

Common Factors, Trends, and Cycles in Large Datasets

Matteo Barigozzi
(London School of Economics)

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10th ECB Workshop on Forecasting Techniques:
Economic Forecasting with Large Datasets
June 18-19, 2018

Disclaimer: the views expressed in this paper are those of the authors and do not necessarily reflect those of the Board of Governors or the Federal Reserve System.

Measuring US Aggregate Output and Output Gap using Large Datasets

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- **Non-parametric Trend-Cycle decomposition**

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- Higher growth has been concentrated in Q1
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- Growth before the GFC was heavily boosted by temporary factors
- Growth after the financial crisis is due primarily to permanent factors
- Our estimate indicates that as of 2017:Q4 there is still slack

Outline

- The non-stationary Dynamic Factor Model

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Let $x_{it} \sim I(1)$

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- $\mathbf{f}_t \sim I(1)$ and $\xi_{it} \sim I(1)$ for some i
- $q - d$ “permanent shocks” and d “transitory shocks”
- $q - d$ common trends drive the dynamics of \mathbf{f}_t
- \mathbf{f}_t has cointegration rank d

Estimation: the “static” representation

Standard practice: estimate different representation

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$$\mathbf{x}_t = \underset{n \times q}{\mathbf{D}(L)} \underset{q \times 1}{\mathbf{f}_t} + \underset{q \times 1}{\boldsymbol{\xi}_t}$$

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$$\underset{r \times r}{\mathbf{A}(L)} \underset{r \times 1}{\mathbf{F}_t} = \underset{r \times q}{\mathbf{G}} \underset{q \times 1}{\mathbf{u}_t}$$

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Stock & Watson, 2005; Bai & Ng, 2007; Forni, Giannone, Lippi & Reichlin, 2009; BLL, 2016b.

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Same constraints on the co-movement of the data

Estimation: quasi maximum likelihood

- ML estimation via EM algorithm with Kalman smoother.
Doz, Giannone & Reichlin, 2011, 2012.
- Initialization: BLL, 2016b & Koopman, 1997

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Constraints:

- 1 $\lambda_{\text{GDP}} = \lambda_{\text{GDI}} \implies \chi_{\text{GDP},t} = \chi_{\text{GDI},t} = \text{GDO}_t$
- 2 The **non-stationary** ξ_{it} are additional states:

$$\xi_{it} = \rho_i \xi_{it-1} + e_{it}, \quad e_{it} \sim \mathcal{N}(0, \sigma_i^2), \quad \rho_i = \begin{cases} 1 & \text{if } \xi_{it} \sim I(1), \\ 0 & \text{if } \xi_{it} \sim I(0). \end{cases}$$

Outline

- **Model Set-up**

Model set-up

- $n = 103$ US macroeconomic time series;
- quarterly from 1960:Q1 to 2017:Q4, sample size $T = 232$
- log of all variables in levels which are not p.p.
- variables that are $I(1)$ are not transformed,
- variables that are $I(2)$ are differenced once
- inflation rates, unemployment rate, interest rates are in levels;
- \mathbf{x}_t are de-trended data—when necessary

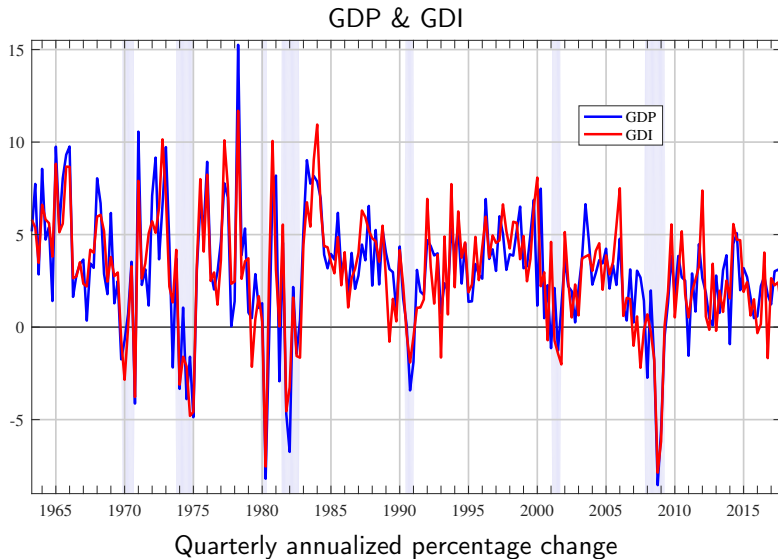
- $q = 3$;
- $q - d = 1$;
- $r = 6$.

- unit-root test on estimated idiosyncratic components;
- idiosyncratic of most aggregated variables are assumed $I(0)$
GDP, GDI, UR, FFR, CPI inflation, PCE inflation.

Outline

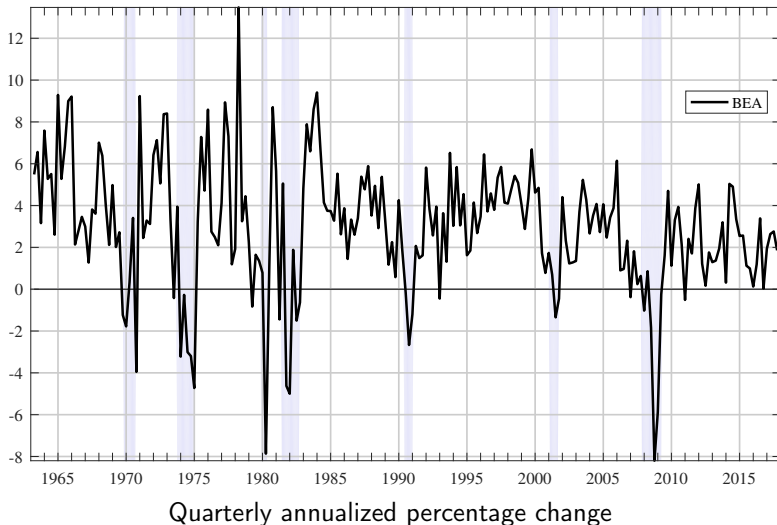
- Gross domestic output

Measures of aggregate output



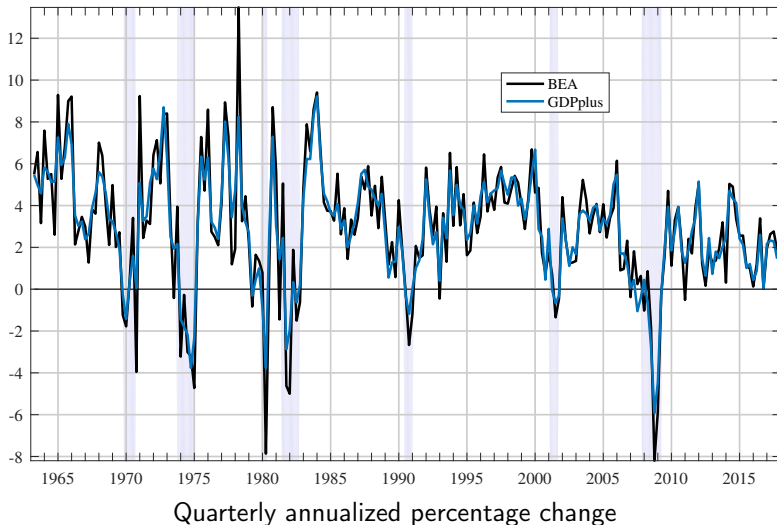
Measures of aggregate output

Average GDP-GDI



Measures of aggregate output

Average GDP-GDI & Philly FED DFM of GDP-GDI



Our measure of GDO

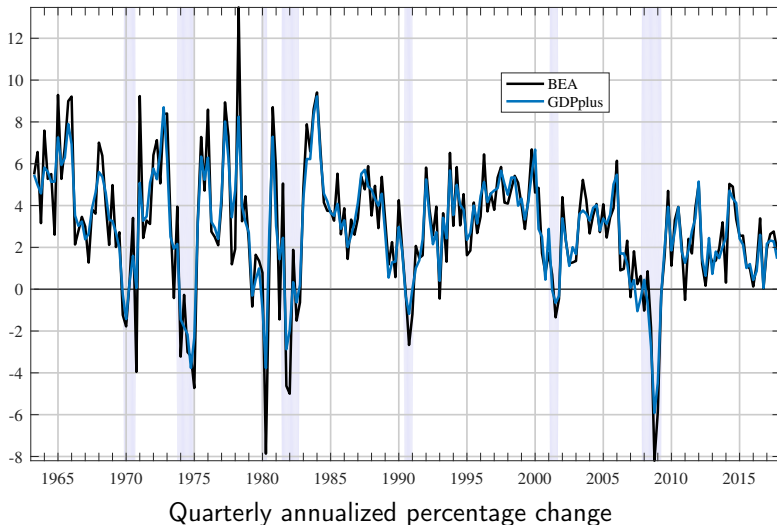
- GDO = part of GDP and GDI driven by u_t

Our measure of GDO

- GDO = part of GDP and GDI driven by \mathbf{u}_t
- Estimation base on two assumptions:
 - 1 GDP and GDI respond to \mathbf{u}_t in the same way
 $\implies \chi_t^{GDP} = \chi_t^{GDI}$
 - 2 the long run dynamics of GDP and GDI are entirely driven by \mathbf{u}_t
 $\implies \xi_t^{GDP}, \xi_t^{GDI} \sim I(0)$

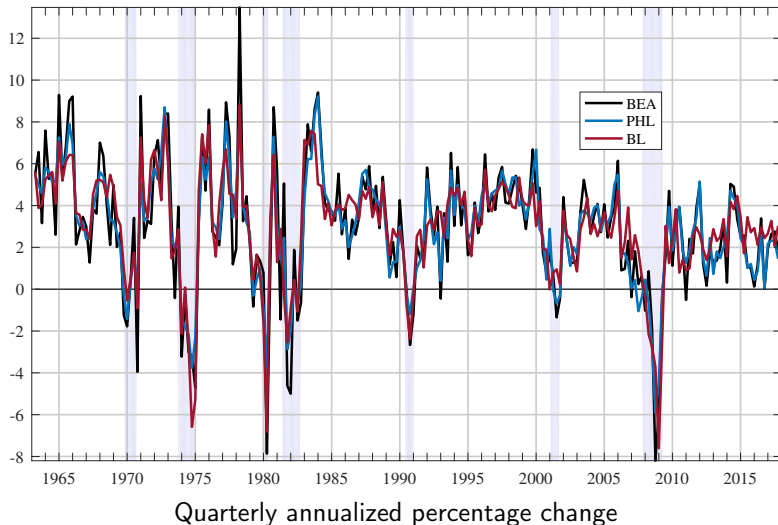
Gross domestic output

Average GDP-GDI & Philly FED DFM of GDP-GDI



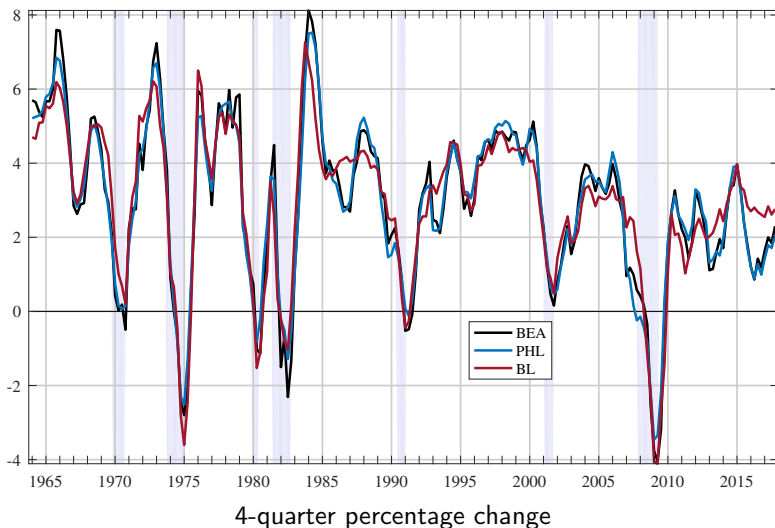
Gross domestic output

Average GDP-GDI & Philly FED DFM of GDP-GDI & BL



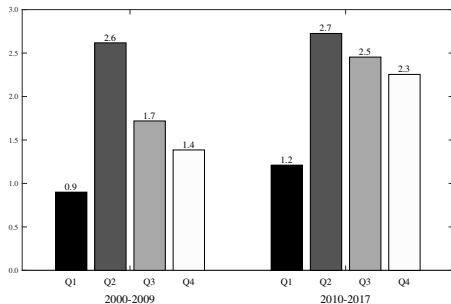
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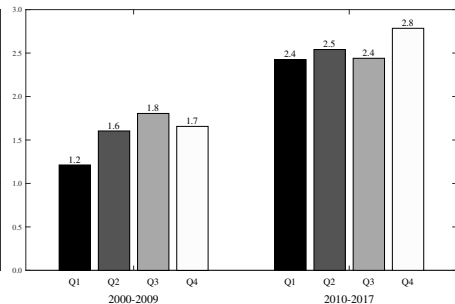


Our estimate does not show residual seasonality

GDP

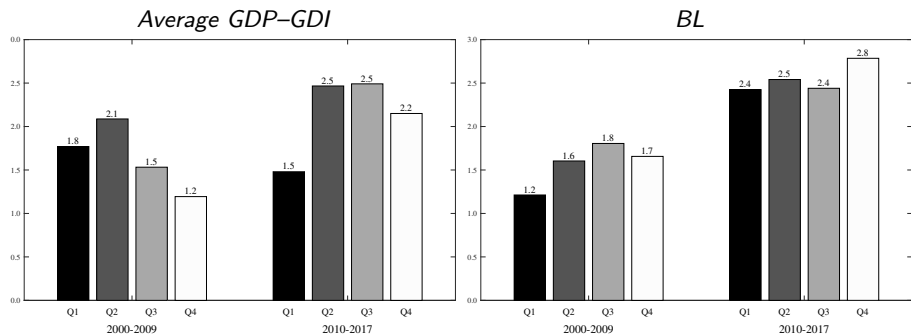


BL



Average quarterly annualized percentage change per quarter 2010–2016

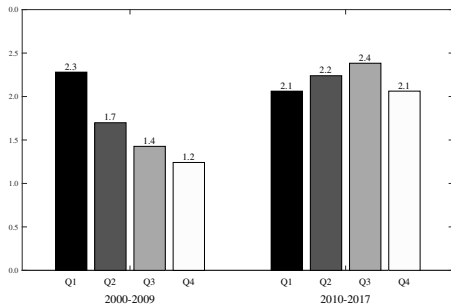
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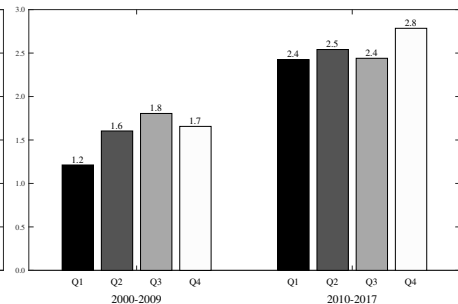
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GDP Plus



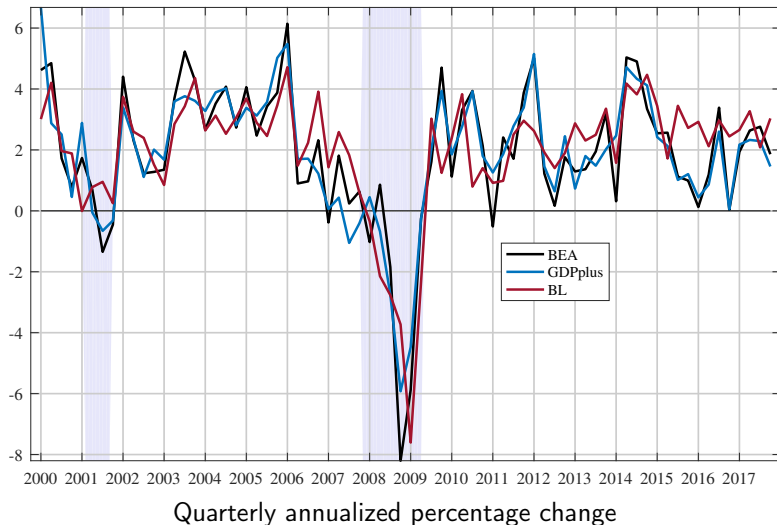
BL



Average quarterly annualized percentage change per quarter 2010–2016

US economy grew faster than NA statistics

Average GDP-GDI & Philly FED DFM of GDP-GDI & BL



Outline

- Output gap

Trend-Cycle decomposition common factors

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Since $\mathbf{F}_t = \mathbf{K}(\mathbf{f}'_t \cdots \mathbf{f}'_{t-s})'$, then:

- \mathbf{F}_t has $(q - d)$ unit roots
- \mathbf{F}_t is with a rank of cointegration c , $d \leq c \leq (r - q + d)$

Barigozzi, Lippi & Luciani, 2016ab

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$$\mathbf{F}_t = \Phi \mathbf{T}_t + \Gamma_t$$

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- Φ is $r \times (q - d)$ with full column rank and
- Γ_t is stationary

Trend-Cycle decomposition via eigen-analysis

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$$S = \frac{1}{T^2} \sum_{t=1}^T F_t F_t'$$

Trend-Cycle decomposition via eigen-analysis

$$\mathbf{S} = \frac{1}{T^2} \sum_{t=1}^T \mathbf{F}_t \mathbf{F}'_t$$

- First $q - d$ eigenvectors of $\mathbf{S} \longrightarrow \Phi$;

Peña & Poncela, 1997, 2006; Bai, 2004; Zhang, Yao & Robinson, 2016

Trend-Cycle decomposition via eigen-analysis

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$$\widehat{\mathbf{T}}_t = \widehat{\Phi}' \widehat{\mathbf{F}}_t$$

$$\widehat{\Gamma}_t = \widehat{\Phi}_\perp \widehat{\Phi}_\perp' \widehat{\mathbf{F}}_t = \widehat{\Phi}_\perp \widehat{\mathbf{G}}_t$$

$$\widehat{\chi}_{it} = \widehat{\lambda}'_i \widehat{\Phi} \widehat{\mathbf{T}}_t + \widehat{\lambda}'_i \widehat{\Phi}_\perp \widehat{\mathbf{G}}_t,$$

Output gap: definition

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Our measure

- Output gap = cyclical component of GDO

$$\hat{\chi}_{GDO,t} = \hat{\lambda}'_{GDO} \hat{\Phi} \hat{\mathbf{T}}_t + \hat{\lambda}'_{GDO} \hat{\Phi}_{\perp} \hat{\mathbf{G}}_t$$

Output gap: definition

Our measure

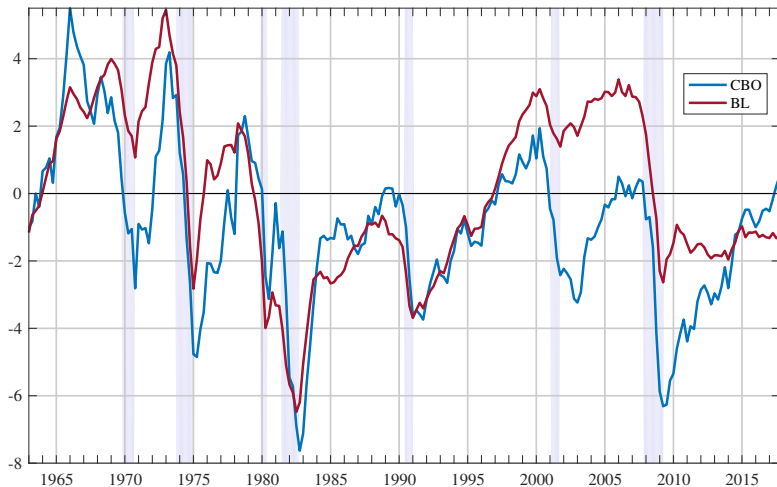
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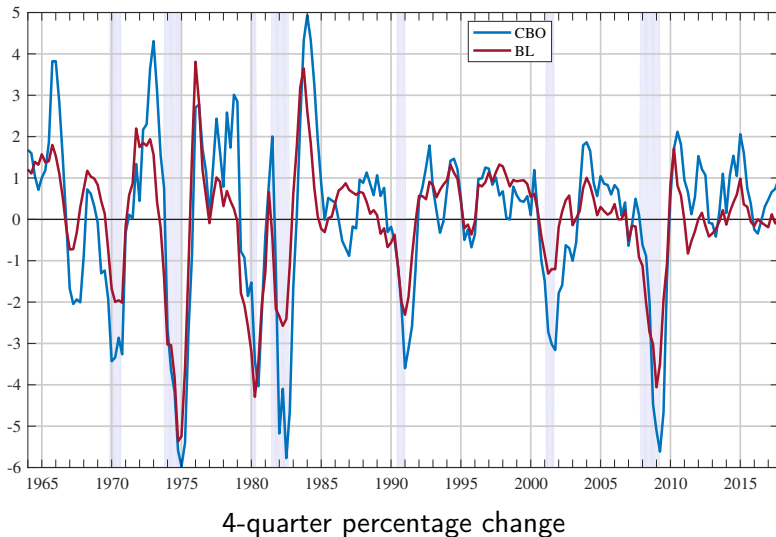
Congressional Budget Office

- Output gap = GDP - potential output
 - Solow growth model
 - Okun's law
 - NAIRU

Output gap

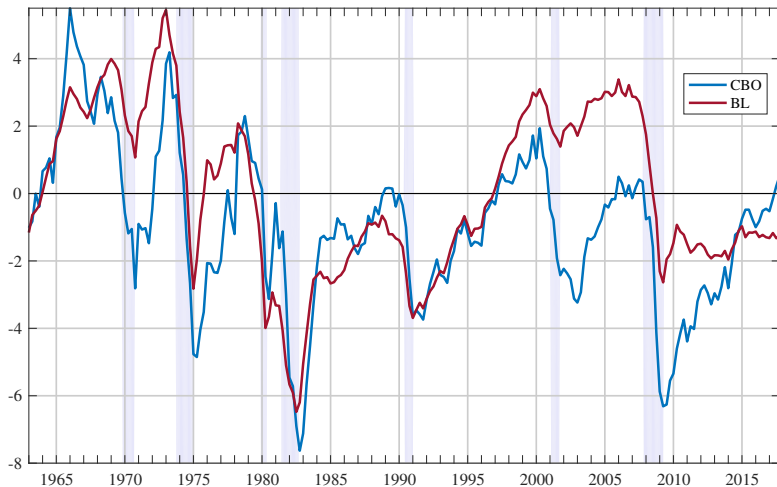


Output gap



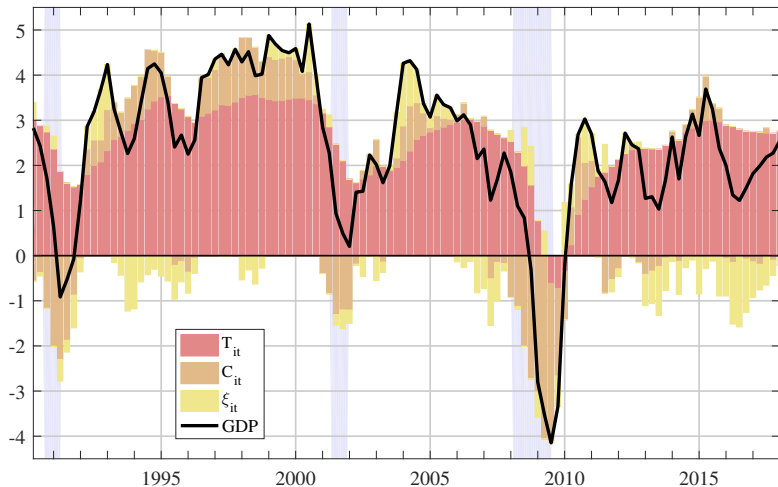
Growth before the GFC was not sustainable

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Growth after the GFC is solid

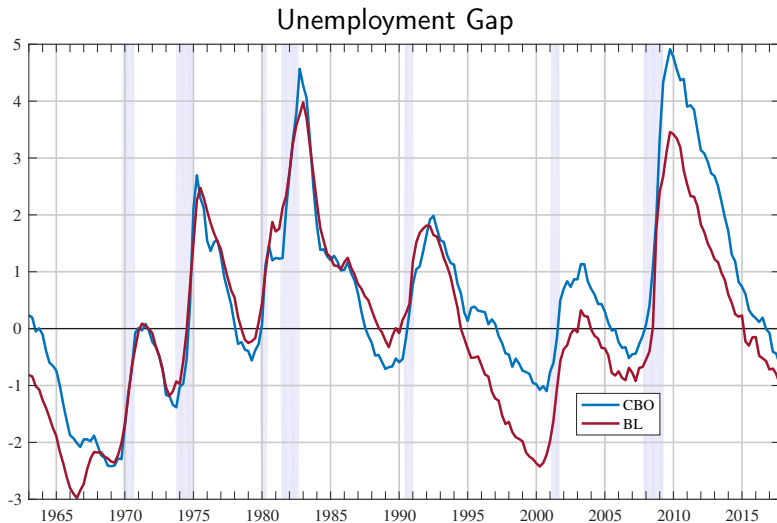
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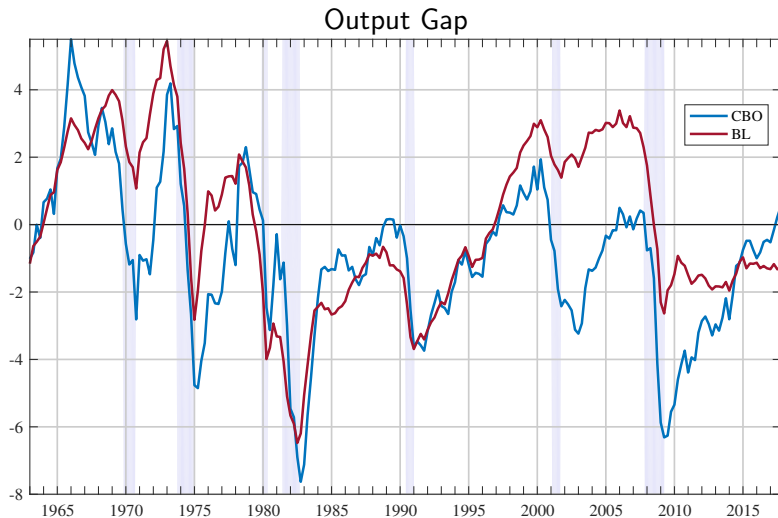
4-quarter percentage change

Labor market sends different signal

Labor market sends different signal



Labor market sends different signal



- **Summary and conclusions**

Summary and conclusions

Aggregate output

Output gap

Summary and conclusions

Aggregate output \Rightarrow Non-Stationary Dynamic Factor Model

Output gap \Rightarrow Non-parametric Trend-Cycle

Summary and conclusions

Aggregate output \Rightarrow Non-Stationary Dynamic Factor Model

- Since 2015 growth was on average 0.4 p.p. higher than GDP
- Higher growth has been concentrated in Q1
- GDP Q1 weakness due to mismeasurement rather than seasonality

Output gap \Rightarrow Non-parametric Trend-Cycle

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