ENVIRONMENTAL SUBSIDIES TO MITIGATE TRANSITION RISK

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ABSTRACT. We explore the role of public subsidies in mitigating the transition risk associated with a climate-neutral objective by 2060. We develop and estimate an environmental dynamic stochastic general equilibrium model for the world economy featuring an endogenous market structure for green products. We show that public subsidies, financed by a carbon tax, are an efficient instrument to promote firm entry into the abatement goods sector by fostering competition and lowering the selling price of such products. We estimate that the subsidy, optimally distributed between startups at 60% and existing companies at 40%, will save nearly $2.9 trillion in world GDP each year by 2060.

JEL: E32, H23, Q50, Q55, Q58.

Keywords: Climate change, E-DSGE model, Bayesian estimation, stochastic growth, endogenous market structure, environment-related products

1 INTRODUCTION

Mitigating climate change is one of the most pressing issues of our time. To maintain the average increase in temperature below 2°C above preindustrial levels, the carbon tax is considered an unavoidable tool, as it provides an incentive for economic agents to reduce their carbon dioxide (CO_2) emissions. Although absolutely necessary, such a policy may lead to large GDP and employment losses and, alone, will not be sufficient to curb world emissions. As emphasized by the Intergovernmental Panel on Climate Change (IPCC) in its Sixth Assessment Report, a net-zero emissions target cannot be reached by 2060 with current green products and requires new technologies, such as large-scale carbon dioxide removal (e.g., bioenergy with carbon capture and storage, biochar, or soil carbon sequestration). Despite commitments by several governments to the Paris Agreement in 2015, little

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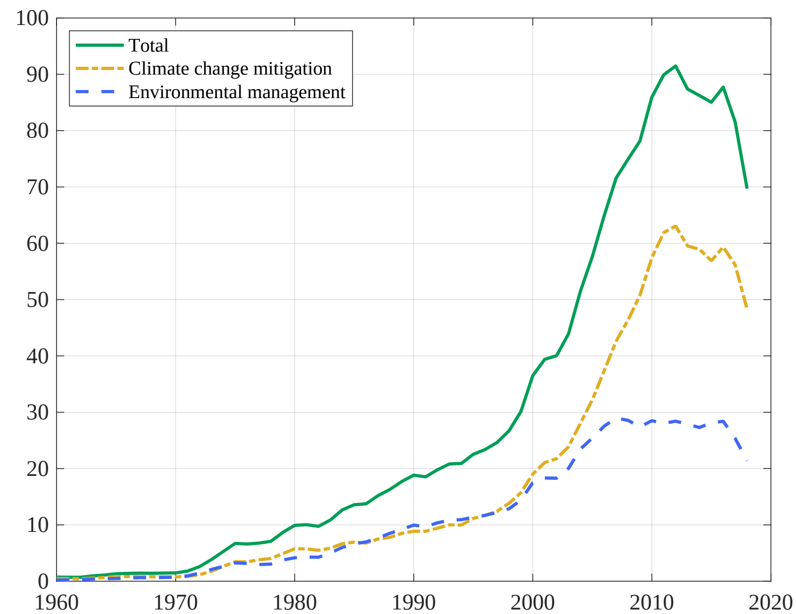
has been done so far, and the climate threat itself seems to be insufficient to encourage firms to invest in green technologies. Indeed, after decades of growth, the number of environment-related patents —a proxy for new green technologies— has decreased since 2012 (Figure 1). Under these conditions, how can we promote the development of the environment-related product market and thus mitigate transition risk?

FIGURE 1. Annual number of world environment-related patents (in thousands)



Note: This figure reports the annual number of environment-related patents from 1960 to 2018 by category (Haščič and Migotto, 2015). The data cover all family sizes worldwide. Source: OECD website.

In this paper, we assess the role of public subsidies in boosting green product creation and reducing the associated economic costs. Theoretically, subsidies can achieve incentive effects similar to those of efficient carbon taxes by distributing equivalent amounts for each unit of carbon abated. They are expected to compensate for the cost of carbon abatement investments and consequently encourage firms to develop more efficient environment-related technologies. In practice, using carbon tax revenues to subsidize the abatement technology industry could trigger a new wave of green technologies. These developments would accelerate the transition and may make negative emissions an additional levy to decarbonize the economy.

For the sake of this analysis, we develop and estimate an environmental dynamic stochastic general equilibrium (E-DSGE) model for the world economy featuring an endogenous market structure for the abatement goods sector. Abatement goods are intermediate inputs purchased by final good firms to lower their carbon footprint. They fall into the so-called environmental goods and services sector, whose activities is to measure, prevent, limit, minimize or correct environmental damage to water, air and soil, as well as problems related to waste, noise and eco-systems. The production of these goods is concentrated on a few worldwide companies and 10% among them account for almost 80% of the operating revenue (Eurostat, 2009; Ecorys, 2009). Industry concentration results from high barriers to entry that prevent potential competitors from challenging incumbent firms. While usually ignored in environmental models, producer entry is however crucial to characterize pricing dynamics as well as the number of green varieties. We thus build on Bilbiie et al. (2012) who endogenize firm entry and the creation of new products in the economy by introducing a clear distinction between *intensive* margins (i.e., changes in the production of existing goods) and *extensive* margins (i.e., changes in the variety of available goods). This framework is well suited to study the dynamics of abatement goods, which is key for the success of the low-carbon transition, and whose availability is actually fostered by the level of competition in abatement technology, as shown by Nesta et al. (2014) and Nicolli and Vona (2016). Indeed, these authors find, respectively, that *(i)* environmental policies are crucial in the generation of more efficient green patents and *(ii)* reducing entry barriers is a very significant driver of renewable energy innovation.

Contrary to the first generation of economic models of climate change proposed by Nordhaus (1992, 1994) under the well-known DICE acronym (for Dynamic Integrated Climate-Economy model), E-DSGE models are well suited for analyzing economic policies.[1] Indeed, E-DSGE models *(i)* formalize the behavior of economic agents based on explicit microfoundations, *(ii)* appropriately control for the effects of policy measures through expectations to respond to the Lucas (1976) critique, and *(iii)* incorporate uncertainty into agent decision-making processes, as suggested by Pindyck (2013). We merge the DICE and DSGE models into a unified framework to examine both the level and growth effects of macroeconomic and climate-related variables on the economy. The resulting framework appropriately controls for the effects of policy measures through expectations, in particular those related to climate change, which imply permanent shifts in macroeconomic time series.

Another novelty is the estimation of a nonlinear E-DSGE model using full-information methods. First, nonlinear estimation is deemed necessary to account for unbalanced growth dynamics originating from climate change, and by nature makes usual perturbation (around a fixed point) methods not suitable for climate issues. Second, by revealing the relative strength of environmental and economic forces and accounting for both parametric and stochastic uncertainties, this estimation strategy reveals essential to properly quantify the effects of climate-oriented policies. To this end, we first use the *extended path solution method* from Fair and Taylor (1983) to numerically solve the model. In summary, the extended path approach uses a perfect foresight solver to obtain endogenous variables that are path

[1] The DICE model is part of Integrated Assessment Models (IAM), which aim to provide insights into global environmental change and sustainable development issues by offering a quantitative description of key processes in the human and earth systems and their interactions.

consistent with the model's equations. Each period, agents are surprised by the realization of shocks but still expect that in the future, shocks are zero on average, consistent with rational expectations. The advantage of this method is that it provides an accurate and fast solution while considering all the nonlinearities of the model. Second, we use an *inversion filter* to calculate the likelihood function. By extracting the sequence of innovations recursively through the inversion of the observation equations for a given set of initial conditions, this filter has recently emerged as a computationally cheap alternative (Guerrieri and Iacoviello, 2017; Atkinson et al., 2020). Finally, using Bayesian techniques, we describe the joint fluctuations of five world's macroeconomic and climate-related time series from 1961 to 2018.

We implement several projection exercises, in line with IPCC (2021). We first present the projections of the E-DSGE under alternative control rates of CO_2 emissions to assess the role of the uncertainty associated with the future evolution of climate risk and to provide insight into the mechanics of the model. The targeted level of CO_2 emissions is attained through a carbon tax proportional to the level of emissions. This policy has a recessive impact on the economy, particularly in the scenario targeting net-zero emissions in 2060 (i.e., Paris Agreement). In this context, we consider two subsidy experiments designed to mitigate the cost of the transition: *(i)* a subsidy to existing firms in the abatement goods sector, and *(ii)* an optimal subsidy to both existing firms and startups. In the latter case, the respective shares of carbon tax revenues given to entrants and established firms are chosen to maximize social welfare. In these exercises, instead of being used for unproductive government expenses, carbon tax revenues are fully used to reduce the price of the abatement technology and help its diffusion to the final good sector. The period of analysis starts in 2019 when the carbon tax and subsidy policies were both announced, and ends in 2100.

We find that if the carbon tax revenues are redistributed through lump-sum transfers to households, the cumulative GDP loss would reach \$258 trillion from 2019 to 2060. Public subsidies, fully financed by the carbon tax, are an efficient instrument to promote firm entry in the abatement goods sector, by fostering competition and lowering the selling price of such technologies. In particular, a subsidy policy targeting startups is more efficient, as it quickly lowers the cost of adopting green production technologies. Allocating 60% of the revenues of the carbon tax to subsidize new firms and 40% to existing firms in the abatement goods sector would lead to a cumulated GDP loss of \$138 trillion from 2019 to 2060. Hence, the optimal subsidy would reduce the GDP loss by nearly \$120 trillion, or equivalently \$2.9 trillion each year. In this scenario, the carbon tax would increase to \$150 per ton of CO_2 by 2040 and \$400 by 2060, the abatement price would be divided by more than 2.5, and the numbers of firms/varieties in the abatement goods sector would substantially increase.

Our paper is related to the scarce literature focusing on climate issues through microfounded structural models. Fischer and Springborn (2011), Heutel (2012), and Angelopoulos et al. (2013) are among the first contributions to introduce pollution in Real Business Cycle models. They assume that pollution stems from production, and adversely impacts utility or has a negative impact in turn on productivity and production. More recent contributions have extended these models in several directions including *(i)* multisector aspects (Golosov et al., 2014; Dissou and Karnizova, 2016), *(ii)* labor market frictions (Gibson and Heutel, 2020; Shapiro and Metcalf, 2021), *(iii)* endogenous entry (Annicchiarico et al., 2018; Shapiro and Metcalf, 2021), or *(iv)* nominal rigidities and monetary policy (Annicchiarico and Di Dio, 2015, 2017; Diluiso et al., 2021; Ferrari and Nispi Landi, 2020; Carattini et al., 2021). These models are mainly used to provide short-run analyzes on the effects of pollution policies, such as pollution taxes or cap-and-trade. However, climate issues, especially the trade-offs between the costs and benefits of reducing emissions, must be assessed from a long-run perspective. Contrary to these papers, we consider long-run trends in CO_2 emissions and macroeconomic variables, which makes our framework well suited for studying environmental policies aiming to mitigate transition risks.

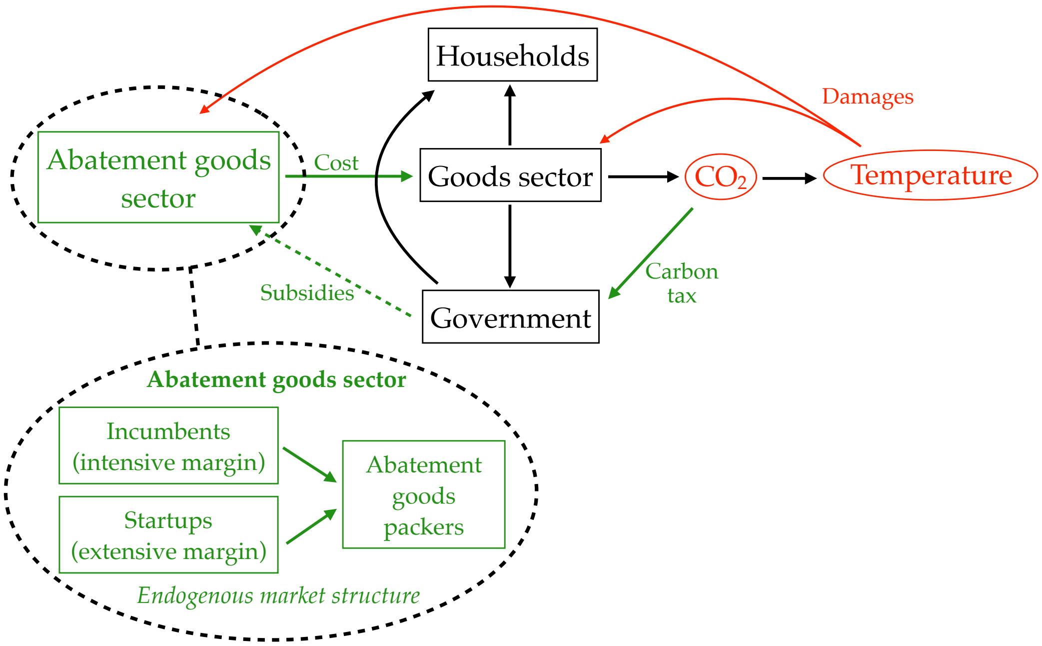
The remainder of the paper is structured as follows. Section 2 describes the environmental dynamic stochastic general equilibrium model. Section 3 reports our data, the estimation methodology, and the parameter estimates. Section 4 provides scenarios in the spirit of the IPCC report and details the economic mechanisms at play. Section 5 quantifies the macroeconomic and climate-related effects of public subsidies. Section 6 concludes the paper.

2 Model

Our model draws on three branches of the economic literature: *(i)* the climate block is derived from DICE models (Nordhaus, 1992, 2018); *(ii)* the macroeconomic block is a real business cycle version of Smets and Wouters (2007); and *(iii)* the innovation sector block has an endogenous market structure, as in Bilbiie et al. (2012).

Figure 2 provides an overview of the main mechanisms at stake in the model. In producing final good for households and the government, firms generate CO_2 emissions, which contribute to increasing the surface air temperature. In turn, global warming adversely impacts the total factor productivity of firms through a damage process. However, this damage does not spontaneously encourage companies to reduce their emissions. Indeed, global warming is the result of the activity of all firms. Therefore, in a decentralized economy, as a "damage taker," each firm would bear the cost of reducing its emissions for a negligible individual impact on global warming. In the absence of any regulation or subsidy, each firm will minimize its mitigation efforts rather than drastically reduce its emissions.

FIGURE 2. Overview of the main mechanisms in the model



To address this free-rider problem, the government imposes a carbon tax (τ) that corrects the market failure. This tax forces firms in the final good sector to acquire technologies that reduce their individual emissions. Although costly, these *abatement technologies* reduce the amount of carbon tax that firms must pay. Abatement goods are produced by specialized firms in an initially immature abatement goods sector. The development of this market is crucial to make the energy transition faster and less costly in terms of output. Indeed, stronger competition may reduce the price of abatement technologies by reducing deadweight losses.[2] From this perspective, policies aiming at fostering competition, such as *public subsidies*, may reduce the abatement price and encourage the use of abatement goods, therefore supporting the reduction of emissions.

2.1 Climate block The climate block relies on Nordhaus (1992, 2018). The law of motion of the atmospheric loading of CO_2 (in gigatons of CO_2) is given by:

$$M_t = M_{1750} + (1 - \delta_M)(M_{t-1} - M_{1750}) + \xi_M E_t, \tag{1}$$

where E_t denotes the anthropogenic carbon emissions in t, $\delta_M \in [0,1]$ represents the rate of transfer of atmospheric carbon to the deep ocean, and $\xi_M \geq 0$ is the atmospheric retention ratio.[3] The term

[2]As evidence of the impact of competition on prices, the average price of solar photovoltaic modules, measured in 2019 U.S. dollars per watt, was reduced by 45% between 1990 and 2000, by 58% between 2000 and 2010, and by 81% between 2010 and 2019, allowing for a relatively fast spread of solar panels (source: Our World In Data).

[3]More advanced climate blocks were developed to better portray the link between temperature and carbon emissions. While this kind of refinement is important in the context of physical risk assessment (Dietz and Venmans, 2019), when it comes to transition risk, this has little added value and would not change the main message from our policy recommendations.

$M_{t-1} - M_{1750}$ represents the excess carbon in the atmosphere net of its (natural) removal, with M_{1750} representing the stock of carbon in the preindustrial era, i.e., the steady-state level in the absence of anthropogenic emissions (see also Traeger, 2014).

The heat received at the Earth's surface F_t (in watts per square meter, W/m^2) is the sum of the forcing caused by atmospheric CO_2 and the non-CO_2 forcing:

$$F_t = \eta \ \log_2 \left(\frac{M_t}{M_{1750}} \right) + F_{EX,t}, \tag{2}$$

where η denotes the effect on temperature from doubling the stock of atmospheric CO_2. As in the latest version of DICE models, the non-CO_2 forcing $F_{EX,t}$ is an exogenous process:

$$F_{EX,t} = \min(F_{EX,t-1} + F_\Delta, F_{\max}), \tag{3}$$

where the parameter F_Δ denotes the fixed increase in exogenous radiative forcing, while $F_{\max}$ is a cap that is met by 2100.

The global mean temperature anomalies of surface T_t and deep oceans T_t^* with respect to the preindustrial period are given by:

$$T_t = \phi_{11} T_{t-1} + \phi_{12} T^*_{t-1} + \xi_T F_t + \varepsilon_{T,t}, \tag{4}$$

$$T^*_t = \phi_{21} T_{t-1} + \phi_{22} T^*_{t-1}, \tag{5}$$

where $\xi_T \geq 0$ is the elasticity of surface temperature to earth surface heat, while parameters ϕ_{11}, ϕ_{12}, ϕ_{21}, and ϕ_{22} capture either persistence or interaction between the temperature of the surface and deep oceans. To disentangle transitory changes in temperature versus permanent drifts, we introduce an exogenous stochastic process, $\varepsilon_{T,t} = \rho_T \varepsilon_{T,t-1} + \eta_{T,t}$ with $\eta_{T,t} \sim \mathcal{N}(0, \sigma_T^2)$, which captures cyclical changes in temperature.[4]

2.2 Household sector The world economy is populated by a mass L_t of atomistic, identical, and infinitely lived households. This mass is time-varying and captures the upward trend of world population observed over the last sixty years. Formally, as in Nordhaus (2014), it is assumed that the world population asymptotically converges to a long-run level $L_T > 0$, such as $L_t = L_{t-1} \left(L_T / L_{t-1} \right)^{\ell_g}$, with $\ell_g \in [0, 1]$ being the geometric rate of convergence to L_T. Each household indexed by $i \in [0, L_t]$ maximizes its sequence of present and future utility flows that depend positively on consumption $c_{i,t}$ and negatively on hours worked $h_{i,t}$:

$$\mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} \beta^\tau \left(\frac{c_{i,t+\tau}^{1-\sigma_c}}{1-\sigma_c} - \psi_t \frac{h_{i,t+\tau}^{1+\sigma_h}}{1+\sigma_h} \right) \right\}, \tag{6}$$

[4]We make the conservative assumption that the volatility of the shock to temperature remains stable over time.

subject to the sequence of real budget constraints

$$c_{i,t} \leq w_t h_{i,t} + \xi_{i,t} + d_{i,t}, \tag{7}$$

where $\mathbb{E}_t$ denotes the expectation conditional upon information available at t, $\beta \in (0,1)$ is the subjective discount factor, $\sigma_c > 0$ is the inverse of the intertemporal elasticity of substitution in consumption, $\sigma_h > 0$ is the inverse of the Frisch labor supply elasticity, and $\psi_t > 0$ is a time-varying parameter that cancels out the effects of the productivity trend on labor supply. Such a feature is necessary to obtain a balance growth path on hours.[5] The household's resources depicted on the right-hand side of the budget constraint are made of real wage w_t, lump-sum transfers from the government $\xi_{i,t}$, and dividend payments received from holding shares of firms in both the intermediate goods and abatement goods sectors $d_{i,t}$. The maximization problem gives the labor supply equation $w_t c_{i,t}^{-\sigma_c} = \psi_t h_{i,t}^{\sigma_h}$. Anticipating symmetry across households, the stochastic discount factor is denoted by $\beta_{t,t+\tau} = \beta^{\tau}(c_{t+\tau}/c_t)^{-\sigma_c}$, i.e., the discount time scheme that converts future payoffs in $t+\tau$ into current consumption equivalents.

2.3 Business sector The business sector is characterized by final good producers that sell a homogeneous final good to households and the government. To produce, they buy and pack differentiated varieties produced by atomistic and infinitely lived intermediate good firms that operate in a monopolistically competitive market. Intermediate good firms contribute to climate change by emitting CO_2 as an unintended result of their production process.

2.3.1 Final good sector At every point in time t, a perfectly competitive sector produces a final good Y_t by combining a continuum of intermediate goods $y_{i,t}$, $i \in [0, L_t]$, according to the technology $Y_t = \left[L_t^{-1/\zeta} \int_0^{L_t} y_{i,t}^{\frac{\zeta-1}{\zeta}} \mathrm{d}i\right]^{\frac{\zeta}{\zeta-1}}$. The number of intermediate good firms owned by households is equal to the size of the population L_t. $\zeta > 1$ measures the substitutability across differentiated intermediate goods. Final good producing firms take their output price, P_t, and their input prices, $P_{i,t}$, as given and beyond their control. Profit maximization implies the demand curve $y_{i,t} = L_t^{-1}(P_{i,t}/P_t)^{-\zeta} Y_t$, from which we deduce the relationship between the price of the final good and the prices of intermediate goods $P_t \equiv \left[L_t^{-1} \int_0^{L_t} P_{i,t}{}^{1-\zeta} \mathrm{d}i\right]^{\frac{1}{1-\zeta}}$.

2.3.2 Intermediate goods sector Intermediate good i is produced by a monopolistic firm using the following production function:

$$y_{i,t} = \Gamma_t h_{i,t}^{I}, \tag{8}$$

[5] Note that ψ_t must grow proportionally to the flow of current consumption. Thus, if Z_t denotes the trend in per capita consumption, then $\psi_t = \psi_h Z_t^{1-\sigma_c}$, with ψ_h as a scaling parameter.

where Γ_t is the Total Factor Productivity (TFP) that affects the labor demand $h^I_{i,t}$.[6] The TFP is actually determined by three components:

$$\Gamma_t = \Phi(T_t) Z_t \varepsilon_{Z,t}, \tag{9}$$

where $\Phi(T_t)$ determines the reduction in TFP due to climate change, Z_t is the deterministic component of productivity, and $\varepsilon_{Z,t}$ is an exogenous productivity shock, which determines the business cycle component of productivity. This shock follows an AR(1) process: $\varepsilon_{Z,t} = (1-\rho_Z) + \rho_Z \varepsilon_{Z,t-1} + \eta_{Z,t}$, with $\eta_{Z,t} \sim N(0, \sigma_Z^2)$. The deterministic component of TFP follows the process $\log Z_t = \log Z_{t-1} + f_Z(Z_{t-1})$, where $f_Z(Z_t) = (1-\exp(\delta_z))(g_{z,t_0}/\delta_z - \log(Z_t/Z_0))$ is the productivity growth rate, g_{z,t_0} is the initial growth rate of productivity, δ_z is the rate of decline in productivity, and t_0 represents the starting date of our simulations. This formulation follows Nordhaus (2018) and indicates that productivity growth decreases over time by a factor δ_z to match the observed slowdown in economic growth over the last sixty years. The damage function $\Phi(T_t)$ represents the impact of climate change on the production process. Additionally, in line with the DICE literature, the damage depends on the atmospheric temperature T_t as $\Phi(T_t) = 1/(1 + aT_t^2)$, where $a > 0$ is a parameter calibrated to match climate-change damage estimates.

A firm's CO_2 emissions stemming from the production process are denoted by $e_{i,t}$. As they are subject to a carbon tax, which aims at internalizing the social cost of carbon emissions, the firm is incentivized to reduce its impact by investing in an emission abatement technology (see Section 2.4). The abatement effort by the firm yields a reduction by $\mu_{i,t}$ (in %) in its CO_2 emissions. A firm's emissions take the following form:

$$e_{i,t} = \sigma_t (1-\mu_{i,t}) y_{i,t} \varepsilon_{E,t}, \tag{10}$$

where σ_t denotes the aggregate carbon intensity of the production sector. Its law of motion is $\log \sigma_t = \log \sigma_{t-1} + f_\sigma(\sigma_{t-1})$, where $f_\sigma(.)$ has a functional form identical to the trend in productivity, $f_\sigma(\sigma_t) = (1-\exp(\delta_\sigma))(g_{\sigma,t_0}/\delta_\sigma - \log(\sigma_t/\sigma_{t_0}))$, where g_{σ,t_0} is the initial decrease rate of emissions-to-output, and δ_σ is the rate of decline of the trend. This trend is set to match the decline in the emissions-to-GDP ratio observed over the last sixty years. Last, the firm's carbon intensity can be temporarily affected by an aggregate exogenous emissions shock, $\varepsilon_{E,t} = (1-\rho_E) + \rho_E \varepsilon_{E,t-1} + \eta_{E,t}$, with $\eta_{E,t} \sim N(0, \sigma_E^2)$, which captures the cyclical changes in the emissions-to-output ratio. A rise in $\varepsilon_{E,t}$ induces a cyclical increase in the carbon intensity of the production sector.

Firms have access to a set of abatement actions. These actions, which consist of substituting carbon-intensive technologies with low-carbon technologies, imply costly changes in the existing lines of production. Hence, following Nordhaus (2018), we assume that the cost of abatement technology (in

[6] Capital can be introduced as an additional factor of production but would further complicate our setup. Given our focus on the abatement goods sector and endogenous firm entry, we have not pursued this generalization.

proportion to output) is given by:

$$\Lambda_{i,t} = (\theta_{1,t}\, \mu_{i,t}^{\theta_2}) y_{i,t}. \tag{11}$$

In this equation, $\theta_{1,t} = (p_b/\theta_2)(1-\delta_{pb})^{t-t_0}\sigma_t$ is the time-varying level of the cost of abatement, where $p_b > 0$ is a parameter determining the initial cost of abatement and $0 < \delta_{pb} < 1$ captures technological progress, which lowers the cost of abatement by a factor δ_{pb} each year. Finally, $\theta_2 > 0$ represents the curvature of the abatement cost function, which typically exhibits increasing returns in IAM's literature.

The intermediate good firm i chooses $\{h_{i,t}^I, \mu_{i,t}\}$ to maximize its one-period profits:

$$p_{i,t} y_{i,t} - w_t h_{i,t}^I - p_t^A \Lambda_{i,t} - \tau_t e_{i,t}, \tag{12}$$

where $p_{i,t} = P_{i,t}/P_t$ is the relative price of intermediate goods, $p_t^A = P_t^A/P_t$ is the relative abatement price, and τ_t is the carbon tax. Importantly, while the relative abatement price is constant in Nordhaus (2018), we rely on an immature market structure of the abatement goods sector that makes the relative abatement price time-varying and higher than unity $p_t^A \geq 1$ (see Section 2.4 for details).

Under imperfect competition, the net profit is the distance between the total gains from selling and the cost of producing, $\Pi_{i,t} = (p_{i,t} - mc_{i,t}) y_{i,t}$, with $mc_{i,t}$ denoting the firm's real marginal cost. Maximizing this profit under the demand curve from final good firms and the production function provides the following pricing scheme: $mc_{i,t}/p_{i,t} = (\zeta - 1)/\zeta$.

Anticipating symmetry across firms, we first rewrite the cost of inputs as follows:

$$w_t = \Gamma_t \left[\frac{\zeta - 1}{\zeta} - p_t^A (\theta_{1,t}\, \mu_t^{\theta_2}) - \tau_t \sigma_t \left(1 - \mu_t\right) \varepsilon_{E,t} \right]. \tag{13}$$

A rise in the carbon tax τ_t results in an increase in the real marginal cost and a decrease in the real wage, which in turn reduces the labor supply and aggregate production. In addition, a rise in the abatement effort μ_t triggers lower growth, as it increases the cost of production. Therefore, an environmental policy reduces carbon emissions at the expense of lower output.

Second, the optimal decision of abatement effort is given by:

$$\mu_t = \left(\frac{\tau_t \sigma_t \varepsilon_{E,t}}{\theta_2 \theta_{1,t} p_t^A} \right)^{1/(\theta_2 - 1)}. \tag{14}$$

Firms are atomistic and have no market power to correctly price CO_2 emissions up to their marginal damage on their profits. As a result, a standard market failure emerges that can be corrected through the introduction of a policy instrument, i.e., a carbon tax. The first-order condition (14) shows that a carbon tax forces firms to internalize the social cost of their emissions on temperature, output, and their profits. Absent this policy instrument ($\tau_t = 0$), firms would not spontaneously consider their externalities. Furthermore, unlike standard IAMs, we allow for market competition to play a role in the determination of the abatement effort. Specifically, the level of market competition affects the relative abatement price p_t^A. In the case of low competition, firms would benefit from rent opportunities. As a consequence, the abatement price would remain high, which may reduce the abatement effort μ_t, as shown by Equation (14), and ultimately impair the emissions reduction. Different policy measures may be introduced to avoid such a situation and lower p_t^A, as we will see later.

2.4 Abatement goods sector Abatement goods are bought by intermediate firms to reduce their emissions. As abatement technologies are supposed to be new, their market structure is initially immature. Its development is crucial for the energy transition. As shareholders of firms operating in this sector, households may decide to create a new abatement good through either *(i)* the introduction of an additional production line in an existing firm (*intensive margin*) or *(ii)* the creation of a startup (*extensive margin*). The adoption of new abatement technologies and the creation of startups are endogenous, following the approach proposed by Bilbiie et al. (2012). In particular, a household will choose to create a startup based on the new firm's expected future profits, which depend on sunk entry costs. Each firm produces one variety of abatement goods, denoted ω, over a continuum of differentiated varieties Ω of abatement goods, the latter reflecting the diversity of abatement solutions available in t. Indeed, in practice, low-carbon production units encompass a large set of goods that are very heterogeneous across industries. Some of these abatement goods are purchased to improve the energy efficiency of production units and buildings, others to improve the internal production process, while the remaining carbon may be captured and stored.[7] Finally, competitive packers buy and transform these varieties into homogeneous abatement goods. Equilibrium conditions in this market determine the abatement price, which is critical in the model, given its influence on the cost of the energy transition. After giving details on packers, we explain each margin of adjustment to carbon taxes in turn.

2.4.1 Abatement goods packers At every point in time t, perfectly competitive packers produce homogeneous abatement goods $y_{i,t}^A$, $i \in [0, L_t]$, by combining a continuum of varieties of abatement goods $y_{i,\omega,t}^A$, $\omega \in \Omega$, according to the technology $y_{i,t}^A = \left[\int_{\omega\in\Omega} (y_{i,\omega,t}^A)^{\frac{\zeta_A - 1}{\zeta_A}} \mathrm{d}\omega\right]^{\frac{\zeta_A}{\zeta_A - 1}}$, where $\zeta_A > 1$ measures the substitutability across varieties. Packers take their output price, $P_{i,t}^A$, and their input prices, $P_{i,\omega,t}^A$, as given and beyond their control. Profit maximization implies the optimal quantity of goods demanded by packer i to each variety of abatement ω, $y_{i,\omega,t}^A = \left(P_{i,\omega,t}^A / P_{i,t}^A\right)^{-\zeta_A} y_{i,t}^A$, from which we deduce the relationship between the price of the homogeneous abatement good and the prices of abatement varieties $P_{i,t}^A = \left[\int_{\omega\in\Omega} \left(P_{i,\omega,t}^A\right)^{1-\zeta_A} \mathrm{d}\omega\right]^{\frac{1}{1-\zeta_A}}$.

[7] For the energy sector, for instance, switching from fossil fuels to renewable energy production requires the purchase of solar panels and wind turbines as abatement goods. For the cemetery sector, abatement goods are typically energy-efficient ovens. For the transport sector, abatement technologies might be hybrid or electric motorization.

2.4.2 Intensive margin Each variety ω from already established firms, *incumbents* for short, is produced using labor, which is subject to the TFP as follows:

$$y^A_{i,\omega,t} = \Gamma_t h^A_{i,\omega,t}, \tag{15}$$

where $h^A_{i,\omega,t}$ is the labor demand from firm ω held by household i. Real profits operating in the abatement goods market are given by:

$$\Pi^A_{i,\omega,t} = p^A_{i,\omega,t} y^A_{i,\omega,t} - w_t h^A_{i,\omega,t} \left(1 - s^A_t\right), \tag{16}$$

where $p^A_{i,\omega,t} = P^A_{i,\omega,t}/P_t$ is the relative price of the abatement good ω and s^A_t is a subsidy rate to incumbents decided by the government, which is expressed as a percentage of the labor input cost. This subsidy rate is not variety/household specific.

Maximizing the profit under the demand curve from abatement goods packers and the production function provides the price of variety ω as:

$$p^A_{i,\omega,t} = \frac{\zeta_A}{\zeta_A - 1} \left(1 - s^A_t\right) \frac{w_t}{\Gamma_t}. \tag{17}$$

Note that in Equation (17), the optimal pricing depends only on aggregate conditions. As a result, in equilibrium all the producers choose the same pricing $p^A_{i,\omega,t} = \tilde{p}^A_t$, regardless of the type of packer i and variety ω, where $\tilde{p}^A_t$ denotes the average selling price of abatement varieties. Consequently, firms operating in the abatement markets are symmetric and exhibit the same output, labor demand, and profits.

2.4.3 Extensive margin While each household manages a continuum of abatement varieties Ω, only a subset of goods $\Omega_t \in \Omega$ is available at any given time t. We denote by $N_{i,t}$ the number of firms owned by household i in the abatement goods sector (a mass of Ω_t) and by $N^E_{i,t}$ the number of startups created by the household. As in Bilbiie et al. (2012), startups at time t start producing only in $t+1$, which features one period of time-to-build. This assumption is necessary to capture the lag between entry and economic growth that is empirically observed. The number of firms owned by household i in the abatement goods sector is given by the following law of motion:

$$N_{i,t} = (1 - \delta_A) \left[N_{i,t-1} + \varepsilon_{N,t-1} \left(1 - f_N \left(\frac{N^E_{i,t-1}}{N^E_{i,t-2}}\right)\right) N^E_{i,t-1} \right], \tag{18}$$

where $\delta_A \in [0, 1]$ is the probability that any firm incurs an exogenous exit-inducing shock. This mechanism forces a fraction of firms to default in every period (Bilbiie et al., 2012). This exit shock means that a fraction of startups default before actually producing abatement goods. In addition to the exit shock, startups face another exit probability $f_N\left(N^E_{i,t-1}/N^E_{i,t-2}\right)$, which is proportional to the growth rate of startups, $N_{i,t-1}^{E}/N_{i,t-2}^{E}$. This term represents a congestion effect that makes startups less likely to enter the market when many of them arrive at the same time (Lewis and Poilly, 2012). The associated function is quadratic and given by $f_N(\varpi) = 0.5\chi(\varpi - 1)^2$ with $\chi \geq 0$, thus capturing the hump-shaped response of startups to macroeconomic shocks at the business cycle frequency. Finally, firms' entry is subject to an exogenous shock $\varepsilon_{N,t}$. This shock stands for possible institutional and financial changes in the conditions driving the creation of firms but also may capture a measurement error between the number of startups in the model and the (highly volatile) change in the number of patents used as an observable variable. This exogenous shock follows an AR(1) process given by $\varepsilon_{N,t} = (1 - \rho_N) + \rho_N \varepsilon_{N,t-1} + \eta_{N,t}$, with $\eta_{N,t} \sim N(0, \sigma_N^2)$.

The decision by a household to create a new firm is based on expected future profits, defined by $\mathbb{E}_t \left\{ (1 - \delta_A)^{t-s} \beta_{t,t+s} \Pi_{t+s}^{A} \right\}$, with $s > t$. For each period t, startups compute their post-entry value v_t, which corresponds to the discounted sum of future profits $v_t = \varepsilon_{N,t} (1 - \delta_A) \mathbb{E}_t \left\{ \beta_{t,t+1} \left(\Pi_{t+1}^{A} + v_{t+1} \right) \right\}$.

Prior to entry, firms face two sunk costs, which are composed of labor inputs and the final good. First, following Bilbiie et al. (2012), $h_{i,t}^{E}$ units of labor must be spent to create a startup, such that the labor demand by household i to create $N_{i,t}^{E}$ firms reads as $h_{i,t}^{E} = \theta_{1,t} X_w N_{i,t}^{E} / \Gamma_t$. This equation can be interpreted as a production function of the $N_{i,t}^{E}$ startups with $X_w \geq 0$ as a productivity parameter that drives the intensity of the sunk cost. Consequently, the household spends $w_t h_{i,t}^{E} \left(1 - s_t^{E}\right)$ of labor cost to create $N_{i,t}^{E}$ new firms, with s_t^{E} denoting the subsidy rate to the labor cost of startups. The second sunk cost is induced by regulatory and administrative barriers to market entry and technological requirements for business creation. To pay this cost, each firm must purchase a quantity $X_q \geq 0$ of a basket of materials in terms of the final good. The marginal sunk cost per new firm is the same across households and is given by

$$X_t = \theta_{1,t} \left[X_w \left(1 - s_t^{E} \right) \frac{w_t}{\Gamma_t} + X_q \right]. \tag{19}$$

To ensure that the effort to enter the market does not asymptotically reach zero, the sunk costs grow proportionally to the level of the cost of abatement $\theta_{1,t}$. As a result, the dynamics of labor in the abatement goods sector are such that both final and abatement goods have the same balanced growth.

Given the symmetry in marginal cost X_t and post-entry firm value v_t, the free-entry condition in the abatement goods sector imposes that the average number of startups is the same across households, $N_{i,t}^{E} = N_t^{E}$. Thus, the resulting free-entry condition is:

$$X_t = v_t - v_t \frac{\partial \left(f_N \left(\frac{N_t^E}{N_{t-1}^E} \right) N_t^E \right)}{\partial N_t^E} - \mathbb{E}_t \left\{ \beta_{t,t+1} v_{t+1} \frac{\partial f_N \left(\frac{N_{t+1}^E}{N_t^E} \right)}{\partial N_t^E} N_{t+1}^E \right\}. \tag{20}$$

Household i establishes startups until the marginal cost of their creation (measured by the left-hand-side term of Equation (20)) reaches its marginal return (measured by the right-hand-side term). The free-entry condition is reached when there are no more profits to take from establishing a new firm. Note that upon entry, new entrants exhibit the same pricing as incumbents and therefore are symmetric with existing firms. As a result, there is no ex post heterogeneity across cohorts of producers that entered the market at different points in time. This condition ensures the model tractability.

2.5 Public sector and environmental policy The government collects the carbon tax from firms' emissions and uses this revenue to *(i)* make some unproductive expenditures, *(ii)* provide some subsidies to the abatement goods sector, and *(iii)* pay a lump-sum transfer to households. The budget constraint is:

$$\tau_t E_t = G_t + s_t^A w_t L_t h_t^A + s_t^E w_t N_t^E L_t h_t^E + \xi_t. \tag{21}$$

Public spending is determined exogenously as $G_t = g_y Y_t \varepsilon_{G,t}$, where $g_y \in [0,1]$ is the steady-state share of public spending to output and $\varepsilon_{G,t}$ is a government spending shock. This shock captures exogenous shifts in aggregate demand and follows $\varepsilon_{G,t} = (1-\rho_G) + \rho_G \varepsilon_{G,t-1} + \eta_{G,t}$, with $\eta_{G,t} \sim \mathcal{N}(0, \sigma_G^2)$. The total lump-sum transfer to households reads as $\xi_t = \int_0^{L_t} \xi_{i,t} \mathrm{d}i$.

2.6 Market clearing and equilibrium conditions First, the annual flow of emissions is given by the total emissions from firms $E_t = \int_0^{L_t} e_{i,t} \mathrm{d}i$, while output is given by $Y_t = \int_0^{L_t} y_{i,t} \mathrm{d}i$. Note that since firms are symmetric, the abatement rate is the same across firms $\mu_{i,t} = \mu_t$. Therefore, the aggregate flow of emissions reads as follows:

$$E_t = \sigma_t (1-\mu_t) Y_t \, \varepsilon_{E,t}. \tag{22}$$

Resource constraints determining the aggregate demand are obtained from the aggregation of household consumption $C_t = L_t c_t = \int_0^{L_t} c_{i,t} \mathrm{d}i$, government spending, and the barrier to entry costs paid in terms of the final good:

$$Y_t = C_t + G_t + N_t^E L_t \theta_{1,t} Z_t X_q. \tag{23}$$

In addition, we define a detrended output as the percentage deviation of output Y_t from productivity and population trends, as follows:

$$\hat{Y}_t = 100 \times \log\left(\frac{Y_t}{Z_t L_t}\right). \tag{24}$$

This metric allows us to compare the dynamics of output more easily than directly focusing on the level of output.[8]

The aggregate demand of abatement goods reads as follows:

$$N_t Y_t^A = \left(\frac{\tilde{P}_t^A}{P_t^A}\right)^{-\zeta_A} L_t \Lambda_t. \tag{25}$$

[8] We do not remove the trend associated with the increase in temperature because it is endogenous and thus would make it impossible to compare different policies.

In this expression, as households are symmetric, the relative price ratio is unchanged at the aggregate level $\tilde{P}^A_{i,t}/P^A_{i,t} = \tilde{P}^A_t/P^A_t$. The aggregate production function reads as follows:

$$N_t Y^A_t = \Gamma_t H^A_t, \tag{26}$$

where Y^A_t is the intensive margin in the abatement goods sector and $H^A_t = L_t h^A_t = \int_0^{L_t}\int_{\omega\in\Omega} h^A_{i,\omega,t}\mathrm{d}\omega\mathrm{d}i$ corresponds to the total demand in labor inputs from incumbents in the abatement goods sector.[9] The aggregate selling price, which takes into account the number of incumbents in the determination of the selling price, is:

$$P^A_t = \tilde{P}^A_t \; N_t^{\frac{1}{1-\zeta_A}}. \tag{27}$$

The labor market is given by the total supply of households $H_t = L_t h_t = \int_0^{L_t} h_{i,t}\mathrm{d}i$, which must equal the demand from firms producing intermediate goods $H^I_t = \int_0^{L_t} h^I_{i,t}\mathrm{d}i$, abatement goods incumbents H^A_t, and startups $H^E_t = L_t h^E_t = \int_0^{L_t} h^E_{i,t}\mathrm{d}i$:

$$H_t = H^I_t + H^A_t + H^E_t, \tag{28}$$

where the aggregate supply of the final good is given by $Y_t = \Gamma_t \; H^I_t$.

Finally, we compute the share of abatement goods in output as:

$$\Psi_t = p^A_t \int_0^{L_t} \left(\frac{\Lambda_{i,t}}{Y_{i,t}}\right) \mathrm{d}i = p^A_t \theta_{1,t} \mu_t^{\theta_2}. \tag{29}$$

3 Bayesian inference and model evaluation

In this section, we estimate the model using Bayesian methods (see An and Schorfheide, 2007, for an overview). The posterior distribution associated with the vector of observable variables is computed numerically using a Markov chain Monte Carlo sampling approach. We first describe how the nonlinear model with trends is solved. We then discuss the selected data and our choice of priors, comment on the posterior distribution of the structural parameters, and discuss the dynamic properties of the model.

3.1 Numerical solution method with stochastic growth We consider the *extended path solution method* from Fair and Taylor (1983) and Adjemian and Juillard (2014) to accurately measure the nonlinear effects of the environmental constraint on growth. In summary, the extended path approach uses a perfect foresight solver to obtain endogenous variables that are path consistent with the model's equations. Each period, agents are surprised by the realization of shocks, but still expect that in the future shocks will be zero on average (consistent with rational expectations). The advantage of this method is that it provides an accurate and fast solution while considering all the nonlinearities of the model.

[9] Aggregated labor demands include the number of firms, as in Bilbiie et al. (2012).

The drawback of the approach is that the Jensen's inequality binds to equality, which means that the nonlinear uncertainty stemming from future shocks is neglected. Note that this drawback also applies to usual linearized DSGE models, such as Smets and Wouters (2007).

Taking nonlinear models to the data is a challenge as nonlinear filters, which are required to form the likelihood function, are computationally expensive. An inversion filter has recently emerged as a computationally cheap alternative (e.g., Guerrieri and Iacoviello, 2017; Atkinson et al., 2020). Initially pioneered by Fair and Taylor (1983), this filter extracts the sequence of innovations recursively by inverting the observation equation for a given set of initial conditions. Unlike other filters (e.g., Kalman or particle filters), the inversion filter relies on an analytic characterization of the likelihood function.[10]

The inversion takes place using the perfect foresight solution proposed by Juillard et al. (1996). The standard approach is to compute the dynamics of the variables given current and future shocks. In the extended path context, the inversion filter *(i)* substitutes current shocks and some endogenous variables when applying the perfect foresight solution, and *(ii)* computes current shocks and nonobservable variable paths given the set of observable variables. Finally, we use the Metropolis-Hastings algorithm as a sampler to draw from the parameter uncertainty. We obtain a random draw of 320,000 from the posterior distribution of the parameters (8 parallel chains simultaneously drawing 40,000 iterations, with a common jump scale parameter to match an acceptance rate of approximately 30%).

3.2 Data description The model is estimated using worldwide annual data from 1961 to 2019.[11] Macroeconomic series are from the *World Bank*. Real GDP and private consumption are expressed in current international dollars, converted by the 2017 purchasing power parity (PPP) conversion factor. The PPP conversion factor is a spatial price deflator and currency converter that eliminates the effects of the differences in price levels among countries. We also include some series that are related to the climate block of the model and are intended to pin down the key parameters of this block. Annual CO_2 emissions correspond to the emissions from the burning of fossil fuels for energy and cement production. For temperature, we use the global average land-sea temperature anomaly relative to the 1961–1990 average temperature. CO_2 emissions are from *Our World In Data*, while temperature anomalies are taken from *NASA*. As pointed out by Nordhaus (2018), CO_2 emissions relative to the world GDP exhibit a quasilinear negative trend with a growth rate equal to -1.26% over the full period. While the rate of decarbonization slightly increased from 2000 onward, the temperature almost continuously increased in the sample. Temperature has increased by 0.8°C over the last 60 years. This

[10] For a presentation of alternative filters to calculate the likelihood function, see Fernández-Villaverde et al. (2016). See also Cuba-Borda et al. (2019) and Atkinson et al. (2020) for details on the relative gains of the inversion filter.

[11] Calibrating the model for a particular country or set of countries would raise the issue that climate change is a worldwide phenomenon. For this reason, a large part of the world carbon would be emitted by regions that are not described by the model. An alternative approach would be to design a multicountry model, as in Kotlikoff et al. (2021). As we focus on environmental policies, this approach is beyond the scope of our paper.

evidence is reflected in the model by the dependence of temperature on the stock, not the annual flow, of CO_2 emissions.

Regarding the abatement goods sector, we use the number of patents in environment-related technologies (see Figure 1), as collected by the *OECD* (Haščič and Migotto, 2015). In the absence of explicit data since 1960 documenting the number of worldwide firms operating in a green sector, patent data appear to be a reasonable alternative to measure the growth rate of green firms. We map the growth rate of environmental-related patents to the model's growth rate of firms, $\Delta \log(N_t^E)$.

Contrary to most of the business cycle literature that uses linearized versions of models to infer structural parameters, as exemplified by Smets and Wouters (2007), our solution method explicitly addresses trends and thus does not impose that variables must return to a steady state.[12] Consequently, we simply use the growth rate (i.e., the first difference of the logarithm) for quantity variables (GDP, consumption, CO_2 emissions, and number of patents) and the variation for temperature anomaly.[13]

3.3 Prior distribution of the parameters A first set of parameters is calibrated. They are reported in Table 1, while the initial conditions are described in Table 2.

As our dynamics for carbon cycles are similar to Nordhaus (1992), we borrow from DICE 1992 the value for the annual rate of transfer $\delta_M = 0.00833$. The initial annual growth rate of the world population is set to 2% ($\ell_g = 0.02$) to replicate the observed dynamics of the world population between 1961 and 2018, which is very close to the calibration in DICE 1992 for a similar period. Most of the other climate parameters and socioeconomic parameters common to the IAM literature are taken from the latest version of DICE in Nordhaus (2018) and Faulwasser et al. (2018). In particular, ϕ_{11}, ϕ_{12}, ϕ_{21}, ϕ_{22}, ξ_M, M_{1750}, L_T, σ_c, δ_σ, θ_2, δ_{pb}, and a are taken directly from DICE-2016R2. For initial values of state variables, as our simulations start sooner than DICE (with $t_0 = 1961$), we backcast/retropolate starting values to reach 1961. The initial cost of abatement θ_{t_0} is 0.7167, the world population L_{t_0} is 3.307 billion people, the emission-to-output ratio σ_{t_0} is 0.5878 (consistent with world data), atmospheric carbon M_{t_0} is 670 Gt, and the surface temperature anomaly T_{t_0} is set to 0.21 (consistent with the mean surface temperature anomaly in 1961 relative to 1750), while the deep ocean temperature anomaly $T^*_{t_0}$ is set to zero. The carbon tax τ_{t_0} is set to match an initial abatement effort of $\mu_{t_0} = 3\%$ as in Nordhaus (2018). Revenues from the environmental policy are redistributed to households via lump-sum transfers. The subsidy rates to incumbents and startups are initially set to zero $s_t^A = s_t^E = 0$ and endogenously vary

[12]Linearization methods approximate any model's decision rules around a fixed point and therefore impose that the model is stationary in the neighborhood of the fixed point. As a result, inference must be assessed based on stationary data; the latter implies a set of transformations (e.g., dividing by the population, business cycle filters, etc.).

[13]Figure A.1 displays the evolution of all observable variables of the model.

in our policy experiments. Finally, the discount factor β is set to 0.985 as in Nordhaus (2018), which is consistent with a real interest rate of 5% (i.e., $\beta = g_{z,t_0}^{\sigma_c}/1.05$).[14]

TABLE 1. Calibrated parameter values (annual basis)

PARAMETER	NAME	VALUE	SOURCE
Panel A: Climate parameters			
CO_2 rate of transfer to deep oceans	δ_M	0.0833/10	Nordhaus (1992)
Climate sensitivity to carbon stock doubling	η	3.68	Nordhaus (2018)
Marginal atmospheric retention ratio	ξ_M	3/11	Nordhaus (2018)
Preindustrial carbon stock (Gt)	M_{1750}	588	Nordhaus (2018)
Atmospheric-Atmospheric temperature	ϕ_{11}	0.8718	Nordhaus (2018)
Atmospheric-Oceans temperature	ϕ_{12}	0.0088	Nordhaus (2018)
Oceans-Atmospheric temperature	ϕ_{21}	0.025	Nordhaus (2018)
Oceans-Oceans temperature	ϕ_{22}	0.975	Nordhaus (2018)
Non-CO_2 forcing change	F_Δ	0.00588	Nordhaus (2018)
Non-CO_2 forcing cap	$F_{\max}$	1	Nordhaus (2018)
Panel B: Socio-economic parameters			
Final population (billion)	L_T	11.500	Nordhaus (2018)
Population growth rate	ℓ_g	0.02	Nordhaus (2018)
Discount factor	β	0.985	Nordhaus (2018)
Curvature of utility of consumption	σ_c	1.45	Nordhaus (2018)
Curvature of disutility of labor	σ_h	1	Galí (2007)
Elasticity of substitution between goods	ζ	6	Galí (2007)
Public spending share in output	g_y	0.16	Authors' calculations
Rate of decline of emission-to-GDP trend	δ_σ	0.001	Nordhaus (2018)
Rate of decline of productivity	δ_Z	0.005	Nordhaus (2018)
Damage cost	a	0.00236	Nordhaus (2018)
Panel C: Abatement goods sector parameters			
Elasticity of substitution between abatement goods	ζ_A	6	Galí (2007)
Entry cost to output ratio	$X_q\ \bar{N}_{t_0}^E/(\bar{y}_{t_0}^A \bar{N}_{t_0})$	0.0385	Cacciatore and Fiori (2016)
Abatement cost parameter	p_b	716.7/1000	Nordhaus (2018)
Curvature of abatement cost	θ_2	2.6	Nordhaus (2018)
Persistence in cost of abatement growth	δ_{pb}	0.025/5	Nordhaus (2018)
Sunk cost labor	X_w	1	Bilbiie et al. (2012)

Concerning parameters that are not common with DICE, we mainly build on Galí (2007). The elasticity of substitution across varieties in each sector is set to 6, which generates a markup of 20%, and the labor curvature σ_h is set to 1. Regarding the technology, the initial output Y_{t_0} is worth 15.917 trillion in 2017 PPP U.S. dollars, while the initial labor supply is normalized to one. The sunk cost paid in terms of the final good represents 3.85% of the abatement goods sector's output as in Cacciatore and Fiori (2016). Finally, we compute using world data the share of public spending in output g_y and find a value of 16% on the sample period. The resulting calibration pins down the shift parameter in the utility function $\psi_h = 1.07$ and the initial productivity level $Z_{t_0} = 4.81$. The rest of the initial steady-state variables (e.g., number of firms) are pinned down by the model's equations.

[14] We rely on Holston et al. (2017), who provide U.S. estimates of the natural rate of interest, i.e., the real short-term interest rate that would prevail in the absence of transitory disturbances, which is the consistent notion within our framework. Notice, however, that the world real interest rate may be above 5% in the 1960s due to significant sovereign risk premia for many countries, especially for emerging ones.

TABLE 2. Initial conditions for state variables in 1961

NAME	PARAMETER	VALUE	SOURCE
Initial period	t_0	1961	Data availability
Emissions-to-output ratio	σ_{t_0}	0.5878	Data
Abatement cost	θ_{1,t_0}	0.162	Authors' calculations
Abatement effort	μ_{t_0}	0.03	Nordhaus (2018)
Hours demand (normalized)	$H^d_{t_0}$	1	Galí (2007)
Population (billion)	L_{t_0}	3.307	Data
Real output (trillion U.S. dollars)	Y_{t_0}	15.917	Data
Initial cost of abating carbon	θ_{t_0}	0.7167	Authors' calculations
Stock of carbon (Gt) in 1961	M_{t_0}	670	Authors' calculations
Atmosphere temperature anomaly	T_{t_0}	0.21	Data
Deep oceans temperature anomaly	$T^*_{t_0}$	0	Data
Non-CO_2 forcing	F_{EX,t_0}	0.235	Authors' calculations
Carbon tax	τ_{t_0}	0.0038	Authors' calculations
TFP level	Z_{t_0}	4.8142	Authors' calculations
Carbon intensity	σ_{t_0}	0.5878	Authors' calculations
Abatement cost	θ_{1,t_0}	0.1750	Authors' calculations
Number of products	N_{t_0}	0.0116	Authors' calculations

Prior distributions of the parameters are reported in Table 3. For exogenous disturbances, the standard deviations are imposed an inverse gamma "type 2" with a prior mean of 0.001 and a standard error of 0.1. Our prior is inspired by Christiano et al. (2014) but with a less informative prior. The persistence of stochastic disturbances is taken from Smets and Wouters (2007) with a beta distribution with a prior mean of 0.5 and a standard error of 0.2. The deterministic growth rate of the TFP in the initial state, g_{z,t_0}, is indirectly measured by the inference of the deterministic growth rate of output $(Y_{t_1}/Y_{t_0} - 1) \times 100$. Its prior is a gamma distribution with a mean of 4 and a standard error of 1. This prior imposes that the initial growth rate is positive and lies roughly between 2% and 6%. This interval includes the observed annual rate of growth that is approximately 5% in real terms during the 1961–1990 period. For the decoupling rate of the emissions-to-output ratio, denoted by $(\sigma_{t_1}/\sigma_{t_0} - 1) \times 100$, a gamma distribution is imposed, with a prior mean of 1 and a standard error of 0.1. This prior imposes that the decoupling rate lies between 0.8% and 1.2% percents, consistent with the rate observed in the 1960s. The effects of radiative forcing on temperature anomalies are measured by elasticity ξ_T, which is typically 0.1005 in the latest DICE model. Instead of calibrating this parameter, we let the data be informative and impose a prior mean of 0.15 and a standard error of 0.02. The exit rate δ_A is typically 10% in Bilbiie et al. (2012) but is not empirically motivated. In particular, as the exit rate of startups may be higher, we assume a beta distribution to bound the parameter in the support [0,1], with a mean of 0.2 and a standard deviation of 0.1, which is a rather diffuse prior. Finally, the entry congestion cost χ is given a prior consistent with the adjustment cost parameter in Smets and Wouters (2007), i.e., a gamma distribution with a mean of 4 and a standard deviation of 1.5.

TABLE 3. Prior and posterior distributions of structural parameters and shock processes

	Parameter	Prior distribution Shape	Mean	Std.	Posterior distribution Mean [0.050;0.950]
Panel A: Structural parameters					
Initial output growth rate	$(Y_{t_1}/Y_{t_0}-1)\times 100$	$\mathcal{G}$	4	1	4.991 *[4.867;5.129]*
Initial emissions-to-output decoupling rate	$-(\sigma_{t_1}/\sigma_{t_0}-1)\times 100$	$\mathcal{G}$	1	0.10	1.132 *[1.028;1.232]*
Temp. elast. to radiating forcing	ξ_T	$\mathcal{B}$	0.15	0.02	0.084 *[0.069;0.108]*
Exit rate	δ_A	$\mathcal{B}$	0.20	0.10	0.060 *[0.028;0.095]*
Entry congestion cost	χ	$\mathcal{G}$	4	1.5	5.626 *[3.660;7.794]*
Panel B: Shock processes					
Std. productivity	σ_Z	$\mathcal{IG}_2$	0.001	0.1	0.015 *[0.012;0.017]*
Std. government spending	σ_G	$\mathcal{IG}_2$	0.001	0.1	0.030 *[0.026;0.036]*
Std. CO_2 emissions	σ_E	$\mathcal{IG}_2$	0.001	0.1	0.015 *[0.013;0.017]*
Std. firm's entry	σ_N	$\mathcal{IG}_2$	0.001	0.1	0.089 *[0.077;0.104]*
Std. temperature	σ_T	$\mathcal{IG}_2$	0.001	0.1	0.132 *[0.111;0.160]*
AR(1) productivity	ρ_Z	$\mathcal{B}$	0.50	0.2	0.949 *[0.903;0.982]*
AR(1) government spending	ρ_G	$\mathcal{B}$	0.50	0.2	0.867 *[0.781;0.940]*
AR(1) CO_2 emissions	ρ_E	$\mathcal{B}$	0.50	0.2	0.940 *[0.886;0.979]*
AR(1) firm's entry	ρ_N	$\mathcal{B}$	0.50	0.2	0.592 *[0.446;0.728]*
AR(1) temperature	ρ_T	$\mathcal{B}$	0.50	0.2	0.181 *[0.051;0.425]*
Log marginal data density					377.2681

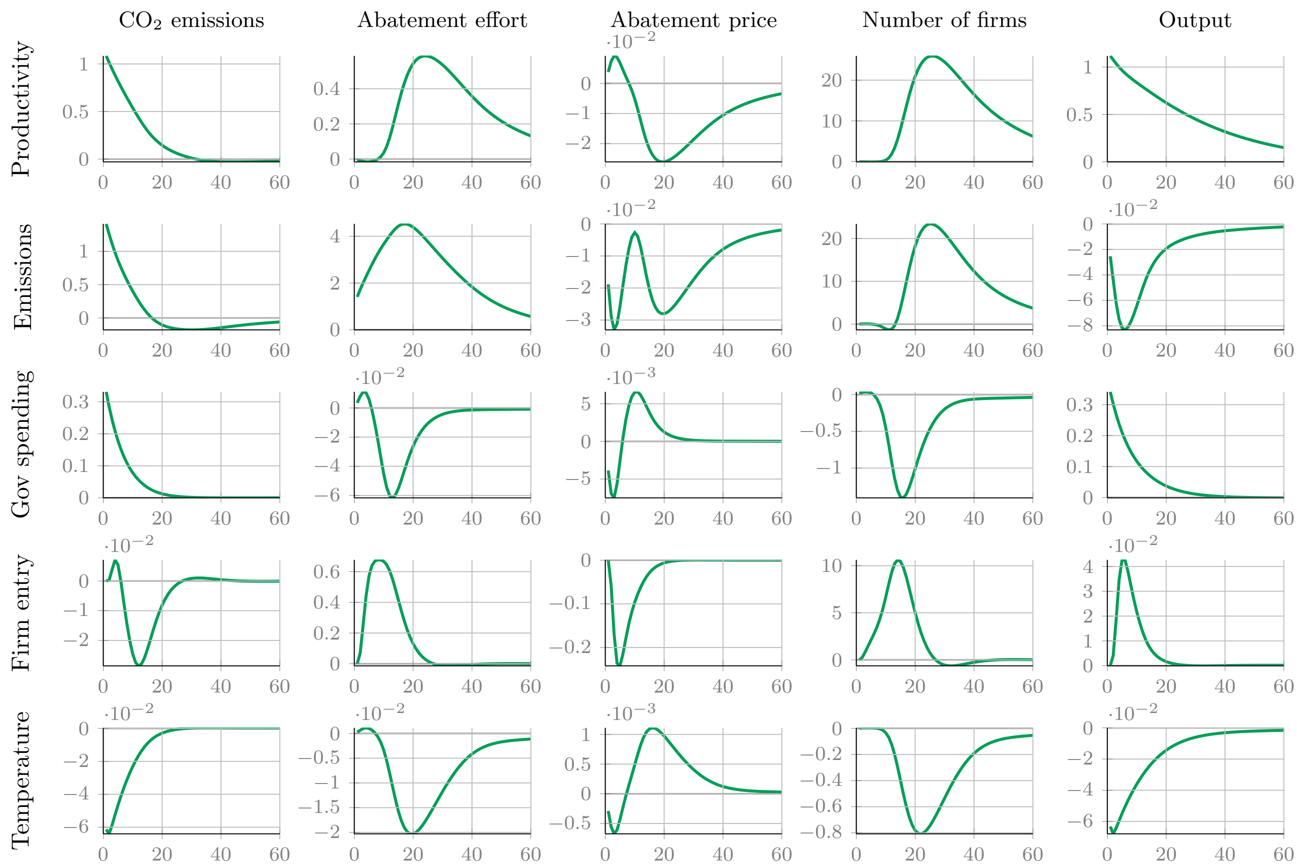
Note: $\mathcal{B}$ denotes the beta, $\mathcal{G}$ the gamma, and $\mathcal{IG}_2$ the inverse gamma (type 2) distributions. The last 160,000 draws are used to compute the posterior mean and 90% confidence interval.

3.4 Posterior estimates of the parameters The last column of Table 3 reports the posterior mean and the 90% confidence interval of the estimated parameters.[15] The first two parameters represent the initial growth rate of the economy and the initial decline rate in the emission-to-output ratio, which are estimated at approximately 4.99% and 1.13% in the initial state, respectively. These values are fairly close to those proposed by Nordhaus (1992) for 1965 (4% and 1.25%, respectively). Regarding the climate block, we estimate the parameter capturing the sensitivity of temperature to radiative forcing (ξ_T in Equation (4)) and obtain a value of 0.084, slightly below the values used in DICE 2013 and DICE 2016. We also estimate two parameters associated with the abatement goods sector. The firm's exit rate δ_A is equal to 0.06, and the entry congestion cost χ is equal to 5.63. The exit rate is lower than in Bilbiie et al. (2012) but captures the observed 7% growth rate of environment-related patents in the sample. Our posterior mean of the entry congestion cost is slightly lower than the estimated value of 9.435 obtained by Lewis and Poilly (2012). Finally, we estimate the parameters pertaining to the dynamics of the five shocks introduced in the model ($\varepsilon_{Z,t}$, $\varepsilon_{G,t}$, $\varepsilon_{E,t}$, $\varepsilon_{N,t}$, $\varepsilon_{T,t}$). As usually found in estimated DSGE models such as Smets and Wouters (2007), the productivity and government spending shocks are highly autocorrelated. This also is the case for the shock on CO_2 emissions, with an autoregressive coefficient of 0.94. The shock on firm entry is less persistent, and the shock on temperature is serially uncorrelated at an annual frequency, with a coefficient of 0.18.

[15] Figure B.1 in the Appendix plots the prior and posterior distributions.

3.5 Model evaluation This section discusses the dynamic properties of the model through *(i)* the impulse response functions of a number of variables of interest to various shocks and *(ii)* the second moments of the observable variables. Both analyses are useful in assessing how shocks to economic variables reverberate through the system and checking if the model correctly captures the statistical properties of the macroeconomic and climate-related data.

FIGURE 3. Generalized impulse response functions



Note: The figure displays the generalized impulse response functions (GIRFs) of several variables to five shocks: productivity, CO_2 emissions, government spending, firm entry, and temperature, in lines 1 to 5, respectively. GIRFs are computed using the value of state variables from 2019, and each GIRF is expressed in percentage deviations from its initial value in 2019. GIRFs are averaged based on 500 exogenous draws.

Figure 3 displays the economy's response to a one percent increase in five shocks: productivity, CO_2 emissions, government spending, firm entry, and temperature, in lines 1 to 5, respectively. They are globally consistent with business cycle theory. For example, a positive productivity shock increases aggregate output, which worsens CO_2 emissions. Hence, the abatement effort increases to meet the emissions target. This effort stimulates the development of the abatement goods sector, with a growing number of firms, which makes the abatement price decrease. Then, the variables smoothly return to their initial values (corresponding to 2019) as the highly inertial productivity shock dissipates. As shown in the second line of plots, an exogenous increase in CO_2 emissions immediately raises the abatement effort to meet the emissions target. This effort again encourages the entry of startups into the abatement goods sector and makes the abatement price decrease as the sector develops. Meanwhile, the damaging effect of emissions on the TFP, the levy of the carbon tax, and the costly abatement effort adversely affects the output, which decreases by almost 5% compared to its initial value in the short run. The third line of plots shows that, as a demand shock, an exogenous increase in government spending stimulates the production of the final good and thus CO_2 emissions, at the expense of abatement goods. Hence, the abatement effort and the number of firms fall by -5% and -1.1%, respectively, while the abatement price rises. Then, these variables return to their respective initial levels, as the stimulating effect of the initial shock on output fades away. The fourth line of Figure 3 indicates that the number of firms in the abatement goods sector rises (by nearly 10% at its peak) because of an exogenous increase in startup entries, which exacerbates competition and thus makes the abatement price drop. Hence, in line with Equation (14), abatement effort is encouraged. Aggregate production benefits from this increase in the number of firms through the increase in revenues paid in the abatement goods sector, but without increasing CO_2 emissions. Finally, the responses to an exogenous and temporary increase in temperature, represented at the bottom of Figure 3, are interesting to assess the economic effects of a climate-related shock. By exacerbating damages to firm productivity, this shock strongly depresses output (by more than 6% initially), which reduces CO_2 emissions accordingly. Consequently, abatement efforts decrease, and the number of new firms in the abatement goods sector shrinks. Last, reduced competition pushes the abatement price up.

TABLE 4. Empirical and model-implied moments

	DATA	E-DSGE Model [5%;95%]	DICE model [5%;95%]
Standard deviations			
Output growth	1.50	[1.21;1.64]	[1.20;1.66]
Consumption growth	1.18	[1.18;1.60]	[1.21;1.64]
Emission growth	2.24	[1.67;2.39]	[1.70;2.43]
Temperature change	0.12	[0.11;0.16]	[0.11;0.17]
Patent growth	10.01	[7.62;13.15]	–
Autocorrelation			
Output growth	0.43	[-0.05;0.43]	[-0.08;0.45]
Consumption growth	0.51	[-0.05;0.43]	[-0.05;0.45]
Emission growth	0.50	[-0.18;0.34]	[-0.20;0.33]
Temperature change	-0.32	[-0.16;0.34]	[-0.19;0.35]
Patent growth	0.63	[0.26;0.73]	–

Note: Model-implied moments are computed across 1,000 random artificial series, each with the same size as the data sample (57). The E-DSGE model corresponds to our benchmark model with firm entry and the DICE model is the alternative version without entry firms.

Table 4 provides the empirical second moments of our five observable variables and the 95% confidence interval, as obtained with our E-DSGE model and an alternative model with perfect competition in the abatement goods sector. The latter model corresponds to DICE-2016R2. The estimation of the

DICE model relies on the same observable variables except for patent growth (and no shock $\eta_{N,t}$). Consequently, likelihood or standard information criteria cannot be employed to discriminate across models. We thus rely on the comparison of second moments. We find that the E-DSGE and DICE models accurately replicate the empirical moments, although both models yield less persistence than in the data. Importantly, the E-DSGE model can reproduce the standard deviation and the autocorrelation of patent growth fairly well.

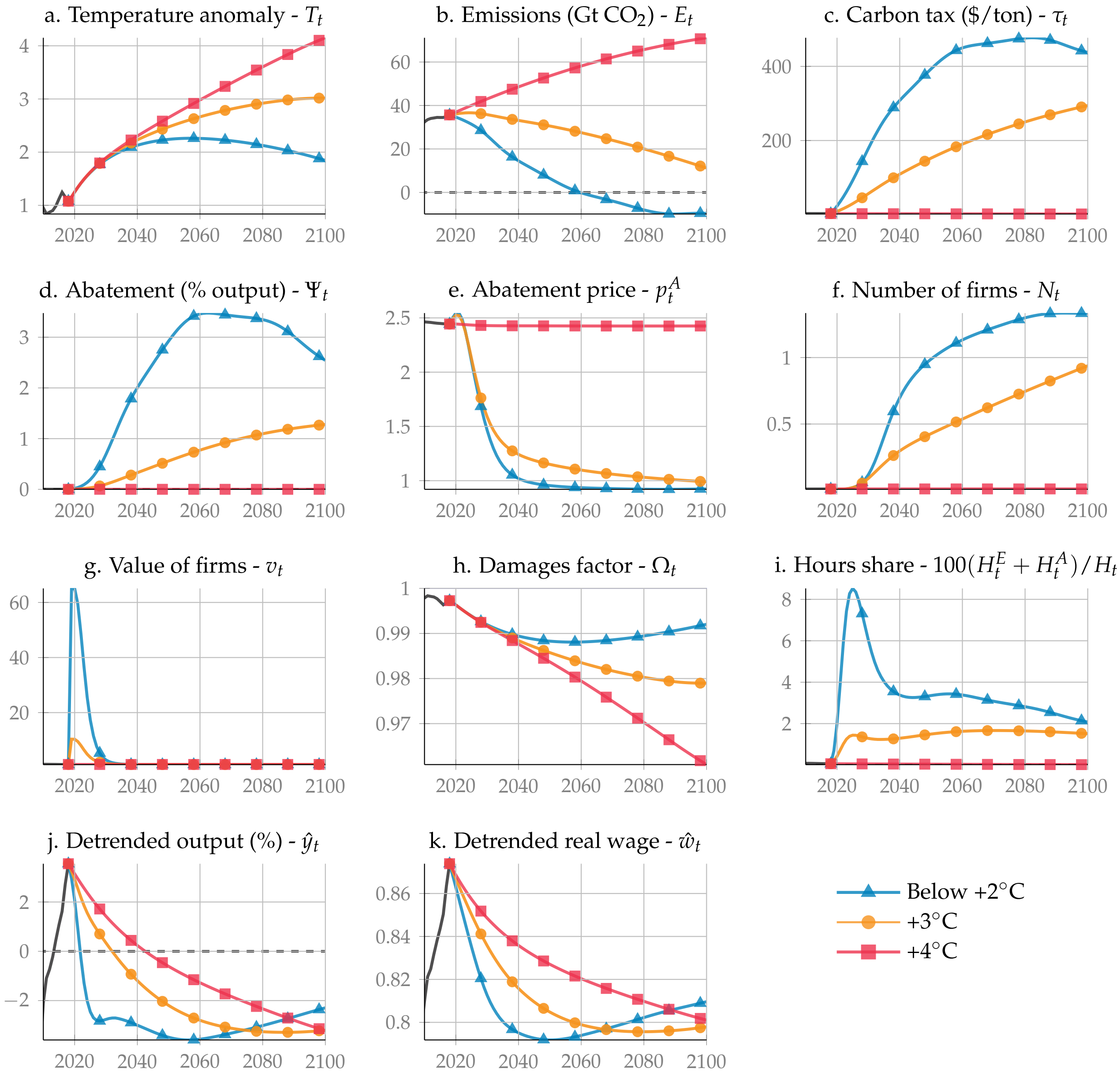
4 Model-implied projections under IPCC CO_2 emissions scenarios

In this section, we present long-term projections derived from our E-DSGE model. There is a large uncertainty about how CO_2 emissions will evolve in the coming decades. Much of the uncertainty concerns future global energy consumption, land-use dynamics, population growth, and technological change. IPCC (2021) addresses this uncertainty through the assessment of five alternative scenarios of evolution of CO_2 emissions. Each scenario is referred to as a Shared Socioeconomic Pathway (SSP). We build on this assessment methodology by considering alternative paths of control rates of emissions $\{\mu_t\}_{2019}^{2500}$ that match CO_2 emissions implied by three of the IPCC (2021)'s SSPs, namely SSP1–1.9, SSP2–4.5, and SSP3–7.0.[16] In SSP1–1.9, it is assumed that carbon neutrality is reached in 2060, while emissions reach -10 Gt by 2100. In the second scenario SSP2–4.5, carbon neutrality is delayed and reached by 2120. In the last scenario SSP3–7.0, there is no carbon control policy, so business remains as usual and CO_2 emissions continue to increase. In our simulations, the value of the carbon tax is determined to match the desired control rate of emissions for each scenario, and the model endogenously generates out-of-sample forecasts based on the posterior mean of the MCMC distribution. The future path of the carbon tax rate is announced in 2019, and expectations adjust in response to this new environment. In this setup, the carbon tax revenues are simply given to households by the government through lump-sum transfers.

4.1 Model-implied projections Figure 4 presents the projections of the main variables of the E-DSGE model under alternative control rates of emissions. The red line corresponds to the laissez-faire trajectory (meaning no environmental policies), which would result in an increase in temperature by 4°C. The yellow and blue lines are associated with the trajectories that would be consistent with temperatures that would remain below 3°C and 2°C above the preindustrial level, respectively. It is worth noting that our scenarios approximately replicate the carbon emissions trajectories of the corresponding scenarios formulated by the IPCC (2021). However, our temperature projections do not exactly match those reported by the IPCC because of the different parameterization of the model.

[16] The time horizon of our simulations is $t = 2500$, as in the DICE-2016R2 model, to ensure that exogenous trends have converged to their asymptotic values.

FIGURE 4. Model-implied projections based on alternative control rates of emissions



Note: This figure displays the projections of the main variables of the E-DSGE model under three scenarios, corresponding to temperature increases of +4°C, below +3°C, and below +2°C relative to preindustrial levels. The +4°C scenario corresponds to the laissez-faire case, and the below +3°C and below +2°C scenarios are associated with a carbon tax that matches IPCC paths of carbon dioxide emissions.

In the +4°C scenario, no policy would be implemented to curb CO_2 emissions, which also is reflected by a carbon tax equal to zero and the absence of abatement. Emissions peak up to 57 Gt in 2060 and 70 Gt in 2100, which induces more atmospheric loadings of CO_2, higher radiative forcing, and finally an increase in temperature by approximately 4°C by 2100. In the medium run of this scenario, there is a recession after 2040 due to the damages induced by climate change. In the long run, damages increase over time and reach a level of 1.5% of GDP per year in 2050 and 4% per year in 2100. The detrended output, currently equal to 3%, decreases slowly to −1% in 2050 and −3% in 2100.

In contrast, the +3°C scenario is associated with a gradual reduction of emissions to reach zero shortly after 2100. This path is consistent with a steady increase in the carbon tax up to $300 per ton in 2100. A reduction in carbon emissions is obtained by an increase in abatement, as producing firms try to reduce the impact of the carbon tax by cutting their emissions. Due to the low number of competitors in the abatement goods sector, firms behave monopolistically and charge a high selling price. When the government announces the introduction of the carbon tax, producing firms reduce their emissions immediately by purchasing abatement goods. The rise in profits of abatement firms boosts their market value and, through the free-entry conditions, incentivizes prospective entrants to establish a startup. The number of firms increases and the resulting competition pushes firms to compress their prices to maintain their market share. The cost of abatement inputs in the total output increases from 0.5% in 2050 to 1.2% in 2100. This cost diverts a fraction of resources from consumption and therefore depresses aggregate demand and leads to an even more negative detrended output, down to −3% in 2060, reflecting the cost of the energy transition.

Finally, the below 2°C scenario requires a more stringent control of carbon emissions: emissions should be negative to reverse the dynamics of the accumulated stock of carbon. To reach this objective, the carbon tax must dramatically increase to a maximum of $480 in 2080, so that emissions turn negative. This scenario is consistent with the IPCC approach to maintain the temperature increase to 1.5°C if the transition is delayed. To reach this objective, the IPCC considers pathways with substantial overshoot, requiring technological innovations such as bioenergy with carbon capture and storage. In this scenario, resources diversion in hours and inputs to the abatement goods sector would have a large negative impact on GDP, with the abatement cost being as large as 3.4% of GDP in 2060. The detrended output would reduce by approximately 4% in the same period. This policy curbs climate change damages in 2100 from 4% in the laissez-faire scenario down to less than 1%.[17] As in the previous scenario, the announcement of a high carbon price strongly increases the value of abatement firms, so the number of prospective entrants is higher and reduces the abatement price, but with enhanced effects here.

In the rest of the paper, we present simulations based on the assumption of a temperature increase below 2°C above preindustrial levels, in line with the Paris Agreement, which is a challenging target. We investigate how subsidizing the abatement goods sector may help reduce the cost of the transition.

4.2 On the role of the endogenous structure of the abatement goods sector While the previous section discusses the dynamics of the model conditional on different climate targets, we now focus on one key and original mechanism of the model, namely, the dynamics of the abatement goods sector.

[17] These results are broadly consistent with the logic of a reduction in the production of consumption goods to curb carbon emissions, as promoted by the Club of Rome (Meadows et al., 1972). Our assessment does not consider that curbing CO_2 emissions and the temperature increase would also reduce physical risks.

The two top plots (*a* and *b*) of Figure 5 display the model-implied paths of the abatement price and the number of new firms following the introduction of a carbon tax under the below 2°C scenario. We observe that our E-DSGE generates a decrease in the abatement price (from an initial value that is 2.5 higher than the price of the final good) as firm entry increases. This inflow of startups is encouraged by the perspective of higher profits, as good producers demand more abatement goods to reduce their carbon tax burden. Initially, when the demand for abatement goods is small, the cost of entry is much higher than the future expected gain. This acts as a barrier to entry and features a detrimental effect on competition. The relative cost of entry becomes proportionally smaller as the market size increases. After 2040, the competition effect stemming from the rise in firm entry is so strong that the abatement price falls below 1. This mechanism highlights that the carbon policy can be less stringent under a more competitive abatement goods sector and thus calls for some policies that would boost competition when starting the energy transition. As the figure illustrates, the confidence intervals are relatively narrow.[18]

Interestingly, the dynamics depicted by the model are in line with the concomitant increase in the number of new environmental-related patents and the drop in their price observed over the last forty years. As an illustration, the two bottom plots (*c* and *d*) of Figure 5 show that the substantial decrease in the cost of solar photovoltaic modules from the late 1970s to the present was associated with an impressive increase in the cumulative number of patents in this sector. We note that the evolution of the solar photovoltaic sector was partly driven by government subsidies in several countries.

In contrast, in standard environmental models, such as the DICE model, the abatement goods sector is supposed to be already mature and competitive; thus, there are no dynamics in the number of firms and in the abatement price, which is equal to one as all the varieties, including abatement goods, are produced by a homogeneous sector.[19]

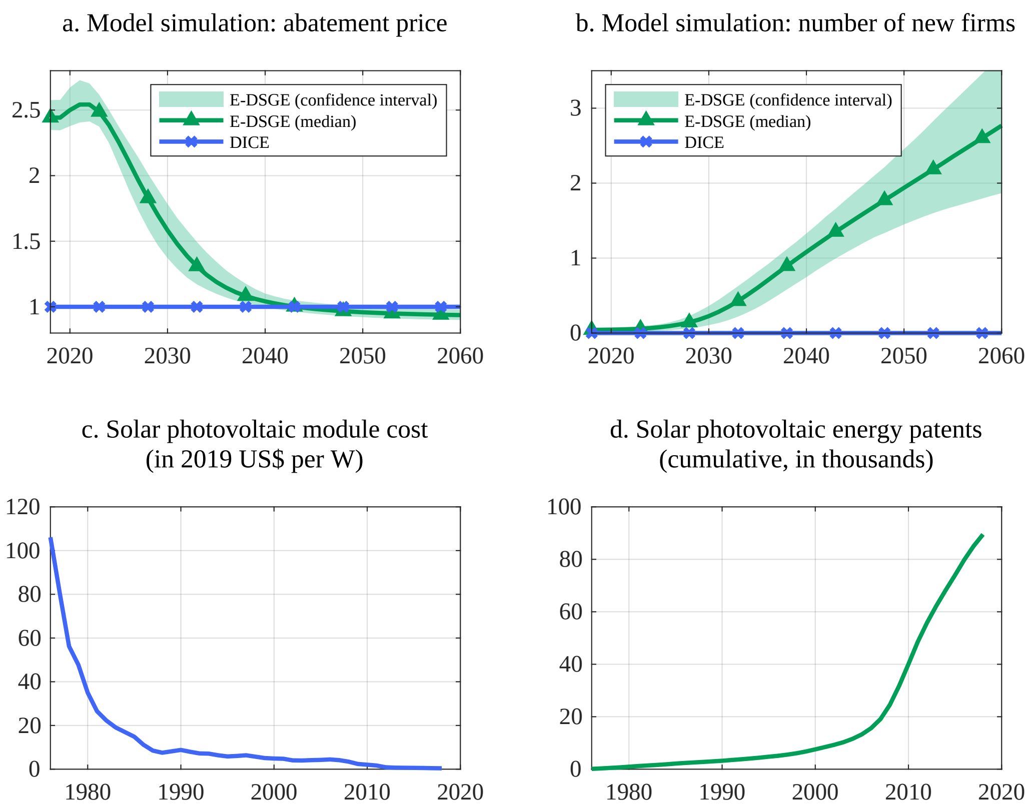
5 Quantifying the effects of environmental subsidies

In typical environment models, the government revenues stemming from a carbon tax are simply redistributed to households through lump-sum transfers. In this section, we question this assumption by assessing the role of government revenues in accelerating the transition to a greener economy. Indeed, using carbon tax revenues to subsidize the abatement technology industry could trigger a second wave of new technologies, switching from the production and storage of renewable electricity to technologies allowing for, among others, green hydrogen production, more efficient battery storage,

[18]See Appendix C for the projection of the observed variables with a confidence interval.

[19]In a DICE environment, a rise in carbon price forces firms to purchase some additional intermediate inputs, the latter being produced at the same selling price as the final good. The carbon tax deteriorates the marginal profit of firms and unintendedly reduces workers' salary. The equilibrium real wage falls proportionally to the carbon tax, so that households are less willing to supply labor. The macroeconomic outcome is a recession resulting from a low labor supply.

FIGURE 5. Model-implied dynamics of the abatement goods sector and historical evidence

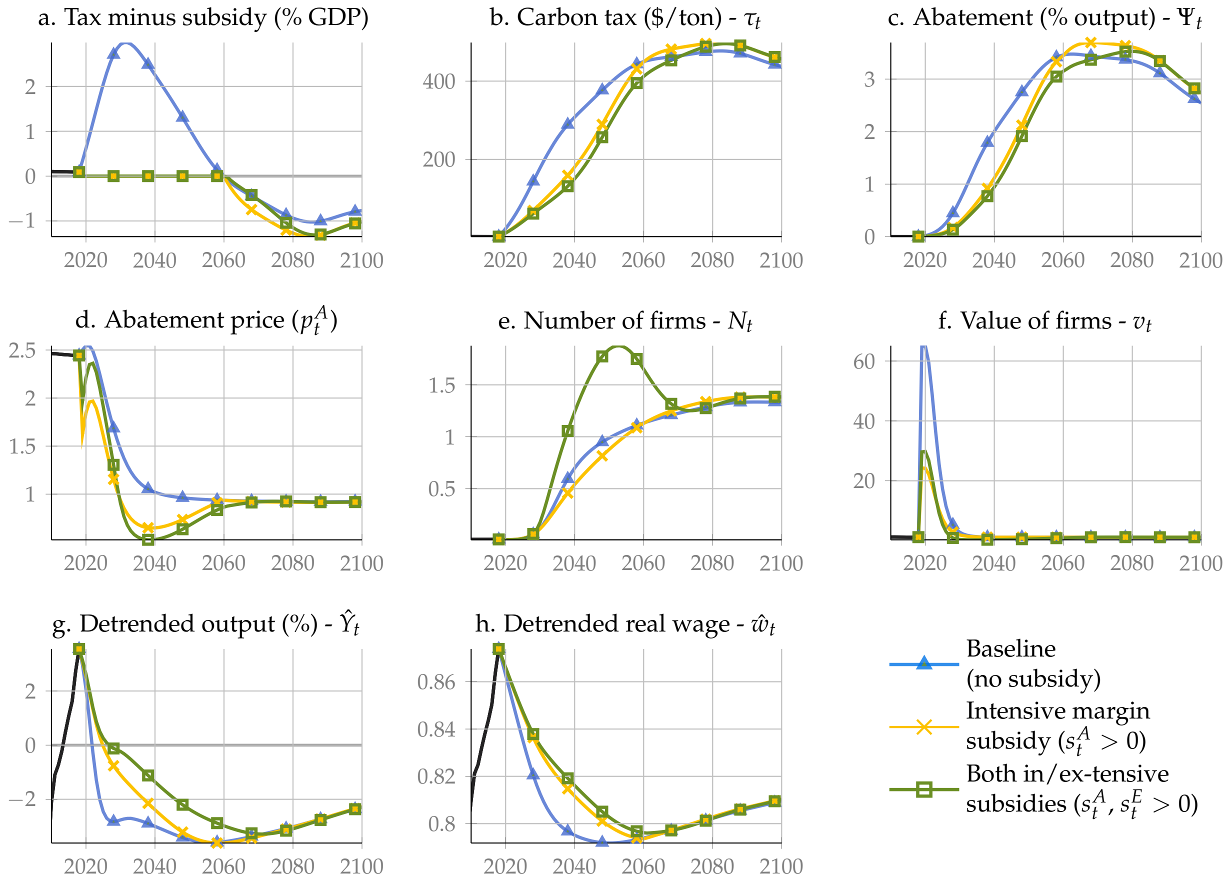


Note: Panels *a* and *b* display the temporal evolution of the abatement price and the cumulative number of new firms under the assumption of a temperature increase below +2°C relative to preindustrial levels. E-DSGE corresponds to a model with imperfect competition in the abatement goods sector (endogenous firm entry), and DICE corresponds to an alternative version with perfect competition on the abatement goods sector. The number of new firms corresponds to the number of additional startups per household. The light green area denotes both parametric and stochastic uncertainties. The 90% confidence intervals are computed from 500 random draws.

floating wind and solar installations, distributed energy aggregation, or large-scale carbon capture and storage. These developments would accelerate the transition and make negative emissions an additional levy to decarbonize the economy.

We focus on two experiments: *(i)* a subsidy to the margin of existing firms in the abatement goods sector and *(ii)* an optimal subsidy to both existing firms and startups. Figure 6 presents the projections of the main variables of our E-DSGE model under both alternative policies in the abatement goods sector. The period of analysis starts in 2019 at the end of the estimation sample when the carbon tax and subsidy policies are both announced and ends in 2100. The blue line corresponds to the trajectory consistent with a policy that contains temperatures below 2°C with a carbon tax only. The yellow line corresponds to the case where the carbon tax is complemented with a subsidy to the margin of abatement firms ($s_t^A > 0$ in Equation (16)). In this scenario, instead of being used for unproductive government expenses, carbon tax revenues are used to reduce the price of the abatement technology

FIGURE 6. Out-of-sample forecasts under alternative subsidy policies in the abatement goods sector



Note: This figure displays the temporal evolution of the main variables of the model under two scenarios, corresponding to a temperature increase of +2°C relative to preindustrial levels. The baseline scenario corresponds to the carbon tax only case and the subsidy scenarios are associated with a carbon tax and a subsidy to the margin of abatement firms with or without a subsidy to entrant firms.

and help its diffusion to the intermediate goods sector. The green line corresponds to the case where the government uses carbon tax revenues to subsidize both incumbents ($s_t^A \geq 0$) and prospective entrants ($s_t^E \geq 0$ in Equation (19)) in the abatement goods sector. In this case, the share of entrants and incumbents is chosen optimally to maximize social welfare. In the following, we discuss each case in turn.

5.1 Environmental subsidies on the intensive margin We first analyze how the proceeds from the carbon tax can be employed to reduce costs in the abatement goods sector and mitigate the recession induced by the carbon tax rise. Formally, the government subsidizes the abatement goods sector proportionally to its input costs as follows:

$$s_t^A H_t^A w_t = \tau_t E_t. \tag{30}$$

In this equation, we impose that the subsidy policy is budget neutral, i.e., the cost of the subsidy policy cannot exceed the net carbon tax revenues.

The introduction of the subsidy massively reduces the selling abatement price (Figure 6, Panel a). The price of the abatement good, relative to the price of the final good, is instantaneously reduced from 2.5 to 1.5. While parity with the price of the final good is reached after 2040 in the baseline case, such parity is obtained before 2030 in the case with the subsidy. Because the diffusion of the abatement technology is much faster than in the baseline case, the aggregate cost of abatement for society is reduced, from 2% of GDP to 0.8% in 2040 and from 2.7% to 2% in 2050. Consequently, the recessive effect of decarbonization on economic growth is substantially attenuated. In 2040, the detrended output is increased from -3% in the baseline scenario to -2% in the subsidy scenario (-3.4% and -3.2% in 2050, respectively). In addition, given the effectiveness of the subsidy, the carbon tax does not increase as much as in the baseline case. While the carbon tax jumps to \$300 per ton in 2040 and \$390 in 2050 when no subsidy mechanism is implemented, the tax increases only to \$160 in 2040 and \$300 in 2050 with subsidies on intensive margins.

Overall, even if the effect of the energy transition on economic growth is largely reduced, this policy deteriorates competition within the abatement goods sector. Incumbents benefit from a subsidy that lowers their cost of production, which increases the equilibrium real wage. As a result, prospective entrants face a higher cost of entry, resulting in a lower number of abatement firms in the transition period.

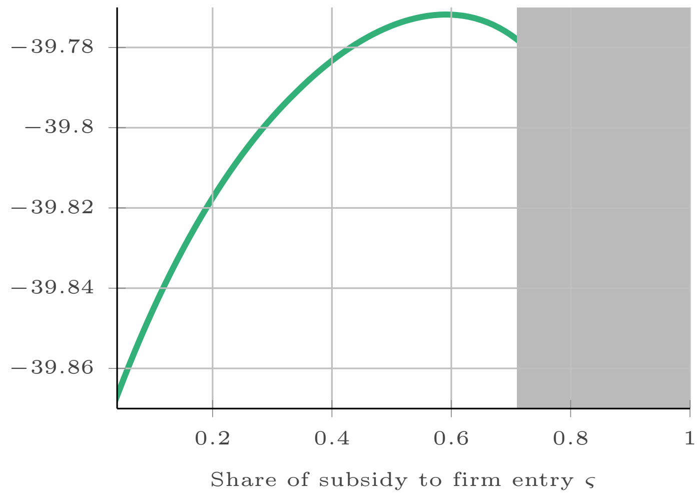
5.2 An optimal environmental subsidy rate to firm entry In the second experiment, we analyze how the government uses carbon tax revenues to subsidize both incumbents and prospective entrants in the abatement goods sector. In a first step, we determine the optimal subsidy, i.e., the share of the proceeds attributed to the incumbents and entrants, to smooth the transition to the low-carbon economy. To explore the policy trade-off faced by policymakers between subsidizing entrants versus incumbents, we denote by $\varsigma \in [0;1]$ the fraction of the carbon tax that is used to subsidize prospective entrants. The value of ς satisfies the following subsidy-sharing rule across firms:

$$s_t^E H_t^E w_t = \varsigma \tau_t E_t \tag{31}$$

$$s_t^A H_t^A w_t = (1-\varsigma) \tau_t E_t, \tag{32}$$

with $s_t^E, s_t^A \geq 0$. Equation (31) defines the subsidy rate to firm entry (s_t^E) such that the fraction ς of the carbon tax revenues is used to reduce the cost of entry in the abatement goods sector. Equation (32) defines the subsidy rate to incumbents (s_t^A) such that incumbent firms receive the complementary $(1-\varsigma)$ of the carbon tax revenues.

FIGURE 7. Social welfare for various subsidy rates to startups



Note: The gray area represents the indeterminacy region that is reached when the value of new entrants v_t becomes negative. Social welfare represents the infinite discounted sum of future utilities. It is given in 2019 when the carbon tax policy is announced and reflects the future path of utilities under the net-zero transition.

The optimal share ς is determined by maximizing the social welfare, defined as

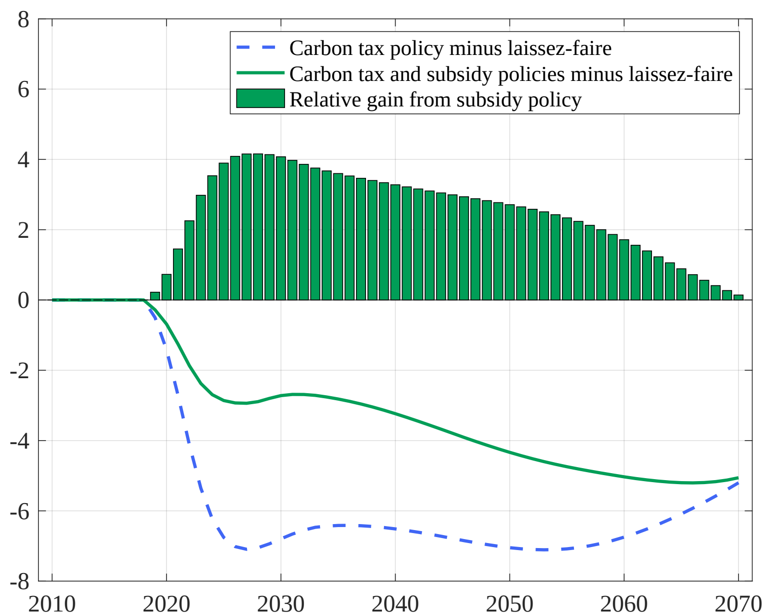
$$\mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} \beta^{\tau} \left(\frac{c_{t+\tau}^{1-\sigma_c}}{1-\sigma_c} - \psi_t \frac{h_{t+\tau}^{1+\sigma_h}}{1+\sigma_h} \right) \right\}. \tag{33}$$

We calculate the welfare value associated with each point of a grid on ς. Figure 7 displays the result and shows that the relationship between the subsidy share and welfare is concave. It takes time for a subsidy to startups to pay off because the benefit of a lower abatement price through firm entry (Panel d. of Figure 6) follows a gradual process. In contrast, subsidizing existing firms immediately reduces the abatement price, but deteriorates competition in the future. Therefore, welfare increases in ς as long as the gradual future gain from firm entry outperforms the immediate loss from a higher abatement price in 2019. The highest welfare value is thus obtained with a subsidy rate to startups reaching 60% of the carbon tax revenues.

We then compute the E-DSGE-implied projections using $\varsigma = 0.6$, which corresponds to the green lines in Figure 6. The total number of firms jumps rapidly because of the sharp increase in the number of entries in the abatement goods sector. Until 2050, the number of firms is almost twice as high as that in the baseline case. Such a boost of competition results in a drop in the abatement price. After 2030, when the number of startups becomes substantial, the reduction in the abatement price exceeds the reduction obtained in the intensive margin subsidy case. Under this subsidy mechanism, at equilibrium, the deadweight loss is lower, as the cost of abatement is weaker. The abatement cost remains slightly lower than in the intensive margin subsidy case until 2080. The recession induced by the transition is substantially dampened because the recessive attenuation effect starts earlier. In 2040, the detrended output is increased from −2% in the scenario with an intensive margin subsidy to −1% in the efficient subsidy scenario (−3.2% and −2% in 2050, respectively). To reach a similar objective of CO_2 emission reduction in 2040, the carbon tax would increase to \$125 instead of \$300 in the baseline scenario and \$150 in the intensive margin subsidy case.

This analysis demonstrates that competition-friendly policies can become, in the decades to come, a serious source of mitigation in the cost of reaching a low-carbon economy. We also conclude that subsidizing abatement firms on the extensive margin, i.e., by reducing the congestion cost for new entrants, has a higher return on investment than subsidizing firms only on the intensive margin, i.e., by increasing their margin benefits.[20]

FIGURE 8. Real GDP loss during the transition (in \$ trillion)



5.3 GDP loss during the transition and subsidy multipliers In this last section, we quantify the effects of environmental subsidies in terms of GDP. The cumulative effect of global climate change will depend on how the world responds to increasing emissions. We therefore compare the evolution of GDP among three scenarios: *(i)* laissez-faire (no policy at all), *(ii)* policy with a carbon tax, and *(iii)* policy action with an optimal subsidy. Relative to the laissez-faire scenario, the carbon tax policy, whose revenue is redistributed through lump-sum transfers to households, implies a cumulative GDP

[20] Acemoglu et al. (2016) also find that research subsidies encourage production and innovation in clean technologies. Their demonstration relies on a microeconomic model in which a continuum of intermediate goods can be produced using either dirty or clean technologies. In our model, the intermediate goods sector reduces its carbon emissions by using abatement goods, such that we focus directly on the dynamics of the eco-industry.

loss reaching $258 trillion between 2019 and 2060. It represents an average annual loss of $6.3 trillion (for illustrative purposes, this amount represents 4.9% of 2019 GDP). This estimation corresponds to the recession implied by the carbon tax burden: firms are incentivized to divert resources from the production of the final good toward the abatement goods sector. The distortionary carbon tax directly affects households who suffer the surge in the relative price of the final good. Lump-sum redistribution of the carbon tax revenue to households appears as a natural solution to address the externality associated with a change in the relative price structure, but may not be the most efficient one.[21] Indeed, allocating the carbon tax revenue to subsidize the abatement goods sector, according to the optimal weight of 60% on startups and 40% on existing firms, leads to a cumulative GDP loss of $141 trillion between 2019 and 2060. According to the green bars in Figure 8, the optimal subsidy saves $123 trillion of GDP, in other words, the average equivalent of $2.9 trillion each year. Importantly, the largest gains are made during the first ten years of the policy, during which the subsidies allow the abatement goods price to be drastically reduced and encourage the entry of new firms into the abatement goods sector. The subsidy policy thus has a double beneficial effect, first by accelerating the development of the abatement goods sector and then by reducing the costs associated with the climate-neutral objective by 2060. Consequently, such a policy substantially mitigates climate transition risk.

Finally, we also compute present value subsidy multipliers, which embody the full dynamics associated with exogenous fiscal actions and properly discount future macroeconomic effects (Fève and Sahuc, 2017, and Leeper et al., 2017):

$$\mathcal{M}(t_0, \mathcal{T}) = \frac{\mathbb{E}_t \left\{ \sum_{t=t_0}^{\mathcal{T}} \tilde{\beta}_{t_0,t} \Delta X_t \right\}}{\mathbb{E}_t \left\{ \sum_{t=t_0}^{\mathcal{T}} \tilde{\beta}_{t_0,t} \Delta S_t \right\}}, \tag{34}$$

where $\tilde{\beta}_{k,t} = \beta^{t-k} g_{z,k}^{\sigma_c} \Pi_{j=k}^{t} g_{z,j}^{-\sigma_c}$, t_0 is the starting date of the fiscal policy experiment, $\mathcal{T}$ is the horizon of interest, and X_t is either Y_t (GDP) or C_t (private consumption). In this formula, ΔX_t is the net GDP (or consumption) gain between the scenario with both carbon tax and subsidy (optimally allocating) policies and the scenario with only the carbon tax policy, and ΔS_t is the related subsidy variation.

TABLE 5. Subsidy multipliers for various policy horizons

	2030	2035	2040	2045	2050	2055	2060
GDP	2.27	2.03	1.89	1.81	1.78	1.80	1.85
Consumption	1.90	1.66	1.53	1.45	1.42	1.44	1.48

Note: Subsidy multipliers are calculated as the present values of additional GDP and consumption over a specific horizon produced by an exogenous change in the present values of public subsidies.

[21] Note that the carbon tax captures transition cost toward a low-carbon economy but does not account for the positive impact of the policy through the reduction in physical risks.

Table 5 displays the subsidy multipliers at various horizons until 2060. We observe that multipliers are large regardless of the horizon, highlighting the interest in implementing a subsidy policy. For GDP, multipliers are above 2 until 2035, when subsidies to startups and existing firms benefit the abatement goods sector the most, consistent with Figure 6.

6 Conclusion

This study has investigated the role of public subsidies in mitigating climate transition risk. The implementation of a pure carbon tax policy to reduce CO_2 emissions would result in substantial GDP losses because firms would divert resources to invest in abatement technologies that are produced in an immature and low-competition sector. Mitigating the recession resulting from the fight against climate change would require a massive subsidizing of the abatement goods sector. This may be done through a joint reduction of labor cost for both entrants and incumbents operating in the abatement goods sector. Such a policy would accelerate the development of the abatement goods sector and offer a large reduction in the selling price of abatement technologies without harming the level of competition in the economy. This subsidy policy would have two main effects on the economy. First, in the transition phase, it would almost halve the distorting effect of the carbon tax compared to the carbon tax policy only. Second, accelerating the development of abatement technologies would significantly reduce GDP losses due to the transition to a low carbon economy. Eventually, the GDP loss would be reduced from $258 trillion between 2019 and 2060 to $141 trillion. Importantly, reducing entry costs in the abatement goods sector would accelerate the transition and reduce the GDP loss mainly at the beginning of the transition.

To the best of our knowledge, this is the first attempt to estimate a nonlinear E-DSGE model including environmental and macroeconomic trends. By combining the extended path solution method to solve the model and the inversion filter to calculate the likelihood function (Fair and Taylor, 1983), we can use Bayesian techniques for the estimation of the model's parameters. Our policy analysis evaluates the ability of a public subsidy policy to reduce the cost of transitioning to a zero-carbon-emissions economy by 2060. The assessment is conditional on a climate scenario (close to the IPCC, 2021's SSP1–1.9) and therefore does not account for the uncertainty about future economic and climate conditions. A promising avenue for future research would be to evaluate the optimal (carbon tax and subsidy) policy by accounting for stochastic economic and climate changes, following the approach proposed by Cai and Lontzek (2019).

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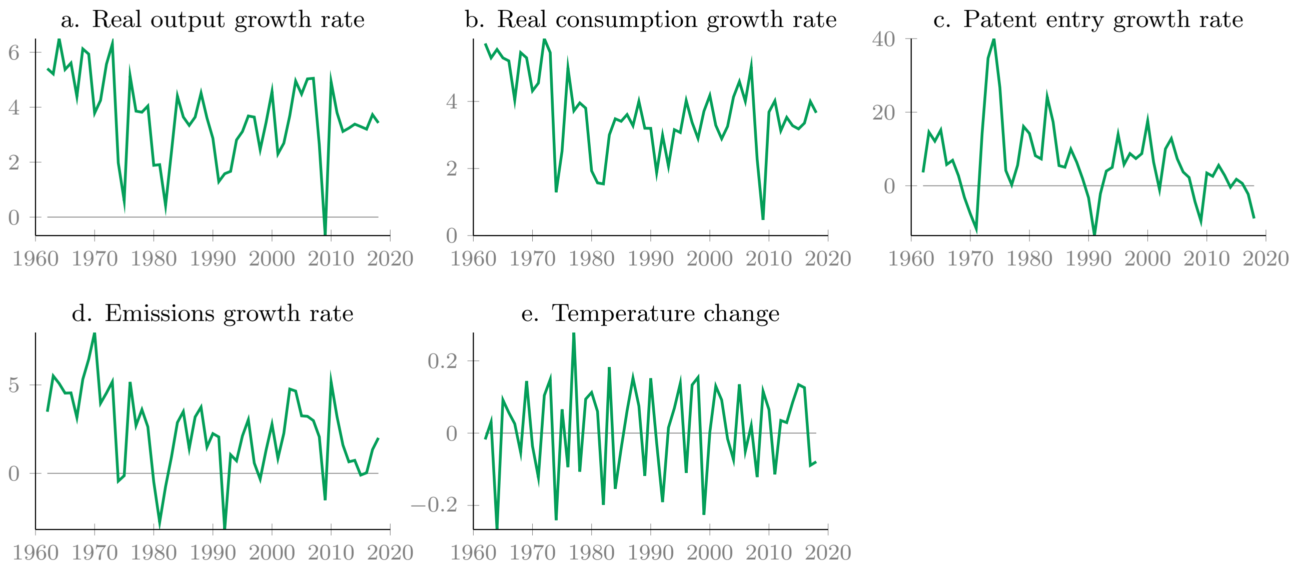
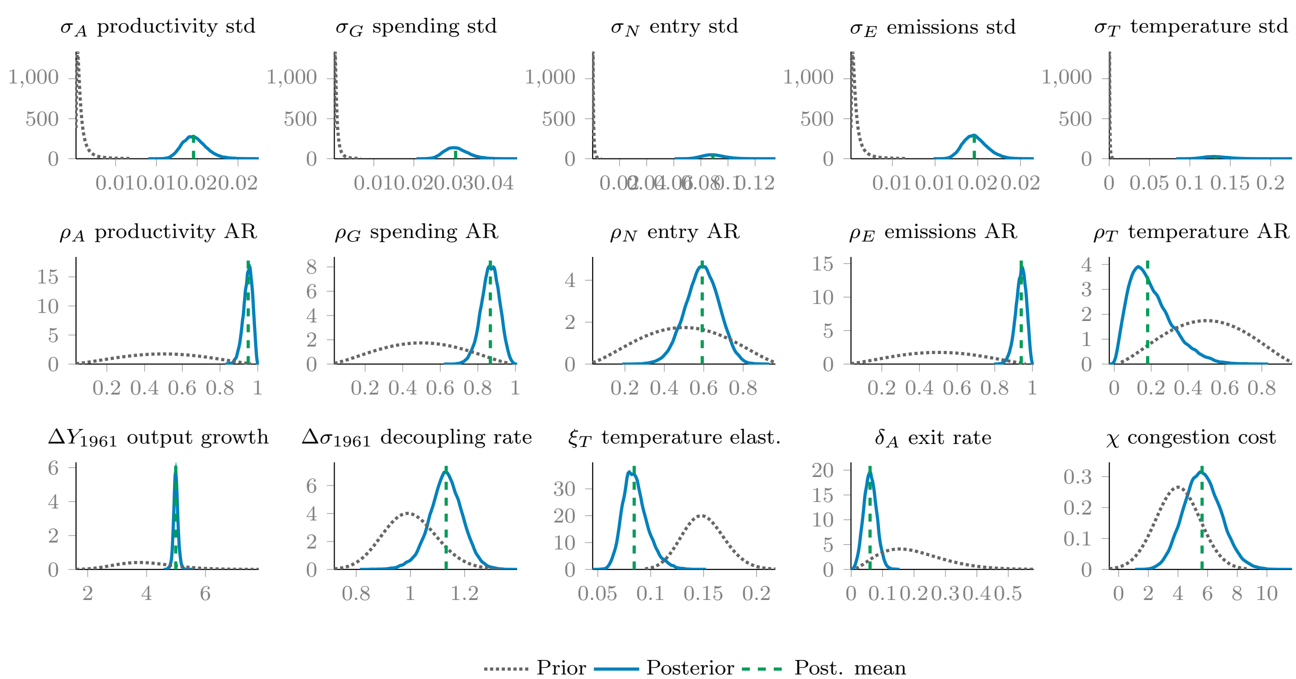
A Data

Figure A.1. Observable variables



B Prior and posterior distributions

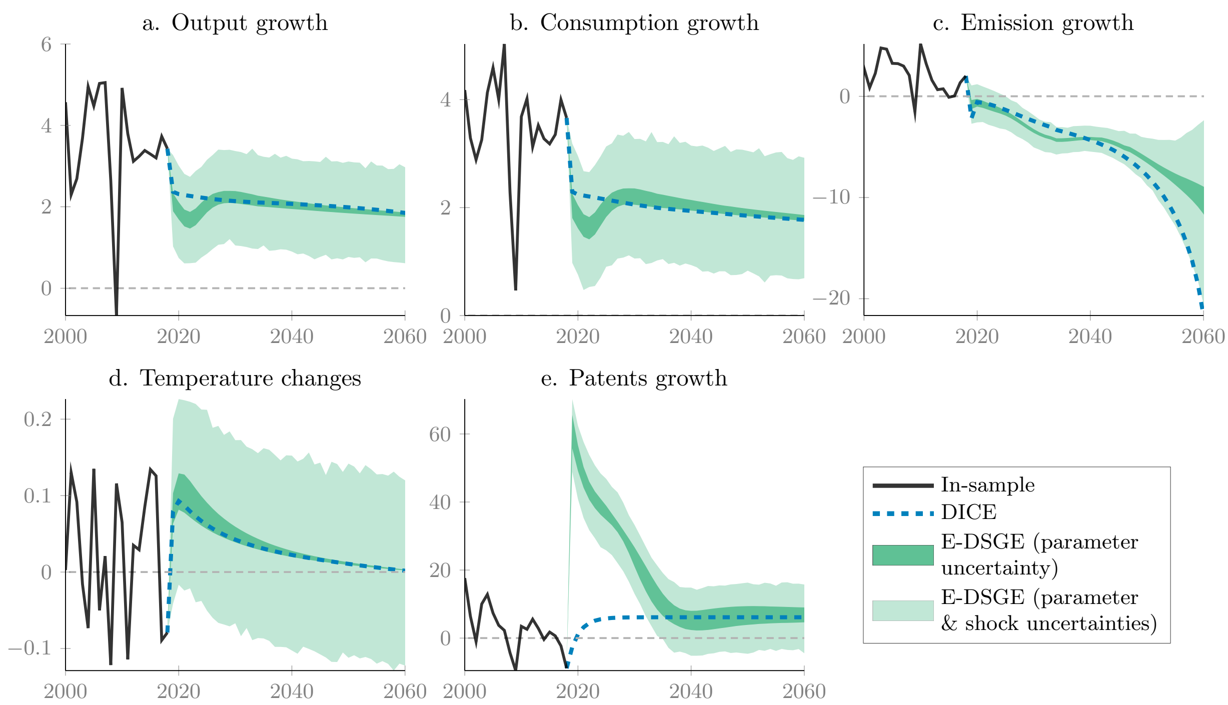
Figure B.1. Prior and posterior distributions



C Projections of the observable variables under uncertainty

Figure C.1 reports the projections of the observable variables under the assumption of a temperature increase below +2°C relative to preindustrial level, incorporating uncertainty. In contrast to IAM literature, using the Bayesian approach allow us to deal with both parametric and stochastic uncertainties. Parametric uncertainty is captured by drawing in the posterior distributions of the structural parameters. This uncertainty is represented by the dark green area in Figure C.1. Stochastic uncertainty concerns the role of stochastic disturbances that generate economic fluctuations. These stochastic disturbances represents cyclical shifts in economic fundamentals (e.g., higher productivity, stronger aggregate demand, etc.) that typically generates stochastic perturbations around a deterministic trend. The future position around the trend is therefore also unknown, but can be captured by drawing in the posterior distributions of both autocorrelations and standard deviations of shocks. This uncertainty is represented by the light green area in Figure C.1. We first observe that this is not parameter uncertainty that generates the strongest deviations around the central scenario, but clearly stochastic uncertainty. The width of the intervals shows how much the uncertainty on the size, sign and the persistence of shocks, outside of any public policy, can modify the trajectory of macroeconomic variables in the future.

Figure C.1. Projections of the observable variables under climate change mitigation policy



Note: The figure displays the out-of-sample forecast of the five observable variables under the assumption of a temperature increase below +2°C above preindustrial levels. The black solid line denotes the in-sample forecast from the estimation, while the dark green area denotes parametric uncertainty, and the light green area denotes both parametric and stochastic uncertainties. Uncertainty intervals are computed from 2,000 random draws of shocks and parameters.

Technical Appendix

Environmental Subsidies to Mitigate Transition Risk

–Not Intended for Publication–

section.1section.2subsection.2.1subsection.2.2subsection.2.3subsubsection.2.3.1subsubsection.2.3.2subsection.2.4

I Data

Our sample is based on the following dataset:

- **GDP, PPP (constant 2017 international \$):** International Comparison Program, World Bank | World Development Indicators database, World Bank | Eurostat-OECD PPP Programme, "NY.GDP.MKTP.PP.KD", (denoted $\mathbf{Y}_t^{\$}$).
- **Households and NPISHs Final consumption expenditure (constant 2017 international \$):** World Bank national accounts data, and OECD National Accounts data files, "NE.CON.PRVT.PP.KD", (denoted $\mathbf{C}_t^{\$}$).
- **Annual CO_2 emissions from fossil fuels, by world region (Gigatonnes):** Global Carbon Project. "Our World in Data", (denoted $\mathbf{E}_t$).
- **Global Land and Ocean Temperature Anomaly:** NASA, degrees Celsius with base period 1901–2000. "data.giss.nasa", (denoted $\mathbf{T}_t$).
- **Patent Environmental Related Technologies:** OECD Environment Directorate. "OECD Stats", (denoted $\mathbf{N}_t^E$).

The observable variable matrix is given by:

$$\begin{bmatrix} \text{Real output growth rate} \\ \text{Real consumption growth rate} \\ CO_2 \text{ emissions growth rate} \\ \text{Temperature anomaly change} \\ \text{Patent growth rate} \end{bmatrix} = \begin{bmatrix} \Delta \log(\mathbf{Y}_t^{\$}) \\ \Delta \log(\mathbf{C}_t^{\$}) \\ \Delta \log(\mathbf{E}_t) \\ \Delta \mathbf{T}_t \\ \Delta \log(\mathbf{N}_t^E) \end{bmatrix}. \tag{ta.1}$$

II Model

II.1 Climate block The law of motion of the atmospheric loading of CO_2 (in gigatons of CO_2) is given by:

$$M_t = M_{1750} + (1 - \delta_M)(M_{t-1} - M_{1750}) + \xi_M E_t, \tag{ta.2}$$

where E_t denotes the anthropogenic carbon emissions in t, $\delta_M \in [0, 1]$ represents the rate of transfer of atmospheric carbon to the deep ocean, and $\xi_M \geq 0$ is the atmospheric retention ratio. The term $M_{t-1} - M_{1750}$ represents the excess carbon in the atmosphere net of its (natural) removal, with M_{1750} the stock of carbon in the preindustrial era, i.e., the steady-state level in the absence of anthropogenic emissions (see also Traeger, 2014).

The heat received at the earth surface F_t (in watts per square meter, W/m^2) is the sum of the forcing caused by atmospheric CO_2 and the non-CO_2 forcing:

$$F_t = \eta \ \log_2 \left(\frac{M_t}{M_{1750}} \right) + F_{EX,t}, \tag{ta.3}$$

where η denotes the effect on temperature from doubling the stock of atmospheric CO_2.

The non-CO_2 forcing $F_{EX,t}$ is an exogenous process:

$$F_{EX,t} = \min(F_{EX,t-1} + F_\Delta, F_{\max}), \tag{ta.4}$$

where the parameter F_Δ denotes the fixed increase in exogenous radiative forcing, while $F_{\max}$ is a cap that is met by 2100.

The global mean temperature anomalies of surface T_t and deep oceans T_t^* with respect to the preindustrial period are given by:

$$T_t = \phi_{11} T_{t-1} + \phi_{12} T_{t-1}^* + \xi_T F_t + \varepsilon_{T,t}, \tag{ta.5}$$

$$T_t^* = \phi_{21} T_{t-1} + \phi_{22} T_{t-1}^*, \tag{ta.6}$$

where $\xi_T \geq 0$ is the elasticity of surface temperature to earth surface heat, while parameters ϕ_{11}, ϕ_{12}, ϕ_{21}, and ϕ_{22} capture either persistence or interaction between temperature of surface and deep oceans. To disentangle transitory changes in temperature versus permanent drifts, we introduce an exogenous stochastic process:

$$\varepsilon_{T,t} = \rho_T \varepsilon_{T,t-1} + \eta_{T,t} \ \text{ with } \ \eta_{T,t} \sim \mathcal{N}(0, \sigma_T^2), \tag{ta.7}$$

which captures cyclical changes in temperature.

II.2 Household sector The world economy is populated by a mass L_t of atomistic, identical, and infinitely lived households. This mass is time-varying and captures the upward trend of world population observed over the last sixty years. Formally, as in Nordhaus (2014), it is assumed that world population asymptotically converges to a long-run level $L_T > 0$, such as:

$$L_t = L_{t-1} \left(\frac{L_T}{L_{t-1}} \right)^{\ell_g}, \tag{ta.8}$$

with $\ell_g \in [0, 1]$ the geometric rate of convergence to L_T.

Each household indexed by $i \in [0, L_t]$ maximizes its sequence of present and future utility flows that depend positively on consumption $c_{i,t}$ and negatively on hours worked $h_{i,t}$:

$$\mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} \beta^{\tau} \left(\frac{c_{i,t+\tau}^{1-\sigma_c}}{1-\sigma_c} - \psi_t \frac{h_{i,t+\tau}^{1+\sigma_h}}{1+\sigma_h} \right) \right\}, \tag{ta.9}$$

subject to the sequence of real budget constraints

$$c_{i,t} + b_{i,t} \leq b_{i,t-1}/\beta_{t-1,t} + w_t h_{i,t} + \xi_{i,t} + d_{i,t}, \tag{ta.10}$$

where $\mathbb{E}_t$ denotes the expectation conditional upon information available at t, $\beta \in (0, 1)$ is the subjective discount factor, $\sigma_c > 0$ is the inverse of the intertemporal elasticity of substitution in consumption, $\sigma_h > 0$ is the inverse of the Frisch labor supply elasticity, and $\psi_t = \psi_h Z_t^{1-\sigma_c}$ is a time-varying parameter that cancels out the effects of the productivity trend on labor supply. The household' resources depicted in the right-hand side of the budget constraint are made of real wage w_t, lump-sum transfers from the government $\xi_{i,t}$, and dividend payments received from holding shares of firms in both the intermediate goods and abatement goods sectors $d_{i,t}$. To derive the analytical expression of the discount factor, let us denote by $b_{i,t}$ the nominal pay-off in period t of a one-period bond bought at the end of period $t-1$. Finally, $\beta_{t-1,t}$ is the stochastic discount factor for one-period ahead nominal pay-offs relevant to the household.

The optimal control problem faced by households is given by:

$$\begin{aligned} \max_{\{c_{i,t}, h_{i,t}, b_{i,t}\}} \mathbb{E}_t \Bigg\{ & \sum_{\tau=0}^{\infty} \beta^{\tau} \left(\frac{c_{i,t+\tau}^{1-\sigma_c}}{1-\sigma_c} - \psi_t \frac{h_{i,t+\tau}^{1+\sigma_h}}{1+\sigma_h} \right) \\ & + \sum_{\tau=0}^{\infty} \beta^{\tau} \lambda_{t+\tau}^{c} \left(w_{t+\tau} h_{i,t+\tau} + \xi_{i,t+\tau} + d_{i,t+\tau} + \frac{b_{i,t-1+\tau}}{\beta_{t-1+\tau,t+\tau}} - c_{i,t+\tau} - b_{i,t+\tau} \right) \Bigg\}, \end{aligned} \tag{ta.11}$$

where λ_t^c denotes the Lagrange multiplier on the constraint given by Equation (ta.10). As long as $\lambda_t^c > 0$, the budget constraint binds to equality.

The first-order conditions are given by:

$$(c_{i,t}) : \lambda_t^c = c_{i,t}^{-\sigma_c}, \tag{ta.12}$$

$$(h_{i,t}) : w_t c_{i,t}^{-\sigma_c} = \psi_t h_{i,t}^{\sigma_h}, \tag{ta.13}$$

$$(b_{i,t}) : \lambda_t^c \, \mathbb{E}_t\{\beta_{t,t+1}\} = \beta \, \mathbb{E}_t\{\lambda_{t+1}^c\}. \tag{ta.14}$$

Substituting the Lagrange multiplier from the previous conditions provides:

$$w_t c_{i,t}^{-\sigma_c} = \psi_t h_{i,t}^{\sigma_h}, \tag{ta.15}$$

$$\beta_{t,t+\tau} = \beta^{\tau}(c_{t+\tau}/c_t)^{-\sigma_c}. \tag{ta.16}$$

II.3 Business sector

II.3.1 Final good sector At every point in time t, a perfectly competitive sector produces a final good Y_t by combining a continuum of intermediate goods $y_{i,t}$, $i \in [0, L_t]$, according to the technology

$$Y_t = \left[L_t^{-1/\zeta} \int_0^{L_t} y_{i,t}^{\frac{\zeta-1}{\zeta}} \mathrm{d}i \right]^{\frac{\zeta}{\zeta-1}}. \tag{ta.17}$$

The number of intermediate good firms, which are owned by households, is equal to the size of the population L_t. $\zeta > 1$ measures the substitutability across differentiated intermediate goods. Final good producing firms take their output price, P_t, and their input prices, $P_{i,t}$, as given and beyond their control.

The final good sector must solve the following problem:

$$\max_{\{Y_t, y_{i,t}\}} P_t Y_t - \int_0^{L_t} P_{i,t} y_{i,t} \mathrm{d}i + \lambda_t^f \left[L_t^{\frac{-1}{\zeta}} \int_0^{L_t} y_{i,t}^{\frac{\zeta-1}{\zeta}} \mathrm{d}i - Y_t^{\frac{\zeta-1}{\zeta}} \right], \tag{ta.18}$$

where λ_t^f is the Lagrangian multiplier on the supply curve.

The first-order conditions are given by:

$$(Y_t) : 0 = P_t - \frac{\zeta-1}{\zeta} \lambda_t^f Y_t^{\frac{-1}{\zeta}}, \tag{ta.19}$$

$$(y_{i,t}) : 0 = -P_{i,t} + \frac{\zeta-1}{\zeta} \lambda_t^f L_t^{\frac{-1}{\zeta}} y_{i,t}^{\frac{-1}{\zeta}}. \tag{ta.20}$$

Profit maximization implies the demand curve:

$$y_{i,t} = \frac{1}{L_t} \left(\frac{P_{i,t}}{P_t} \right)^{-\zeta} Y_t, \tag{ta.21}$$

from which we deduce the relationship between the price of the final good and the prices of intermediate goods

$$P_t \equiv \left[\frac{1}{L_t} \int_0^{L_t} P_{i,t}^{1-\zeta} \mathrm{d}i \right]^{\frac{1}{1-\zeta}}. \tag{ta.22}$$

II.3.2 Intermediate good sector Intermediate good i is produced by a monopolistic firm using the following production function:

$$y_{i,t} = \Gamma_t h_{i,t}^I, \tag{ta.23}$$

where Γ_t is the total factor productivity (TFP) that affects the labor demand $h_{i,t}^I$.

The TFP is actually determined by three components:

$$\Gamma_t = \Phi\left(T_t\right) Z_t \varepsilon_{Z,t}, \tag{ta.24}$$

where

$$\Phi(T_t) = 1/(1 + aT_t^2), \tag{ta.25}$$

$$\varepsilon_{Z,t} = (1-\rho_Z) + \rho_Z \varepsilon_{Z,t-1} + \eta_{Z,t}, \text{ with } \eta_{Z,t} \sim N(0, \sigma_Z^2), \tag{ta.26}$$

$$\log Z_t = \log Z_{t-1} + (1 - \exp(\delta_z)) \left(\frac{g_{z,t_0}}{\delta_z} - \log\left(\frac{Z_t}{Z_0}\right)\right). \tag{ta.27}$$

Firm's emissions take the following form

$$e_{i,t} = \sigma_t \left(1 - \mu_{i,t}\right) y_{i,t} \varepsilon_{E,t}, \tag{ta.28}$$

where:

$$\log \sigma_t = \log \sigma_{t-1} + (1 - \exp(\delta_\sigma)) \left(\frac{g_{\sigma,t_0}}{\delta_\sigma} - \log\left(\frac{\sigma_t}{\sigma_0}\right)\right),$$

$$\varepsilon_{E,t} = (1-\rho_E) + \rho_E \varepsilon_{E,t-1} + \eta_{E,t}, \text{ with } \eta_{E,t} \sim N(0, \sigma_E^2)$$

Firms have access to a set of abatement actions. Following Nordhaus (2018), we assume that the cost of abatement technology (in proportion to output) is given by:

$$\Lambda_{i,t} = \left(\theta_{1,t}\, \mu_{i,t}^{\theta_2}\right) y_{i,t}, \tag{ta.29}$$

where

$$\theta_{1,t} = \frac{p_b}{\theta_2}(1-\delta_{pb})^{t-t_0}\sigma_t.$$

Here, $p_b > 0$ is a parameter determining the initial cost of abatement, $0 < \delta_{pb} < 1$ captures technological progress, $\theta_2 > 0$ represents the curvature of the abatement cost function.

The intermediate good firm i maximizes its one-period profits:

$$\max_{\{y_{i,t}, h_{i,t}^I, \mu_{i,t}, e_{i,t}\}} p_{i,t} y_{i,t} - w_t h_{i,t}^I - p_t^A \Lambda_{i,t} - \tau_t e_{i,t}, \tag{ta.30}$$

where $p_{i,t} = P_{i,t}/P_t$ is the relative price of intermediate goods, $p_t^A = P_t^A/P_t$ the relative abatement price, τ_t is the carbon tax.

The problem can be rewritten as follow:

$$\max_{\{h_{i,t}^I, \mu_{i,t}\}} \left[p_{i,t}\Gamma_t - w_t - p_t^A \left(\theta_{1,t}\, \mu_{i,t}^{\theta_2}\right) \Gamma_t - \tau_t \sigma_t \left(1-\mu_{i,t}\right) \Gamma_t \varepsilon_{E,t} \right] h_{i,t}^I \tag{ta.31}$$

The first-order conditions read as follows:

$$(h^I_{i,t}) : p_{i,t} = \frac{w_t}{\Gamma_t} + p^A_t \left(\theta_{1,t}\, \mu^{\theta_2}_{i,t}\right) + \tau_t \sigma_t \left(1 - \mu_{i,t}\right) \varepsilon_{E,t}, \tag{ta.32}$$

$$(\mu_{i,t}) : p^A_t \left(\theta_{1,t}\, \theta_2 \mu^{\theta_2 - 1}_{i,t}\right) = \tau_t \sigma_t \varepsilon_{E,t}. \tag{ta.33}$$

Under imperfect competition, the net profit is the distance between the total gains from selling and the cost of producing,

$$\max_{\{p_{i,t}\}} (p_{i,t} - mc_{i,t}) y_{i,t} \tag{ta.34}$$

$$s.t.\;\; y_{i,t} = \frac{1}{L_t} \left(\frac{P_{i,t}}{P_t}\right)^{-\zeta} Y_t$$

with $mc_{i,t}$ denoting the firm's real marginal cost.

Maximizing this profit under the demand curve from final good firms and the production function provides the following pricing scheme:

$$\frac{mc_{i,t}}{p_{i,t}} = \frac{\zeta - 1}{\zeta}. \tag{ta.35}$$

Combining (ta.32) and (ta.35) and assuming asymmetry lead to the expression of the real wage offered by firms:

$$w_t = \Gamma_t \left[\frac{\zeta - 1}{\zeta} - p^A_t \left(\theta_{1,t}\, \mu^{\theta_2}_t\right) - \tau_t \sigma_t \left(1 - \mu_t\right) \varepsilon_{E,t}\right]. \tag{ta.36}$$

II.4 Abatement goods sector

II.4.1 Abatement goods packers At every point in time t, perfectly competitive packers produce homogeneous abatement goods $y^A_{i,t}$, $i \in [0, L_t]$, by combining a continuum of varieties of abatement goods $y^A_{i,\omega,t}$, $\omega \in \Omega$, according to the technology

$$y^A_{i,t} = \left[\int_{\omega \in \Omega} (y^A_{i,\omega,t})^{\frac{\zeta_A - 1}{\zeta_A}} \mathrm{d}\omega\right]^{\frac{\zeta_A}{\zeta_A - 1}}, \tag{ta.37}$$

where $\zeta_A > 1$ measures the substitutability across varieties.

Packers take their output price, $P^A_{i,t}$, and their input prices, $P^A_{i,\omega,t}$, as given and beyond their control. These packers solve the following optimal control problem:

$$\max_{\{y^A_{i,t}, y^A_{i,\omega,t}\}} P^A_{i,t} y^A_{i,t} - P^A_{i,\omega,t} y^A_{i,\omega,t} + \lambda^\omega_t \left[\int_{\omega \in \Omega} (y^A_{i,\omega,t})^{\frac{\zeta_A - 1}{\zeta_A}} \mathrm{d}\omega - (y^A_{i,t})^{\frac{\zeta_A - 1}{\zeta_A}}\right]. \tag{ta.38}$$

The first-order conditions are given by:

$$(y_{i,t}^A) : 0 = P_{i,t}^A - \frac{\zeta_A - 1}{\zeta_A} \lambda_t^\omega (y_{i,t}^A)^{\frac{-1}{\zeta_A}}, \tag{ta.39}$$

$$(y_{i,\omega,t}^A) : 0 = -P_{i,\omega,t}^A + \frac{\zeta_A - 1}{\zeta_A} \lambda_t^\omega (y_{i,t})^{\frac{-1}{\zeta_A}}. \tag{ta.40}$$

Profit maximization implies the optimal quantity of goods demanded by packer i to each variety of abatement ω,

$$y_{i,\omega,t}^A = \left(\frac{P_{i,\omega,t}^A}{P_{i,t}^A} \right)^{-\zeta_A} y_{i,t}^A, \tag{ta.41}$$

from which we deduce the relationship between the price of the homogeneous abatement good and the prices of abatement varieties $P_{i,t}^A = \left[\int_{\omega \in \Omega} \left(P_{i,\omega,t}^A \right)^{1-\zeta_A} \mathrm{d}\omega \right]^{\frac{1}{1-\zeta_A}}$.

II.4.2 Intensive margin Each variety ω from already established firms, *incumbents* for short, is produced using labor, which is subject to the TFP as follows:

$$y_{i,\omega,t}^A = \Gamma_t h_{i,\omega,t}^A, \tag{ta.42}$$

where $h_{i,\omega,t}^A$ is the labor demand from firm ω held by household i.

Real profits operating in the abatement goods market are given by:

$$\Pi_{i,\omega,t}^A = \frac{P_{i,\omega,t}^A}{P_t} y_{i,\omega,t}^A - w_t h_{i,\omega,t}^A \left(1 - s_t^A \right), \tag{ta.43}$$

where s_t^A is a subsidy rate to incumbents.

Combining Equation (ta.41) and Equation (ta.42) in Equation (ta.43), the problem reads as follows:

$$\max_{\{P_{i,\omega,t}^A\}} \left[\frac{P_{i,\omega,t}^A}{P_t} - \frac{w_t}{\Gamma_t} \left(1 - s_t^A \right) \right] \left(\frac{P_{i,\omega,t}^A}{P_{i,t}^A} \right)^{-\zeta_A} y_{i,t}^A. \tag{ta.44}$$

The first-order condition reads as:

$$\left(1 - \zeta_A \right) \frac{P_{i,\omega,t}^A}{P_t} = -\zeta_A \frac{w_t}{\Gamma_t} \left(1 - s_t^A \right).$$

Using $p_{i,\omega,t}^A = P_{i,\omega,t}^A / P_t$ and isolating the price, we find:

$$p_{i,\omega,t}^A = \frac{\zeta_A}{\zeta_A - 1} \left(1 - s_t^A \right) \frac{w_t}{\Gamma_t}. \tag{ta.45}$$

II.4.3 Extensive margin While each household manages a continuum of abatement varieties Ω, only a subset of goods $\Omega_t \in \Omega$ is available at any given time t. We denote by $N_{i,t}$ the number of firms owned by household i in the abatement goods sector (a mass of Ω_t) and by $N_{i,t}^E$ the number of startups created by the household. As in Bilbiie et al. (2012), startups at time t only start producing in $t+1$, which features one period of time-to-build. This assumption is necessary to capture the lag between entry and economic growth that is empirically observed. The number of firms owned by household i in the abatement goods sector is given by the following law of motion:

$$N_{i,t} = (1-\delta_A)\left[N_{i,t-1} + \varepsilon_{N,t-1}\left(1 - f_N\left(\frac{N^E_{i,t-1}}{N^E_{i,t-2}}\right)\right)N^E_{i,t-1}\right], \tag{ta.46}$$

where $\delta_A \in [0,1]$ is the probability that any firm incurs an exogenous exit-inducing shock, startup also face another exit probability $f_N\left(N^E_{i,t-1}/N^E_{i,t-2}\right) = 0.5\chi(N^E_{i,t-1}/N^E_{i,t-2} - 1)^2$, entry is subject to an exogenous shock follows an AR(1) process given by:

$$\varepsilon_{N,t} = (1-\rho_N) + \rho_N \varepsilon_{N,t-1} + \eta_{N,t}, \text{ with } \eta_{N,t} \sim N(0, \sigma^2_N). \tag{ta.47}$$

Following Bilbiie et al. (2012), setting up a new firm requires labor services, such as

$$\Gamma_t h^E_{i,t} = X_{w,t} N^E_{i,t}, \tag{ta.48}$$

where $h^E_{i,t}$ is the number of hours worked necessary to establish a startup (subject to total factor productivity Γ_t), while $X_{w,t}$ is a sunk cost subject to trends $\theta_{1,t}$ and Z_t to ensure that the barrier to entry is trend-neutral, $X_{w,t} = \theta_{1,t} Z_t X_w$.

The balance sheet of the investor/household is given by:

$$(1-\delta_A)(\Pi^A_t + v_t)\left[x_{i,t-1} + \varepsilon_{N,t-1}\left(1 - f_N\left(\frac{N^E_{i,t-1}}{N^E_{i,t-2}}\right)\right)N^E_{i,t-1}\right]$$
$$= d^E_{i,t} + \left(1 - s^E_t\right)h^E_{i,t} w_t + X_{q,t} N^E_{i,t} + x_{i,t} v_t. \tag{ta.49}$$

The RHS of this equation represents the expenditure side of the household-investor, composed of the labor cost ($h^E_{i,t} w_t$) used in Equation (ta.48) for the creation of startups subject to subsidy policy $\left(1 - s^E_t\right)$. Shares purchase ($x_{i,t}$) valued at market price (v_t) pays dividends equal to the next-period profits (Π^A_t). Firm's entry is also subject to a barrier to entry, $X_{q,t} = X_q Z_t$, which also grows at the same rate as TFP and the cost of abatement in order to make sure the entry barrier remains the same across time in relative terms. Investing in existing firms and startups provides profits denoted by $d^E_{i,t}$, which go to zero under perfect competition across investor.

An investor/household willing to establish a new startup solves the following optimization problem:

$$\max_{\left\{h^E_{i,t}, N^E_{i,t}, x_{i,t}\right\}} \mathbb{E}_t \left\{\sum_{\tau=0}^{\infty} \beta_{t,t+\tau} d^E_{i,t+\tau}\right\}, \tag{ta.50}$$

which can rewritten as follows:

$$\max_{\left\{N^E_{i,t},x_{i,t}\right\}} \mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} \beta_{t,t+\tau} \left[(1-\delta_A)(\Pi^A_{t+\tau}+v_{t+\tau}) \left(x_{i,t-1+\tau} + (1-f_{N,t-1})\, N^E_{i,t-1+\tau}\right) \right.\right.$$
$$\left.\left. - \left(N^E_{i,t+\tau} X_{i,t+\tau} + x_{i,t+\tau}\, v_{t+\tau}\right)\right] \right\},$$

where $f_{N,t-1} = f_N\left(N^E_{i,t-1}/N^E_{i,t-2}\right)$ and $X_{i,t} = X_t = \left[X_w(1-s^E_t)w_t/\Gamma_t + X_q\right]$.

The first-order conditions solving the optimal control problem are:

$$(x_{i,t}) : v_t = \mathbb{E}_t \left\{\beta_{t,t+1}(1-\delta_A)(\Pi^A_{t+1}+v_{t+1})\right\}, \tag{ta.51}$$

$$(N^E_{i,t}) : X_{i,t} = \mathbb{E}_t \left\{\beta_{t,t+1}(1-\delta_A)(\Pi^A_{t+1}+v_{t+1}) \left(1 - \frac{\partial f_{N,t} N^E_{i,t}}{\partial N^E_{i,t}}\right)\right\} \tag{ta.52}$$
$$- \mathbb{E}_t \left\{\beta_{t,t+2}(1-\delta_A)(\Pi^A_{t+2}+v_{t+2}) \frac{\partial f_{N,t+1}}{\partial N^E_{i,t}} N^E_{i,t+1}\right\}.$$

To rewrite the system in a state-space form, one needs to get rid off $t+2$ terms in the first-order condition associated with $(N^E_{i,t})$ by exploiting forward recursion in the first-order condition associated with $(x_{i,t})$. To do so, consider first:

$$\lambda_t v_t = \beta \lambda_{t+1}(1-\delta_A)(\Pi^A_{t+1}+v_{t+1}).$$

Iterating forward:

$$\lambda_{t+1} v_{t+1} = \beta \lambda_{t+2}(1-\delta_A)(\Pi^A_{t+2}+v_{t+2}).$$

Therefore, the term in $t+2$ of the second first-order condition can be rewritten as:

$$\begin{aligned}
\mathbb{E}_t \left\{\beta_{t,t+2}(1-\delta_A)(\Pi^A_{t+2}+v_{t+2}) \frac{\partial f_{N,t+1}}{\partial N^E_{i,t}} N^E_{i,t+1}\right\} &= \mathbb{E}_t \left\{\beta^2 \frac{\lambda_{t+2}}{\lambda_t}(1-\delta_A)(\Pi^A_{t+2}+v_{t+2}) \frac{\partial f_{N,t+1}}{\partial N^E_{i,t}} N^E_{i,t+1}\right\} \\
&= \mathbb{E}_t \left\{\beta \frac{1}{\lambda_t} \beta \lambda_{t+2}(1-\delta_A)(\Pi^A_{t+2}+v_{t+2}) \frac{\partial f_{N,t+1}}{\partial N^E_{i,t}} N^E_{i,t+1}\right\} \\
&= \mathbb{E}_t \left\{\beta \frac{1}{\lambda_t} \lambda_{t+1} v_{t+1} \frac{\partial f_{N,t+1}}{\partial N^E_{i,t}} N^E_{i,t+1}\right\} \\
&= \mathbb{E}_t \left\{\beta_{t,t+1} v_{t+1} \frac{\partial f_{N,t+1}}{\partial N^E_{i,t}} N^E_{i,t+1}\right\}
\end{aligned}$$

Combining these two first-order conditions allows us to get:

$$Z_t \theta_{1,t} \left[\left(1-s^E_t\right) X_w N^E_{i,t} \frac{w_t}{\Gamma_t} + X^q\right] = v_t \left(1 - \frac{\partial f_{N,t} N^E_{i,t}}{\partial N^E_{i,t}}\right) - \mathbb{E}_t \left\{\beta_{t,t+1} v_{t+1} \frac{\partial f_{N,t+1}}{\partial N^E_{i,t}} N^E_{i,t+1}\right\}, \tag{ta.53}$$

where:

$$\frac{\partial f_{N,t} N_{i,t}^E}{\partial N_{i,t}^E} = 0.5\chi \left(3 \left(\frac{N_{i,t}^E}{N_{i,t-1}^E} \right)^2 + 1 - 4\frac{N_{i,t}^E}{N_{i,t-1}^E} \right),$$

$$\frac{\partial f_{N,t+1}}{\partial N_{i,t}^E} N_{i,t+1}^E = -\chi \left(\frac{N_{i,t+1}^E}{N_{i,t}^E} \right)^2 \left(\frac{N_{i,t+1}^E}{N_{i,t}^E} - 1 \right).$$

II.5 Public sector and environmental policy The government collects the carbon tax from firms' emissions and uses this revenue to *(i)* make some unproductive expenditures, *(ii)* provide some subsidies to the abatement goods sector, and *(iii)* pay a lump-sum transfer to households. The budget constraint is:

$$\tau_t E_t = G_t + s_t^A w_t L_t h_t^A + s_t^E w_t N_t^E L_t h_t^E + \xi_t. \tag{ta.54}$$

Public spending is determined exogenously as $G_t = g_y Y_t \varepsilon_{G,t}$, where $g_y \in [0,1]$ is the steady-state share of public spending to output and $\varepsilon_{G,t}$ is a government spending shock.

This shock captures exogenous shifts in aggregate demand and follows

$$\varepsilon_{G,t} = (1-\rho_G) + \rho_G \varepsilon_{G,t-1} + \eta_{G,t}, \text{ with } \eta_{G,t} \sim \mathcal{N}(0, \sigma_G^2). \tag{ta.55}$$

The total lump-sum transfer to households reads as $\xi_t = \int_0^{L_t} \xi_{i,t} \mathrm{d}i$.

II.6 Market clearing and equilibrium conditions First, the annual flow of emissions is given by the total emissions from firms $E_t = \int_0^{L_t} e_{i,t} \mathrm{d}i$, while output is given by $Y_t = \int_0^{L_t} y_{i,t} \mathrm{d}i$. Note that since firms are symmetric, the abatement rate is the same across firms $\mu_{i,t} = \mu_t$. Therefore, the aggregate flow of emissions reads as:

$$E_t = \sigma_t (1-\mu_t) Y_t \, \varepsilon_{E,t}. \tag{ta.56}$$

Resource constraints determining the aggregate demand are obtained from the aggregation of households' consumption $C_t = L_t c_t = \int_0^{L_t} c_{i,t} \mathrm{d}i$, government spending, and the barrier to entry costs paid in terms of the final good:

$$Y_t = C_t + G_t + N_t^E L_t \theta_{1,t} Z_t X_q. \tag{ta.57}$$

In addition, we define a detrended output as the percentage deviation of output Y_t from productivity and population trends, as follows:

$$\hat{Y}_t = 100 \times \log \left(\frac{Y_t}{Z_t L_t} \right). \tag{ta.58}$$

This metric allows us to compare the dynamics of output more easily than directly focusing on the level of output.[22]

[22] We do not remove the trend associated with the increase in temperature because it is endogenous and would thus make it impossible to compare different policies.

The aggregate demand of abatement good reads as:

$$N_t Y_t^A = \left(\frac{\tilde{p}_t^A}{p_t^A}\right)^{-\zeta_A} L_t \Lambda_t. \tag{ta.59}$$

with $p_t^A = P_t^A / P_t$ and $\tilde{p}_t^A = \tilde{P}_t^A / P_t$. In this expression, as households are symmetric, the relative price ratio is unchanged at the aggregate level $\tilde{P}_{i,t}^A / P_{i,t}^A = \tilde{P}_t^A / P_t^A$. The aggregate production function reads as:

$$N_t Y_t^A = \Gamma_t H_t^A, \tag{ta.60}$$

where Y_t^A is the intensive margin in the abatement goods sector and $H_t^A = L_t h_t^A = \int_0^{L_t} \int_{\omega \in \Omega} h_{i,\omega,t}^A \mathrm{d}\omega \mathrm{d}i$ corresponds to the total demand in labor inputs from incumbents in the abatement goods sector.[23] The aggregate selling price, which takes into account the number of incumbents in the determination of the selling price, is:

$$p_t^A = \tilde{p}_t^A \left(N_t\right)^{\frac{1}{1-\zeta_A}}, \tag{ta.61}$$

with $p_t^A = P_t^A / P_t$ and $\tilde{p}_t^A = \tilde{P}_t^A / P_t$.

The labor market is given by the total supply of households $H_t = L_t h_t = \int_0^{L_t} h_{i,t} \mathrm{d}i$, which must equal the demand from firms producing intermediate goods $H_t^I = \int_0^{L_t} h_{i,t}^I \mathrm{d}i$, abatement goods incumbents H_t^A, and startups $H_t^E = L_t h_t^E = \int_0^{L_t} h_{i,t}^E \mathrm{d}i$:

$$H_t = H_t^I + H_t^A + H_t^E, \tag{ta.62}$$

where the aggregate supply of the final good is given by $Y_t = \Gamma_t \, H_t^I$.

Finally, we compute the share of abatement goods in output as:

$$\Psi_t = p_t^A \int_0^{L_t} \frac{\Lambda_{i,t}}{Y_{i,t}} \mathrm{d}i = p_t^A \theta_{1,t} \mu_t^{\theta_2}. \tag{ta.63}$$

II.7 Model's summary This section reports the first-order conditions for the agents' optimizing problems and the other relationships that define the equilibrium of the model.

II.7.1 Exogenous trends

$$F_{EX,t} = \min(F_{EX,t-1} + F_\Delta, F_{\max}),$$

$$\log Z_t = \log Z_{t-1} + (1 - \exp(\delta_z)) \left(\frac{g_{z,t_0}}{\delta_z} - \log\left(\frac{Z_t}{Z_0}\right)\right)$$

$$\log \sigma_t = \log \sigma_{t-1} + (1 - \exp(\delta_\sigma)) \left(\frac{g_{\sigma,t_0}}{\delta_\sigma} - \log\left(\frac{\sigma_t}{\sigma_0}\right)\right)$$

[23] Aggregated labor demands include the number of firms, as in Bilbiie et al. (2012).

$$\theta_{1,t} = \frac{p_b}{\theta_2}(1-\delta_{pb})^{t-t_0}\sigma_t$$

$$L_t = L_{t-1}\left(\frac{L_T}{L_{t-1}}\right)^{\ell_g}$$

II.7.2 Exogenous shocks

$$\varepsilon_{T,t} = \rho_T \varepsilon_{T,t-1} + \eta_{T,t}$$

$$\varepsilon_{Z,t} = (1-\rho_Z) + \rho_Z \varepsilon_{Z,t-1} + \eta_{Z,t}$$

$$\varepsilon_{E,t} = (1-\rho_E) + \rho_E \varepsilon_{E,t-1} + \eta_{E,t}$$

$$\varepsilon_{N,t} = (1-\rho_N) + \rho_N \varepsilon_{N,t-1} + \eta_{N,t}$$

$$\varepsilon_{G,t} = (1-\rho_G) + \rho_G \varepsilon_{G,t-1} + \eta_{G,t}$$

II.7.3 Climate block

$$M_t = M_{1750} + (1-\delta_M)(M_{t-1} - M_{1750}) + \xi_M E_t \quad \text{(ta.64)}$$

$$F_t = \eta \ \log_2\left(\frac{M_t}{M_{1750}}\right) + F_{EX,t} \quad \text{(ta.65)}$$

$$T_t = \phi_{11} T_{t-1} + \phi_{12} T^*_{t-1} + \xi_T F_t + \varepsilon_{T,t} \quad \text{(ta.66)}$$

$$T^*_t = \phi_{21} T_{t-1} + \phi_{22} T^*_{t-1} \quad \text{(ta.67)}$$

II.7.4 Household sector

$$w_t c_t^{-\sigma_c} = \psi_h Z_t^{1-\sigma_c} h_t^{\sigma_h} \quad \text{(ta.68)}$$

$$\beta_{t,t+1} = \beta\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma_c} \quad \text{(ta.69)}$$

II.7.5 Business sector

$$\Gamma_t = \frac{Z_t \varepsilon_{Z,t}}{(1+aT_t^2)} \quad \text{(ta.70)}$$

$$\frac{w_t}{\Gamma_t} = \frac{\zeta-1}{\zeta} - p_t^A(\theta_{1,t}\ \mu_t^{\theta_2}) - \tau_t \sigma_t\,(1-\mu_t)\,\varepsilon_{E,t}$$

$$E_t = \sigma_t\,(1-\mu_t)\,Y_t \quad \text{(ta.71)}$$

$$Y_t = \Gamma_t H_t^I \quad \text{(ta.72)}$$

$$\mu_t = \left(\frac{\tau_t \sigma_t \varepsilon_{E,t}}{\theta_{1,t}\theta_2 p_t^A}\right)^{1/(\theta_2-1)} \quad \text{(ta.73)}$$

II.7.6 Abatement goods sector

$$\tilde{p}_t^A = \frac{\zeta_A}{\zeta_A - 1}\left(1 - s_t^A\right)\frac{w_t}{\Gamma_t} \tag{ta.74}$$

$$N_t Y_t^A = \Gamma_t H_t^A \tag{ta.75}$$

$$\Gamma_t h_t^E = \theta_{1,t} Z_t X_w N_t^E \tag{ta.76}$$

$$Z_t\theta_{1,t}\left[(1 - s_t^E)X_w N_t^E w_t/\Gamma_t + X_q\right] = v_t\left(1 - \frac{\partial f_{N,t} N_t^E}{\partial N_t^E}\right) - \mathbb{E}_t\left\{\beta_{t,t+1} v_{t+1}\frac{\partial f_{N,t+1}}{\partial N_t^E} N_{t+1}^E\right\} \tag{ta.77}$$

$$N_t = (1 - \delta_A)\left[N_{t-1} + \varepsilon_{N,t-1}\left(1 - \frac{\chi}{2}\left(\frac{N_{t-1}^E}{N_{t-2}^E} - 1\right)^2\right)N_{t-1}^E\right] \tag{ta.78}$$

$$v_t = \mathbb{E}_t\left\{\beta_{t,t+1}(1 - \delta_A)(\Pi_{t+1}^A + v_{t+1})\right\} \tag{ta.79}$$

$$N_t L_t \Pi_t^A = \tilde{p}_t^A Y_t^A - w_t H_t^A\left(1 - s_t^A\right) \tag{ta.80}$$

II.7.7 Equilibrium conditions

$$L_t h_t = H_t^I + H_t^A + L_t h_t^E$$

$$N_t Y_t^A = \left(\frac{\tilde{p}_t^A}{p_t^A}\right)^{-\zeta_A}(\theta_{1,t}\,\mu_t^{\theta_2})Y_t$$

$$p_t^A = \tilde{p}_t^A\, N_t^{\frac{1}{1-\zeta_A}}$$

$$Y_t = L_t c_t + g_y Y_t \varepsilon_{G,t} + N_t^E Z_t L_t \theta_{1,t} X_q$$

Our system is thus composed of 17 economic variables/equations, $\{c_t,\ h_t,\ \beta_{t,t+1},\ \Gamma_t,\ w_t,\ \mu_t,\ E_t,\ Y_t,\ H_t^I,\ \tilde{p}_t^A,\ p_t^A,\ H_t^A,\ Y_t^A,\ N_t,\ N_t^E,\ h_t^E,\ v_t,\ \Pi_t^A\}$, four climate variables $\{M_t,\ F_t,\ T_t,\ T_t^*\}$, five deterministic trends $\{F_{EX,t}, Z_t, \sigma_t, \theta_{1,t}, L_t\}$ and five exogenous disturbances $\{\varepsilon_{T,t}, \varepsilon_{Z,t}, \varepsilon_{E,t}, \varepsilon_{N,t}, \varepsilon_{G,t}\}$.

III Detrending the model

The estimation procedure requires a large number of resolutions of the model. It is therefore necessary to reduce the time dedicated to the resolution as much as possible. To this end, we remove trends from macroeconomic variables, while we let climate-related variables in level. Removing trends reduces the magnitude of the residuals in the dynamic equations when using Newton optimization routines. In particular, the extended-path method that we use (see Appendix IV) requires less iterations to get residuals below tolerance value when the model is detrended.

Let us first define the growth rates of labor productivity Z_t and cost of abatement $\theta_{1,t}$ as follows:

$$g_{z,t} = \frac{Z_t}{Z_{t-1}} \quad \text{and} \quad g_{\theta,t} = \frac{\theta_{1,t}}{\theta_{1,t-1}}.$$

III.1 Household sector

- **Detrended Euler equation:**

$$w_t \tilde{c}_t^{-\sigma_c} = \psi_h h_t^{\sigma_h}, \tag{ta.81}$$

with $\tilde{c}_t = c_t / Z_t$, $\tilde{w}_t = w_t / Z_t$.

- **Detrended stochastic discount factor:**

$$\tilde{\beta}_{t,t+1} = g_{z,t+1}^{-\sigma_c} \beta \left(\frac{\tilde{c}_{t+1}}{\tilde{c}_t} \right)^{-\sigma_c}, \tag{ta.82}$$

with $\tilde{\beta}_{t,t+1} = g_{z,t+1}^{-\sigma_c} \beta_{t,t+1}$.

III.2 Business sector

- **Detrended TFP:**

$$\tilde{\Gamma}_t = \frac{\varepsilon_{Z,t}}{1 + a T_t^2}, \tag{ta.83}$$

with $\tilde{\Gamma}_t = \Gamma_t / Z_t$.

- **Detrended real wage:**

$$\frac{\tilde{w}_t}{\tilde{\Gamma}_t} = \frac{\zeta - 1}{\zeta} - \theta_{1,t} \left[p_t^A \; \mu_t^{\theta_2} + \tilde{\tau}_t \left(1 - \mu_t\right) \varepsilon_{E,t} \right], \tag{ta.84}$$

with $\tilde{\tau}_t = \tau_t \sigma_t / \theta_{1,t}$.

- **Emissions (as a function of detrended output):**

$$E_t = \sigma_t \left(1 - \mu_t\right) L_t Z_t \tilde{Y}_t, \tag{ta.85}$$

where $\tilde{Y}_t = Y_t / (L_t Z_t)$. Note that we do not detrend E_t as it is a direct input for climate-related variables.

- **Detrended production:**

$$\tilde{Y}_t = \tilde{\Gamma}_t \tilde{H}_t^I, \tag{ta.86}$$

where $\tilde{H}_t^I = \tilde{H}_t^I / L_t$.

- **Detrended abatement share:**

$$\mu_t = \left(\frac{\tau_t \sigma_t \varepsilon_{E,t}}{\theta_{1,t} \theta_2 p_t^A} \right)^{1/(\theta_2 - 1)} \tag{ta.87}$$

III.3 Abatement goods sector

- **The price of each variety:**

$$\tilde{p}_t^A = \frac{\zeta_A}{\zeta_A - 1} \left(1 - s_t^A \right) \frac{\tilde{w}_t}{\tilde{\Gamma}_t} \tag{ta.88}$$

- **Detrended production in the abatement goods sector:**

$$N_t \tilde{Y}_t^A = \tilde{\Gamma}_t \tilde{H}_t^A, \tag{ta.89}$$

with $\tilde{Y}_t^A = Y_t^A / (\theta_{1,t} L_t Z_t)$ and $\tilde{H}_t^A = H_t^A / (\theta_{1,t} L_t)$.

- **Detrended labor in startup creation:**

$$\tilde{\Gamma}_t \tilde{h}_t^E = X_w N_t^E, \tag{ta.90}$$

with $\tilde{h}_t^E = h_t^E / \theta_{1,t}$.

- **Detrended free-entry condition:**

$$\left(1 - s_t^E \right) X_w N_t^E \frac{\tilde{w}_t}{\tilde{\Gamma}_t} + X_q = \tilde{v}_t \left(1 - \frac{\partial f_{N,t} N_t^E}{\partial N_t^E} \right) - \mathbb{E}_t \left\{ g_{\theta,t+1} g_{z,t+1}^{1-\sigma_c} \tilde{v}_{t+1} \tilde{\beta}_{t,t+1} \frac{\partial f_{N,t+1}}{\partial N_t^E} N_{t+1}^E \right\}, \tag{ta.91}$$

with $v_t = \tilde{v}_t / (Z_t \theta_{1,t})$.

- **Law of motion of firms:**

$$N_t = (1 - \delta_A) \left[N_{t-1} + \varepsilon_{N,t-1} \left(1 - \frac{\chi}{2} \left(\frac{N_{t-1}^E}{N_{t-2}^E} - 1 \right)^2 \right) N_{t-1}^E \right] \tag{ta.92}$$

- **Detrended firm value:**

$$\tilde{v}_t = (1 - \delta_A) \mathbb{E}_t \left\{ g_{\theta,t+1} g_{Z,t+1}^{1-\sigma_c} \tilde{\beta}_{t,t+1} (\tilde{\Pi}_{t+1}^A + \tilde{v}_{t+1}) \right\}, \tag{ta.93}$$

with $\tilde{\Pi}_t^A = \Pi_t^A / (Z_t \theta_{1,t})$.

- **Detrended profit per firm:**

$$\tilde{\Pi}_t^A = \frac{\tilde{p}_t^A \tilde{Y}_t^A - \tilde{w}_t \tilde{H}_t^A \left(1 - s_t^A \right)}{N_t} \tag{ta.94}$$

III.4 Equilibrium conditions

- **Detrended equilibrium on the labor market:**

$$h_t = \tilde{H}_t^I + \theta_{1,t} \left(\tilde{H}_t^A + \tilde{h}_t^E \right) \tag{ta.95}$$

Note that the fraction of the abatement goods sector is not constant, even in the detrended version of the model.

- **Detrended equilibrium on the abatement goods market:**

$$N_t \tilde{Y}_t^A = \left(\frac{\tilde{p}_t^A}{p_t^A} \right)^{-\zeta_A} (\theta_{1,t} \, \mu_t^{\theta_2}) Y_t \tag{ta.96}$$

- **Relative abatement price:**

$$p_t^A = \tilde{p}_t^A \, (N_t)^{\frac{1}{1-\zeta_A}} \tag{ta.97}$$

- **Detrended resource constraint:**

$$\tilde{Y}_t = \tilde{c}_t + g_y \tilde{Y}_t \varepsilon_{G,t} + N_t^E \theta_{1,t} X_q \tag{ta.98}$$

IV Estimation method

IV.1 Solution method Our simulations are based on the extended path method as initially proposed by Fair and Taylor (1983). Recent work towards taking into account future uncertainty is described in Adjemian and Juillard (2014). In this paper, we use the deterministic version of the extended path that assumes certainty equivalence, but offers a computationally cheap method to estimate the model.

Let us consider the solution of a set of nonlinear deterministic equations:

$$\mathbb{E}_t\{f_\Theta\left(y_{t-1}, y_t, y_{t+1}, \epsilon_t\right)\} = 0, \tag{ta.99}$$

where y_t is a $N \times 1$ vector of endogenous variables in time period t, ϵ_t is a $M \times 1$ vector of exogenous variables, f_Θ is a set of N nonlinear equations based on a vector of parameters θ.

The perfect foresight algorithm. Let y_0 and y_T denote initial and terminal states. Let also Y denote the matrix of endogenous variables and X the matrix of exogenous variables:

$$Y_{1:T-1} = \begin{pmatrix} y_1 \\ y_2 \\ \dots \\ y_{T-1} \end{pmatrix} \quad \text{and} \quad X_{1:T-1} = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \dots \\ \epsilon_{T-1} \end{pmatrix}.$$

The solution system (ta.99) can be represented by a set of N nonlinear equations over $T-1$ time periods. Stacking those equations over all time periods produces a set of $N \times (T-1)$ equations:

$$R\left(Y_{1:T-1}, X_{1:T-1}, y_{t-1}, y_T\right) = \begin{pmatrix} f_\Theta\left(y_0, y_1, y_2, \epsilon_1\right) & = 0 \\ f_\Theta\left(y_1, y_2, y_3, \epsilon_2\right) & = 0 \\ \dots & \\ f_\Theta\left(y_{T-3}, y_{T-2}, y_{T-1}, \epsilon_{T-2}\right) & = 0 \\ f_\Theta\left(y_{T-2}, y_{T-1}, y_T, \epsilon_{T-1}\right) & = 0 \end{pmatrix}. \tag{ta.100}$$

A perfect foresight simulation simply solves:

$$Y^*_{1:T} = \arg \min_{\{Y_{1:T-1}\}} |R\left(Y_{1:T-1}, X_{1:T-1}, y_0, y_T\right)|, \tag{ta.101}$$

with residual matrix $R\left(Y^*_{1:T-1}, X_{1:T-1}, y_0, y_T\right)$ reaching some tolerance threshold.

In this paper, we use the relaxation algorithm developed by Laffargue (1990), Boucekkine (1995), and Juillard et al. (1996).

The extended path algorithm. The extended path approach is simply a perfect foresight solution that is consistent with rational expectations, i.e., $\mathbb{E}_t\left\{\epsilon_{t+s}\right\} = 0$ for $s > 0$. Consider the system given by Equation (ta.100) under rational expectations, to get the first simulation period y^*_1, the corresponding stacked equations reads as follows:

$$R\left(Y_{1:T-1},\epsilon_1,y_0,y_T\right)=\begin{pmatrix} \mathbb{E}_t\{f_\theta\left(y_0,y_1,y_2,\epsilon_1\right)\} & =0 \\ \mathbb{E}_t\{f_\theta\left(y_1,y_2,y_3,0\right)\} & =0 \\ ... & \\ \mathbb{E}_t\{f_\theta\left(y_{T-3},y_{T-2},y_{T-1},0\right)\} & =0 \\ \mathbb{E}_t\{f_\theta\left(y_{T-2},y_{T-1},y_T,0\right)\} & =0 \end{pmatrix}.$$

Using Equation (ta.101), we can find an initial value for y_1 consistent with both contemporaneous surprises ϵ_1 and rational expectations $\mathbb{E}_t\left\{\epsilon_{t+s}\right\}=0$. Note also that the path of expected variables $\mathbb{E}_t\left\{y_{t+s}\right\}$, $s=\{1,T\}$, is also updated.

The extended path solves recursively:

$$Y^*_{t:T-1}=\arg\min_{\{Y_{t:T-1}\}}\mathbb{E}_t\left\{R\left(Y_{t:T-1},\epsilon_t,Y^*_{t-1},y_T\right)\right\} \text{ for } t=\{1,T-1\}, \tag{ta.102}$$

with sequences of surprises ϵ_t, assuming that $\mathbb{E}_t\left\{\epsilon_{t+s}\right\}=0$, for $s>0$, and $Y^*_{t-1}=y_0$, for $t=1$.

IV.2 Inversion filter The inversion filter from Fair and Taylor (1983) consists in solving Equation (ta.102) by interverting structural shocks with a subset of endogenous variables that are observable. In the context of the extended path, the endogenous variables are unknown and are computed given a set of exogenous disturbances. Consider an inference based on a sample $\mathcal{Y}$ of size $T^*\times N^*$, with T^* the number of periods and N^* the number of observable variables. Let ω denote a selection matrix that picks some observable variables within the endogenous variable vector (ωy_t) and z_t denote unobserved variables.

The new set of unknown variables that has to be numerically computed each period is given by:

$$W_t=\begin{pmatrix} w_t \\ y_{t+1} \\ ... \\ y_{T-1}\end{pmatrix}, \text{ with } w_t=\begin{pmatrix}\epsilon_t \\ z_t\end{pmatrix}.$$

In this expression, the vector w_t stacks both current shocks and unobserved variables.

The new stacked residuals in t reads as:

$$F\left(W_{t:T^*},\mathcal{Y}_t,\hat{y}_{t-1},y_T\right)=\begin{pmatrix} \mathbb{E}_t\{f_\Theta\left(\hat{y}_{t-1},y_t,y_{t+1},\epsilon_t\right)\} & =0 & \text{with } \mathcal{Y}_t=\omega y_t \\ \mathbb{E}_t\{f_\Theta\left(y_1,y_2,y_3,0\right)\} & =0 & \\ ... & & \\ \mathbb{E}_t\{f_\Theta\left(y_{T-3},y_{T-2},y_{T-1},0\right)\} & =0 & \\ \mathbb{E}_t\{f_\Theta\left(y_{T-2},y_{T-1},y_T,0\right)\} & =0 & \end{pmatrix},$$

where $\hat{y}_t$ are smoothed endogenous variables. Shocks are now unknown, but are inferred through the constraint $\mathcal{Y}_t = \omega y_t$. As in the case of linearized models in Cuba-Borda et al. (2019) and Kollmann (2017), the number of shocks must equal the number of observable variables in order to have the same number of variables in both w_t and y_t, otherwise the system is indeterminate.

The inversion filter for extended path solves recursively the following optimization scheme:

$$W^*_{t:T-1} = \arg\min_{\{W_{t:T-1}\}} \mathbb{E}_t\{F\left(W_{t:T-1}, \mathcal{Y}_t, \hat{y}_{t-1}, y_T\right)\} \quad \text{for} \quad t = \{1, T^*\}, \tag{ta.103}$$

with $T^* \leq T-1$. Smoothed shocks $\hat{\epsilon}_t$ and $\hat{y}_t$ are obtained recursively from Equation (ta.103).

IV.3 Likelihood function We use the inversion filter to extract the sequence of N^* shocks and T^* periods. When the structural innovations are drawn from a multivariate normal distribution with covariance matrix Σ, the log-likelihood is given by:

$$\mathcal{L}_t = -\frac{T^* N^*}{2}\log\left(2\pi\right) - \frac{T^*}{2}\log(\det\left(\Sigma\right)) - \frac{1}{2}\Sigma_{t=1}^{T^*}\hat{\epsilon}_t\Sigma^{-1}\hat{\epsilon}_t' + \frac{1}{2}\Sigma_{t=1}^{T^*}\log\left(|\det(\mathcal{J}_t)|\right), \tag{ta.104}$$

where $\mathcal{J}_t$ is the Jacobian matrix of the transformation of observable variables in innovations $\hat{\epsilon}_t$. We thus get the Jacobian of endogenous variables as $-B_t^{-1}D_t$, where $B_t = \frac{\partial f_\Theta(.)}{\partial y_t}$ and $D_t = \frac{\partial f_\Theta(.)}{\partial \epsilon_t}$. We next use the selection matrix ω to pick some observable variables, such that:

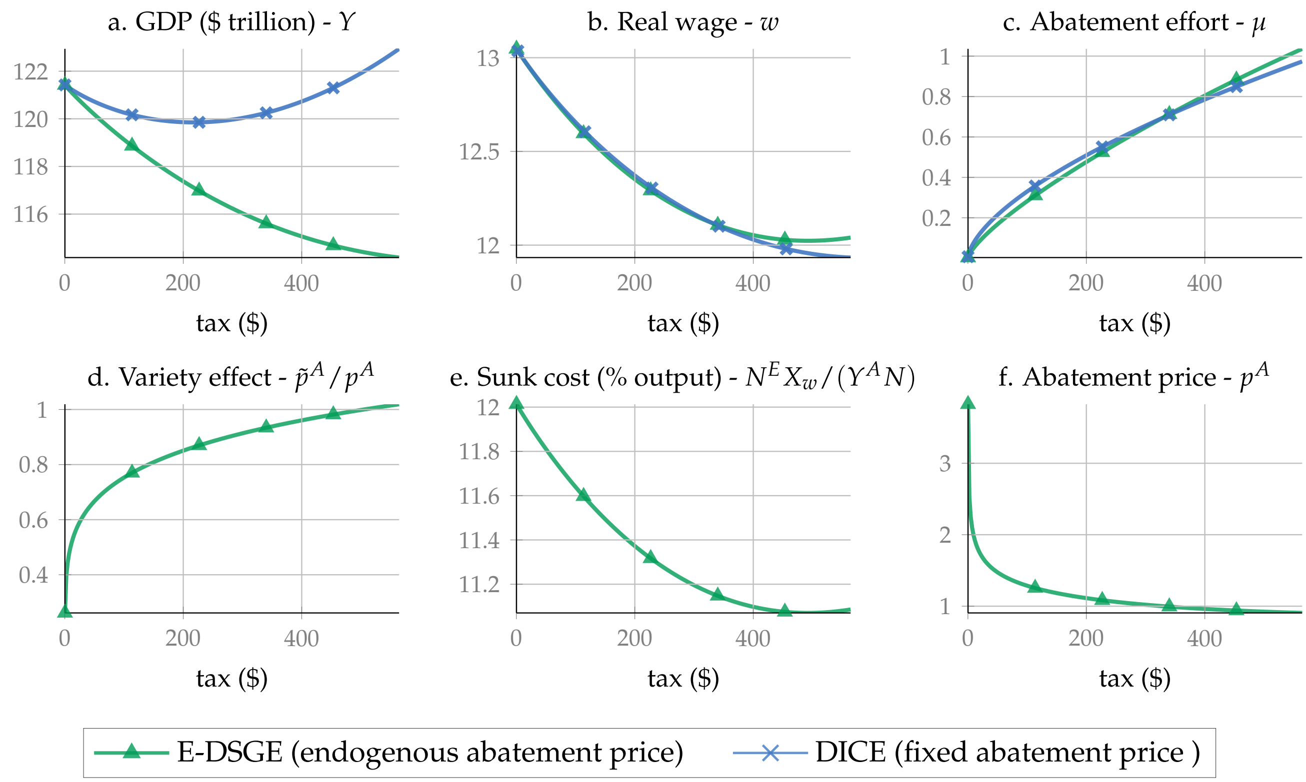
$$\mathcal{J}_t = -\omega B_t^{-1} D_t. \tag{ta.105}$$

While the Jacobian is not time-varying and can be computed directly from the policy function in linearized models, it is state-dependent and must be calculated each period over the sample in nonlinear models.

V LONG-TERM INTERACTION BETWEEN FIRM ENTRY AND CARBON TAXING

This section offers additional details on the transmission of a carbon tax on GDP. To this end, we consider a static version of our E-DSGE model in which trends are set to their 2019 values. The carbon price τ is the only exogenous variable, and we measure how the other variables respond to a permanent change in the carbon price.[24] Figure V.1 displays the responses in both E-DSGE and DICE models.

FIGURE V.1. Static effects of firm entry and carbon price



In typical DICE models, sectors are perfectly homogeneous. A rise in the price of carbon emissions forces firms to purchase some additional intermediate inputs, the latter being produced at the same selling price as the final good. The carbon tax deteriorates the marginal profit of firms and unintendedly reduces the labor income received by households. The real wage falls proportionally to the carbon tax, which makes households less willing to supply labor. The resulting macroeconomic outcome is a recession.

The presence of firm entry breaks the sectoral symmetry, and increases the abatement price with respect to the price of the final good. To maintain the total supply of abatement goods Ny^A, outflows from exiting firms must be compensated by a proportional number of entries. Therefore, households

[24] Climate effects on TFP are not considered in this exercise in order to isolate only the dynamic of the abatement goods sector following a change in the carbon price. In addition, the fixed entry cost F^q is set to zero to have simple closed-form expressions.

must spend $N^E F$ in entry costs, in addition to the standard labor costs wh^A. Such costs inefficiently divert a fraction of labor resources to maintain the same number of firms.

The intensive margin is given by the following expression:

$$y^A = \theta_1 X_w \left(\zeta_A - 1\right) \left(\frac{r - (1 - \delta_A)}{1 - \delta_A}\right), \tag{ta.106}$$

where $r = g_z^{\sigma_c} / \beta$ is the real interest rate.

Note that the intensive margin is not determined by the carbon tax. Therefore, environmental policies cannot influence the amount produced by each firm. The intensive margin in Equation (ta.106) has three determinants. The first term is the entry cost in labor X_w, which represents a barrier preventing new competitors from entering the market. It means that a high entry cost X_w depresses competition and rises the market share of existing firms. The second term $(\zeta_A - 1)$ originates from the pricing decisions in the wake of CES preferences. When goods are more substitutable (i.e., when ζ_A is high), margins of existing firms are reduced, which discourages new competitors from entering the market. Finally, the last term originates from the valuation of firms: if the opportunity cost of establishing a new firm rises (i.e., when r is high) or if the exit rate δ_A increases, financial markets reduce the financing of startups. As a result, existing firms are favored with respect to prospective entrants, so the intensive margin increases.

Let us now consider what drives the number of firms in the abatement goods sector. In a static case, the market structure is summarized by the number of existing firms:

$$N = \left(\theta_1 \mu(\tau)^{\theta_2} \frac{Y}{y^A}\right)^{\zeta_A - 1}. \tag{ta.107}$$

Interestingly, the number of firms operating in the abatement goods sector is driven by the demand for abatement goods. As the abatement effort $\mu(\tau)$ is proportional to the carbon tax policy, the cost of entry – embedded into y^A – is much higher than the future expected gain when the demand for abatement goods is low (i.e, small $\theta_1 \mu(\tau)^{\theta_2} Y$). It acts as a barrier to entry and features detrimental effect on competition. When the market size increases, the number of firms tends to increase, as the relative cost of entry is proportionally smaller.

To understand the benefit from an increased competition, let us consider the condition characterizing the market maturity:

$$\frac{\tilde{p}^A}{p^A} = \left(\theta_1 \mu^{\theta_2} \frac{Y}{y^A}\right)^{-1}. \tag{ta.108}$$

The variety effects exhibits negative marginal returns. The more (resp. less) immature the market, the higher (resp. lower) the marginal benefits of having an additional variety of abatement good. Given this negative marginal return, the variety effects decrease with the size of the market and $\tilde{p}^A / p^A$ converges to one.