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A structural model to assess the impact of bank capitalization changes conditional on a bail-in versus bail-out regime

Abstract

We develop a structural model for valuing bank balance sheet components such as the equity and debt value, the value for the government when the bank is operated by private shareholders including the present value of a possible future bailout, the bailout value incurred by the government following the abandonment of the private shareholders, and, moreover, some price and risk parameters, including the funding cost spread and the banks' probability of default. The structural model implies an abandonment threshold, at which if total income drops below this threshold, private shareholders abandon the bank. In this case, the shareholders lose part (or all) of the capital that they hold in the bank, the creditors lose part or all of their debt, and the government receives a portion (or all) of the capital and all of the debt that is not recovered by creditors. Hence, we assume that part of the capital can be lost due to financial distress or to cover bankruptcy costs. We use the model framework to assess the impact of capital-based macro-prudential policy measures and focus in particular on assessing the difference that an assumed bail-in as opposed to bail-out regime can make.

Keywords: Structural model, abandonment trigger, bank bailout, capital-based macro-prudential policy.

JEL codes: G21, G28, H81

Non-technical summary

In this paper, we construct a structural model in which shareholders run a bank if the expected future cumulative income exceeds costs. When this net worth becomes negative, shareholders ultimately abandon the bank, forcing a governmental intervention.

Under such modelling assumptions an abandonment threshold is implied. If total income drops below this threshold, private shareholders abandon the bank. In such distress moment the shareholders lose part or all of the capital that they hold in the bank. The creditors lose part or all of their debt and the government, since it operates the bank under unlimited liability, is responsible for all the fixed claims of a bank upon its abandonment by the original equity holders. In particular, the government receives a portion or all of the capital and all of the debt that is not covered by creditors.

Using this model we determine the value of some bank balance sheet components. These range from the value of equity and debt, the value for the government when the bank is operated by the private shareholders, including the present value of a possible future government bailout, the bailout value incurred by the government following the abandonment by the private shareholders, the bank spread and finally the bank probability of default.

Our structural model lets us quantify and monitor distance to default for banks over time. It can therefore indicate whether supervisory action in terms of recapitalization via equity increases in the market or through public participation is needed at certain point in time. Furthermore it allows for quantification of benefits to the government and the debt holders in monetary terms, through a decrease in funding cost spreads and the probability of default of the banks conditional on an assumed capital increase.

We use the model to assess the impact of capital-based macro-prudential policy measures. We examine the funding cost impact and change in the probabilities of default under shocks introduced to banks' capital ratios.

Finally, we also illustrate the use of the model by relating it to the notions of bail-in and bail-out. According to the model, the bank's value for shareholders, the probability of default, the bailout threshold and the time to bailout are independent of the prevailing (bail-in or bail-out) regime. However, since under the bail-in regime not only the government is responsible for bank losses, both the bank (negative) value for the government and the bailout cost decrease when we move from a bail-out to a bail-in regime. Such a finding supports the rationale of the recent bail-in directive (Directive 2014/59/EU).

1.   Introduction

The current crisis saw the largest intervention of governments in the banking sector through a massive transfer of resources from the taxpayers to the banking sector to restore confidence in the financial system as noted, for instance, by US Treasury Secretary Henry M. Paulson.[1]

The literature confirms that banking crises are commonly associated with major micro- and macroeconomic adverse effects. Concretely, banking crises are associated with recessions and severe reductions in the current and future GDP (Friedman and Schwartz, 1963 and Boyd, Kwak and Smith, 2005), stock market declines (Boyd, Kwak and Smith, 2005 and Dermine and Schoenmaker, 2010), currency crises (Kaminsky and Reinhart, 1999), an overall loss of collateral value due to asset fire sales (Diamond and Dybvig, 1983), and contagion effects in national financial institutions due to herding behavior (Calvo, 2005) or are the result of bank runs or counterparty risks (Heider, Hoerova, and Holthausen, 2008), the costs of financial distress (Korteweg, 2010) or underinvestment (Veronesi and Zingales, 2010).

Until recently, following years of financial stability, the topic of banking crises was not included in public and research agendas, and widespread belief prevailed that more developed economies were not quite at risk from experiencing banking crises. Recent evidence indicates, of course, that no economy is immune to a banking crisis, and the historical frequency of crises is similar for developed and developing countries (see Reinhart and Rogoff, 2013). Moreover, it also shows that the negative effects of a banking crisis are substantial even when banking crises are non-systemic (Boyd, Kwak and Smith, 2005). With recent developments in mind, governments are now highly sensitive to banking crises and are ready to intervene even when there is no clear evidence of a market failure that requires government intervention.

Most of the recent research being developed is still highly focused on analyzing the different costs incurred in a banking crisis (e.g., Reinhart and Rogoff, 2013, and Correia et al., 2016) as well as the costs and benefits of the different policy instruments to address them (e.g., Philippon and Schnabl, 2009, and Veronesi and Zingales, 2010). However, we believe that more effort should also be devoted to predicting future problems, with the model that we develop in the following sections being one step in that direction.

We develop a structural model for valuing bank balance sheet components, such as the value of equity and debt, the value for the government when the bank is operated by the private shareholders, including the present value of a possible future bailout, the bailout value incurred by the government following the abandonment of the private shareholders, the bank spread and the bank probability of default.

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[1] Statement by Secretary Henry M. Paulson, Jr. on Actions to Protect the U.S. Economy, October 14, 2008. http://www.financialstability.gov/latest/hp1205.html.

Merton's (1974) model remains the workhorse framework that drives all major developments in the area of structural credit risk modeling; however, it has been developed since its inception quite materially to make it a more realistic model for firms. One such development was the emergence of first-time-passage models by Black and Cox (1976) and Leland (1994). This methodology does not model equity like a European option on the assets of the firm but instead as a down-and-out-option that creates a default boundary and allows modeling of the more realistic case in which default may be triggered by the inability of the firm to honor its coupon payments. In a Merton-style approach, the default is more narrowly defined and may only occur at the maturity of the debt obligations. Our choice of a first-time-passage model framework is based on our belief that it represents a more general model and is better suited and more widely applicable for the analysis of the distress of financial institutions.

Regarding the choice of the underlying process, we choose an income-based process (as in Decamps et al., 2004) instead of the classic market value of an asset-based model framework for several reasons. First, cash and earnings flows are directly observable variables, whereas the market value of assets is not. This is useful with respect to the calibration of a model, which is less subject to uncertainty in that particular respect. Second, the changes in asset values result from changes in cash or earnings flow expectations. By operating directly with cash flow or earnings changes, it is possible to more realistically model the effects of different exogenous shocks (e.g., see work on business cycles). Several academics recognize the advantages of working directly with cash or earnings flows, and this modeling choice is becoming increasingly more popular, as exemplified by the seminal papers by Goldstein, Ju and Leland (2001) and Hackbarth, Miao and Morellec (2006).

Our model, although limited in the sense that it does not take into account negative externalities such as the increase in public debt (see Reinhart and Rogoff, 2013) or the introduction of competitive distortions in the banking sector (see Hakenes and Schnabel, 2010 and Gropp, Hakenes and Schnabel, 2011), provides equal and comparable measures across different financial institutions in which every financial institution is treated homogeneously, avoiding the distorting results that may arise from the use of particular policy solutions that are made to fit the interests of influential and powerful incumbents (see Hart and Zingales, 2008 and Johnson, 2009). The use of a structural model also allows us to estimate the expected distress moment of a financial institution because we model the abandonment option of the original private shareholders.

We apply our model to an array of more than one hundred EU banks, and as a first result, we find a clear, nonlinear, monotonous, positive relationship between the funding cost spread and the probability of default (reflecting the structure of the model), along with a less clear negative relationship between the banks' capital ratios and their estimated probability of default. The latter finding is expected because the probability of default is not simply a function of only leverage but also the variance, i.e., the risk, surrounding the banks' income and expense streams that reflect their risk.

In addition to the base case estimates of all model parameters, we apply a +1pp shock to the banks' capital ratios, and we see that, as expected, the value for the government increases and the bailout cost falls. The value for creditors increases, whereas the value for shareholders decreases. The funding cost spreads impact, and the change in the probabilities of the default is positive and quite visible. The objective of simulating the capital ratio shocks is twofold overall: first, to shape our understanding of how the variables of interest move in directional terms, and second, to concretely quantify the effects for a concrete sample of banks.

Moreover, we use one parameter from the model to relate it to the notion of bail-in and bail-out, and in that context, also repeat the capital ratio shock simulations. The value for shareholders remains unaffected, as with the probability of default as well as the bailout threshold and the time to bailout. Under the capital raising scenario, there is an incremental impact for the bank value for the government and the bailout cost decreases when we move from the bail-out toward the bail-in regime.

In Section 2, we present our theoretical model. Section 3 is devoted to the data that we employ to calibrate the model. Section 4 presents the main results, including the "base" estimates for the key variables from the model, as well as the results from an assessment as to how the banks' parameters change in response to capital-based macroprudential policies. Section 5 concludes the paper.

2.   The model

Our model comprises a government and banks in which the government is committed to keeping banks operating even if private shareholders decide to abandon the bank. Therefore, we model banks operated by private shareholders and banks operated by the government following the abandonment of the private shareholders. Private shareholders follow a strict wealth maximization objective, do not bear any social responsibilities and are free to abandon the banking business when it is no longer profitable. In turn, the government is a welfare-maximizing agent and therefore assumes control of the bank when the total income of the bank drops below a given threshold that reflects the abandonment trigger of the private shareholders. However, before assuming control of a bank, the government assumes a passive role by merely enforcing its tax schedule and not taking any preemptive action in anticipation of the future bailout.

Our model assumes full symmetry of information in which the government, private shareholders and the different creditors of the bank possess all relevant information and rationally anticipate each other's actions. This assumption hypothesizes why there is never the risk of a bank run or the emergence of underinvestment problems; however, to anticipate a bailout may expose the bank to moral hazard problems.[2]

The bank has uncertain total income, x, and cost, c_e, both following a Geometric Brownian motion (GBM, that is, $dx = \mu x dt + \sigma x dz$ and $dc_e = \mu c_e dt + \sigma c_e dz$), where μ is the instantaneous growth rate, σ is the standard deviation, and dz is the increment of a standard Wiener process under the real world measure P. For the sake of coherence – as otherwise, their values can differ significantly

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[2] We focus on the estimation of the strict bailout costs for the government; however, our model can be easily extended to include investment decisions by the banks and analyze the effect of these potential distortions.

in the long term – we assume that total income, x, and cost, c_e, both follow the exact same Geometric Brownian motion (same μ, σ and dz). We also assume that $\mu < r$, with r being the constant and known interest rate, which allows us to obtain finite solutions.

Considering the existence of an appropriate exogenous factor Λ_t to discount x and following Thijssen (2010), we assume that Λ_t also follows a GBM, that is, $d\Lambda_t = r\Lambda_t dt + \sigma_\Lambda \Lambda_t dz$ in which the drift term r represents the return on a riskless asset and σ_Λ is the constant standard deviation of Λ_t under the real world measure P, which represents the exogenous price of total income risk.

Assuming the existence of complete markets and no-arbitrage arguments, we employ a contingent claims analysis (CCA) to value the different claims on x. As a result, under the risk-neutral measure Q, the dynamics of x and c_e are described by $dx = (\mu - \sigma\sigma_\Lambda)xdt + \sigma x dz$ and $dc_e = (\mu - \sigma\sigma_\Lambda) c_e dt + \sigma c_e dz$ in which the drift parameter is reduced by the market risk premium $\sigma\sigma_\Lambda$ and in which dz now represents an increment of a standard Wiener process under the risk-neutral measure Q. We define the return shortfall with uncertainty as δ_1 and $\delta_1 = r + \sigma\sigma_\Lambda - \mu$.

2.1 Private shareholders operating the bank

Private shareholders operating the bank represent the "normal" state of the financial system. The role of the government in this case is limited to collecting taxes and 'returning' taxes because the tax system is assumed to be symmetrical. It is important to model the value accruing for private shareholders because their abandonment is what triggers the government bailout.

Private shareholders operate the bank and collect dividends as long as $x > c_e$, the dividend is equal to $(x - c_e)(1 - \tau)$, and we assume a residual dividend policy. Whenever $x < c_e$, shareholders make the necessary cash injections to avoid default and are willing to inject cash in the bank as long as these cash injections do not exceed the market value of their shares.[3]

There are also creditors that lend money to the bank (wholesale funding). This debt is constant, and perpetual debt pays a coupon "c". The bank has an amount of money inside which capital (K) is constant while the bank is run by the shareholders. This capital belongs to the shareholders.

2.2 The government bails out the bank

We have assumed that shareholders can abandon the bank whenever they want. When this happens, the government is committed to keeping banks operating or paying the restructuring costs. Depositors cannot lose their money.

We assume that when shareholders abandon the bank, they lose all of the capital (K) and creditors lose part (or all) of the value of their claims, in line with the bail-in regulation. Consequently, if shareholders abandon the bank, the creditors obtain porc2*(c/r), where 0<=porc2<=1.

Moreover, we assume as well that due to financial distress and/or bankruptcy costs, the government does not receive all of the capital but part of it; that is, if shareholders abandon the bank, the

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[3] We assume that neither the government nor the private shareholders are financially constrained.

government receives porc1*K + (1-por2)*c/r, where 0<=porc1<=1. If porc1 is not equal to one, the financial distress and/or bankruptcy costs are assumed to be (1 – porc1)*K.[4]

2.3   Contingent claims

For both the government and private investors, we determine the values of four different claims: the value for the private shareholders when operating the bank, $E(x)$; the value for the creditors, $D(x)$; the value for the government when the bank is operated by the private shareholders, including the present value of a possible future bailout, $G(x)$; and the bailout value incurred by the government following the abandonment of the private shareholders, $B(x)$. We briefly describe the value of a general claim A, in which $A = E, D, G, B$ for the cases of the government, private shareholders and bailout costs, respectively. The following Ordinary Differential Equation - ODE - describes the value of this general claim.

$$0.5\sigma^2 x^2 A_{xx} + (\mu - \sigma\sigma_\Lambda)x A_x - rA + \pi = 0 \tag{1}$$

where $\pi = a(x - c_e) + b$ (see Table 1) represents the total income and cost accruing for each claim, comprising a variable component a associated with the evolution of x and c_e and a fixed component b that is independent of x and c_e. In determining the claim value, there is another factor that should be taken into account: L_A, which represents the terminal value when the shareholders abandon the bank, and shareholders abandon the bank when $x = x_a$. Table 1 defines a, b and L_A for the different claims considered. The government runs a symmetrical corporate tax scheme to which it is committed and in which firms are taxed at a rate denoted as τ.

[Table 1 here]

The general solution to ODE (1) is:

$$A(x) = \begin{cases} \frac{ax}{\delta_1} + \frac{b}{r} + B_1 x^{\beta_1} + B_2 x^{\beta_2} & \text{if } x > x_a \\ L_A & \text{if } x \leq x_a \end{cases} \tag{2}$$

with a, b and L_A given in Table 1, where $\delta_1 = r + \sigma\sigma_\Lambda - \mu$ and β_1 and β_2 are the roots of the following characteristic polynomial:

$$(0.5\sigma^2\beta^2 + \beta(\mu - \sigma\sigma_\Lambda - 0.5\sigma^2) - r)x^\beta = 0, \tag{3}$$

yielding

$$\beta_1 = \frac{1}{2} - \frac{\mu - \sigma\sigma_\Lambda}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu - \sigma\sigma_\Lambda}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1, \tag{4}$$

$$\beta_2 = \frac{1}{2} - \frac{\mu - \sigma\sigma_\Lambda}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu - \sigma\sigma_\Lambda}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0. \tag{5}$$

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[4] For the sake of simplicity, we assume that there are no financial distress and/or bankruptcy costs with respect to debt claims.

The constants B_1 and B_2 are determined given appropriate boundary conditions (see Dixit and Pindyck, 1994). From boundary condition (6), we obtain B_1 equal to zero. Boundary (6) simply states that when the total income is considerably high, the bank will never be abandoned.

$$lim_{x \to \infty} A(x) = \frac{ax}{\delta_1} + \frac{b}{r} \tag{6}$$

The constant B_2 is determined with the abandonment value matching conditions that we present next.

Following equation (6), we know that absent the possibility of abandonment, the value of the private shareholders' claim is equal to

$$E(x) = \left(\frac{x - c_e}{\delta_1} - \frac{c}{r} \right)(1 - \tau) \tag{7}$$

Shareholders, however, benefit from limited liability and therefore, following a fall in x, they are not forced to assume the losses associated with the operations of the bank. Equation (7) represents the value of the shareholders' claim if they did not benefit from limited liability. Because they do, there is an earnings level defined as x_a, at which shareholders decide to abandon the banking business; however, as residual claimants, they receive nothing and, as stated above, they lose their capital under the abandonment scenario. The constant B_2 takes into account the value generated by the abandonment possibility, and this is reflected in the following abandonment value matching condition:

$$E(x_a) = L_E = -K \Leftrightarrow \left(\frac{x_a - c_e}{\delta_1} - \frac{c}{r} \right)(1 - \tau) + B_2 x_a^{\beta_2} = L_E. \tag{8}$$

Equation (8) allows us to determine B_2 and replace it in equation (2) with the appropriate values for a, b_1 and b_2. We have the value for the private shareholders of the bank, taking into account the abandonment option.

$$E(x) = \begin{cases} \left(\frac{x - c_e}{\delta_1} - \frac{c}{r} \right)(1 - \tau) - \left[\left(\frac{x_a - c_e}{\delta_1} - \frac{c}{r} \right)(1 - \tau) - L_E \right] \left(\frac{x}{x_a} \right)^{\beta_2} & if \ x \geq x_a \\ L_E = -K & if \ x \leq x_a \end{cases} \tag{9}$$

The second term of equation (9) represents the abandonment option that adds value to the claim of the private shareholders. Naturally, the equivalent term in the value equation of the government will represent the bailout costs, as we will outline next.

We can calculate the value of the bailout, defined as $B(x)$, at the time it occurs ($x = x_a$) as

$$B(x_a) = min \left\{ \frac{x_a - c_e}{\delta_1} - \frac{c}{r} + porc1 * K + (1 - porc2) * \frac{c}{r}, 0 \right\} \tag{10}$$

Because the government is not allowed to abandon the bank, the value function of the government once it assumes control of the bank does not include any option values. Equation (10) represents the value function for the government running the bank, which is the present value when the bailout occurs for all future capitalization costs $(x - c_e)$ associated with the honoring of all financial obligations of the bank to its creditors, therefore representing the true measure of the bailout costs

of the bank by the government. Equation (10) also includes the value of capital and debt received by the government when shareholders abandon the bank. The minimum function means that if the first term is positive, not all of the creditors' funds are needed to cover the bank losses. An inequality that always holds is $\frac{x_a-c_e}{\delta_1} - \frac{c}{r} + \text{porc1} * \text{K} < 0$; otherwise, shareholders never abandon the bank.

While the private shareholders run the bank, the role of the government is merely to collect taxes when $x > c_e$ and 'return' taxes when $x < c_e$. Again, following equation (6), we know that, absent the possibility of abandonment, the value of the government claim is equal to

$$G(x) = \left(\frac{x-c_e}{\delta_1} - \frac{c}{r}\right)\tau \tag{11}$$

Contrary to the shareholders that benefit from limited liability and are free to abandon operations when they are no longer profitable, the position of the government in the model is one of unlimited liability. Governmental responsibility in reality takes many forms, such as deposit insurance, debt guarantees and equity injections, among many other forms of ensuring bank solvency. Such interventions reflect an ongoing concern approach for the bank, eliminating uncertainty and calming creditors (depositors and other creditors such as bondholders), thereby preventing bank runs or the imposition of constraints by debt holders. In our model, the imposition of unlimited liability on the government following the abandonment of the private shareholders encapsulates the effects of all the different types of governmental intervention in a single measure, aiming to reduce the risk of creditors. Naturally, unlimited liability will only be effective assuming full symmetry of information in which the different stakeholders rationally anticipate that, upon abandonment by the private shareholders, the government will step in and assume control of the bank, covering all cash flow shortfalls and fulfilling all of the bank's financial obligations.

When we consider that private shareholders may abandon the bank, we must include the cost of unlimited liability provided by the government through the B_2 coefficient. The following abandonment value-matching condition reflects the incorporation of unlimited liability following abandonment.

$$G(x_a) = L_G = \frac{x_a-c_e}{\delta_1} - \frac{c}{r} + \text{porc1} * \text{K} + (1 - \text{porc2})\frac{c}{r} \Leftrightarrow \left(\frac{x_a-c_e}{\delta_1} - \frac{c}{r}\right)\tau + B_2 x_a^{\beta_2} = min\left\{\frac{x_a-c_e}{\delta_1} - \frac{c}{r} + \text{porc1} * \text{K} + (1 - \text{porc2}) * \frac{c}{r}, 0\right\} \tag{12}$$

From equation (12), we determine the B_2 parameter and replace it in equation (2) with the appropriate values for a, b_1 and b_2, and we have the value of the bank for the government, taking into account the possible future bailout costs.

$$G(x) =$$
$$\begin{cases} \left(\frac{x-c_e}{\delta_1} - \frac{c}{r}\right)\tau - \left(\left(\frac{x_a-c_e}{\delta_1} - \frac{c}{r}\right)\tau - min\left\{\frac{x_a-c_e}{\delta_1} - \frac{c}{r} + \text{porc1} * \text{K} + (1 - \text{porc2}) * \frac{c}{r}, 0\right\}\right)\left(\frac{x}{x_a}\right)^{\beta_2} & if\ x \geq x_a \\ L_G = min\left\{\frac{x_a-c_e}{\delta_1} - \frac{c}{r} + \text{porc1} * \text{K} + (1 - \text{porc2}) * \frac{c}{r}, 0\right\} & if\ x \leq x_a \end{cases}$$
$$\tag{13}$$

Finally, if there were no possibilities of bail-out and if shareholders run the bank forever, injecting money when necessary, the value of the perpetual debt should be

$$D(x) = \frac{c}{r} \tag{14}$$

However, because we know that when the total income reaches the x_a level, the value of this claim for creditors is L_D; thus, we have

$$D(x_a) = L_D = \text{porc2} * \left(\frac{c}{r}\right) + max\left\{\frac{x_a - c_e}{\delta_1} - \frac{c}{r} + \text{porc1} * K + (1 - \text{porc2}) * \frac{c}{r}, 0\right\} \Leftrightarrow \frac{c}{r} + B_2 x_a^{\beta_2} =$$

$$\text{porc2} * \left(\frac{c}{r}\right) + max\left\{\frac{x_a - c_e}{\delta_1} - \frac{c}{r} + \text{porc1} * K + (1 - \text{porc2}) * \frac{c}{r}, 0\right\} \tag{15}$$

From equation (15), we determine the B_2 parameter and replace it in equation (2) with the appropriate values for b_1 and b_2, and we have the value of the debt claim.

$$D(x) =$$

$$\begin{cases} \frac{c}{r} - \left(\frac{c}{r} - \left(\text{porc2} * (c/r) + max\left\{\frac{x_a - c_e}{\delta_1} - \frac{c}{r} + \text{porc1} * K + (1 - \text{porc2}) * \frac{c}{r}, 0\right\}\right)\right)\left(\frac{x}{x_a}\right)^{\beta_2}) & if\ x \geq x_a \\ \\ L_D = \text{porc2} * (c/r) + max\left\{\frac{x_a - c_e}{\delta_1} - \frac{c}{r} + \text{porc1} * K + (1 - \text{porc2}) * \frac{c}{r}, 0\right\} & if\ x \leq x_a \end{cases}$$

$$\tag{16}$$

The maximum term is needed in the event that not all funds from creditors are needed to bail out the bank. As stated above, an inequality that always holds is $\frac{x_a - c_e}{\delta_1} - \frac{c}{r} + \text{porc1} * K < 0$; otherwise, shareholders will never abandon the bank.

The default spread on a risky bond represents the difference between the required return on the bond, defined as r_D and the risk-free rate as r. The required return of the bond r_D is simply the ratio between the promised coupon c and the present value of the bond D(x) ($r_D = c/D(x)$). Consequently,

$$Spread = \frac{c}{D(x)} - r \tag{17}$$

Finally, for the default probabilities and following Thijssen (2010), we determine the probabilities under the real world measure P, in which the default may take place within T periods.

$$Prob(Sup_{0 \leq t \leq T}\ x \leq x_a)$$

$$= 1 + \Phi\left(\frac{ln\left(\frac{x_a}{x}\right) + \left(\mu - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right) e^{-\frac{2\left(\mu - \frac{\sigma^2}{2}\right)ln\left(\frac{x_a}{x}\right)}{\sigma^2}} - \Phi\left(-\frac{ln\left(\frac{x_a}{x}\right) - \left(\mu - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right)$$

$$\tag{18}$$

in which $\phi(.)$ is the cumulative distribution function of the standard normal distribution.

We now have all of the value equations required for our analysis; however, we still have to determine the expected time in which the bailout will occur. The government bailout follows the abandonment of the private shareholders; therefore, we have to determine when the private

shareholders optimally abandon the operations of the bank. The abandonment trigger is obtained from the following smooth pasting condition.

$$\frac{\partial E(x)}{\partial x}\bigg|_{x=x_a} = 0 \tag{19}$$

Upon inserting equation (9) into equation (19) and solving for x_a, we have

$$x_a = \frac{\beta_2}{\beta_2 - 1}\left(c_e + \frac{c\delta_1}{r} - \frac{K\delta_1}{(1-\tau)}\right) \tag{20}$$

We may easily convert the abandonment trigger into an expected time to abandonment (θ_a); following the procedure of Shackleton and Wajokowski (2002), we obtain

$$\theta_a = \frac{1}{\mu - 0.5\sigma^2} ln\left(\frac{x_a}{x}\right) = \frac{1}{\mu - 0.5\sigma^2} ln\left(\frac{\beta_2}{\beta_2 - 1}\left(c_e + \frac{c\delta_1}{r} - \frac{K\delta_1}{(1-\tau)}\right)\right) \tag{21}$$

2.4   Static analysis

Following the logic of the model, we have nine parameters (μ, σ, σ_Λ, c_e*, c, K, τ, Porc1 and Porc2) that drive the eight main endogenous outcomes of the model: the bailout threshold (x_a), the expected time to bailout (θ_a), the value of the private shareholders claim (E), the value of the creditor claim (D), the value of the government claim (G), the bailout cost (B), the spread and the probability of default.

Table 2 reports how these measures change in response to changes in the parameter values. Regarding x_a, we can observe that the measures increase with μ because total income and costs increase with it. As expected, x_a grows with c_e and c and decreases with σ, because if costs or coupon payments increase, the bank is less profitable and shareholders have an incentive to abandon early. With respect to σ, because abandonment is an option for shareholders, the abandonment threshold decreases with volatility; in other words, if volatility increases, the possibility of further profit increases and shareholders wait longer before abandoning. σ_Λ is the discount rate volatility (risk premium) and tends to follow the same direction as σ in almost all cases because both parameters are volatilities. Because shareholders lose the capital (K) if they abandon the bank, the abandonment point decreases as the capital requirements increase, which is a desirable property in this model. In the case of taxes, because we assume a fully symmetrical tax system, an increase in taxes makes the abandonment less likely since the government faces a higher amount of losses.

[Table 2 here]

Regarding θ_a, in general, as expected, if x_a increases, then θ_a decreases because the higher x_a is, the easier it is for the total income to reach the threshold. The only exceptions are the cases of μ and σ. The issue is the following related to μ: θ_a tends to decrease when x_a increases, as explained before, because when μ increases, it is less likely for the total income to reach the threshold. The role of σ can also be explained following the same logic of the option pricing theory: if the volatility is higher, on the one hand, x_a decreases, and consequently, it is more difficult for the total income to reach the threshold. However, on the other hand, the underlying asset value moves faster, increasing the

likelihood that the exercise threshold will be reached, and that is the reason why there is no clear movement for θ_a when σ increases.

As expected, the value claim for equity holders, E(x), grows with μ and decreases with volatility: if volatility is higher, the underlying asset value moves faster, increasing the likelihood that the exercise threshold will be reached. It is also clear that this claim value decreases with costs, taxes or coupon payments since the bank is less profitable. Because shareholders lose the capital (K) if they abandon the bank, if capital requirements increase, then the value of the claim decreases.

In terms of the bailout cost B(x_a), the first aspect to recall is that it represents a cost; it is a negative number, and consequently, we should analyze the results in Table 2 accordingly. As expected, if c_e increases, B(x_a) decreases because the bailout cost increases. For what concerns volatility, we have to continue following the logic of the option pricing theory: if the volatility increases, the value of the option for the buyer increases and decreases for the seller. In other words, the bailout cost increases; that is, B(x_a) decreases or becomes more negative. Related to c, it is clear that as the government receives part of the debt and the government continues paying the remaining debt if a bailout occurs, then the higher c is, the higher the bail-out cost will be (B(x_a) decreases). The opposite logic could apply to K; the higher the amount of capital that is transferred from shareholders to the government, the lower the bail-out cost is (B(x_a) increases). When we increase taxes, this causes x_a to decrease and bail-out occurs at a lower total income level, while the bail-out cost increases (B(x_a) decreases). For what concerns Porc1 and Porc2, as expected, the bail-out cost decreases with Porc1 and increases with Porc2 (B(x_a) has the opposite sign). Finally, the role of μ is slightly less obvious to see as it affects income and costs; however, the bail-out cost increases with μ as B(x_a) decreases.

Once the bail-out cost is analyzed, the government claim is straightforward to assess. In all cases except for μ, σ_Λ and τ, we have the same results as when we analyzed the bail-out cost and for mainly the same reasons. In the case of taxes, the reason for changing the sign is clear: conditional on a bail-out, the fact that banks pay taxes is not included, and the higher the tax rate is, the higher the government benefit is. As stated above, σ_Λ is the discount rate volatility (risk premium) and tends to follow the same direction as σ except in the case of bail-out, which is just one part of the government claim. However, when we consider the whole government claim, again, σ_Λ and σ follow the same logic. As stated above in the case of μ, it is somewhat more complicated as it affects income and costs; however, in this case, as the opposite to bail-out costs, G(x) increases with μ as it does not include bail-out costs but also revenue from taxes before the bail-out.

The debt claim, D(x), grows in value as c and Porc2 increase and c_e decreases. As in the case of equity, it decreases with volatility because if volatility is higher, the underlying asset value moves faster, increasing the likelihood that the exercise threshold will be reached. However, in this case, we see that the claim value grows with capital requirements and taxes because both make the bail-out less likely. The fact that the value of the debt grows with the capital requirement is an important result. In this case, D(x) increases with μ for the same reason as equity.

Finally, the spread and the default probability grow with σ, c_e and c and decrease with K and τ. The spread, as expected, decreases with Porc2. Regarding μ, as in previous cases, we see some

ambiguous results because total income and costs grow at a rate μ: the spread decreases with it, whereas the probability of default increases (but mainly remains unchanged) with it.

To conclude, one of the main results from the model is the fact that when capital requirements in a bank increase (and to the extent that actual capital ratios follow suit), the bail-out occurs at a lower total income threshold because the bank is better hedged. This implies a cost for shareholders, while for the rest of the agents in the economy, the government and other creditors, it implies a gain. In other words, on the side of creditors, we can say that debtors claim that value increases and the credit spread decreases. For the government, we obtain a smaller bail-out cost when it occurs, and it occurs less frequently. Consequently, the value of the bank for the government increases.

3.   Data and model calibration

The empirical application of the model rests on a quarterly data sample for 101 banks from Europe (from a subset of 20 countries) covering the 2006Q1-2015Q4 period. All relevant data were sourced from Capital IQ and SNL. The inputs that are required include operating income ("total income"), operating expenses and asset write-downs (for the sum that we refer to as "total cost"), total capital and total debt. For the calculations related to the impact of capital-based macroprudential policy, the amount of total assets is also a required input.

As stated above, we assume that the bank faces an income stream x along with a cost c_e that are surrounded by uncertainty, and we thus let them both follow the Geometric Brownian motion (same μ and σ). However, at the estimation stage we process the two separately and compute μ and σ for income and cost but then take the average of the two parameters for each bank.

In particular, we estimate μ and σ first based on the time series for total income, then based on total costs. Both are assumed to follow a geometric Brownian motion, i.e. their log differences have a Gaussian distribution, which allows for defining an exact likelihood function. We employ some standard Bayesian estimation methodology that is known in the literature to infer the parameters of these distributions (Robert and Casella, 2004; Gelman et al., 2013).

We assume a truncated[5] normal distribution as a prior for μ and a gamma distribution as a prior for σ. At the data cleaning and pre-calibration phase, the prior predictive distributions are used to remove outliers from total income and total cost quarterly time series (Hoff, 2009; Gelman et al., 2013).

Our Bayesian MCMC simulation algorithm is based on two single-component Metropolis-Hastings type updates (Robert and Casella, 2004; Gelman et al., 2013). In the first step, based on the truncated[5] normal distribution, we update μ and in the second step, involving the lognormal distribution, we update σ. Eventually, we obtain sequences of posterior drift and variance estimates and choose their means as our final parameter estimates. Some additional details that we omit here can be found in the appendix.

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[5] We introduce a truncation scheme because, by assumption, r > μ.

4. Empirical application

4.1. Baseline estimates of banks' probability of default and funding cost spreads

Figure 1 is a scatter that visualizes the *ex post* relationship between the estimated funding cost spread and the probability of default based on the cross-section of 101 banks, which confirms a monotonous positive relationship between the two and a very nonlinear shape.

[Figure 1 here]

Figure 2 shows a scatter plot of the capital ratios of the banks against their estimated probability of default. Here, the relationship is less visible (albeit negative), which is, in fact, expected to be less visible because the probability of default is not simply a function of only leverage but also the variance, i.e., the risk surrounding the banks' income and expense streams that reflect their risk.

[Figure 2 here]

4.2. Capital-based macroprudential policy

We now apply +1pp shocks (denoted as $+\Delta CAPR$) to the banks' capital ratios, starting from their ratios as of the end-sample position (2015Q4). We assume that the banks move to the higher capital ratio by raising equity capital, i.e., not through shrinking the size of their balance sheet. The corresponding absolute amount of additional capital ΔC (with the same additional amount being added to total assets) can be computed using the following formula:

$$\Delta C = -C \left(\frac{(\Delta CAPR + CAPR) \times D}{C \times (\Delta CAPR + CAPR - 1)} + 1 \right) \tag{22}$$

Moreover, we do not make any assumptions at this point about possible implications for loan pricing and the associated volume effects, i.e., outstanding loan stocks of the banks remain constant without new loans and deposits being created (or loans being allowed to mature without being replaced by new business) by assumption. It is an instantaneous assessment in that sense. We also abstain from quantifying the intensity of the pass-through of changes in capital requirements to actual capital ratio changes (macroprudential policy would operate via the former), and instead start from assuming a capital ratio shift.

Figure 3 shows the results of a +1pp capital ratio shift for all banks. The impacts are expressed in relative terms to the underlying base case estimates without the policy shift.

[Figure 3 here]

In line with the earlier discussion about the differential impact of higher capital, we see that the value for equity holders moves down (i.e., the *E*/E* ratios are lower than 1) in all cases since, as stated before, the capital is taken into account as a penalization for equity holders if they decide to abandon the bank. Note that the *E*/E* ratios in Figure 3, at the right end where they become negative, mean that an initial positive base estimate of *E* has moved into negative territory after the

application of the capital ratio shift, which is the case for eight banks. The value for the government increases, i.e., the negative estimates for *G* become less negative (and the *G*/G* ratios are higher than 1) for G to increase by approximately 8% on average across all banks. Moreover, the bailout cost *B* falls (corresponding *B*/B* ratios lower than 1) by an average of 2%, with up to 7%, for one bank in the same. Because we assume in the base case that only 90% of the capital moves to the government when shareholders abandon the bank (porc1 = 90%), it can happen, and we do see the bail-out cost increasing along with the capital increase for two banks.[6] In this respect, we shall note that *B* represents the cost for the government when the bank is abandoned for shareholders. The cost arises from bank losses but also takes into account the funds that the government receives from shareholders and bondholders when the shareholders decide to abandon the bank. The same can be said for *G* because *B* is the terminal value for *G*.

Because the portion of debt recovered by debt holders is high (80%) in this base case, the value for creditors *D* remains rather unaffected, although if anything, it increases (*D*/D* ratios are higher than 1) by up to 1.5% for one bank in the sample. If we modify this assumption, however, the sensitivity of the value for debtholders changes.

The funding cost spread impact and the change in probabilities of default are quite notable, with a maximum impact in absolute terms for some banks in the sample amount to down to -11bps for the funding spread and -15bps for default probabilities. The bailout thresholds fall by up to 26% for one bank in the sample and on average across all banks by 5%. Concerning the time to bailout measure, we measure an average prolongation by approximately 8%, with the maximum being observed for one bank in the sample for which the estimated time to bailout amounts to 68%.

4.3. Bail-in versus Bail-out

We can use one parameter from the model to relate it to the notion of bail-in and bail-out. By modifying the value of "1-porc2" (the percentage of the amount of funds that flow from creditors to the government in the case of shareholders abandoning the bank), we can move gradually from a "bail-out" (porc2 = 1) to a "bail-in" regime (porc2 = 0). Since 2014, when the Directive 2014/59/EU had entered into force, supposedly, all shareholders, creditors and depositors above a certain threshold will cover some portion of losses in case of a bank default.[7]

We use a range of values for the porc2 parameter to reflect different extents of a bail-in regime, amounting to [20, 40, 60, 80]%. The porc1 parameter, the portion of capital that the government receives after a bank's default, we keep at 100%. The value for shareholders remains unaffected by the porc2 parameter change, just as the probability of default as well as the bailout threshold and the time to bailout because the assumption about the porc1 does not affect the actual fundamental structure of the bank balance sheet or its flow evolution over time in any way, nor the shareholders' abandonment decision.

Figure 4 shows the differential, relative impact of a +1pp capital ratio shift for G, B, D, and the cost of funding spread.

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[6] If we assume porc1 = 100%, in all cases, the bail-out cost decreases when we increase capital.

[7] The directive has not yet been effectively applied, hence the terminology "supposedly".

[Figure 4 here]

FThe value for the government keeps decreasing (becoming more negative), along with a move toward the bail-out setting, as expected. From the porc2=20% to the porc2=80%, the value for the government more than doubles from approximately EUR -2.2 trillion to EUR -4.5 trillion in sum over all banks in the sample. The combined bailout cost across all banks would increase by approximately 44%, from EUR 7.5 trillion to EUR 10.8 trillion, when moving from a 20% to an 80% "bailout portion". For some banks, the bail-out cost (B) approaches zero when we move from bail-out (porc2 = 1) to bail-in (porc2 = 0), which means that for some banks under the bail-in regime, there is no cost for the government.

With regard to the influence of the bail-in/out setting on the differential impact of the +1pp capital ratio shock, with respect to *E,* we can observe that, as before, all ratios are less than 1, and they are in fact equal and therefore independent of the porc2 parameter for the same reasons as explained above. With regard to *G*, we can see that the absolute *G*-G* difference is positive in general, while it becomes more positive with a move toward the bailout regime. G*-G is negative when the bail-out cost is zero. In other words, if the bail-out cost is already zero because of the assumed bail-in regime, the higher the capital is, the lower the abandonment point is, and consequently, the more time there is during which the government should pay negative taxes. The sums of the differential impacts across all banks under porc1=20% and porc1=80% settings equal EUR 109 billion and EUR 212 billion, implying a 94% increase. The *G*/G* ratios in Figure 4, moreover, suggest that the relative sensitivity of the government cost becomes more visible the closer the porc2 parameter moves to zero, i.e., closer to the bail-in regime, as expected.

The incremental impact of the capital ratio on the bailout cost in absolute terms is, in general, not affected by the bail-in/out parameter setting. It remains positive and equal across all settings, except for a few cases for which the difference drops to zero for smaller values of porc2. The reason for this is that the bail-out cost decreases when the capital increases, but this decrease is a linear function of the increase in capital, and therefore, it is not related to the bail-in/out regime. An exception arises when the bail-out cost approaches zero because in this case, the bail-out function becomes nonlinear. Because the decrease does not change in absolute terms, in relative terms for the *B*/B* ratios in Figure 4, it does, and we see a gradual fall under the porc2=80% setting at the right end of the bank distribution. At the left end, the blue bars (i.e., for porc2=20%) do not appear for 8 banks in total, which is because the bailout cost was zero for these banks under that setting in the base simulation.

Concerning the value for debt holders *D*, we see that the capital ratio increase has less of an effect on the value for debt holders when moving toward a bailout regime. The reason is that in a full bail-out regime, debt holders do not lose anything in case of a bank default, with a capital ratio increase not changing this fact. However, this is not true in a bail-in regime, which is why the sums under porc1=20% and 80% equal EUR 131 billion and EUR 28 billion, representing a decrease of -79%. Moreover, the relative changes in the ratio form for *D* in Figure 4 suggest that the impact of the capital ratio shift on the value of D is becoming more pronounced in relative terms the closer we move to a bail-in regime.

Finally, with respect to the impact of the capital increasing the funding cost, we can note that under the 20% and 80% bail-out-bail-in parameter setting, the average (across all banks again) funding cost relief amounts to -33bps and -2bps, respectively. The reason again is that in a bail-out regime (porc2 =1), debt holders do not lose anything, whereas in a bail-in regime (porc2 = 0 in a *full* bail-in regime), they lose all, and the funding cost spread related to debt in the sequel does not change in response to a capital increase in the full bail-out regime, while it does in the bail-in case.

5.   Conclusions

The purpose of the structural model presented in this paper is to value the bank balance sheet components such as the value of equity and debt, the value for the government when the bank is operated by private shareholders including the present value of a possible future bailout, the bailout value incurred by the government following the abandonment of the private shareholders, the bank spread and the bank probability of default. This paper does so by assuming that a government operates the bank under unlimited liability, being responsible for all of the bank's fixed claims upon abandonment of the original equity holders.

The model allows its user to quantify and monitor the distance to default for the banks to which it is applied over time. It can therefore inform decisions about the need for supervisory action in terms of a possible recapitalization via equity increases in the market or public participation. In that context, the model can not only help inform decisions but also be of avail to quantify the benefit in monetary terms for the government and debtholders through a decrease in the funding cost spread and the probabilities of default of the bank, conditional on an assumed capital increase.

Furthermore, we have illustrated the use of the model by relating it to the notion of bail-in and bail-out. The value for shareholders remains unaffected by the bail-in/bail-out regime setting, similar to the probability of default as well as the bailout threshold and the time to bailout. There is an incremental impact that we can measure, however, for the bank (negative) value for the government and the bailout cost, which both decrease when we move from a bail-out to a bail-in regime. Such a finding supports the rationale of the recent bail-in directive (Directive 2014/59/EU).

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Appendix

Since total income x and total cost c_e follow, by assumption, geometric Brownian motions, the calibration procedure is the same for both. Hence, we will outline the methodology for total income x only. The dynamics of the latter are governed by the following stochastic differential equation (SDE)

$$dx = \mu x dt + \sigma x dz$$

where μ is the instantaneous growth rate of total income (also referred to as "drift"), σ is the standard deviation (also referred to as "risk"), and dz is the increment of a standard Wiener process. Hence, the distribution of its logarithmic differences $y_t = \log\left(\frac{x_t}{x_{t-1}}\right)$ is Gaussian

$$y_t \sim \mathrm{N}(m, s)$$

where $t = 2, \ldots, T$, m is the mean and s is the standard deviation.

Since in our model $r > \mu$, by assumption, we assume a truncated normal distribution (TN) as prior for μ:

$$\mu \sim \mathrm{TN}\big((-4r, r); m_\mu, s_\mu\big)$$

where $r = 0.06$ is truncation at the right led by this assumption and $-4r$ is left truncation, introduced for practical reasons after experimenting with the data. We choose the mean of this prior distribution for μ to be $m_\mu = 0$ and its standard deviation to equal $s_\mu = 0.1$.

Further we assume a gamma distribution as prior for σ

$$\sigma \sim \mathrm{G}\left(\frac{v_0}{2}, \frac{2m_\sigma}{v_0}\right)$$

where its mean is chosen to be $m_\sigma = 0.3$ and its dispersion parameter $v_0 = 5$.

Before actual calibration we clean financial statements data using prior predictive distributions in order to remove outliers from the quarterly time series of total income and total cost (Hoff, 2009; Gelman et al., 2013). We exclude those observations which are outside a 95% confidence interval.

We employ a Bayesian MCMC algorithm which comprises two single-component Metropolis-Hastings type updates (Robert and Casella, 2004; Gelman et al., 2013). In the first step, we update μ, which involves a truncated normal distribution.

$$\mu_i \sim \mathrm{TN}\left((-4r, r); \mu_{i-1}, s_\mu^p\right)$$

where we set $\mu_0 = 0$ and $s_\mu^p = 0.4$ (across banks from SNL and 0.1 across those from Capital IQ) to thereby obtain sufficient acceptance rates and reasonably low autocorrelations of posterior sequences (across all banks).

In the second step, we update σ, which involves a lognormal distribution, which is equivalent to sampling from a normal distribution on a log scale:

$$\log(\sigma_i) \sim N\left(\log\left(\frac{\sigma_{i-1}}{1 + \sqrt{\frac{(s_\sigma^p)^2}{\sigma_{i-1}^2}}}\right), \sqrt{\log\left(1 + \frac{(s_\sigma^p)^2}{\sigma_{i-1}^2}\right)}\right)$$

where we set $\sigma_0 = 0.2$ and $s_\sigma^p = 0.125$ (across banks from SNL and 0.05 across banks from Capital IQ) which results in sufficient acceptance rates and acceptable autocorrelations of posterior sequences (across all banks). In both steps, $i = 1, \dots, 10\text{^}6$, the first half of which is defined as a burn-in period.

Consequently, we obtain sequences of posterior drift and risk parameter estimates for total income *x* and choose their means as our final estimates of μ and σ for this variable. We repeat the entire procedure for total cost c_e. Eventually, we average each of the two parameters for each bank to complete our structural model calibration procedure.

Tables

Table 1: Specification of the ODE

This table defines a, b and L_A for the different claims considered.

A(x)	a	b	L_A
E(x)	(1-τ)	-c	-K
D(x)	0	c	$porc2 * (c/r) + max\left\{\dfrac{x_a - c_e}{\delta_1} - \dfrac{c}{r} + porc1 * K + (1 - porc2) * \dfrac{c}{r}, 0\right\}$
G(x)	τ	-c	$B(x_a) = min\left\{\dfrac{x_a - c_e}{\delta_1} - \dfrac{c}{r} + porc1 * K + (1 - porc2) * \dfrac{c}{r}, 0\right\}$
B(x)	1	-c	-

Table 2: Sensitivity analysis

This table reports how the bailout threshold (x_a), the expected time to bailout (θ_a), the value of the private shareholders' claim (E), the value of the creditors' claim (D), the bailout cost (B), the value of the government claim (G), the spread and the probability of default change in response to changes in the parameter values (μ, σ, c_e*, c, K, τ, Porc1 and Porc2)).

	x_a	θ_a	E(x)	G(x)	$B(x_a)$	D(x)	Spread	Def. Prob.
μ	↑	↑	↑	↑	↓	↑	↓	=↑
σ	↓	↓↑	↓	↓	↓	↓	↑	=↑
σ_Λ	↓	↑	↓	↓	↑	↓	↑	=↓
c_e	↑	↓	↓	↓	↓	↓	↑	↑
c	↑	↓	↓	↓	↓	↑	↑	↑
K	↓	↑	↓	↑	↑	↑	↓	↓
τ	↓	↑	↓	↑	↓	↑	↓	↓
Porc1	-	-	-	↑	↑	-	-	-
Porc2	-	-	-	↓	↓	↑	↓	-

Figures

Figure 1: Probabilities of default versus funding cost spreads

The chart shows a scatter of the estimated probabilities of default and funding cost spread estimates of the underlying 101 banks.
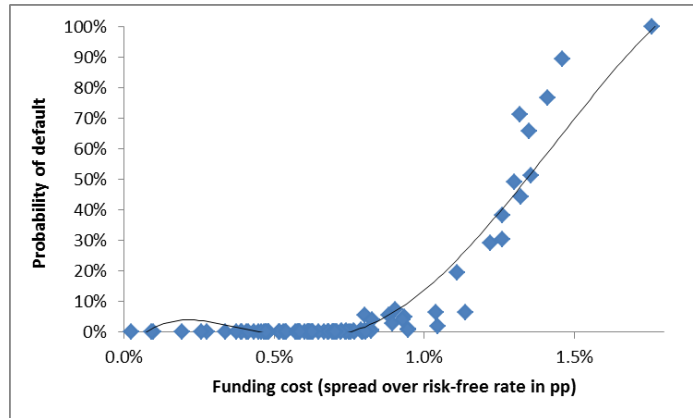
Figure 2: Probabilities of default versus initial capital ratios

The chart shows a scatter of the estimated probabilities of default and the observed capital ratios (as of year-end 2015) of the underlying 101 banks.
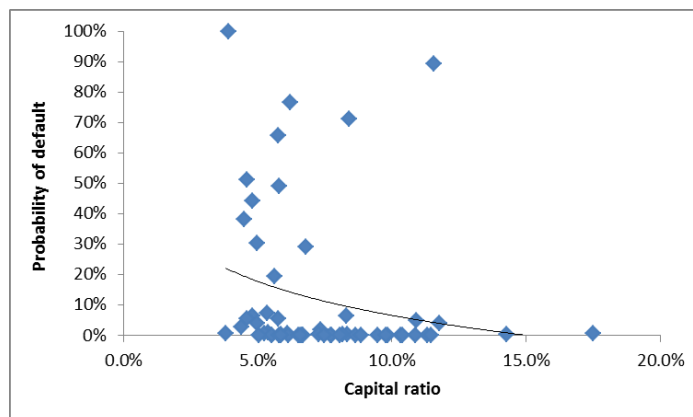
Figure 3: Differential impact of +1pp capital ratio shift for all model variables

The bar charts visualize the impact of a +1pp shift of the capital ratios of all banks on E, G, B, and B, expressed in relative form. The banks are sorted in each graph in descending order based on the respective ratios. The horizontal bar at 1 indicates a 'no impact' position.
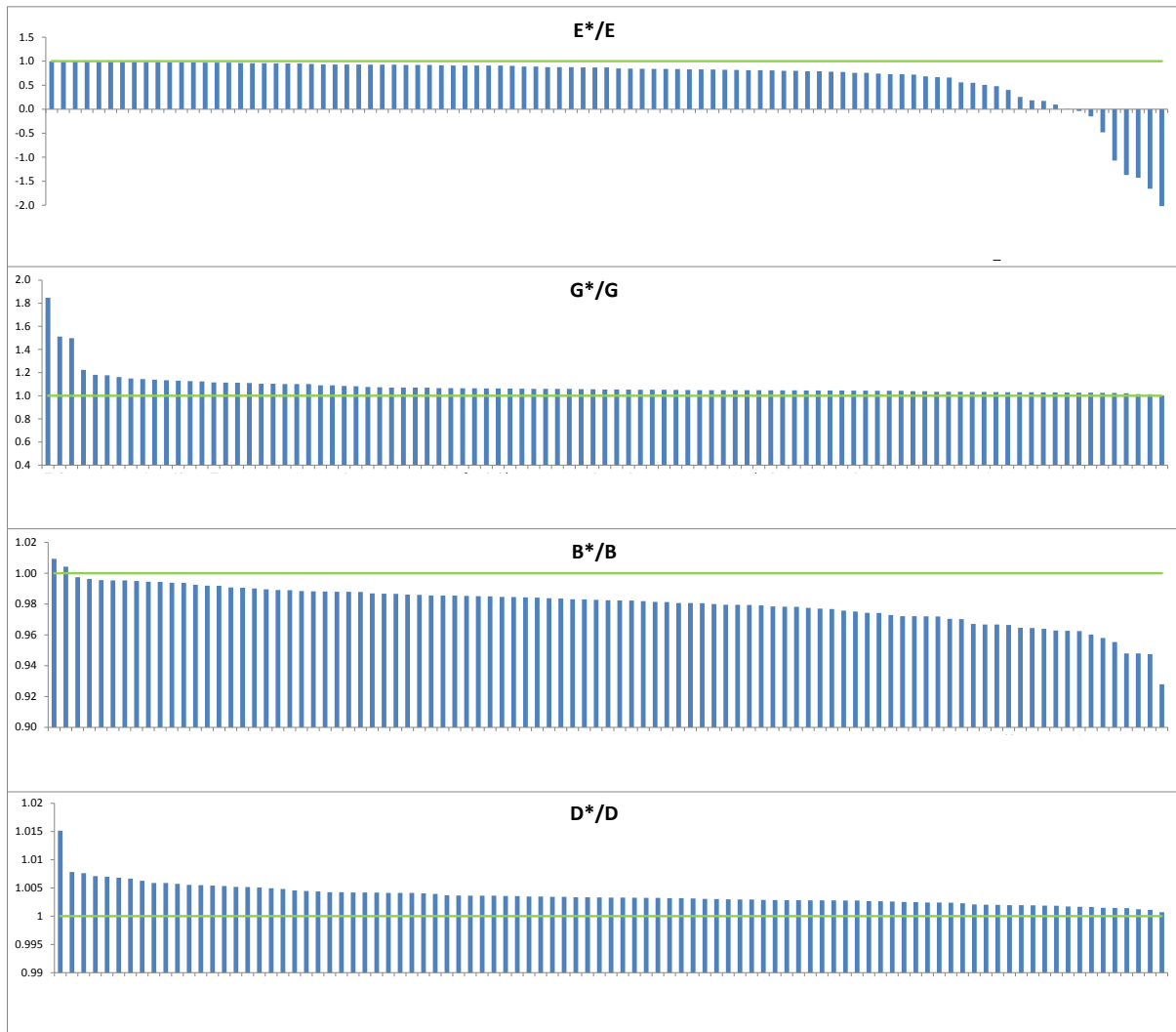
Figure 4: Differential impact of +1pp capital ratio shift under bail-in versus bail-out regimes

The bar charts visualize the impact of a +1pp shift of the capital ratios of all banks, for all model variables (E, G, B, and D) in relative form. The banks are sorted in each graph in descending order based on the respective ratios. The horizontal bar at 1 indicates a 'no impact' position. The porc2=20% setting (blue bars) corresponds to a regime that is closer to bail-in setting, while the porc2=80% is closer to a bail-out regime.
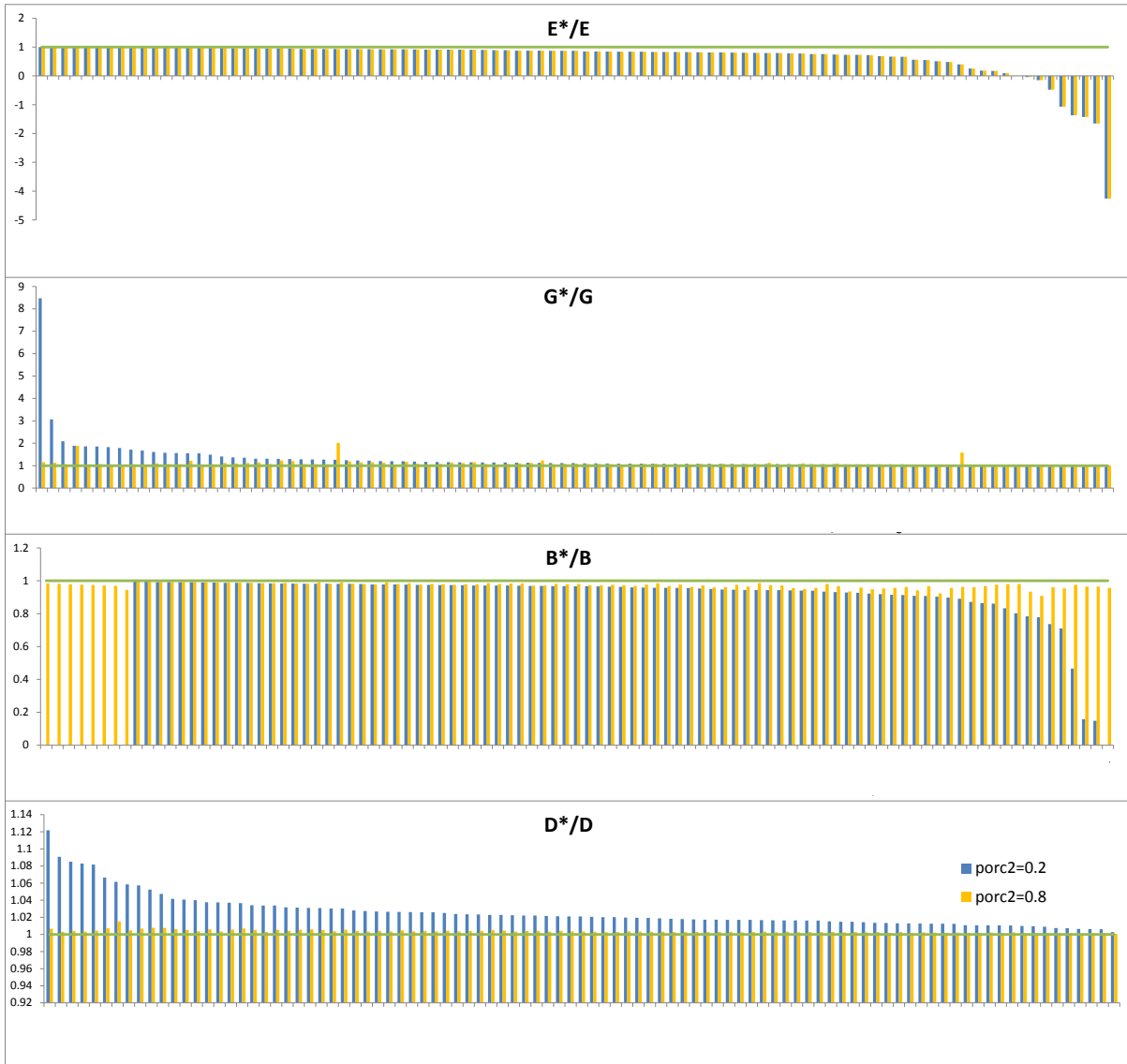
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