EUROPEAN CENTRAL BANK

EUROSYSTEM

Working Paper Series

Alexander Rodnyansky,
Alejandro Van der Ghote, Daniel Wales

Product quality, measured inflation and monetary policy

No 2680 / July 2022

Abstract

This paper proposes a tractable New Keynesian (NK) economy with endogenous adjustment in product quality that nests the canonical framework. Endogenous quality choice reduces the slope of the traditional NK Phillips curve and amplifies the economy's response to productivity shocks. This leads to a less reactionary monetary policy where model misspecification of imperfectly observable quality adjustments matters more for macroeconomic stabilization than the mismeasurement of those adjustments. With no misperception of product quality by the monetary authority, the principles for optimal monetary policy are, nonetheless, unchanged as the quality extensions to the canonical NK model preserve divine coincidence.

JEL Codes: E31, E32, E52, E58.

Key Words: Product Quality, Inflation Indexes, Monetary Policy.

Non-technical Summary

The classical literature on incomplete pass-through in international finance suggests that firms are slow and reluctant to increase prices when confronted with rising intermediate input costs. More recently, a growing body of empirical work has documented that product quality adjustments play a prominent role in the overall response of firms to a wide array of economic shocks. In this paper, we investigate how the addition of firms with a product quality choice into an otherwise standard New-Keynesian framework influences macroeconomic outcomes and the transmission mechanism of monetary policy.

Specifically, we propose a New-Keynesian model augmented for endogenous firm quality choices, which nests the canonical framework. This approach shows how firms may endogenously determine the optimal product quality level alongside prices and production quantity. Intuitively, two opposing forces are simultaneously operative: firms can choose to produce higher quality products and face larger marginal costs of production, reducing profits, but enjoy higher demand and hence revenue. In an equilibrium where symmetric firms adjust the price, quantity, and quality of their products sluggishly due to Rotemberg (1982) adjustment costs, we thus add one additional condition to determine quality, thereby obtaining dynamics for the quality of output inherently different from the usual quantity dynamics.

It turns out that a New Keynesian model with endogenous quality adjustments has several different implications for inflation dynamics, shock propagation, and monetary transmission. First, the quality augmented model implies a smaller slope in the traditional Phillips curve. This result arises because inflation and the output gap now have to be measured in quality-adjusted terms, and there is an omitted variable capturing contemporaneous quality choices. The slope of the Phillips curve also decreases in the relative ratio of the cost of price adjustment to that of quality adjustment. That is, if quality adjusts relatively more quickly after a positive productivity shock, this will pull down the aggregate measures of inflation.

Second, endogenous quality choices further amplify the response of the baseline economy to productivity (or cost) shocks as changes in productivity now cause firms to alter both the price and quality of their products. After an initial positive shock to productivity, current marginal costs fall. Firms then seek to re-optimize by both cutting prices, to re-establish an optimal markup, and also by increasing quality, which becomes cheaper to produce. Both of those effects lead to an increase in demand for each variety.

Third, for calibrated versions of the model, and assuming the monetary policy authority responds to shocks according to a standard Taylor rule, we show that the amplification result leads to larger changes in the nominal interest rate. We also analyze the empirically relevant scenarios of *bad measurement*, when the monetary authority cannot accurately measure quality adjustments, as well as the extreme case where the central bank neglects quality changes altogether, which we will refer to as *bad model*. We find that bad measurement of quality almost does not change the monetary policy reaction, whereas, in the case of a bad model, the central bank tends to stabilize the economy significantly more.

1 Introduction

The classical literature on incomplete pass-through in international finance suggests that firms are slow and reluctant to increase prices when confronted with rising intermediate input costs. More recently, a growing body of empirical work has documented that product quality adjustments play a prominent role in the overall response of firms to a wide array of economic shocks.[1] In this paper, we investigate how the addition of firms with a product quality choice into an otherwise standard New-Keynesian framework influences macroeconomic outcomes and the transmission mechanism of monetary policy.

Specifically, we propose a New-Keynesian model augmented for endogenous firm quality choices, which nests the canonical framework. This approach shows how firms may endogenously determine the optimal product quality level alongside prices and production quantity. Intuitively, two opposing forces are simultaneously operative: firms can choose to produce higher quality products and face larger marginal costs of production, reducing profits, but enjoy higher demand and hence revenue. In an equilibrium where symmetric firms adjust the price, quantity, and quality of their products sluggishly due to Rotemberg (1982) adjustment costs, we thus add one additional condition to determine quality, thereby obtaining dynamics for the quality of output inherently different from the usual quantity dynamics.

Product quality plays a central role in many areas of economics. For instance, it explains why some firms are more successful than others, how households respond to business cycles, the patterns of international trade, and deepens our understanding of the production process.[2] Those empirical results on the importance of quality serve as motivating evidence for our analysis. Despite a rich literature and a long tradition, particularly in trade and industrial organization, most macroeconomic workhorse models do not feature any quality decisions and assume that firms only adjust the price and quantity of their products. We augment the standard New Keynesian model to understand how quality choices mediate the transmission of shocks and, in particular, we show how the transmission of monetary policy shocks depends critically upon the relative

[1] For instance, Goetz and Rodnyansky (2020) show that exchange rate devaluations can cause importers to drop high-quality goods more quickly than low-quality items; Medina (2020) finds that an increase in trade-induced competition leads firms to change the quality of their output.

[2] Hottman, Redding, and Weinstein (2016) show firm appeal (quality) explains 76% of variation in firm sales; Jaimovich, Rebelo, and Wong (2019) document that consumers traded down in the quality of goods consumed during the Great Recession; Manova and Zhang (2012) propose a model with heterogeneous quality to explain patterns in the universe of Chinese customs data; Jaimovich, Rebelo, and Wong (2019) and Jaimovich, Rebelo, Wong, and Zhang (2019) find that high-quality goods are more labor-intensive, particularly for high skilled workers.

rigidity of price and quality adjustments.

It turns out that a New Keynesian model with endogenous quality adjustments has several different implications for inflation dynamics, shock propagation, and monetary transmission. First, the quality augmented model implies a smaller slope in the traditional Phillips curve. This result arises because inflation and the output gap now have to be measured in quality-adjusted terms, and there is an omitted variable capturing contemporaneous quality choices. The slope of the Phillips curve also decreases in the relative ratio of the cost of price adjustment to that of quality adjustment. That is, if quality adjusts relatively more quickly after a positive productivity shock, this will pull down the aggregate measures of inflation.

Second, endogenous quality choices further amplify the response of the baseline economy to productivity (or cost) shocks as changes in productivity now cause firms to alter both the price and quality of their products. After an initial positive shock to productivity, current marginal costs fall. Firms then seek to re-optimize by both cutting prices, to re-establish an optimal markup, and also by increasing quality, which becomes cheaper to produce. Both of those effects lead to an increase in demand for each variety.

Third, for calibrated versions of the model, and assuming the monetary policy authority responds to shocks according to a standard Taylor rule, we show that the amplification result leads to larger changes in the nominal interest rate. We also analyze the empirically relevant scenarios of *bad measurement*, when the monetary authority cannot accurately measure quality adjustments, as well as the extreme case where the central bank neglects quality changes altogether, which we will refer to as *bad model*. We find that bad measurement of quality almost does not change the monetary policy reaction, whereas, in the case of a bad model, the central bank tends to stabilize the economy significantly more.

In building a theoretical economy, our framework allows for an explicit analytical formulation for the difference between the true rate of price inflation and a potentially biased or mismeasured value captured by statistical authorities, which does not account for quality changes. Discussions around this topic have a long history dating back at least to Boskin, Dulberger, Gordon, Griliches, and Jorgenson (1996) and, more recently, the ESA 2010 and the UK's Bean Review (2016), which document how statistical agencies in the U.S., EA, and UK, respectively, have struggled with those issues. We show how the difference in variation between the two measures has important implications for the optimal conduct of monetary policy.

If monetary policymakers misperceive the level of the output gap or inflation (perhaps due to mismeasurement), the economy responds in a similar fashion. However, if the monetary policy authority misperceives the true model and reacts to productivity shocks assuming quality does not adjust, it will stabilize the economy by more than intended. The principle for optimal monetary policy is nonetheless unchanged under no misperception as “divine coincidence” also applies in this broader setting, with monetary policymakers thus seemingly able to simultaneously stabilize the output, inflation, and quality movements.

1.1 Related Literature

Our work integrates several strands of the literature. Empirically, there are at least two channels through which product quality may matter for the macroeconomy, as a source of a product’s price (Nakamura and Steinsson (2008)) or sales (Hottman, Redding, and Weinstein (2016)) variation, and as a key component in heterogeneous product replacement (or upgrading) decisions. The product life-cycle is likewise important as innovative higher-quality products replace outdated ones (Argente, Lee, and Moreira (2020)).

Our paper is also related to work by Adam and Weber (2019) that studies monetary policy when firms face both stickiness in setting quality and prices. The authors show that the optimal level of trend inflation depends on the number of firms entering the market, and their relative productivity (and hence pricing) differences to incumbents. Schmitt-Grohé and Uribe (2012) compute a theoretical level of optimal trend inflation which varies depending on whether prices or quality-adjusted prices are subject to an adjustment rigidity. As opposed to those papers, quality adjustments are endogenous in our setting, and as such depend on monetary policy, and the focus is on business cycle fluctuations rather than on long-term trends. In contrast to much of the literature on quality choice (Khandelwal (2010), Fajgelbaum, Grossman, and Helpman (2011), Jaimovich, Rebelo, and Wong (2019)), our results do not rely upon non-homothetic utility function assumptions, although this would reinforce our main findings.

We are not the first paper to consider quality movements in the context of macroeconomics, and the consequences of new products replacing existing ones as part of an ongoing process of innovation. Bils and Klenow (2001) used the U.S. Consumer Expenditure Survey to quantify the quality bias in consumer durable goods, and found that quality grew on average 3.8 percent per year in the 1980-1996 period. More recently, Garcia-Macia, Hsieh, and Klenow (2019) show that most productivity growth in the U.S. is generated by incumbent firms improving their existing products rather than the creation of new varieties, and that this force is more important than creative destruction. Other work has focused on mismeasurement of growth and inflation due to unaccounted quality changes (Aghion, Bergeaud, Boppart, Klenow, and Li (2019), Broda and Weinstein (2010), Nakamura and Steinsson (2012)).

Our paper connects to a rapidly growing theoretical and empirical literature on the "disappearance" of the traditional Phillips curve relationship. The lack of sensitivity of inflation to changes in employment over the recent decades, and especially during the Great Recession, has led many to believe that the Phillips curve has flattened. This expanding body of work offers a number of explanations for those patterns, such as shifting inflation expectations or globalization and intermediate input flows (Ball and Mazumder, 2011; Simon, Matheson, and Sandri, 2013; Ball and Mazumder, 2019; Hall, 2013; Blanchard, Cerutti, and Summers, 2015; Coibion and Gorodnichenko, 2015; Geerolf, 2019; Stock and Watson, 2019; Barnichon and Mesters, 2020; Del Negro, Lenza, Primiceri, and Tambalotti, 2020; Hazell, Herreño, Nakamura, and Steinsson, 2020; Rubbo, 2020). We contribute to this stand by showing that the apparent flattening of the Phillips curve does not have to imply a flaw in the New Keynesian model once it is augmented to account for endogenous quality changes.[3]

Finally, we relate to studies on the role of quality in international finance and trade. Linder (1961) first noted the role of quality as a determinant of the direction of trade; Rodriguez (1979); Hummels and Klenow (2005) use quantities exported and proxies for the number of varieties to argue that quality differences are necessary to explain (at least part of) the observed differences in unit values. Levchenko, Lewis, and Tesar (2011), Chen and Juvenal (2018) and Bems and di Giovanni (2016) have found evidence that the disproportionate drop in the value of trade after the global negative income shock in 2008 was caused by the higher quality of traded goods combined with non-homotheticity of demand. Previous work also examined the relationship between trade distances and quality (Alchian and Allen (1964), Hummels and Skiba (2004), and Feenstra and Romalis (2014)). Recent contributions have also shown that firms may choose to up- or downgrade their product quality in response to exchange rate movements (Goetz and Rodnyansky, 2020; Bastos, Silva, and Verhoogen, 2018), or because competing with inexpensive imports induces companies to upgrade, as in Medina (2020).[4]

[3]De Ridder (2019) shows that U.S. investments in corporate research and development have steadily increased by 61% as a fraction of national income over the last 30 years, reflecting the ever increasing desire on the part of firms to produce higher quality versions of goods than their counterparts.

[4]Other trade shocks that can drive firms to quality upgrade include rising competition from low-wage countries (as in Martin and Mejean (2014)), cheaper intermediate inputs (see Verhoogen (2008), Fieler, Eslava, and Xu (2014) and Bas and Strauss-Kahn (2015)) or access to larger markets (see Bustos (2011), Lileeva and Trefler (2010), and Aw,

The remainder of the paper proceeds as follows. Section 2 provides the model, section 3 discusses the implications of quality adjustments on the slope of the Phillips curve and the business cycle fluctuations of the natural rate of interest, section 4 solves a calibrated version of the model and conducts the monetary policy exercises, and section 5 concludes.

2 Model

2.1 Household Problem

A set of identical households seek to maximize their utility, given by:

$$\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{C_s^{1-\sigma}}{1-\sigma} - \frac{L_s^{1+\eta}}{1+\eta} \right),$$

where $0 < \beta < 1$ denotes their subjective discount factor, C_t denotes their consumption of a composite good and L_t denotes their labor supply. Households have access to one period nominal bonds which they may use to smooth consumption over time. Thus households face a nominal budget constraint given by:

$$P_t C_t + B_{t+1} = (1 + i_t) B_t + W_t L_t + P_t T_t + P_t \Pi_t.$$

B_{t+1} represents the bonds purchased by households in period t. These pay out the nominal interest rate, $1 + i_t$, in period $t + 1$. The nominal wage is given by W_t and T_t denotes the real level of lump sum transfers (or taxes when negative) given to households by the government fiscal authority while Π_t are the lump sum value of real profits given to households by some intermediate goods firms (see subsection 2.3 for details on these firms).

The solution of this problem then yields the standard set of Euler conditions, defining the relationship between marginal utility across time periods, as well as the standard intratemporal trade-off between current consumption and current labor supply:

$$\frac{W_t}{P_t} = C_t^{\sigma} L_t^{\eta},$$
$$1 = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1 + i_t}{1 + \pi_{t+1}} \right],$$

Roberts, and Xu (2011)).

where we define the aggregate gross rate of household price inflation as $1 + \pi_t \equiv P_t / P_{t-1}$.

2.2 Final Good Producers

The composite good is a CES aggregate of a continuum of intermediate goods:

$$Y_t = \left[\int_0^1 [\mathcal{G}(F_t(j))]^{\frac{1}{\varepsilon}} Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} \, \mathrm{d}j \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $\mathcal{G}(\cdot)$ is a generalized demand shifter with the property $\mathcal{G}_F(\cdot) \equiv \partial \mathcal{G}(\cdot)/\partial F_t(j) > 0$ while its argument, $F_t \geq 0$, represents the number of "features" a given product or variety has—i.e., its quality. Parameter $\varepsilon > 1$ is the elasticity of substitution across varieties, and a measure of competition in the economy. Having noted that the function $\mathcal{G}(F_t(j))$ may be idiosyncratic across the intermediate goods depending upon their level of quality, we will suppress this as $\mathcal{G}_t(j) \equiv \mathcal{G}(F_t(j))$ for notational convenience.

The cost minimization problem of the final goods producers results in the standard demand function for each intermediate variety given by:

$$Y_t(j) = \mathcal{G}_t(j) \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} Y_t,$$

while the aggregate "hedonic" (welfare relevant, quality adjusted) price level may be found as:

$$P_t = \left[\int_0^1 \mathcal{G}_t(j) P_t(j)^{1-\varepsilon} \, \mathrm{d}j \right]^{\frac{1}{1-\varepsilon}}.$$

This formulation is consistent with both the existing macroeconomics literature[5] and the trade literature[6]. The cost-of-goods index (i.e., the price level that is not adjusted for product quality) is instead:

$$P_t^S = \left[\int_0^1 P_t(j)^{1-\varepsilon} \, \mathrm{d}j \right]^{\frac{1}{1-\varepsilon}}.$$

[5] As in Adam and Weber (2019) and Schmitt-Grohé and Uribe (2012).

[6] As in Hallak (2006), Kugler and Verhoogen (2012), Feenstra and Romalis (2014).

2.3 Intermediate Firms

A typical intermediate firm, j, faces a production function given by:

$$Y_t(j) = N_t(j)\mathcal{H}(A_t, F_t(j)),$$

where $N_t(j)$ is the level of labor input and the function $\mathcal{H}(\cdot)$ takes the place of the productivity of labor. This function shifts the production process depending on both the economy-wide level of productivity, A_t, and the level of quality firm j chooses to produce, $F_t(j)$. Again we take as given the arguments of this function, and suppress the notation as $\mathcal{H}_t(j) \equiv \mathcal{H}(A_t, F_t(j))$ for convenience. Throughout we will assume that $\mathcal{H}_{F,t}(j) \equiv \partial\mathcal{H}(\cdot)/\partial F_t(j) < 0$, such that producing goods with a higher quality reduces the level of labor productivity, and $\mathcal{H}_{A,t}(j) \equiv \partial\mathcal{H}(\cdot)/\partial A_t > 0$, as is standard. Given the economy-wide level of productivity and level of quality produced by firm j real marginal costs can be written as:

$$MC_t(j) = \frac{W_t}{P_t}\frac{1}{\mathcal{H}_t(j)}.$$

This potentially differs across firms through heterogeneous quality levels.

2.3.1 Optimality Under Flexible Price and Quality Adjustments

Assuming these relationships for the intermediate goods firms, under monopolistic competition in the intermediate goods market, the static profit maximization problem then takes the form:

$$\max_{P_t(j),F_t(j)} \left[\frac{(1+\tau_t)P_t(j)}{P_t} - MC_t(j)\right]\mathcal{G}_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t,$$

where firms are able to choose both prices, $P_t(j)$, and quality, $F_t(j)$. The variable $1+\tau_t$ represents a production subsidy given to intermediate goods firms by the fiscal authority. This subsidy will be set to eliminate distortions from monopoly power (see subsection 2.4).

The first order condition with respect to prices, $P_t(j)$, reveals the standard condition for the equilibrium level of prices:

$$\frac{P_t(j)}{P_t} = \frac{\varepsilon}{\varepsilon-1}\frac{MC_t(j)}{1+\tau_t},$$

which shows that in this framework the optimal relative price remains a constant mark-up above marginal costs, also accounting for the additional loss in profits when producing goods with a higher level of quality. Next, the equilibrium level of quality in each product may be determined from the first order condition of the profit function, which is:

$$\underbrace{\frac{\partial MC_t(j)}{\partial F_t(j)} Y_t(j)}_{\text{Marginal Cost Increasing}} = \underbrace{\left[\frac{(1+\tau_t)P_t(j)}{P_t} - MC_t(j)\right]}_{\text{Markup}} \underbrace{\frac{\partial Y_t(j)}{\partial F_t(j)}}_{\text{Demand Shifting}} ,$$

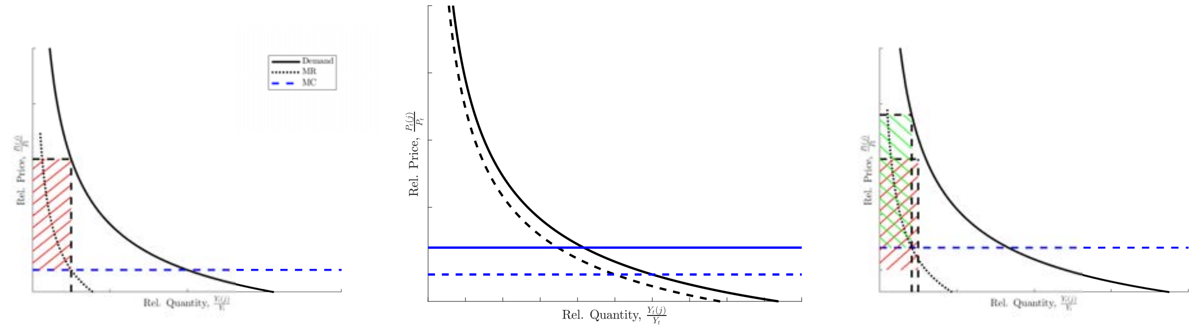
such that, after optimally setting prices to achieve the optimal markup, firms are able to further increase profits through the quality of their products and the potential production subsidy. This clearly shows the two influences:

1. Profits fall when producing a higher level of quality, which increases marginal costs.

2. Higher quality products increase profits through their influence as a "demand shifter."

2.3.2 Graphical Interpretation

This situation is shown graphically in Figure 1. Due to imperfect competition in the intermediate goods sector, individual firms face a downward sloping demand curve and, given both the quality of their products and aggregate prices, marginal costs are constant in the quantity produced. This situation is shown in panel (a) where the production point for the monopolist of each variety arises at the intersection of marginal cost and marginal revenue. In this situation, prices are a fixed markup over marginal costs, and profits are then the shaded area. An increase in the quality of products supplied is then shown in panel (b) as the movement from the dashed toward the solid lines. As quality improves, we observe two effects on the equilibrium outcome. Demand increases, as higher quality increases the demand for a given product, but marginal costs also increase, as higher quality products are more costly to produce from a given level of factor inputs (the effective productivity of inputs falls asymmetrically). The resultant profit maximization condition is then shown in panel (c), where profits are once more given as the shaded region between the demand schedule and marginal costs at equilibrium prices and quantities. For reference, the initial level of profits is also shown on the diagram to highlight how the firm will use this simultaneous shift in marginal costs and demand to determine an equilibrium level of quality (or features), which maximize profits. The firm chooses the new level of quality if the new area is larger than the initial one.



(a) Initial Case (b) Higher Quality (c) Profit Comparison

Figure 1: **Introduction of quality under monopolistic competition**
Notes: A representation of how increasing product quality shifts both demand and costs, and potentially increases profits. Panel (a) shows the initial situation, panel (b) shows the demand and marginal cost movements. Panel (c) shows the resultant change in profits.

2.3.3 Optimality Under Price and Quality Rigidities

Having outlined the benchmark case for the optimality conditions for intermediate firms' under flexible prices and quality, we now turn to optimal decisions in a New-Keynesian framework under Rotemberg (1982) adjustment costs for both prices and quality, which generate clear analytical results. The real profit flow for a given intermediate goods producer is given by:

$$\Pi_t(j) = (1+\tau_t)\frac{P_t(j)}{P_t}Y_t(j) - \frac{W_t}{P_t}N_t(j) - \frac{\theta_P}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t - \frac{\theta_F}{2}\left(\frac{F_t(j)}{F_{t-1}(j)} - 1\right)^2 Y_t,$$

where, again, $1+\tau_t$ are the production subsidy given to firms by the fiscal authority. For tractability, we assume quality improvements incur adjustment costs in a form symmetric to the standard case with prices. Under monopolistic competition, intermediate firms seek to maximize the present discounted flow of profits by choosing the price and quality of goods they produce. Appendix A shows how the two first order conditions for this problem may therefore be written as:

$$\begin{aligned}(\varepsilon-1)(1+\tau_t) &= \varepsilon MC_t \mathcal{G}_t^{-\frac{1}{\varepsilon-1}} - \theta_P \pi_t^S(1+\pi_t^S) \\ &\quad + \beta\theta_P \mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\pi_{t+1}^S(1+\pi_{t+1}^S)\right], \qquad \text{(NKPC P)}\\ (1+\tau_t)\mathcal{G}_{F,t}\mathcal{G}_t^{-1} &= MC_t\mathcal{G}_t^{-\frac{1}{\varepsilon-1}}\left(\mathcal{G}_{F,t}\mathcal{G}_t^{-1} - \frac{\mathcal{H}_{F,t}}{\mathcal{H}_t}\right) + \theta_F\pi_t^F(1+\pi_t^F) \\ &\quad - \beta\theta_F\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\pi_{t+1}^F(1+\pi_{t+1}^F)\right], \qquad \text{(NKPC F)}\end{aligned}$$

where our definition of consumer price inflation as $1 + \pi_t \equiv P_t/P_{t-1}$ has been used, and we similarly define the growth rate of quality as $1+\pi_t^F \equiv F_t/F_{t-1}$ and denote sticker price inflation, π_t^S (the price inflation of an individual firm), as $1+\pi_t^S \equiv [\mathcal{G}_t/\mathcal{G}_{t-1}]^{\frac{1}{\varepsilon-1}}(1+\pi_t)$. Symmetry of the problem across all firms enables firm indexation to be suppressed.

Together, these equations form the New-Keynesian Phillips curve for Prices (NKPC P) and quality (NKPC F). The first states that firms optimally chose sticker prices based upon current real marginal costs and their expectations for future price levels. Similarly, the second equation states that quality is set optimally based upon the current level of profit available for higher quality, and a firm's expectations for the evolution of future quality. It is therefore clear within this framework that the standard (NKPC P) condition, derived from the optimal condition of individual firms, is usually cast in terms of sticker price inflation, rather than the rate of change of aggregate consumer prices, as these are what is chosen by the symmetric intermediate goods firms.

2.4 Government

The government fiscal authority finances sales subsidies by taxing households to maintain a balanced budget:

$$0 = T_t + \tau_t \int_0^1 \frac{P_t(j)}{P_t} Y_t(j)\,\mathrm{d}j$$

The aim of government fiscal policy is assumed to be to offset the equilibrium impact of imperfect competition in distorting pricing decisions of intermediate goods firms. The subsidy, τ_t, is therefore set to deliver the efficient level of marginal costs in equilibrium. In a steady state with zero inflation, NKPC P shows:

$$(\varepsilon - 1)(1+\bar{\tau}) = \varepsilon \bar{MC}\bar{\mathcal{G}}^{-\frac{1}{\varepsilon-1}},$$

where a bar denotes equilibrium levels. In the efficient allocation, derived in Appendix B, $\bar{MC}\bar{\mathcal{G}}^{-\frac{1}{\varepsilon-1}} = 1$. The production subsidy will therefore ensure the steady state production level is undistorted by monopolistic competition and delivers the first best allocation whenever:

$$(1+\tau_t) = (1+\bar{\tau}) \equiv \frac{\varepsilon}{\varepsilon - 1}.$$

2.5 Monetary Policy

The monetary authority sets the reference one-period nominal interest rate which in equilibrium is perfectly arbitraged with the nominal bond rate. To do so the authority follows a standard Taylor rule. As the baseline scenario (subsection 4.2), we consider the case in which quality adjustments are perfectly measurable, but afterwards (subsection 4.3) we also consider cases in which the authority cannot measure quality adjustments properly or, even worse, neglects quality adjustments altogether. We present the baseline Taylor rule in section 3 after deriving the log-linear solution to the model. We present the other Taylor rules in section 4 where we conduct the monetary policy analysis.

2.6 Aggregate

The resource constraint in this economy is given as:

$$Y_t = C_t + \frac{\theta_P}{2}\left(\pi_t^S\right)^2 Y_t + \frac{\theta_F}{2}\left(\pi_t^F\right)^2 Y_t.$$

Output is either consumed or spent on the wasteful adjustment costs of prices and quality. Given the symmetry between firms and labor market clearing, the aggregate production function may be found as:

$$Y_t = \left[\int_0^1 \left[\mathcal{G}_t(j)\right]^{\frac{1}{\varepsilon}} \left[N_t(j)\mathcal{H}_t(j)\right]^{\frac{\varepsilon-1}{\varepsilon}} \, \mathrm{d}j\right]^{\frac{\varepsilon}{\varepsilon-1}} = \mathcal{G}_t^{\frac{1}{\varepsilon-1}} L_t \mathcal{H}_t$$

The model is then completed, up to the monetary policy rule, with an AR(1) process for technology:

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_t,$$

where $\rho_A < 1$ ensures convergence and ε_t are *iid* shocks. The full set of model equations are outlined in Appendix C along with a method to calculate equilibrium prices and quantities.

3 Quality Adjustments, the Phillips Curve, and the Natural Rate

This section discusses the implications of quality adjustments on the slope of the Phillips curve and the business cycle fluctuations of the natural rate of interest. To keep the analysis tractable, the section assigns specific functional forms to the demand shifter, $\mathcal{G}_t(j)$, and to labor productivity, $\mathcal{H}_t(j)$. The precise steps, log linearization and model solution method are presented in detail in Appendix D.

Notation. Bars denote steady state levels, $\bar{x}$, hats denote percentage deviation from steady state level, $\hat{x}$ ($\approx$ to log deviations), and tildes denote differences from natural rates, $\tilde{x}$. Variables are written in upper case with their logarithm in lower case.

3.1 Functional Forms

The functional forms for $\mathcal{G}_t(j)$ and $\mathcal{H}_t(j)$ are specified to enable the firm block of the model to be log-linearized. The generalized demand shifter, $\mathcal{G}_t(j)$, is set as:

$$\mathcal{G}_t(j) \equiv [F_t(j)]^{\phi}, \qquad \text{with} \qquad \mathcal{G}_{F,t}(j) \equiv \phi\, [F_t(j)]^{\phi-1},$$

such that the parameter $\phi > 0$ may be used to control the elasticity of aggregate demand to changes in quality.[7] The impact of quality on the production process is set as:

$$\mathcal{H}_t(j) \equiv \left[A_t^{\frac{\nu-1}{\nu}} - \kappa[F_t(j)]^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}, \quad \text{with} \quad \mathcal{H}_{F,t}(j) \; = -\kappa\left[A_t^{\frac{\nu-1}{\nu}} - \kappa[F_t(j)]^{\frac{\nu-1}{\nu}}\right]^{\frac{1}{\nu-1}} [F_t(j)]^{-\frac{1}{\nu}}$$

where $\nu > 0$ and $\kappa > 0$ are positive parameters, such that the firm level productivity index is a CES aggregate of economy-wide productivity and the decision of which level of quality to produce.[8] In this setting the level of quality in the steady state is determined as:

$$\frac{1}{\varepsilon-1} = -\frac{\bar{\mathcal{H}}_F}{\bar{\mathcal{H}}}\frac{\bar{\mathcal{G}}}{\bar{\mathcal{G}}_F}, \qquad \Rightarrow \qquad \bar{F} = \bar{A}\left(\frac{1}{\kappa}\right)^{\frac{\nu}{\nu-1}}\left[\frac{\phi}{\varepsilon-1+\phi}\right]^{\frac{\nu}{\nu-1}}, \tag{1}$$

which implies that quality depends crucially upon the economy-wide level of productivity. In the steady state, moreover,

$$\bar{\mathcal{H}} = \bar{A}\left[\frac{\varepsilon-1}{\varepsilon-1+\phi}\right]^{\frac{\nu}{\nu-1}} \qquad \text{and} \qquad \bar{\mathcal{H}}_F = -\kappa\left[\frac{\kappa(\varepsilon-1)}{\phi}\right]^{\frac{1}{\nu-1}}.$$

[7] This closely follows the formulation in Hallak (2006); Kugler and Verhoogen (2012); Feenstra and Romalis (2014) and Jaimovich, Rebelo, and Wong (2019) among others.

[8] This is again a similar formulation to Kugler and Verhoogen (2012) and Feenstra and Romalis (2014).

3.2 Model Solution

The model is then given as the solution to:

$$
\begin{aligned}
\tilde{y}_t &= \mathbb{E}_t[\tilde{y}_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\tilde{\pi}_{t+1}] - r^n_t), && \text{(Dynamic IS)}\\
\tilde{\pi}^S_t &= \frac{\varepsilon(\sigma+\eta)}{\theta_P}\tilde{y}_t + \beta\mathbb{E}_t\left[\tilde{\pi}^S_{t+1}\right], && \text{(Log Linearized NKPC P)}\\
\tilde{\pi}^F_t &= \frac{\omega_1}{\theta_F}(\sigma+\eta)\tilde{y}_t + \frac{\omega_2-\omega_1}{\theta_F}\tilde{f}_t + \beta\mathbb{E}_t\left[\tilde{\pi}^F_{t+1}\right], && \text{(Log Linearized NKPC F)}\\
i_t &= r^n_t + \phi_\pi\tilde{\pi}_t + \phi_y\tilde{y}_t. && \text{(Taylor rule)}
\end{aligned}
$$

where $\omega_1 > 0$, $\omega_2 \in \mathbb{R}$, $\phi_\pi \geq 0$, and $\phi_y \geq 0$ are constants. Together these conditions determine the endogenous variables, i_t, $\tilde{y}_t$, $\tilde{\pi}^S$, and $\tilde{f}_t$ as a function of the exogenous block of definitions for r^n_t, π^F_t, π_t and a_t.

$$
r^n_t = \rho + \sigma\psi_a\mathbb{E}_t[\Delta a_{t+1}], \tag{2}
$$

$$
\tilde{\pi}^F_t = \tilde{f}_t - \tilde{f}_{t-1}, \tag{3}
$$

$$
\tilde{\pi}^S_t = \frac{\phi}{\varepsilon-1}\tilde{\pi}^F_t + \tilde{\pi}_t, \tag{4}
$$

$$
a_t = \rho_A a_{t-1} + \varepsilon_t. \tag{5}
$$

By inspection, this simplified model shares many characteristics with the canonical NK model. Both the dynamic IS curve and NKPC P are unchanged, though as "sticker" prices are in general different to "hedonic" prices, the latter equation makes this distinction and holds only for "sticker" prices. The AR(1) technology process and closing the model with a Taylor rule for nominal interest rates are also standard.

In contrast to the standard setting, the equilibrium now contains an additional equation, namely the simplified form of the Log Linearized NKPC F, which describes how the quality of goods evolves through time. Current quality choices depends on two factors:

1. Expected discounted future quality choices, through a Rotemberg effect which arises as quality is costly for firms to adjust, whenever $\theta_F > 0$.

2. The contemporaneous trade-off between higher demand and higher marginal costs as described in section 2.3.2 above. In particular, when the Rotemberg effect is turned off, with costless quality adjustment $\theta_F = 0$, the reduced form of the Log Linearized NKPC F be-

comes:

$$\tilde{f}_t = \omega_1 \frac{\sigma + \eta}{\omega_1 - \omega_2} \tilde{y}_t. \tag{6}$$

This states that changes in product quality may be either procyclical or countercyclical in the context of this model.

3.3 The Phillips Curve

Theorem 3.1 (Phillips Curve)**.** *The reduced form relationship between aggregate prices and the output gap may be represented as a Phillips curve:*

$$\tilde{\pi}_t = \frac{\varepsilon(\sigma + \eta)}{\theta_P} \left(1 - \frac{\phi \omega_1}{\varepsilon(\varepsilon - 1)} \frac{\theta_P}{\theta_F}\right) \tilde{y}_t + \frac{\phi}{\varepsilon - 1} \frac{\omega_1 - \omega_2}{\theta_F} \tilde{f}_t + \beta \mathbb{E}_t \left[\tilde{\pi}_{t+1}\right]. \tag{7}$$

Proof. The relationship between "sticker" and "hedonic" price inflation—i.e., the definition given in equation 4—may be used alongside the Log Linearized NKPC P and the Log Linearized NKPC F to derive this equation. □

The red font in equation 7 highlights the elements that do not appear in the canonical New Keynesian framework. Two corollaries are thus immediately apparent as a consequence of theorem 3.1.

Corollary 3.1.1 (Slope)**.** *The Phillips curve in equation 7 has a smaller slope than the cannonical NKPC.*

This arises as the new component $1 - \frac{\phi \omega_1}{\varepsilon(\varepsilon-1)} \frac{\theta_P}{\theta_F} < 1$ is below 1, in particular as $\omega_1 > 0$. In some circumstances the slope may even be negative, especially when the costs of price adjustment far outweigh those of quality adjustment $\theta_P \gg \theta_F$. This arises as quality adjusts quickly after a positive (negative) shock dragging aggregate inflation measures down (up).

Corollary 3.1.2 (Bias)**.** *Empirical reduced form estimation of the Phillips curve must control for quality adjustments or suffer omitted variable bias.*

The second term in red in equation 7 shows how the current level of quality matters for reduced form Phillips curve estimation.

3.4 The Natural Rate

Theorem 3.2 (Amplification)**.** *Quality amplifies the response of the natural real interest rate to productivity shocks.*

Proof. The natural real interest rate evolves according to:

$$r_t^n = \rho + \sigma\psi_a \mathbb{E}_t[\Delta a_{t+1}],$$

where $\psi_a \equiv \frac{\eta+1}{\sigma+\eta}\frac{(\varepsilon-1)+\phi}{\varepsilon-1}$, which with $\phi > 0$ is larger than the coefficient in the canonical case. $\square$

The amplification channel described in Theorem 3.2 is depicted graphically in Figure 2. After an initial increase in productivity, current marginal costs fall. Firms then seek to re-optimize by both cutting prices, to re-establish an optimal markup, and also by increasing quality which is relatively cheaper to produce, given the functional form of $\mathcal{H}_t(\cdot)$. Both of these effects cause the aggregate goods firm to increase demand for each variety. In a model with flexible prices and persistent productivity shocks this causes the real interest rate to fall, in general equilibrium, to prevent households from smoothing the higher current level of consumption through time.

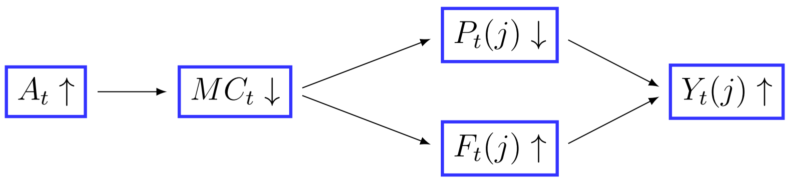


Figure 2: **Amplification of productivity shocks**
Notes: A representation of how a positive productivity shock impacts current demand.

4 Monetary Policy Analysis

This section first investigates positive aspects of monetary policy and then focuses on normative considerations. To do so the section calibrates the model.

4.1 Calibration

By design, our model with endogenous quality adjustment nests the workhorse New Keynesian model. We therefore choose a number of parameters to match standard values found in the literature.[9] The parameters chosen for our baseline specification of the Taylor rule are also standard in the literature. (See Table 1 for details.) Investigating alternative levels of productivity persistence leaves the results unchanged. This leaves five parameters to specify.

In our baseline specification we choose the parameter controlling Rotemberg (1982) price adjustment costs, θ_P, so that the evolution of marginal costs under a standard New Keynesian model with Rotemberg (1982) pricing coincides with Calvo (1983) and Yun (1996) pricing with a probability of price adjustment of $\theta = 2/3$, taken from Galí (2015). The Rotemberg and Calvo specifications arise whenever:

$$\theta_P = \frac{(\varepsilon - 1)\theta}{(1-\theta)(1-\theta\beta)},$$

such that $\theta_P \approx 29.41$ given our other parameter specifications. For tractability, our baseline specification sets the Rotemberg cost of quality adjustment equal to that of prices $\theta_P = \theta_F$.

Alternative strategies to calibrate θ_F suggest lower values are plausible. Nakamura and Steinsson (2008) calculate the mean implied duration for the prices of goods contained within the CPI basket between 1998-2005 as 9 months (for goods excluding substitutions) this would also generate a quarterly Calvo (1983) and Yun (1996) pricing parameter of $\theta = 1 - 3/9 = 2/3$. In addition, they show this falls to 7.7 months when including product substitutions. This may indicate a relative ease of adjusting quality, rather than prices with an associated quarterly Calvo (1983) and Yun (1996) parameter of $\theta = 1 - 3/7.7 \approx 0.61$ corresponding to a Rotemberg (1982) cost of quality adjustment of $\theta_F \approx 19.78$. A second method, employed by Adam and Weber (2019), would set the probability of quality adjustment as 11.5 percent per year. This is the midpoint between the average establishment birth rate (12.4 percent) and the average establishment exit rate (10.7 percent) over the period 1977 to 2015 reported in the Business Dynamics Statistics (BDS) of the US Census Bureau and is similar to the product entry and exit rates observed in Argente, Lee, and Moreira (2020). This would generate a quarterly Calvo parameter of $\theta = 1 - 0.115^{0.25} = 0.42$. This would also generate a lower estimated cost of quality adjustment at $\theta_F \approx 6.20$.

Turning to the elasticity of demand with respect to quality, ϕ. In the trade literature, ϕ is often used as a term relative to the United States. This has a very different interpretation than in our model where the absolute level of ϕ matters. We therefore use a baseline specification with $\phi = 1$ for unitary demand as our starting point. This is supported by Khandelwal (2010) who regresses estimates of the quality of imported goods on per capita GDP from the exporting country and

[9] For example, see the calibration in Chapter 3 of Galí (2015), as shown in Table 1. We set σ_i^2 such that a one standard deviation shocks will generate a 25bps (100bps annualized) shock to the nominal interest rate on impact.

finds a demand elasticity of quality with $\phi = 0.8$. He also provides a range of plausible estimates for the elasticity of a change in TFP on quality vary between 1.05 and 0.83, depending on the regressions specification. Fan, Li, and Yeaple (2014) use a similar international setting with quality ladders and estimate the elasticity of a change in TFP on quality in a lower range between 0.30 and 0.24.

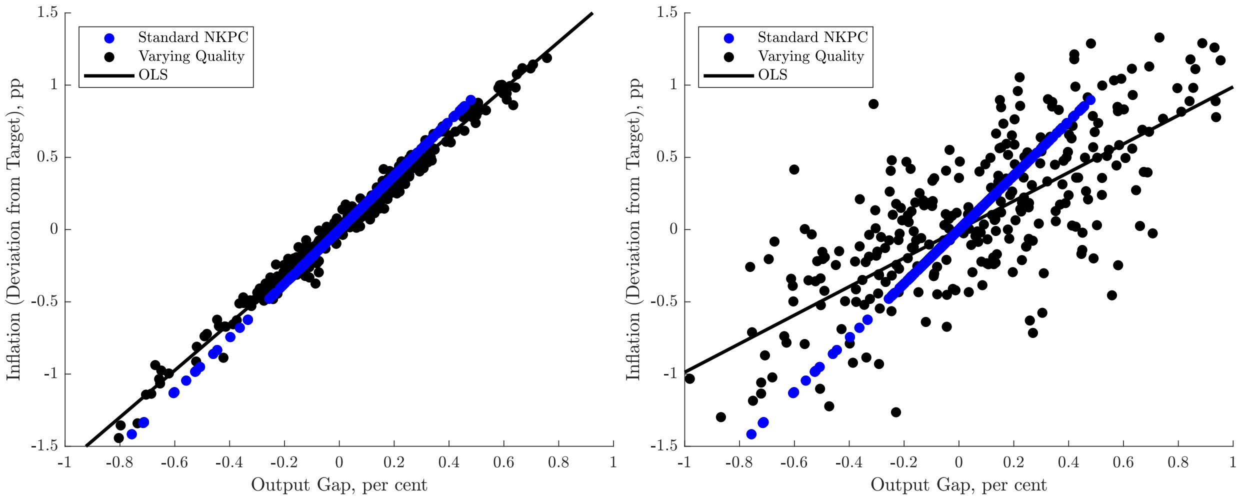
Finally, Feenstra and Romalis (2014) suggest $\nu \in [0.42, 1.31]$, with a mean of $\nu = 0.63$. We set $\nu = 0.8$ is set to ensure productivity and quality are weak complements, while $\kappa = 0.38$ is used as a normalization to target $\bar{F} = 1$. Again, varying these parameters leaves the main results unchanged.

Table 1: Baseline Parameter Calibration

Description	Parameter	Value	Target
Time-discount factor	β	0.99	Standard value
Coefficient of relative risk aversion	σ	1.00	Standard value
Frisch elasticity of labor substitution	$1/\eta$	∞	Standard value
Elasticity of sub. across intermediate varieties	ε	6.00	Standard value
Inflation target (gross)	$1 + \bar{\pi}$	1.00	Standard value
Productivity level	$\ln \bar{A}$	1.00	Standard value
Productivity persistence	ρ_A	0.90	Standard value
Productivity shock variance	σ_A^2	0.01^2	Standard value
Interest rate shock persistence	ρ_i	0.50	Standard value
Interest rate shock variance	σ_i^2	0.0025^2	Standard value
Taylor rule inflation coefficient	ϕ_π	1.50	Baseline
Taylor rule output gap	ϕ_y	0.50/4	Baseline
Rotemberg price parameter	θ_P	29.41	Match Calvo with $\theta = \frac{2}{3}$
Rotemberg quality parameter	θ_F	29.41	Match price rigidities
Elasticity of demand to quality	ϕ	1.00	Baseline
Elasticity of sub. across productivity and quality	ν	0.8	Baseline
Quality cost	κ	0.38	Baseline

4.2 No Misspecification in Inflation Behavior

To highlight Corollary 3.1.1, the baseline model is simulated for 500 periods and the output gap and inflation are plotted against one another. This is shown in Figure 3. Panel (a) shows that even under the the case of unitary elasticity of demand with respect to quality, the Phillips curve has a lower slope when firms are able to adjust the quality of their products. This is demonstrated further in Panel (b) of the same diagram, which shows that as the elasticity of demand with respect to quality increases, to $\phi = 5$, the difference between the slope of the Phillips curve in the canonical NK model increases. Of course in the standard model this is generally ambiguous, as the response of output depends critically upon the elasticity of labor supply and the weight on output gap stabilization assigned by the central bank, as outlined in Galí (2015).



(a) Baseline, with $\phi = 1$

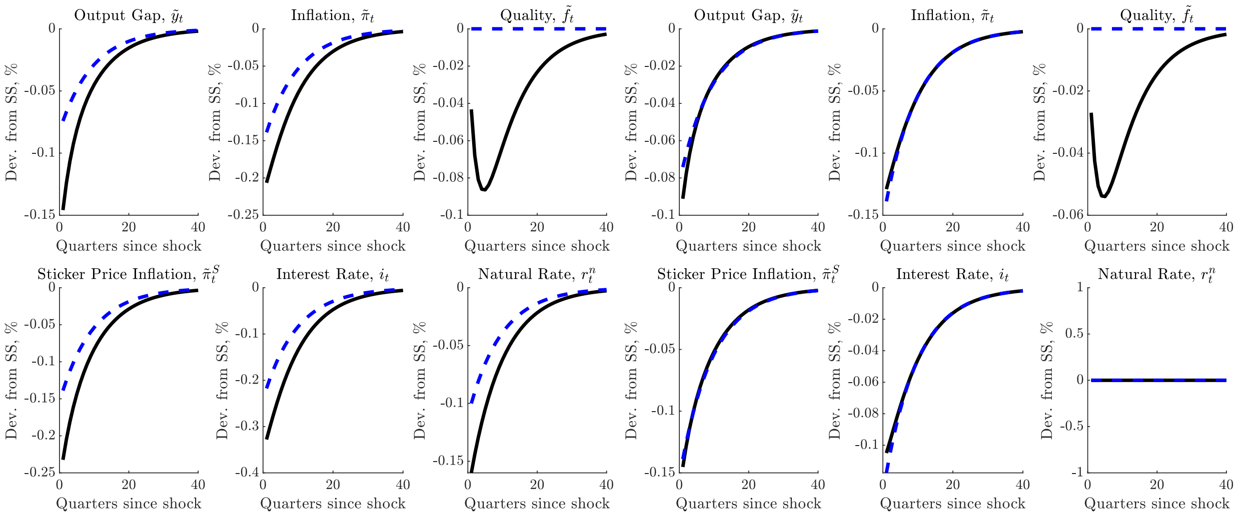
(b) Alternative, with $\phi = 5$

Figure 3: **Simulated Phillips curve relationship**
Notes: Baseline model simulated for 500 periods. ***Blue*** *dots show model implied NKPC relationship with constant quality.* ***Black*** *dots show response with adjustable quality. Panel (b) shows an alternative simulation with adjustable quality with a higher elasticity of quality demand,* $\phi = 5$.

To illustrate Theorem 3.2 for the baseline model the impulse response functions to a productivity and monetary policy shocks are shown in Figure 4. In Panel (a) productivity increases by 1%. This causes the real interest rate, inflation and the output gap to fall. The output gap falls as the current level of output rises by less than the natural level of output. The standard response of the canonical NK model are shown in **blue dashed**, while the response allowing firms to update the quality of their products is shown in **black solid** lines. Both models use a common Taylor rule. In concordance with Theorem 3.2, the impulse responses are larger when firms are able to update quality. In contrast, for monetary policy shocks, which do not alter the real interest rate the distinction between boths models is small. This is shown in Panel (b).

4.3 Monetary Policy under Mismeasurement or Model Misspecification

Until now, we have implicitly assumed that monetary policymakers are able to correctly ascertain the influence model variables have upon the economy. However, in reality, there are major empirical impediments to quality-adjusting price data at national statistical offices that limit the ability



(a) Productivity shock, $A_t \uparrow$ (b) Monetary shock, $i_t \uparrow$

Figure 4: **Impulse response functions**

*Notes: Baseline calibration. Response to a 1% increase in A_t or i_t. **Black solid** lines show response with quality. **Blue dashed** lines show response without quality.*

of policymakers to understand movements in "hedonic" terms. Ultimately these considerations can be summarized as either:

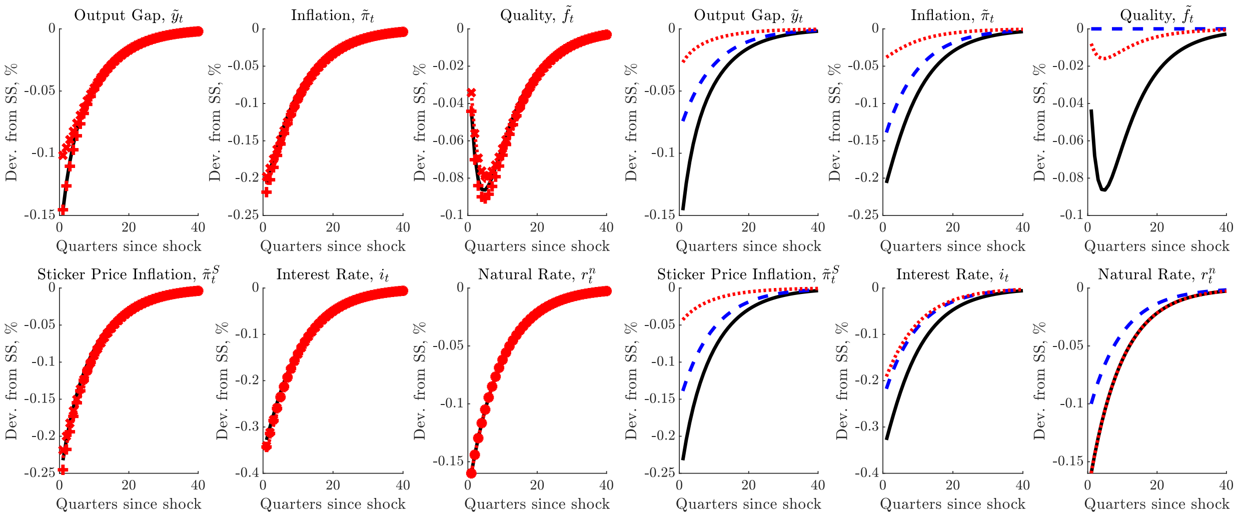
1. Bad measurement of $\tilde{\pi}_t$, $\tilde{y}_t$, or both.
2. Bad model, not accounting for quality movements.

Each case is now considered in turn.

Bad Measurement. This may arise whenever quality adjusted price indices are unavailable or unreliable. In such instances, policymakers may replace their Taylor rule with a function such as:

$$i_t = r_t^n + \phi_\pi \tilde{\pi}_t^S + \phi_y \tilde{y}_t^S, \tag{8}$$

where ϕ_π and ϕ_y are the same as before but premultiply movements in quality unadjusted variables. Figure 5, panel (a) shows that using quality unadjusted variables to replace components of the monetary policy rule has little impact on the response of the economy following productivity shocks. In particular the three **red** lines in the figure align almost exactly, and on top of the **black solid** lines. This shows that when policymakers respond to quality unadjusted variables, the economy responds in largely the same way, no matter the source of misperception. The initial response of the output gap differs when this variable is misperceived. Bad measurement of economic variables may therefore not be problem for policymakers.



(a) Bad Measurement (b) Bad Model

Figure 5: **Impulse response functions to productivity shock under misperception**
*Notes: Baseline calibration. Response to a 1% increase in A_t. **Black solid** lines show response with quality. In Panel (a) three **red** lines denote various misperceptions due to bad measurement. Respectively dots, x's and +'s correspond to mismeasurements in inflation, the output gap and both. In Panel (b) **blue dashed** lines show response without quality while **red** lines denote model misspecification.*

Bad Model. This case arises whenever policymakers follow the canonical NK model, but in reality firms are able to update both the price and quality of their goods. The relevant impulse responses for this situation are shown in **red** lines in Figure 5, panel (b), alongside the standard responses from the NK model and model with quality. After a productivity shock policymakers respond in the same way as they would in the canonical NK model. This is seen as the red and blue lines broadly align with one another for the nominal interest rate, i_t.

However, the economy responds in the same way as in the model with quality adjustment. This is seen as the red and black lines broadly align with one another for the natural real interest rate, r^n_t. This results in an over-stabilization of the economy by the policymaker. Thus the output gap and inflation both respond by less than intended, as evidenced by the fact that the red line is above blue line (which is what policymakers *intended* to do) for $\tilde{y}_t$ and $\tilde{\pi}_t$.

Although the use of a bad model leads to an unintended consequence of an over-stabilized economy, this analysis remains positive in nature. This statement says nothing about the welfare implications of such moments, which we turn to for our normative results in the next subsection.

4.4 Normative Analysis

The introduction of product quality into the otherwise standard NK model does not break the classic Blanchard and Galí (2007) divine coincidence. This is, perhaps, not surprising given that the equilibrium level of quality is stationary, and does not depend on the level of productivity. This means that the introduction of quality does not change the equilibrium path for the real interest rate which policymakers desire under full flexibility. By inspection of the reduced model, the optimal policy solution remains $i_t = r_t^n$ and is consistent with both inflation at target and a closed welfare relevant output gap.

However, this policy may not be attainable, and represents a problematic solution due to indeterminacy. Instead, conjecture that monetary policy follows a rule of the form:

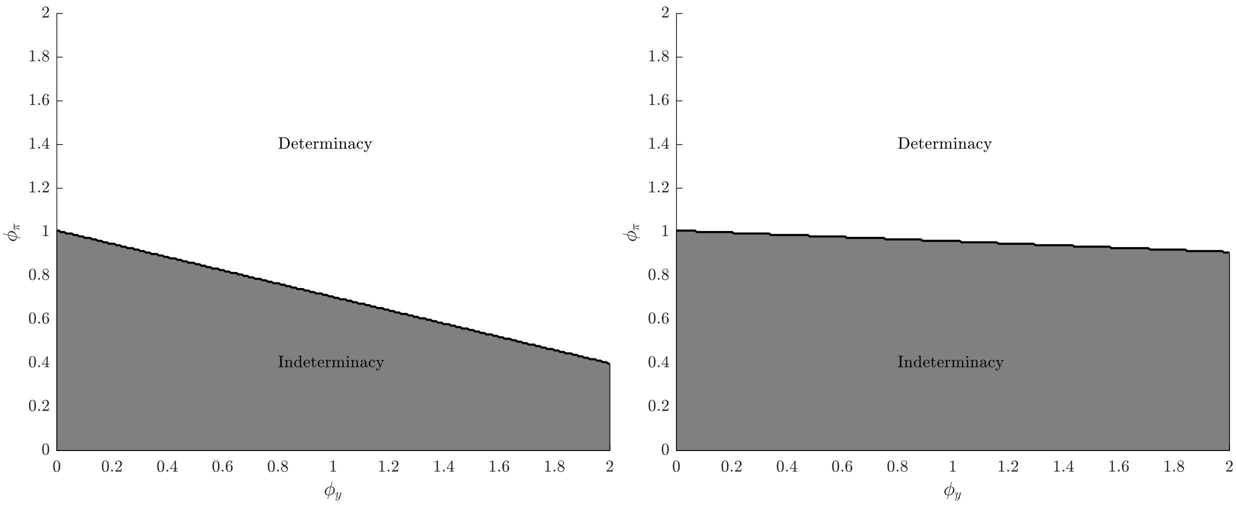
$$i_t = r_t^n + \phi_\pi \tilde{\pi}_t + \phi_y \tilde{y}_t,$$

which may restore a determinant solution to the model provided the values for ϕ_π and ϕ_y ensure policymakers respond sufficiently to any change in the natural real interest rate, r_t^n.

In a model where firms are able to update both the price and quality of their goods, the issue of interest rate rule determinacy is more problematic. This is shown graphically by comparing the determinacy regions in Figure 6. For the baseline parameterization, the inclusion of a mechanism whereby firms face costs, but are able to adjust quality reduces the severity of the indeterminacy problem facing monetary policymakers. Policymakers who would usually choose to place a relatively high weight on output gap stabilization may face an indeterminate solution whenever firms are also able to adjust quality.

5 Conclusion

In this paper we develop a tractable NK economy with sluggish adjustments in both product price and product quality. We find that relative to the canonical NK model with adjustments in product price alone, quality adjustments reduce the slope of the Phillips curve and also amplify the economy's response to productivity shocks. As a consequence, in general, monetary policy responds less to economic shocks where model misspecification of imperfectly observable quality adjustments matters more for macroeconomic stabilization than the mismeasurement of the adjustments. Under no misperceptions in product quality, nonetheless, the principles for optimal



(a) Canonical NK Model (b) Model with Quality Adjustment

Figure 6: **Determinacy regions**
Notes: Baseline calibration. Panel (a) highlights regions of model indeterminacy for a standard NK model while panel (b) shows these regions for a model with endogenous quality adjustment. Appendix E gives the explicit system to be solved for determinacy.

monetary policy are unchanged, with monetary policymakers seemingly able to jointly stabilize the output, inflation and quality movements.

References

Adam, K., and H. Weber (2019): "Optimal Trend Inflation," *American Economic Review*, 109(2), 702–737.

Aghion, P., A. Bergeaud, T. Boppart, P. J. Klenow, and H. Li (2019): "Missing Growth From Creative Destruction," *American Economic Review*, 109(8), 2795–2822.

Alchian, A. A., and W. R. Allen (1964): *University Economics.* Wadsworth.

Argente, D., M. Lee, and S. Moreira (2020): "The Life Cycle of Products: Evidence and Implications," Working Paper.

Aw, B. Y., M. J. Roberts, and D. Y. Xu (2011): "R&D Investment, Exporting, and Productivity Dynamics," *American Economic Review*, 101(4), 1312–44.

Ball, L., and S. Mazumder (2011): "Inflation Dynamics and the Great Recession," *Brookings Papers on Economic Activity*, pp. 337–81.

——— (2019): "A Phillips Curve with Anchored Expectations and Short-Term Unemployment," *Journal of Money, Credit and Banking*, 51(1), 111–137.

Barnichon, R., and G. Mesters (2020): "Identifying Modern Macro Equations With Old Shocks," *Quarterly Journal of Economics*, 135(4), 2255–2298.

Bas, M., and V. Strauss-Kahn (2015): "Input-Trade Liberalization, Export Prices and Quality Upgrading," *Journal of International Economics*, 95(2), 250–262.

Bastos, P., J. Silva, and E. Verhoogen (2018): "Export Destinations and Input Prices," *American Economic Review*, 108(2), 353–92.

Bems, R., and J. di Giovanni (2016): "Income-Induced Expenditure Switching," *American Economic Review*, 106(12), 3898–3931.

Bils, M., and P. J. Klenow (2001): "Quantifying Quality Growth," *American Economic Review*, 91(4), 1006–1030.

Blanchard, O., E. Cerutti, and L. Summers (2015): "Inflation and Activity—Two Explorations and their Monetary Policy Implications," NBER Working Paper Series.

Blanchard, O., and J. Galí (2007): "Real Wage Rigidities and the New Keynesian Model," *Journal of Money, Credit and Banking*, 39(1), 35–65.

Boskin, M., E. Dulberger, R. Gordon, Z. Griliches, and D. Jorgenson (1996): “The Boskin Commission Report: Toward a More Accurate Measure of the Cost of Living,” *Washington DC: Senate Finance Committee.*

Broda, C., and D. E. Weinstein (2010): “Product Creation and Destruction: Evidence and Price Implications,” *American Economic Review*, 100(3), 691–723.

Bustos, P. (2011): “Trade Liberalization, Exports, and Technology Upgrading: Evidence on the Impact of MERCOSUR on Argentinian Firms,” *American Economic Review*, 101(1), 304–40.

Calvo, G. A. (1983): “Staggered Prices in a Utility-Maximizing Framework,” *Journal of Monetary Economics*, 12, 383–398.

Chen, N., and L. Juvenal (2018): “Quality and the Great Trade Collapse,” *Journal of Development Economics*, 135, 59–76.

Coibion, O., and Y. Gorodnichenko (2015): “Is the Phillips Curve Alive and Well After All? Inflation Expectations and the Missing Deflation,” *American Economic Journal: Macroeconomics*, 7(1), 197–232.

De Ridder, M. (2019): “Market Power and Innovation in the Intangible Economy,” Working Paper.

Del Negro, M., M. Lenza, G. E. Primiceri, and A. Tambalotti (2020): “What’s up With the Phillips Curve?,” Working Paper.

Fajgelbaum, P., G. M. Grossman, and E. Helpman (2011): “Income Distribution, Product Quality, and International Trade,” *Journal of Political Economy*, 119(4), 721–765.

Fan, H., Y. A. Li, and S. R. Yeaple (2014): “Trade Liberalization, Quality and Export Prices,” *Review of Economics and Statistics*, 97(5), 1033–1051.

Feenstra, R. C., and J. Romalis (2014): “International Prices and Endogenous Quality,” *Quarterly Journal of Economics*, 129(2), 477–527.

Fieler, A. C., M. Eslava, and D. Xu (2014): “Trade, Skills, and Quality Upgrading: A Theory with Evidence From Colombia,” Discussion paper, National Bureau of Economic Research.

Galí, J. (2015): *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications.* Princeton University Press, 2 edn.

Garcia-Macia, D., C.-T. Hsieh, and P. J. Klenow (2019): “How Destructive Is Innovation?,” *Econometrica*, 87(5), 1507–1541.

Geerolf, F. (2019): "The Phillips Curve: A Relation Between Real Exchange Rate Growth and Unemployment," 2018 Meeting Papers.

Goetz, D., and A. Rodnyansky (2020): "Exchange Rate Shocks and Quality Adjustments," *Review of Economics and Statistics*, forthcoming.

Hall, R. E. (2013): "The Routes Into and Out of the Zero Lower Bound," *Paper presented at the Global Dimensions of Unconventional Monetary Policy Federal Reserve Bank of Kansas City Symposium, Jackson Hole, Wyoming*, pp. 1–33.

Hallak, J. C. (2006): "Product Quality and the Direction of Trade," *Journal of International Economics*, 68(1), 238–265.

Hazell, J., J. Herreño, E. Nakamura, and J. Steinsson (2020): "The Slope of the Phillips Curve: Evidence From U.s. States," Working Paper.

Hottman, C. J., S. J. Redding, and D. E. Weinstein (2016): "Quantifying the Sources of Firm Heterogeneity," *Quarterly Journal of Economics*, pp. 1291–1364.

Hummels, D., and P. J. Klenow (2005): "The Variety and Quality of a Nation's Exports," *American Economic Review*, 95(3), 704–723.

Hummels, D., and A. Skiba (2004): "Shipping the Good Apples Out? An Empirical Confirmation of the Alchian-Allen Conjecture," *Journal of Political Economy*, 112(6), 1384–1402.

Jaimovich, N., S. Rebelo, and A. Wong (2019): "Trading Down and the Business Cycle," *Journal of Monetary Economics*, 102, 96–121.

Jaimovich, N., S. Rebelo, A. Wong, and M. B. Zhang (2019): "Trading Up and the Skill Premium," *NBER Macroeconomics Annual*.

Khandelwal, A. (2010): "The Long and Short (of) Quality Ladders," *Review of Economic Studies*, 77(4), 1450–1476.

Kugler, M., and E. A. Verhoogen (2012): "Prices, Plant Size, and Product Quality," *Review of Economic Studies*, 79(1), 307–339.

Levchenko, A. A., L. T. Lewis, and L. L. Tesar (2011): "The "Collapse in Quality" Hypothesis," *American Economic Review: Papers & Proceedings*, 101(3), 293–297.

Lileeva, A., and D. Trefler (2010): "Improved Access to Foreign Markets Raises Plant-Level Productivity for Some Plants," *Quarterly Journal of Economics*, 125(3), 1051–1099.

Linder, S. B. (1961): *An Essay on Trade and Transformation.* Almqvist & Wiksell Stockholm.

Manova, K., and Z. Zhang (2012): "Export Prices Across Firms and Destinations," *Quarterly Journal of Economics*, 127(1), 379–436.

Martin, J., and I. Mejean (2014): "Low-Wage Country Competition and the Quality Content of High-Wage Country Exports," *Journal of International Economics*, 93(1), 140–152.

Medina, P. (2020): "Import Competition, Quality Upgrading and Exporting: Evidence from the Peruvian Apparel Industry," Working Paper.

Nakamura, E., and J. Steinsson (2008): "Five Facts About Prices: A Reevaluation of Menu Cost Models," *Quarterly Journal of Economics*, 123(4), 1415–1464.

——— (2012): "Lost in Transit: Product Replacement Bias and Pricing to Market," *American Economic Review*, 102(7), 3277–3316.

Rodriguez, C. A. (1979): "The Quality of Imports and the Differential Welfare Effects of Tariffs, Quotas, and Quality Controls as Protective Devices," *Canadian Journal of Economics*, 12(3), 439–449.

Rotemberg, J. J. (1982): "Sticky Prices in the United States," *Journal of Political Economy*, 90(6), 1187–1211.

Rubbo, E. (2020): "Networks, Phillips Curves, and Monetary Policy," Working Paper.

Schmitt-Grohé, S., and M. Uribe (2012): "On Quality Bias and Inflation Targets," *Journal of Monetary Economics*, 59(4), 393–400.

Simon, J., T. Matheson, and D. Sandri (2013): "The Dog That Didn't Bark: Has Inflation Been Muzzled or Was It Just Sleeping?," *World Economic Outlook*, pp. 79–95.

Stock, J. H., and M. W. Watson (2019): "Slack and Cyclically Sensitive Inflation," Working Paper.

Verhoogen, E. A. (2008): "Trade, Quality Upgrading, and Wage Inequality in the Mexican Manufacturing Sector," *Quarterly Journal of Economics*, 123(2), 489–530.

Yun, T. (1996): "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles," *Journal of Monetary Economics*, 37, 345–370.

Appendix

A NKPC Derivations

Under Rotemberg (1982) pricing the real profit flow for a given intermediate goods producer is given by:

$$\begin{aligned}\Pi_t(j) = (1+\tau_t)\frac{P_t(j)}{P_t}Y_t(j) - \frac{W_t}{P_t}N_t(j) \\ - \frac{\theta_P}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t - \frac{\theta_F}{2}\left(\frac{F_t(j)}{F_{t-1}(j)} - 1\right)^2 Y_t,\end{aligned}$$

This may be rewritten as:

$$\begin{aligned}\Pi_t(j) = (1+\tau_t)\left[\frac{P_t(j)}{P_t} - MC_t(j)\right]\mathcal{G}_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t \\ - \frac{\theta_P}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t - \frac{\theta_F}{2}\left(\frac{F_t(j)}{F_{t-1}(j)} - 1\right)^2 Y_t\end{aligned}$$

Under monopolistic competition, the full optimization problem facing an intermediate firms is then:

$$\begin{aligned}\max_{\{P_s(j),F_s(j)\}_{s=t}^{\infty}} \mathbb{E}_t\Bigg[\sum_{s=t}^{\infty}\beta^{s-t}\frac{u_C(C_s)}{u_C(C_t)}\Bigg(\left[(1+\tau_s)\frac{P_s(j)}{P_s} - MC_s(j)\right]\mathcal{G}_s(j)\left(\frac{P_s(j)}{P_s}\right)^{-\varepsilon} Y_s \\ - \frac{\theta_P}{2}\left(\frac{P_s(j)}{P_{s-1}(j)} - 1\right)^2 Y_s - \frac{\theta_F}{2}\left(\frac{F_s(j)}{F_{s-1}(j)} - 1\right)^2 Y_s\Bigg)\Bigg],\end{aligned}$$

The first order optimality condition for price setting is then given as:

$$\begin{aligned}(1+\tau_t)\mathcal{G}_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon}\frac{Y_t}{P_t} - \varepsilon\left[(1+\tau_t)\frac{P_t(j)}{P_t} - MC_s(j)\right]\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon-1}\mathcal{G}_t(j)\frac{Y_t}{P_t} \\ - \theta_P\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)\frac{Y_t}{P_{t-1}(j)} + \beta\theta_P\mathbb{E}_t\left[\frac{u_C(C_{t+1})}{u_C(C_t)}\left(\frac{P_{t+1}(j)}{P_t(j)} - 1\right)Y_{t+1}\frac{P_{t+1}(j)}{[P_t(j)]^2}\right] = 0,\end{aligned}$$

while the first order optimality condition for quality may be given as:

$$\left[(1+\tau_t)\frac{P_t(j)}{P_t} - MC_t(j)\right]\mathcal{G}_{F,t}(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t - MC_{F,t}(j)\mathcal{G}_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t$$
$$- \theta_F\left(\frac{F_t(j)}{F_{t-1}(j)} - 1\right)\frac{Y_t}{F_{t-1}(j)} + \beta\theta_F\mathbb{E}_t\left[\frac{u_C(C_{t+1})}{u_C(C_t)}\left(\frac{F_{t+1}(j)}{F_t(j)} - 1\right)Y_{t+1}\frac{F_{t+1}(j)}{[F_t(j)]^2}\right] = 0,$$

where $MC_{F,t}(j) \equiv \frac{\partial MC_t(j)}{\partial F_t(j)} = -\frac{W_t}{P_t}\frac{\mathcal{H}_{F,t}(j)}{[\mathcal{H}_t(j)]^2} = -MC_t(j)\frac{\mathcal{H}_{F,t}(j)}{\mathcal{H}_t(j)}$. The final equality is used in the above to simplify the expression. We also simplify by multiply the first expression by $\frac{P_t(j)}{Y_t}$ and the second by $\frac{F_t(j)}{Y_t}$, in addition to using the functional form of the utility function to give $u_C(C_t) = C_t^{-\sigma}$. We then observe these expressions as:

$$(1+\tau_t)\mathcal{G}_t(j)\left(\frac{P_t(j)}{P_t}\right)^{1-\varepsilon} - \varepsilon\left[(1+\tau_t)\frac{P_t(j)}{P_t} - MC_t(j)\right]\mathcal{G}_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon}$$
$$- \theta_P\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)\frac{P_t(j)}{P_{t-1}(j)} + \beta\theta_P\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\left(\frac{P_{t+1}(j)}{P_t(j)} - 1\right)\frac{P_{t+1}(j)}{P_t(j)}\right] = 0,$$
$$\left[(1+\tau_t)\frac{P_t(j)}{P_t} - MC_t(j)\right]\mathcal{G}_{F,t}(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} + MC_t(j)\frac{\mathcal{H}_{F,t}(j)}{\mathcal{H}_t(j)}\mathcal{G}_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon}$$
$$- \theta_F\left(\frac{F_t(j)}{F_{t-1}(j)} - 1\right)\frac{F_t(j)}{F_{t-1}(j)} + \beta\theta_F\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\left(\frac{F_{t+1}(j)}{F_t(j)} - 1\right)\frac{Y_{t+1}}{Y_t}\frac{F_{t+1}(j)}{F_t(j)}\right] = 0.$$

Faced with symmetric first order conditions, all firms behave identically in equilibrium. All intermediate firms will charge the same price and produce the same output at the same level of quality. We may therefore write the aggregate price level as a function of the individual level of prices:

$$P_t = [\mathcal{G}_t(j)]^{\frac{1}{1-\varepsilon}}P_t(j).$$

Eliminating individual level prices, as $P_t(j) = [\mathcal{G}_t(j)]^{\frac{1}{\varepsilon-1}}P_t$, and dropping the individual indexation on quality the two first order conditions may be written as:

$$(1-\varepsilon)(1+\tau_t) + \varepsilon MC_t\mathcal{G}_t^{-\frac{1}{\varepsilon-1}} - \theta_P\left(\left[\frac{\mathcal{G}_t}{\mathcal{G}_{t-1}}\right]^{\frac{1}{\varepsilon-1}}\frac{P_t}{P_{t-1}} - 1\right)\left[\frac{\mathcal{G}_t}{\mathcal{G}_{t-1}}\right]^{\frac{1}{\varepsilon-1}}\frac{P_t}{P_{t-1}}$$
$$+ \beta\theta_P\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\left(\left[\frac{\mathcal{G}_{t+1}}{\mathcal{G}_t}\right]^{\frac{1}{\varepsilon-1}}\frac{P_{t+1}}{P_t} - 1\right)\left[\frac{\mathcal{G}_{t+1}}{\mathcal{G}_t}\right]^{\frac{1}{\varepsilon-1}}\frac{P_{t+1}}{P_t}\right] = 0,$$
$$(1+\tau_t)\mathcal{G}_{F,t}\mathcal{G}_t^{-1} - MC_t\mathcal{G}_{F,t}\mathcal{G}_t^{-\frac{\varepsilon}{\varepsilon-1}} + MC_t\frac{\mathcal{H}_{F,t}}{\mathcal{H}_t}\mathcal{G}_t^{-\frac{1}{\varepsilon-1}}$$

$$
-\theta_F\left(\frac{F_t}{F_{t-1}}-1\right)\frac{F_t}{F_{t-1}}+\beta\theta_F\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\left(\frac{F_{t+1}}{F_t}-1\right)\frac{F_{t+1}}{F_t}\right]=0.
$$

Next, returning to our definition of price inflation as $1+\pi_t \equiv \frac{P_t}{P_{t-1}}$ and defining the growth rate of quality similarly as $1+\pi_t^F \equiv \frac{F_t}{F_{t-1}}$. The two equations then become:

$$
\begin{aligned}
(\varepsilon-1)(1+\tau_t) &= \varepsilon MC_t\mathcal{G}_t^{-\frac{1}{\varepsilon-1}}-\theta_P\left(\left[\frac{\mathcal{G}_t}{\mathcal{G}_{t-1}}\right]^{\frac{1}{\varepsilon-1}}(1+\pi_t)-1\right)\left[\frac{\mathcal{G}_t}{\mathcal{G}_{t-1}}\right]^{\frac{1}{\varepsilon-1}}(1+\pi_t)\\
&+\beta\theta_P\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\left(\left[\frac{\mathcal{G}_{t+1}}{\mathcal{G}_t}\right]^{\frac{1}{\varepsilon-1}}(1+\pi_{t+1})-1\right)\left[\frac{\mathcal{G}_{t+1}}{\mathcal{G}_t}\right]^{\frac{1}{\varepsilon-1}}(1+\pi_{t+1})\right], \qquad \text{(NKPC P)}\\
(1+\tau_t)\mathcal{G}_{F,t}\mathcal{G}_t^{-1} &= MC_t\mathcal{G}_t^{-\frac{1}{\varepsilon-1}}\left(\mathcal{G}_{F,t}\mathcal{G}_t^{-1}-\frac{\mathcal{H}_{F,t}}{\mathcal{H}_t}\right)+\theta_F\pi_t^F(1+\pi_t^F)\\
&-\beta\theta_F\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\pi_{t+1}^F(1+\pi_{t+1}^F)\right]. \qquad \text{(NKPC F)}
\end{aligned}
$$

Together these equations will form the New-Keynesian Phillips curve for Prices (NKPC P) and quality (NKPC F). Notice that our equilibrium condition linking aggregate and individual level prices, $P_t(j)=[\mathcal{G}_t(j)]^{\frac{1}{\varepsilon-1}}P_t$, may also be written as a rate of change to then we denote sticker price inflation, π_t^S, (the price inflation of an individual firm) as :

$$
1+\pi_t^S \equiv \left[\frac{\mathcal{G}_t}{\mathcal{G}_{t-1}}\right]^{\frac{1}{\varepsilon-1}}(1+\pi_t),
$$

The NKPC P condition may then be cast in terms of sticker price inflation, rather than the rate of change of aggregate consumer prices as.

$$
\begin{aligned}
(\varepsilon-1)(1+\tau_t) &= \varepsilon MC_t\mathcal{G}_t^{-\frac{1}{\varepsilon-1}}-\theta_P\pi_t^S(1+\pi_t^S)\\
&+\beta\theta_P\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\pi_{t+1}^S(1+\pi_{t+1}^S)\right]. \qquad \text{(NKPC P)}
\end{aligned}
$$

B The Social Planners Problem

Under flexible prices and quality, the first best allocation will solve the problem of maximising household utility, given production constraints. The Lagrangian for this allocation is therefore:

$$\mathcal{L} = \max_{C,F,L} \left\{ \frac{C^{1-\sigma}}{1-\sigma} - \frac{L^{1+\eta}}{1+\eta} - \lambda \left(C - \mathcal{G}^{\frac{1}{\varepsilon-1}} \mathcal{H} L \right) \right\},$$

with efficient solution:

$$\begin{aligned} \frac{1}{\varepsilon - 1} &= -\frac{\mathcal{H}_F}{\mathcal{H}} \frac{\mathcal{G}}{\mathcal{G}_F}, \\ L &= \left(\mathcal{G}^{\frac{1}{\varepsilon-1}} \mathcal{H} \right)^{\frac{1-\sigma}{\sigma+\eta}}, \\ C &= \left(\mathcal{G}^{\frac{1}{\varepsilon-1}} \mathcal{H} \right)^{\frac{1+\eta}{\sigma+\eta}}. \end{aligned}$$

which may be solved once the functions are $\mathcal{G}$ and $\mathcal{H}$ are specified. Given that the household optimality condition for consumption and leisure determines the equilibrium real wage, the efficient level of intermediate firms marginal costs are therefore given as:

$$MC = \frac{W}{P} \frac{1}{\mathcal{H}} = \frac{C^{\sigma} L^{\eta}}{\mathcal{H}} = \mathcal{G}^{\frac{1}{\varepsilon-1}}.$$

C Full Set of Model Equations

Table 2 sets out the complete list of 12 endogenous model variables, which are solved by the set of equations that follow.

Table 2: Endogenous Variables

Variable Type	Variable List					Count
Household	C_t	L_t				2
Firms	Y_t	MC_t	π_t^S	F_t	π_t^F	5
Equ/Ex	$\frac{W_t}{P_t}$	i_t	A_t	π_t	τ_t	5
Total						12

Household block:

$$\frac{W_t}{P_t} = C_t^{\sigma} L_t^{\eta}, \tag{9}$$

$$1 = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1 + i_t}{1 + \pi_{t+1}} \right]. \tag{10}$$

Firm block:

$$Y_t = \mathcal{G}_t^{\frac{1}{\varepsilon - 1}} L_t \mathcal{H}_t, \tag{11}$$

$$MC_t = \frac{W_t}{P_t} \frac{1}{\mathcal{H}_t}, \tag{12}$$

$$\begin{aligned}(\varepsilon - 1)(1 + \tau_t) = {} & \varepsilon MC_t \mathcal{G}_t^{-\frac{1}{\varepsilon - 1}} - \theta_P \pi_t^S (1 + \pi_t^S) \\ & + \beta \theta_P \mathbb{E}_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{Y_{t+1}}{Y_t} \pi_{t+1}^S (1 + \pi_{t+1}^S) \right],\end{aligned} \tag{13}$$

$$\begin{aligned}(1 + \tau_t) \mathcal{G}_{F,t} \mathcal{G}_t^{-1} = {} & MC_t \mathcal{G}_t^{-\frac{1}{\varepsilon - 1}} \left(\mathcal{G}_{F,t} \mathcal{G}_t^{-1} - \frac{\mathcal{H}_{F,t}}{\mathcal{H}_t} \right) + \theta_F \pi_t^F (1 + \pi_t^F) \\ & - \beta \theta_F \mathbb{E}_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{Y_{t+1}}{Y_t} \pi_{t+1}^F (1 + \pi_{t+1}^F) \right],\end{aligned} \tag{14}$$

$$1 + \pi_t^F = \frac{F_t}{F_{t-1}}. \tag{15}$$

Market clearing and exogenous block:

$$Y_t = C_t + \frac{\theta_P}{2}(\pi_t^S)^2 Y_t + \frac{\theta_F}{2}(\pi_t^F)^2 Y_t, \tag{16}$$

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_t, \tag{17}$$

$$1 + \pi_t^S = \left[\frac{\mathcal{G}_t}{\mathcal{G}_{t-1}}\right]^{\frac{1}{\varepsilon-1}} (1 + \pi_t), \tag{18}$$

$$1 + \tau_t = \frac{\varepsilon}{\varepsilon - 1}. \tag{19}$$

Finally, the model is closed by specifying a Taylor rule for the equilibrium movements of the nominal interest rate. This is assumed to be of the form:

$$i_t - \bar{\pi} - \rho = \phi_\pi(\pi_t - \bar{\pi}), \tag{20}$$

where $\rho \equiv -\ln\beta$ and $\phi_\pi > 1$, obeying the Taylor principle.

C.1 Steady State Solution

The steady state of the model may be computed as follows. Together the Euler condition and the policy rule determine that inflation is the target value, which is assumed to be $\bar{\pi} = 0$, and the nominal interest rate as $\bar{i} = \bar{\pi} + \rho$. In a steady state $\bar{\pi}^S = \bar{\pi}^F = 0$ and technology is at its equilibrium value, $\bar{A}$, while the production subsidy is constant at $\bar{\tau} = \frac{\varepsilon}{\varepsilon-1}$. The market clearing condition then infers $\bar{Y} = \bar{C}$. Altogether this reduces the model to the following system:

$$\frac{\bar{W}}{\bar{P}} = \bar{Y}^\sigma \bar{L}^\eta, \tag{21}$$

$$\bar{Y} = \bar{\mathcal{G}}^{\frac{1}{\varepsilon-1}} \bar{L} \bar{\mathcal{H}} \tag{22}$$

$$\bar{MC} = \frac{\bar{W}}{\bar{P}} \frac{1}{\bar{\mathcal{H}}}, \tag{23}$$

$$\bar{MC} = \bar{\mathcal{G}}^{\frac{1}{\varepsilon-1}}, \tag{24}$$

$$\varepsilon \bar{\mathcal{G}}_F \bar{\mathcal{G}}^{-1} = (\varepsilon - 1)\bar{MC}\bar{\mathcal{G}}^{-\frac{1}{\varepsilon-1}} \left(\bar{\mathcal{G}}_F \bar{\mathcal{G}}^{-1} - \frac{\bar{\mathcal{H}}_F}{\bar{\mathcal{H}}}\right) \tag{25}$$

Simultaneously solving the final two equations together:

$$\frac{1}{\varepsilon - 1} = -\frac{\bar{\mathcal{H}}_F}{\bar{\mathcal{H}}} \frac{\bar{\mathcal{G}}}{\bar{\mathcal{G}}_F},$$

such that in equilibrium quality is at the efficient level. This may depend upon the steady state level of productivity, $\bar{A}$.

The remainder of the model may then be solved recursively as:

$$
\begin{aligned}
\bar{MC} &= \bar{\mathcal{G}}^{\frac{1}{\varepsilon-1}}, \\
\frac{\bar{W}}{\bar{P}} &= \bar{MC}\bar{\mathcal{H}}, \\
\bar{Y} &= \left[\left(\frac{\bar{W}}{\bar{P}}\right)^{\frac{1}{\eta}} \bar{\mathcal{G}}^{\frac{1}{\varepsilon-1}} \bar{\mathcal{H}}\right]^{\frac{\eta}{\eta+\sigma}}, \\
\bar{L} &= \left(\frac{\bar{W}}{\bar{P}}\right)^{\frac{1}{\eta}} \bar{Y}^{-\frac{\sigma}{\eta}},
\end{aligned}
$$

thus replicating the efficient allocations.

D Derivation of a 4 Equation Model

This appendix firstly log-linearises the model before combining equations to present a model of only four equations similar to the canonical New Keynesian model. For clarity, wherever possible derivations follow those presented in Galí (2015).

Notation. Bars denote steady state levels, $\bar{x}$, hats denote percentage deviation from steady state level, $\hat{x}$, ($\approx$ to log deviations). Tildes denote differences from natural rates, $\tilde{x}$. Variables are written in upper case with their logarithm in lower case.

D.1 Household Block

Written in logs the first order condition for consumption and labor trade-off, equation 9, is:

$$\ln W_t - \ln P_t = \sigma \ln C_t + \eta \ln L_t,$$
$$w_t = \sigma c_t + \eta \ell_t.$$

Note the defined of log real wages as $w_t \equiv \ln W_t - \ln P_t$. A log-linear approximation around the steady state of equation 10 shows:

$$\begin{aligned} 1 = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1+i_t}{1+\pi_{t+1}} \right] &= \mathbb{E}_t \left[e^{(\ln \beta - \sigma \ln C_{t+1} + \sigma \ln C_t + \ln(1+i_t) - \ln(1+\pi_{t+1}))} \right] \\ &= \mathbb{E}_t \left[e^{(i_t - \rho - \sigma \Delta c_{t+1} - \pi_{t+1})} \right], \end{aligned}$$

where in the equality defines $\rho \equiv -\ln \beta$, and use a first order Taylor approximation which gives that for small values of i_t and π_{t+1}, we have $\ln(1+i_t) \approx i_t$ and $\ln(1+\pi_{t+1}) \approx \pi_{t+1}$. In an economy with perfect foresight, constant inflation, π, and constant consumption growth, γ, this equation would determines the equilibrium nominal interest rate as:

$$i = \rho + \sigma\gamma + \pi.$$

A first-order Taylor approximation of $\exp(i_t - \rho - \sigma \Delta c_{t+1} - \pi_{t+1})$ around this steady state results in:

$$\exp(i_t - \sigma \Delta c_{t+1} - \pi_{t+1} - \rho) \approx 1 + (i_t - i) - \sigma(\Delta c_{t+1} - \gamma) - (\pi_{t+1} - \pi),$$

$$\approx 1 + i_t - \sigma\Delta c_{t+1} - \pi_{t+1} - \rho,$$

which, when used in the first order condition to yields the log-linear approximation.

$$\begin{aligned}
1 &= \mathbb{E}_t\left[e^{(i_t-\rho-\sigma\Delta c_{t+1}-\pi_{t+1})}\right],\\
1 &\approx \mathbb{E}_t\left[1 + i_t - \sigma\Delta c_{t+1} - \pi_{t+1} - \rho\right],\\
i_t &\approx \mathbb{E}_t[\sigma\Delta c_{t+1} + \pi_{t+1}] + \rho,\\
c_t &\approx \mathbb{E}_t[c_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] - \rho).
\end{aligned}$$

D.2 Firm Block

Given the functional forms specified in section 3.1 of the main text, the aggregate production function (equation 11), marginal costs (equation 12) and definition of quality improvement (equation 15) may be specified as:

$$\begin{aligned}
Y_t &= F_t^{\frac{\phi}{\varepsilon-1}} L_t \left[A_t^{\frac{\nu-1}{\nu}} - \kappa F_t^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}},\\
MC_t &= \frac{W_t}{P_t}\left[A_t^{\frac{\nu-1}{\nu}} - \kappa F_t^{\frac{\nu-1}{\nu}}\right]^{-\frac{\nu}{\nu-1}},\\
1 + \pi_t^F &= \frac{F_t}{F_{t-1}}.
\end{aligned}$$

Each equation may then be written down directly, in logs, to give:

$$\begin{aligned}
y_t &= \ell_t + \frac{\phi}{\varepsilon-1} f_t + \frac{\nu}{\nu-1}\ln\left[e^{\frac{\nu-1}{\nu}a_t} - \kappa e^{\frac{\nu-1}{\nu}f_t}\right],\\
mc_t &= w_t - \frac{\nu}{\nu-1}\ln\left[e^{\frac{\nu-1}{\nu}a_t} - \kappa e^{\frac{\nu-1}{\nu}f_t}\right],\\
\pi_t^F &= f_t - f_{t-1}.
\end{aligned}$$

where the definition for the log of real wages is used, introduced above, is used again and an approximation that for small values $\ln(1+\pi_t^F) \approx \pi_t^F$. The first two equations are still non-linear, but around the steady state approximately become:

$$\hat{y}_t \approx \hat{\ell}_t + \frac{\phi}{\varepsilon-1}\hat{f}_t + \frac{\hat{a}_t - \kappa e^{\frac{\nu-1}{\nu}\bar{f}}\hat{f}_t}{1 - \kappa e^{\frac{\nu-1}{\nu}\bar{f}}},$$

$$\hat{mc}_t \approx \hat{w}_t - \frac{\hat{a}_t - \kappa e^{\frac{\nu-1}{\nu}\bar{f}}\hat{f}_t}{1 - \kappa e^{\frac{\nu-1}{\nu}\bar{f}}}.$$

These may be simplified using equation 1 to note that $\bar{f} = \frac{\nu}{\nu-1}\ln\left[\frac{\phi}{\kappa(\varepsilon-1)+\kappa\phi}\right]$, such that $\kappa e^{\frac{\nu-1}{\nu}\bar{f}} = \frac{\kappa\phi}{\kappa(\varepsilon-1)+\kappa\phi}$ and $1 - \kappa e^{\frac{\nu-1}{\nu}\bar{f}} = \frac{\kappa(\varepsilon-1)}{\kappa(\varepsilon-1)+\kappa\phi}$ to show:

$$\begin{aligned}
\hat{y}_t &\approx \hat{\ell}_t + \frac{(\varepsilon-1)+\phi}{(\varepsilon-1)}\hat{a}_t,\\
\hat{mc}_t &\approx \hat{w}_t - \frac{(\varepsilon-1)+\phi}{(\varepsilon-1)}\hat{a}_t + \frac{\phi}{(\varepsilon-1)}\hat{f}_t.
\end{aligned}$$

Next attention turns to log-linearization of the two NKPC equations, performed around the point $\pi_t = \pi_t^S = \pi_t^F = \bar{\pi} = \bar{\pi}^S = \bar{\pi}^F = 0$. Starting with NKPC P, and using the functional forms from section 3.1 of the main text, this approximation may be written as:

$$\begin{aligned}
(\varepsilon-1)(1+\tau_t) &= \varepsilon MC_t F_t^{-\frac{\phi}{\varepsilon-1}} - \theta_P \pi_t^S(1+\pi_t^S) + \beta\theta_P \mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\pi_{t+1}^S(1+\pi_{t+1}^S)\right],\\
\ln[(\varepsilon-1)(1+\tau_t)] &= \ln\Bigg[\varepsilon MC_t F_t^{-\frac{\phi}{\varepsilon-1}} - \theta_P \pi_t^S(1+\pi_t^S)\\
&\qquad + \beta\theta_P \mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\pi_{t+1}^S(1+\pi_{t+1}^S)\right]\Bigg],\\
(\varepsilon-1)(\tau_t - \bar{\tau}) &\approx \varepsilon\bar{MC}\bar{F}^{-\frac{\phi}{\varepsilon-1}}\left(\hat{MC}_t - \frac{\phi}{\varepsilon-1}\hat{F}_t\right)\\
&\quad - \theta_P\bar{\pi}^S(1+\pi_t^S - 1 - \bar{\pi}^S) - \theta_P(\pi_t^S - \bar{\pi}^S)(1+\bar{\pi}^S)\\
&\quad + \beta\theta_P\bar{\pi}^S\mathbb{E}_t\left[1+\pi_{t+1}^S - 1 - \bar{\pi}^S\right] + \beta\theta_P\mathbb{E}_t\left[\pi_{t+1}^S - \bar{\pi}^S\right](1+\bar{\pi}^S) + 0,\\
0 &\approx \varepsilon\bar{MC}\bar{F}^{-\frac{\phi}{\varepsilon-1}}\left(\hat{mc}_t - \frac{\phi}{\varepsilon-1}\hat{f}_t\right) - \theta_P\hat{\pi}_t^S + \beta\theta_P\mathbb{E}_t\left[\hat{\pi}_{t+1}^S\right].
\end{aligned}$$

In the steady state, when fiscal policymakers implement the revenue subsidy policy $\bar{MC}\bar{\mathcal{G}}^{-\frac{1}{\varepsilon-1}} = 1$, such that the above expression may be rewritten as:

$$\hat{\pi}_t^S \approx \frac{\varepsilon}{\theta_P}\left(\hat{mc}_t - \frac{\phi}{\varepsilon-1}\hat{f}_t\right) + \beta\mathbb{E}_t\left[\hat{\pi}_{t+1}^S\right]. \qquad \text{(Log Linearized NKPC P)}$$

When quality is constant at the steady state level $\hat{f}_t = 0$ and $\pi_t^S = \pi_t$, which infers the Rotemberg (1982) version of the New Keynesian Phillips curve is a specific case of the general form, above.

We may also see this by rewriting the first expressions as:

$$\hat{\pi}_t^S = \frac{\varepsilon}{\theta_P}\sum_{s=t}^{\infty}\beta^{s-t}\mathbb{E}_t\left[\hat{mc}_s + \frac{\phi}{\varepsilon-1}\hat{f}_s\right].$$

This is once more consistent with deviations of price inflation being equal to the discounted value of marginal cost deviations, whenever the model is simplifies with quality remaining at the steady state value.

Finally, an approximation of NKPC F given these functional forms follows as:

$$\begin{aligned}
\phi(1+\tau_t)F_t^{-1} &= MC_t F_t^{-\frac{\phi}{\varepsilon-1}}\left(\frac{\phi}{F_t} + \frac{\kappa}{F_t^{\frac{1}{\nu}}A_t^{\frac{\nu-1}{\nu}} - \kappa F_t}\right) + \theta_F \pi_t^F(1+\pi_t^F) \\
&\quad - \beta\theta_F \mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\pi_{t+1}^F(1+\pi_{t+1}^F)\right], \\
\ln[\phi(1+\tau_t)] - f_t &= \ln\Bigg[MC_t F_t^{-\frac{\phi}{\varepsilon-1}}\left(\frac{\phi}{F_t} + \frac{\kappa}{F_t^{\frac{1}{\nu}}A_t^{\frac{\nu-1}{\nu}} - \kappa F_t}\right) + \theta_F \pi_t^F(1+\pi_t^F) \\
&\quad - \beta\theta_F \mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\pi_{t+1}^F(1+\pi_{t+1}^F)\right]\Bigg] \\
-\left(\ln[\phi(1+\bar{\tau})] - \bar{f}\right)\hat{f}_t &\approx \bar{MC}\bar{F}^{-\frac{\phi}{\varepsilon-1}}\left(\frac{\phi}{\bar{F}} + \frac{\kappa}{\bar{F}^{\frac{1}{\nu}} - \kappa\bar{F}}\right)\left(\hat{mc}_t - \frac{\phi}{\varepsilon-1}\hat{f}_t\right) \\
&\quad + \bar{MC}\bar{F}^{-\frac{\phi}{\varepsilon-1}}\left(-\frac{\phi}{\bar{F}}\hat{f}_t - \frac{\kappa}{(\bar{F}^{\frac{1}{\nu}} - \kappa\bar{F})^2}(\hat{a}_t + \bar{F}^{\frac{1}{\nu}}\hat{f}_t - \kappa\bar{F}\hat{f}_t)\right) \\
&\quad - \theta_F\hat{\pi}_t^F + \beta\theta_F\mathbb{E}_t\left[\hat{\pi}_{t+1}^F\right].
\end{aligned}$$

Using the equilibrium observation $1 = \bar{MC}\bar{F}^{-\frac{\phi}{\varepsilon-1}}$ can simplify the above expression to:

$$\begin{aligned}
-\left(\ln[\phi(1+\bar{\tau})] - \bar{f}\right)\hat{f}_t &\approx \left(\frac{\phi}{\bar{F}} + \frac{\kappa}{\bar{F}^{\frac{1}{\nu}} - \kappa\bar{F}}\right)\left(\hat{mc}_t - \frac{\phi+(\varepsilon-1)}{\varepsilon-1}\hat{f}_t\right) \\
&\quad - \frac{\kappa}{(\bar{F}^{\frac{1}{\nu}} - \kappa\bar{F})^2}\hat{a}_t - \theta_F\hat{\pi}_t^F + \beta\theta_F\mathbb{E}_t\left[\hat{\pi}_{t+1}^F\right], \\
\theta_F\hat{\pi}_t^F &\approx \left(\frac{\phi}{\bar{F} - \kappa\bar{F}^{2-\frac{1}{\nu}}}\right)\left(\hat{mc}_t - \frac{\phi+(\varepsilon-1)}{\varepsilon-1}\hat{f}_t\right) \\
&\quad + \left(\ln[\phi(1+\bar{\tau})] - \bar{f}\right)\hat{f}_t - \frac{\kappa}{(\bar{F}^{\frac{1}{\nu}} - \kappa\bar{F})^2}\hat{a}_t + \beta\theta_F\mathbb{E}_t\left[\hat{\pi}_{t+1}^F\right].
\end{aligned}$$

(Log Linearized NKPC F)

D.3 Market Clearing and Exogenous Block

The aggregate goods market condition may be written as

$$
\begin{aligned}
e^{\ln Y_t} &= e^{\ln C_t} + \frac{\theta_P}{2}(\pi_t^S)^2 e^{\ln Y_t} + \frac{\theta_F}{2}(\pi_t^F)^2 e^{\ln Y_t}, \\
e^{y_t} &= e^{c_t} + \frac{\theta_P}{2}(\pi_t^S)^2 e^{y_t} + \frac{\theta_F}{2}(\pi_t^F)^2 e^{y_t}, \\
e^{\bar{y}} + e^{\bar{y}}(y_t - \bar{y}) &\approx e^{\bar{c}} + e^{\bar{c}}(c_t - \bar{c}),
\end{aligned}
$$

where we note that the approximation is about the point $\pi_t = \pi_t^S = \pi_t^F = \bar{\pi} = \bar{\pi}^S = \bar{\pi}^F = 1$. Since in equilibrium $\bar{Y} = \bar{C}$ we then have:

$$\hat{y}_t \approx \hat{c}_t.$$

The technology process is already written in logs and becomes

$$a_t = \rho_A a_{t-1} + \varepsilon_t,$$

while, in deviations, the definition of "sticker" price inflation becomes:

$$\hat{\pi}_t^S = \frac{\phi}{\varepsilon - 1}\hat{\pi}_t^F + \hat{\pi}_t.$$

The monetary policy rule remains unchanged.

D.4 Full Set of Approximated Equations

The full set of exact and approximated equations is thus given as follows:

Household block:

$$w_t = \sigma c_t + \eta \ell_t, \tag{26}$$

$$c_t \approx \mathbb{E}_t[c_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] - \rho). \tag{27}$$

Firm block:

$$\hat{y}_t \approx \hat{\ell}_t + \frac{(\varepsilon - 1) + \phi}{(\varepsilon - 1)}\hat{a}_t, \tag{28}$$

$$\hat{mc}_t \approx w_t - \frac{(\varepsilon-1)+\phi}{(\varepsilon-1)}\hat{a}_t + \frac{\phi}{(\varepsilon-1)}\hat{f}_t, \tag{29}$$

$$\hat{\pi}_t^S \approx \frac{\varepsilon}{\theta_P}\left(\hat{mc}_t - \frac{\phi}{\varepsilon-1}\hat{f}_t\right) + \beta\mathbb{E}_t\left[\hat{\pi}_{t+1}^S\right], \tag{30}$$

$$\begin{aligned}\theta_F\hat{\pi}_t^F \approx & \left(\frac{\phi}{\bar{F}-\kappa\bar{F}^{2-\frac{1}{\nu}}}\right)\left(\hat{mc}_t - \frac{\phi+(\varepsilon-1)}{\varepsilon-1}\hat{f}_t\right) \\ & + \left(\ln[\phi(1+\bar{\tau})]-\bar{f}\right)\hat{f}_t - \frac{\kappa}{(\bar{F}^{\frac{1}{\nu}}-\kappa\bar{F})^2}\hat{a}_t + \beta\theta_F\mathbb{E}_t\left[\hat{\pi}_{t+1}^F\right],\end{aligned} \tag{31}$$

$$\hat{\pi}_t^F = \hat{f}_t - \hat{f}_{t-1}. \tag{32}$$

Market clearing and exogenous block:

$$\hat{y}_t \approx \hat{c}_t, \tag{33}$$

$$a_t = \rho_A a_{t-1} + \varepsilon_t, \tag{34}$$

$$\hat{\pi}_t^S = \frac{\phi}{\varepsilon-1}\hat{\pi}_t^F + \hat{\pi}_t. \tag{35}$$

Along with the monetary policy and tax rate rules these 10 equations solve fully describe the model.

D.5 Model Reduction

Combining the market clearing condition and household optimality condition for labor gives:

$$\hat{w}_t = \sigma\hat{y}_t + \eta\hat{\ell}_t,$$

which may then be used alongside the marginal cost function to give:

$$\hat{mc}_t = \sigma\hat{y}_t + \eta\hat{\ell}_t - \frac{(\varepsilon-1)+\phi}{(\varepsilon-1)}\hat{a}_t + \frac{\phi}{(\varepsilon-1)}\hat{f}_t,$$

The aggregate production function may then be used to eliminate labor to give:

$$\hat{mc}_t = \sigma\hat{y}_t + \eta\left[\hat{y}_t - \frac{(\varepsilon-1)+\phi}{(\varepsilon-1)}\hat{a}_t\right] - \frac{(\varepsilon-1)+\phi}{(\varepsilon-1)}\hat{a}_t + \frac{\phi}{(\varepsilon-1)}\hat{f}_t,$$

$$\hat{mc}_t = (\sigma+\eta)\hat{y}_t - (\eta+1)\frac{(\varepsilon-1)+\phi}{(\varepsilon-1)}\hat{a}_t + \frac{\phi}{\varepsilon-1}\hat{f}_t.$$

This relationship holds in the economy no matter what degree of price, and quality, stickiness. In particular, defining the deviation of the flexible equilibrium solution to the model as $\{\hat{a}_t, \hat{y}_t^n, \hat{f}_t^n, \cdots\}$, shows that since:

$$\begin{aligned} f_t^n &= a_t + \frac{\nu}{\nu-1} \ln\left[\frac{\phi}{\kappa(\varepsilon-1)+\kappa\phi}\right], \\ mc_t^n &= \frac{\phi}{\varepsilon-1} f_t^n. \end{aligned}$$

Therefore, given that $\hat{f}_t^n = \hat{a}_t$ and thus $\hat{mc}_t^n = \frac{\phi}{\varepsilon-1}\hat{a}_t$, this implies:

$$\begin{aligned} \hat{mc}_t^n &= (\sigma+\eta)\hat{y}_t^n - (\eta+1)\frac{(\varepsilon-1)+\phi}{(\varepsilon-1)}\hat{a}_t + \frac{\phi}{\varepsilon-1}\hat{f}_t^n, \\ \frac{\phi}{\varepsilon-1}\hat{a}_t &= (\sigma+\eta)\hat{y}_t^n - (\eta+1)\frac{(\varepsilon-1)+\phi}{(\varepsilon-1)}\hat{a}_t + \frac{\phi}{\varepsilon-1}\hat{a}_t, \\ 0 &= (\sigma+\eta)\hat{y}_t^n - (\eta+1)\frac{(\varepsilon-1)+\phi}{(\varepsilon-1)}\hat{a}_t, \\ \hat{y}_t^n &= \frac{\eta+1}{\sigma+\eta}\frac{(\varepsilon-1)+\phi}{(\varepsilon-1)}\hat{a}_t. \end{aligned}$$

Alternatively:

$$y_t^n = \psi_a a_t + \psi.$$

where $\psi_a \equiv \frac{\eta+1}{\sigma+\eta}\frac{(\varepsilon-1)+\phi}{(\varepsilon-1)}$ and $\psi \equiv \bar{y}$. This is a larger parameter than in the previous case. This then gives:

$$mc_t - mc^n \equiv \tilde{mc}_t = \hat{mc}_t - \hat{mc}_t^n = (\sigma+\eta)\tilde{y}_t + \frac{\phi}{\varepsilon-1}\tilde{f}_t.$$

Using this in both of the NKPCs gives:

$$\begin{aligned} \tilde{\pi}_t^S &= \frac{\varepsilon}{\theta_P}(\sigma+\eta)\tilde{y}_t + \beta\mathbb{E}_t\left[\tilde{\pi}_{t+1}^S\right], \\ \tilde{\pi}_t^F &= \frac{1}{\theta_F}\left(\frac{\phi}{\bar{F}-\kappa\bar{F}^{2-\frac{1}{\nu}}}\right)\left((\sigma+\eta)\tilde{y}_t - \tilde{f}_t\right) + \frac{1}{\theta_F}\left(\ln[\phi(1+\bar{\tau})] - \bar{f}\right)\tilde{f}_t + \beta\mathbb{E}_t\left[\tilde{\pi}_{t+1}^F\right]. \end{aligned}$$

A more convenient form of the NKPC P may be found, using equation 1, with $\bar{A}=1$:

$$\bar{F} = \left[\frac{\phi}{\kappa(\varepsilon-1)+\kappa\phi}\right]^{\frac{\nu}{\nu-1}}, \tag{36}$$

to set: $$\omega_1 \equiv \frac{\phi}{\bar{F}-\kappa\bar{F}^{2-\frac{1}{\nu}}} = \frac{[\kappa(\varepsilon-1)+\kappa\phi]\phi}{\kappa(\varepsilon-1)\bar{F}} = \frac{[\kappa(\varepsilon-1)+\kappa\phi]^{\frac{2\nu-1}{\nu-1}}}{\kappa(\varepsilon-1)\phi^{\frac{1}{\nu-1}}} > 0, \tag{37}$$

$$\text{and} \qquad \omega_2 \equiv \ln\left[\frac{\phi(1+\bar{\tau})}{\bar{F}}\right] = \frac{\nu}{\nu-1}\ln\left[\kappa\varepsilon\left(\frac{\varepsilon-1}{\phi\varepsilon}\right)^{\frac{1}{\nu}} + \kappa\left(\frac{\phi\varepsilon}{\varepsilon-1}\right)^{\frac{\nu-1}{\nu}}\right] \tag{38}$$

and hence:

$$\tilde{\pi}_t^F = \frac{\omega_1}{\theta_F}(\sigma+\eta)\tilde{y}_t + \frac{\omega_2-\omega_1}{\theta_F}\tilde{f}_t + \beta\mathbb{E}_t\left[\tilde{\pi}_{t+1}^F\right].$$

The intertemporal IS curve is formed by combining the market clearing and Euler conditions to give:

$$\hat{y}_t - \mathbb{E}_t[\hat{y}_{t+1}] = -\frac{1}{\sigma}(i_t - \mathbb{E}_t[\hat{\pi}_{t+1}] - \rho),$$

which, when expressed as deviations from their natural rates gives:

$$\tilde{y}_t - \mathbb{E}_t[\tilde{y}_{t+1}] = -\frac{1}{\sigma}(i_t - \mathbb{E}_t[\tilde{\pi}_{t+1}] - r_t^n), \tag{DIS}$$

which is referred to as the Dynamic IS relationship where r_t^n is the natural real interest rate defined as:

$$r_t^n \equiv \rho + \sigma\mathbb{E}_t[\Delta y_{t+1}^n] = \rho + \sigma\psi_a\mathbb{E}_t[\Delta a_{t+1}]$$

where the second equality uses the above relationship for y_t^n.

The reduced model is then given as the solution to:

$$\tilde{y}_t = \mathbb{E}_t[\tilde{y}_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\tilde{\pi}_{t+1}] - r_t^n), \tag{Dynamic IS}$$

$$\tilde{\pi}_t^S = \frac{\varepsilon(\sigma+\eta)}{\theta_P}\tilde{y}_t + \beta\mathbb{E}_t\left[\tilde{\pi}_{t+1}^S\right], \tag{NKPC P}$$

$$\tilde{\pi}_t^F = \frac{\omega_1}{\theta_F}(\sigma+\eta)\tilde{y}_t + \frac{\omega_2-\omega_1}{\theta_F}\tilde{f}_t + \beta\mathbb{E}_t\left[\tilde{\pi}_{t+1}^F\right], \tag{NKPC F}$$

$$i_t = r_t^n + \phi_\pi\tilde{\pi}_t + \phi_y\tilde{y}_t, \tag{Taylor rule}$$

$$r_t^n = \rho + \sigma\psi_a\mathbb{E}_t[\Delta a_{t+1}], \tag{39}$$

$$\tilde{\pi}_t^F = \tilde{f}_t - \tilde{f}_{t-1}, \tag{40}$$

$$\tilde{\pi}_t^S = \frac{\phi}{\varepsilon-1}\tilde{\pi}_t^F + \tilde{\pi}_t, \tag{41}$$

$$a_t = \rho_A a_{t-1} + \varepsilon_t. \tag{42}$$

E Determinacy

Given a policy rule of the form:

$$i_t = r_t^n + \phi_\pi \tilde{\pi}_t + \phi_y \tilde{y}_t,$$

the system of equations for the reduced model may be written as:

$$0 = \begin{bmatrix} 1 & \frac{1}{\sigma} & 0 & 0 \\ 0 & \beta & 0 & \frac{\phi}{\varepsilon-1}\frac{(\omega_1-\omega_2)}{\theta_F} \\ 0 & 0 & \beta & \frac{(\omega_2-\omega_1)}{\theta_F} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbb{E}_t[\tilde{y}_{t+1}] \\ \mathbb{E}_t[\tilde{\pi}_{t+1}] \\ \mathbb{E}_t[\tilde{\pi}^F_{t+1}] \\ \tilde{f}_t \end{bmatrix} + \begin{bmatrix} -1-\frac{\phi_y}{\sigma} & -\frac{\phi_\pi}{\sigma} & 0 & 0 \\ \frac{\varepsilon(\sigma+\eta)}{\theta_P}\left(1-\frac{\phi\omega_1}{\varepsilon(\varepsilon-1)}\frac{\theta_P}{\theta_F}\right) & -1 & 0 & 0 \\ \frac{\sigma+\eta}{\theta_F}\omega_1 & 0 & -1 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} \tilde{y}_t \\ \tilde{\pi}_t \\ \tilde{\pi}^F_t \\ \tilde{f}_{t-1} \end{bmatrix}$$

This is in the form:

$$0 = \mathbf{A}\mathbb{E}_t[\mathbf{x}_{t+1}] + \mathbf{B}\mathbf{x}_t,$$
$$\mathbb{E}_t[\mathbf{x}_{t+1}] = -\mathbf{A}^{-1}\mathbf{B}\mathbf{x}_t,$$

which is permissible as, by inspection, $\mathbf{A}^{-1}$ exists. As $\tilde{f}_{t-1}$ is predetermined, uniqueness requires (exactly) 3 eigenvalues from the matrix $-\mathbf{A}^{-1}\mathbf{B}$ to be outside the unit circle.

Acknowledgements

We have benefited from comments by Klaus Adam, Markus Brunnermeier, Vasco Carvalho, Giancarlo Corsetti, Jordi Gali, Petra Geraats, Nobuhiro Kiyotaki, Kurt Mitman, Sergio Rebelo, Christina Romer, David Romer, and seminar participants at the University of Cambridge.
The views expressed in this article are those of the authors and do not necessarily reflect those of the European Central Bank or Eurosystem.

Alexander Rodnyansky
University of Cambridge, Cambridge, United Kingdom; email: a.rodnyansky@gmail.com

Alejandro Van der Ghote
European Central Bank, Frankfurt am Main, Germany; email: Alejandro.Van_der_Ghote@ecb.europa.eu

Daniel Wales
University of Cambridge, Cambridge, United Kingdom; email: ddgw2@cam.ac.uk

Postal address 60640 Frankfurt am Main, Germany
Telephone +49 69 1344 0
Website www.ecb.europa.eu

This paper can be downloaded without charge from www.ecb.europa.eu, from the Social Science Research Network electronic library or from RePEc: Research Papers in Economics. Information on all of the papers published in the ECB Working Paper Series can be found on the ECB's website.

PDF ISBN 978-92-899-5264-4 ISSN 1725-2806 doi: 10.2866/6355 QB-AR-22-045-EN-N