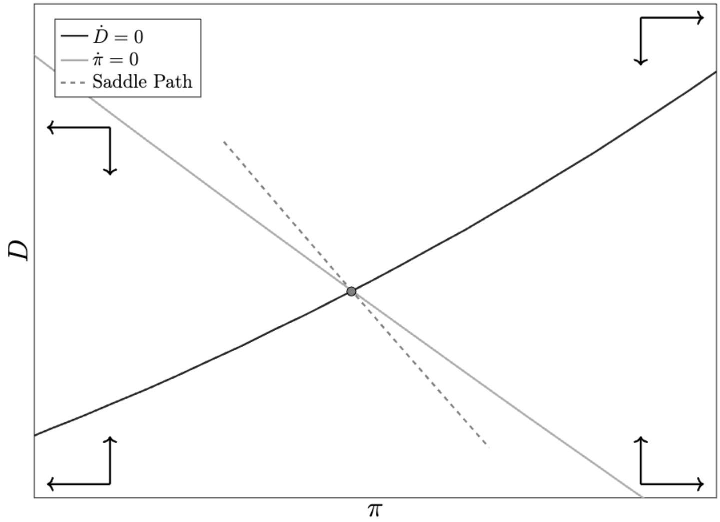
Transition Dynamics

Proposition

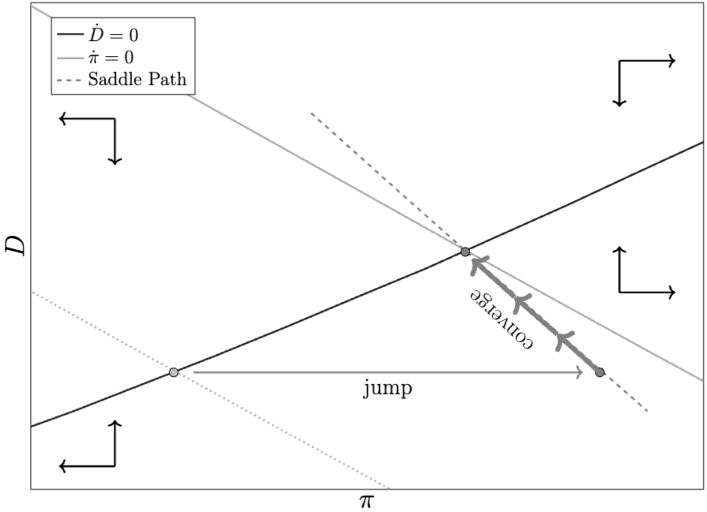
Take economy at steady state at t_0. In transition to new steady state $\{D', \pi'\}$ following unanticipated permanent increase in τ or decrease in σ (for low enough τ), inflation overshoots (i.e., there exists $t' \geq t_0$ with $\pi_t > \pi' \; \forall t > t'$)

- Result proved by analyzing the three-dimensional non-linear system
- Saddle-path stability with a one-dimensional stable saddle-path
- Special case: $\sigma \to 1$ while adjusting τ to hold markup $\frac{\sigma(1+\tau)}{\sigma-1}$ fixed
  $\implies \delta_t = \delta_{ss}, \forall t \geq 0$ and the system is two-dimensional phase diagram

Phase Diagram



Phase Diagram: Unanticipated Increase in Labor Wedge



Special Case of $\sigma \to 1$

- Closed-form solution $\implies$ inflation and log-dispersion decay at the rate of λ:

$$\ln D_t = \ln D_{ss} - \ln\left(\frac{D_{ss}}{D_0}\right) e^{-\lambda t}$$

$$\pi_t = \pi_{ss} + \lambda \ln\left(\frac{D_{ss}}{D_0}\right) e^{-\lambda t}$$

- Saddle path:

$$\pi(D) = \pi_{ss} - \lambda\left(\ln D - \ln D_{ss}\right)$$

- Cumulative overshooting of inflation along the transition path:

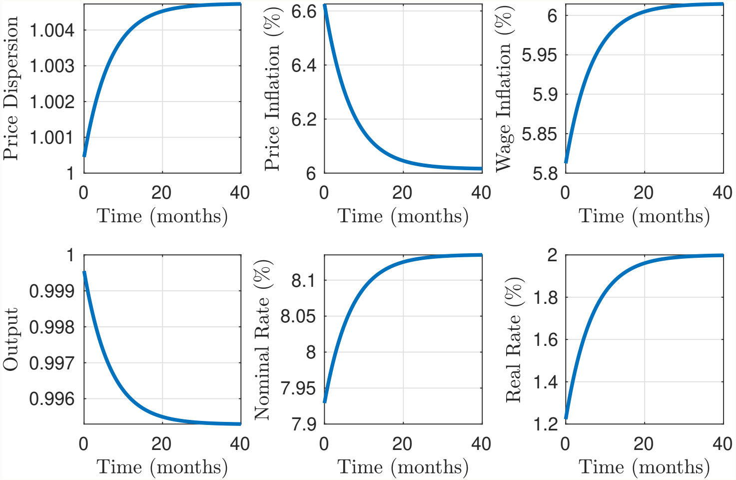
$$\int_0^1 (\pi_t - \pi_{ss}) dt = \ln\left(\frac{D_{ss}}{D_0}\right)$$

Table 1: Parameters

Parameter	Value	Target
Discount factor, β	$(1.02)^{-1/12}$	2% annual real interest rate
Fraction of sticky-price firms, θ	0.86	Nakamura and Steinsson (2008)
Elasticity of substitution, σ	7	Coibion, Gorodnichenko, and Wieland (2012)
Inverse Frisch elasticity, ψ	2.5	Chetty, Guren, Manoli, and Weber (2011)
Labor wedge, τ	-0.1427	2% annual inflation without commitment

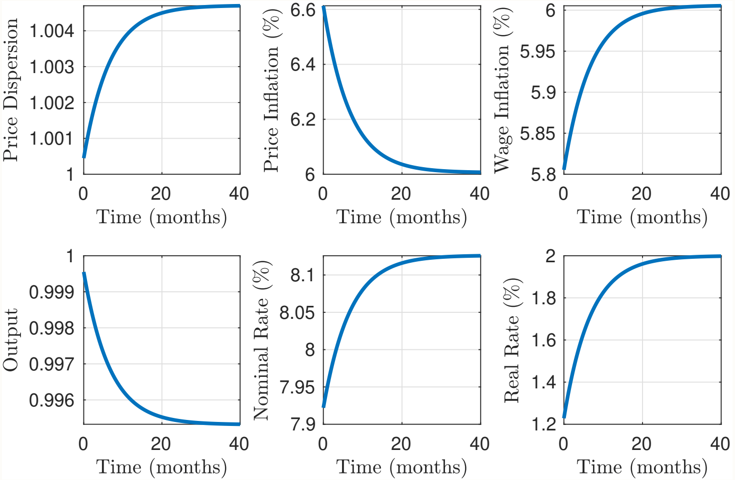
Unanticipated Increase in Labor Wedge

Figure 1: Response to Unanticipated Increase in Labor Wedge



Unanticipated Decrease in Elasticity of Substitution

Figure 2: Response to Unanticipated Decrease in Elasticity of Substitution



Inflation Targeting Versus No Commitment

Table 2: Inflation Targeting versus No Commitment

Scenario	Welfare under Targeting	Welfare under No Commitment	Welfare Difference
Labor Wedge Shock	0.981	0.922	0.059
Elasticity of Substitution Shock	0.981	0.921	0.060

Discussion of Quantitative Magnitudes

- Large magnitudes are a robust feature of the model
- Emerge because long-run Phillips curve is almost vertical

$$\mu = \frac{\sigma - 1}{\sigma(1+\tau)} \left[1 + (1-\beta) \frac{\theta \Pi^{\sigma-1}(\Pi - 1)}{(1-\theta\Pi^{\sigma})(1-\beta\theta\Pi^{\sigma-1})} \right]$$

  - Small changes in $\tau \rightarrow$ Large changes in Π (to keep μ fixed)
- Implications for models with flatter long-run Phillips curves
  - Smaller magnitudes in response to shocks
  - Smaller value of commitment to inflation targeting
  - Meaningful economic benefits from increasing long run inflation

Conclusion

Conclusion

- Analysis of lack of commitment in non-linear NK model
    - Long-run and transition dynamics in response to permanent shocks
- Framework for interpreting past and future inflation (Afrouzi et al, 2024)
    - Tailwinds that drove inflation down: globalization, Washington consensus
    - Headwinds likely driving it up: deglobalization, industrial policy
- Framework for assessing the value of commitment
    - Commitment to IT quantitatively large