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MONETARY POLICY AND FEARS OF FINANCIAL INSTABILITY

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BY VINCENT BROUSSEAU AND CARSTEN DETKEN[1]

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Abstract

Exploiting a specific sunspot equilibrium in a standard forward-looking New Keynsian model, we present an example of a possible conflict between short-term price stability and financial stability. We find a conflict because the sunspot process consists of a self-fulfilling belief linking the stability of inflation to the smoothness of the interest rate path. A policy focusing only on a fixed-horizon inflation forecast neglects the potential effects of this belief on the variance of inflation. The nature of the conflict case is interpreted as evidence for the occasional relevance as well as the general tenuousness of the conflict case. The implementation of our example has led us, furthermore, to illustrate the lack of general applicability of the Bellman principle in dynamic programming for forward-looking models. Our result holds with respect to a more general (Nash-type) concept of optimality.

JEL classification system: C61, C62, E52, E58

Keywords: financial stability, price stability, optimal monetary policy, sunspot equilibria, Bellman principle, time inconsistency

Non-technical summary

Recently the question whether a central bank should exclusively conduct policy with the aim to keep its inflation forecast right at target or whether it should occasionally deviate in the short-term in order to account for problems in the financial sector has received a lot of attention (see Bernanke and Gertler (1999) and Cecchetti et al. (2000)). Nevertheless, there is a surprising scarcity of theoretical analysis combining both a well-specified macroeconomic environment with a sound foundation for financial instability (Clarida, Gali and Gertler (1999)).

Clarida, Gali and Gertler (1999) presented a stylised, entirely forward-looking version of the well-known New Keynsian model, that has become widely accepted as a reference for analysing optimal monetary policy. We use this model, but exploit its inherent indeterminacy by introducing a possibly irrational, self-fulfilling fear of financial instability. This allows us to reduce the necessary ad hoc assumptions to explore the aforementioned short term conflict arising between price stability and financial stability as well as to circumvent the problem of finding a well-specified reason for financial instability.

We find, contrary to our interpretation of Bernanke and Woodford (1997), that optimal monetary policy should explicitly react to fears of financial instability. The three major general assumptions, which drive our result, are:

1. crises encompass a self-fulfilling element, possibly unrelated to economic fundamentals.
2. crises have significant effects on future inflation and output variability.
3. crises (or rather the self-fulfilling dynamics driving the crises) can actually be contained by central bank monetary policy.

Situations, in which these assumptions hold, albeit being rather rare events, should not be disregarded in the conduct of monetary policy.

1. Introduction and definitions

The optimal monetary policy strategy with respect to financial stability issues has since long been a contentious issue. The fact that the primary objective of a central bank should be price stability is widely accepted. But a commonly held view argues that the financial system is inherently fragile and that a central bank has occasionally to compromise its objective of price stability (PS) when financial stability (FS) is threatened[2].

The opposite view claims that by always pursuing the goal of price stability central banks will in fact best promote financial stability (the "Schwartz-hypothesis"[3]). In accordance with the latter view a separate weight on financial stability considerations in the monetary policy objective is likely to rather destabilise the economy[4]. Note that the "Schwartz-hypothesis" is consistent with a central bank using information about the state of the financial system to improve the inflation forecast. The contentious issue is rather whether the financial system deserves attention over and above its importance for the immediate inflation prospect The debate whether to include asset prices in a Taylor rule spelled out by Bernanke and Gertler (1999; arguing against) and Cecchetti et al. (2000; arguing in favour) is the most prominent recent example for the conflict debate[5].

Or to cast it differently, the issue is whether the aim of minimising deviations from the inflation target and financial stability are two mutually consistent, complementary objectives or whether there could actually be a conflict between the two goals? The aim of the present paper is to analyse whether monetary policy could be faced with a *conflict* between *price stability in the short-term* and *financial stability* due to *sunspot* financial crises.

In the following a financial crisis[6] is defined as a major disruption to the efficient channelling of savings to investment opportunities, which might be triggered, e.g. by the failure of a major bank and the associated snowballing contagion effects[7]. *Financial stability* refers to a system capable of avoiding financial crises. Deviations from *price stability* are defined as the deviation from the target rate of inflation[8]. After having defined financial stability and price stability it remains to explain what is our understanding of conflict and a sunspot financial crisis.

[2] See Kent and Debelle (1998).
[3] Schwartz (1995), Bordo and Wheelock (1998).
[4] Bernanke and Gertler (1999), Cogley (1999).
[5] Goodfriend and King (1988) and McCallum (1994) are other examples of no-conflict findings, while Kent and Lowe (1997) and Goodhart and Huang (1999) report conflict results.
[6] The definition resembles the one in Mishkin (1991).
[7] See De Bandt and Hartmann (2000) for a survey on systemic risk.
[8] An inflation targeting central bank is defined as a monetary policy authority, which uses the interest rate as an instrument to minimise the discounted sum of expected squared deviations of inflation from the target rate and output from natural output. The weight of output in the loss function is not crucial for the arguments in this paper.

It is useful to distinguish the conflict from the trade-off debate. A trade-off between two objectives exists when changing relative weights in the utility function will lead to an optimal policy, which eventually achieves more of one and less of the other objective. In other words, a trade-off between price stability and financial stability would exist when the efficient policy frontier in the PS/FS plane had a negative slope. This is the traditional definition of a trade-off, as exposed for example, in Clarida, Gali and Gertler (1999, p. 1672) in the context of the two goals of price and output stability.[9] In order to assess the existence of a trade-off, the two objectives have to explicitly enter preferences.

In this paper we focus on a different debate, as we do not include any financial stability objective in the loss function, as the central bank only cares about the traditional objectives of price and output stability.[10] This is why we will not refer to a trade-off but define the term "conflict" below. The reasons for excluding a financial stability objective are twofold. First, we are mainly interested in the Bernanke/Gertler versus Cechetti debate. The main issue in this debate is the role of financial stability for the general goal of macroeconomic stabilisation. Second, we intend to use the standard New Keynsian model, for which the micro-foundations of the simple central bank loss function omitting financial stability objectives are well known and accepted, which cannot be said for an extension including the elusive concept of financial stability.

Within an inflation-targeting rule[11] framework we define a *conflict* to exist when the policy rule, which is optimal from the point of view of overall price stability (which by construction takes into account all future effects of a financial crisis on current and future states of the economy) leads to inflation deviations from target in the short-run. Thus the question here is whether a central bank only caring for overall price stability, would optimally chose to refrain from keeping inflation right at target in the short term in order to stabilise the financial system. We argue that the separation between short-term price stability (i.e. in our context the equivalence of next period's unconditional[12] expected inflation and the target rate of inflation) and overall price stability (defined as the minimum value for the

[9] Cukierman (1990) and Illing (2000) provide examples of trade-off cases between PS and FS. In their models either lower interest rates (Cukierman) or higher inflation (Illing) benefits banking profits. Thus the weight given to the profits of the banking sector – used as proxy for financial stability - in the central bank loss function, negatively determines the optimal achievement in terms of price stability. In Cukiermann (1990) the trade-off only exists in the short-run, as moral hazard behaviour will eliminate all gains in financial stability in the long run. The Schwartz-hypothesis is also the best counter-example to a trade-off result as it claims that optimal financial stability is achieved by pursuing the only goal of price stability.

[10] In practice central banks' responsibilities concerning financial stability vary a lot. The Eurosystem, for example, does not have an independent objective to safeguard financial stability. However, it has the task to contribute to policies pursued by the competent authorities relating to the stability of the financial system [art. 105 (5) of the Treaty, art. 3.3 and 25.1 of the Statute]. In addition it should promote the smooth functioning of payment systems [art. 105 (2) fourth indent of the Treaty, art. 3.1 and 22 of the Statute]. This lack of direct responsibility would not prevent the Eurosystem to take into account information concerning financial instability in the conduct of its monetary policy if considered helpful to fulfil the primary objective of maintaining price stability.

[11] Minimising a central bank's loss function with respect to the chosen instrument of monetary policy derives a targeting rule.

infinitely discounted, expected squared deviation of inflation from its target rate) is of interest, in this particular debate. The reason is that the standard optimal policy rule (omitting the possibility of sunspot crises as in Clarida, Gali and Gertler (1999)) as well as the instrument rule in Bernanke and Gertler (1999) postulate an interest rate reaction function only reacting to the fixed-horizon inflation forecast.

A recent example of a conflict finding in this vein is Kent and Lowe (1997). They show how an inflation targeting central bank's attempt to burst an asset price bubble as early as possible can lead to next period's expected inflation to be below the target rate of inflation.

The policy implications derived from the finding of a conflict or a trade-off are very distinct[13]. A conflict finding would warn against pursuing the goal of price stability by focusing on an inflation forecast with a too short horizon. A trade-off result instead highlights the importance of clearly defining the mandate of the central bank based on society's optimal weights for price stability and financial stability[14]. Evaluating the relative weights of both objectives is completely irrelevant when there is no trade-off between them in the first place. In our terminology, a trade-off necessarily implies a conflict while the reverse is not true.

There are two distinct views on financial instability in general and on bank runs in particular. The first, the *sunspot view*, explains crises with panics based on psychology or self-fulfilling prophecies while the second, the *business cycle view*[15], claims that crises are triggered by recessions in the presence of a specific financial friction[16]. The best known example of the sunspot view is the Diamond and Dybvig (1983) bank run model, where due to the sequential service constraint and costly early liquidation of assets, a bank run will happen when depositors believe it is about to occur. The sunspot view relies on arbitrary but self-fulfilling believes, which might significantly change the normal economic mechanisms. We will explore how the presence of such self-fulfilling beliefs can be employed to derive a conflict case.

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[12] Unconditional means not conditional on a constant interest rate policy, but with optimal policy.

[13] See Detken (2001) for a literature survey including a classification into the conflict and trade-off results.

[14] Allen and Gale (2000c) and Diamond and Rajan (2000) show that the inherent fragility of an intermediary system based on non-contingent deposit contracts and the maturity mismatch of bank's deposits and liabilities can resemble the optimal risk sharing arrangement. Thus while bankruptcies are a natural consequence of this system and can be optimal, a financial crisis as defined above certainly is not. This leaves open the question about the desired degree of financial (in)stability.

[15] Examples for the business cycle view are Allen and Gale (2000a) and Morris and Shin (2000) who develop models explaining bank runs due to the receipt of possibly business cycle related bad signals deteriorating future return expectations in combination with the non-contingent nature of deposit contracts.

[16] See Allen and Gale (2000c; p. 273).

In order to analyse potential conflict situations we need to deal with a technical issue, i.e. the non applicability of the Bellman principle in forward-looking models of a kind described below. The issue is a general one and is not created by our use of sunspot equilibria.

Section 2 will set up the standard New Keynsian macro model and explain in which sense we view our sunspot equilibria as short cuts to financial instability. Section 3 will deal with a technical issue, mentioned above and define a time consistent concept of optimality, suited for our problem. Section 4 will demonstrate that we encounter a conflict case. Section 5 discusses and section 6 summarises the results.

2. The new Keynsian model with sunspot equilibria

Large asset price movements are often considered a possible source of financial crises. Nevertheless, there are at least six well-known arguments why a central bank should avoid reacting mechanically to asset price fluctuations. These arguments are

1) the creation of moral hazard problems, i.e. the possibility that the expected safety net provided by the central bank will trigger more risky behaviour on part of the financial sector participants (e.g., Goodhart and Huang, 1999),
2) the undesirability of very volatile interest rate decisions (e.g., Cukierman, 1990),
3) the possible problem of indeterminacy of equilibrium and thus possibly arbitrarily large inflation volatility (Bernanke and Woodford, 1997),
4) the uncertainty in determining the fundamental value of financial assets (e.g., Issing, 1998)
5) the demand/supply shock signal extraction problem of asset price fluctuations (Smets, 1997)
6) the possibly destabilising effects of an asset price augmented Taylor rule in a forward-looking new Keynsian macro-model including an asset price bubble (Bernanke and Gertler, 1999).

Although several of the above arguments are not undisputed, see for example Cechetti et al. (2000) with respect to the merits of an asset price augmented Taylor rule, we would still tend to conclude that a *mechanical* reaction is very unlikely to be optimal due to the arguments listed above. But, this does not exclude the possibility that at times there might exist situations of conflict between the goal of price stability and the reaction necessary to further financial stability. Unfortunately there is little theoretical work to support this view, especially if one would like to rely on results obtained from first principles. Kent and Lowe (1997) derive a conflict with an exogenous asset price bubble exhibiting ad-hoc asymmetric effects on the rate of inflation. Goodhart and Huang (1999) show a conflict in the sense that optimally the central bank allows some banks to fail which creates some variance in inflation today. This will stabilise the financial system in the future as the moral hazard problem is contained and increase overall price stability. Both models expose conflict cases but lack a well-specified macroeconomic environment derived from first principles.

We will use a standard, micro-founded new Keynsian model of the economy, in which we will nest the possibility of sunspot equilibria, to explore the possibility of conflict situations. We exploit the degrees of freedom resulting from the indeterminacy of the forward-looking model in selecting a special sunspot process, which we link to financial instability. In our case the sunspot link between macroeconomics and financial instability is ad-hoc too, but the beauty of it is that - in contrast to standard modelling attempts - this is exactly the point of the sunspot idea.

In the following we will use the new IS and the forward-looking Phillip's curve featuring imperfect competition and nominal rigidities in a dynamic general equilibrium framework. As we base our arguments on the most standard version of this by now widespread model, we only refer to, e.g. Gali (2000) for an outline of the derivation from first principles and a discussion of the properties of this model. The exact specification and the notation are the same as in Clarida, Gali and Gertler's (1999) survey article on state-of-the-art optimal monetary policy. All variables are expressed as deviations from their long run levels.

$$(1) \qquad x_t = -\varphi\left[i_t - \mathrm{E}_t\, \pi_{t+1}\right] + \mathrm{E}_t\, x_{t+1} + g_t$$

$$(2) \qquad \pi_t = \lambda x_t + \beta\, \mathrm{E}_t\, \pi_{t+1} + u_t$$

$$(3) \qquad Loss = \mathrm{E}_t\left(\sum_{n=0}^{\infty} \beta^n \left[\pi_{t+n}^2 + \alpha\, x_{t+n}^2\right]\right)$$

Equation (1) is the new IS curve, where the output gap x depends negatively on the real interest rate and positively on next periods expected output gap. E_t denotes expectations in period t, i is the nominal short term interest rate (the policy instrument), π stands for inflation, and g is a demand shock relative to natural output. Equation (2) is a forward-looking Phillips curve explaining current inflation with today's output gap, next period's expected inflation and a shock u, which sometimes is interpreted as a cost-push shock or a mark-up shock[17]. The central bank's loss function is given in equation (3), which specifies that the central bank prefers inflation at the target rate zero as well as a closed output gap.[18]

The weight given to the output gap in the loss function is α and the discount factor is denoted β [19]. While letting the central bank minimise loss function (3) we avoid to introduce an explicit concern for financial stability. None of the below results depends on α being different from zero. The disturbance terms u and g are supposed to follow the autoregressive processes $g_t = \mu\, g_{t-1} + \varepsilon_t$ and $u_t = \rho\, u_{t-1} + \upsilon_t$, where ε_t and υ_t are mean zero, constant variance, white noise variables. As Clarida, Gali and Gertler (1999) demonstrate the optimal reaction function of the central bank under discretion takes the form:

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[17] See Clarida, Gali and Gertler (1999, p. 1667).

[18] Note that the stability-oriented two-pillar monetary policy strategy of the ECB differs from flexible inflation targeting. An attempt to formalise a monetary policy strategy closer to the ESCB strategy is found in Smets (2000).

(4) $i_t = \gamma_1 E_t\pi_{t+1} + \gamma_2 g_t$ *with* $E_t\pi_{t+1}=\gamma_3 u_t$

where γ_1 exceeds 1 and $\gamma_1\gamma_3 = \dfrac{\lambda - \lambda\rho + \alpha\rho\varphi}{(\alpha + \lambda^2 - \alpha\beta\rho)\varphi}$ and $\gamma_2 = 1/\varphi$.

Equation (4) shows that the central bank should perfectly offset the inflationary effects of a demand shock, not react to a shock to potential output, which simultaneously creates its own demand ($g = 0$), and accommodate a supply shock, which does not ($g < 0$). Inflationary effects of cost-push or mark-up shocks will not be completely offset as they create a short-term trade-off between inflation and output variability (as long as $\alpha > 0$), but nominal rates should change sufficiently to move the real interest rate to stabilise inflation.

Bernanke and Woodford (1997) were the first to point out the hazard of indeterminacy and sunspot equilibria in the New Keynsian IS-AS model. They derive the conditions for the weights of an instrument rule (Taylor rule) where the central bank reacts to private sector inflation forecasts and the output gap to obtain a unique bounded solution[20]. Also Svensson and Woodford (1999) discuss the indeterminacy problem in a model close to the one above and demonstrate how a Taylor rule with large enough weights on the inflation forecast or the output gap can avoid the indeterminacy problem. With regard to optimal policy Svensson and Woodford (1999, p. 26) conclude, that "any purely forward-looking decision problem implies a reaction function that results in indeterminacy of equilibrium if the central bank is committed to this procedure". Clarida, Gali and Gertler (1999, p. 1683) claim otherwise, i.e. because the weight on expected inflation in reaction function (4) is large enough (i.e. exceeding 1) no problem of indeterminacy would arise under optimal discretionary policy. Their claim has been refuted by Svensson and Woodford (1999, p. 4) by noticing that the inflation expectations term in reaction function (4) is an unconditional expectation, depending on the instrument itself. Thus eventually the instrument reacts only to the state variables u and g and indeterminacy necessarily results.

Thus while the appropriate choice of weights for an instrument rule might solve the problem of indeterminacy, the issue remains relevant under optimal monetary policy and is unavoidable in the standard forward-looking model employed here[21].

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[19] The notational equivalence of the discount factor in (3) and the coefficient of expected inflation in (2) is no coincidence, as the latter is derived from Calvo pricing.

[20] In their model problems of indeterminacy arise as well when the central bank additionally reacts to forecasts of the instrument itself.

[21] But see Svensson and Woodford (1999) for a discussion of hybrid (targeting/instrument) rules making use of the targeting part to secure optimal policy and the instrument part to safeguard determinacy.

Our system of equations (1) and (2) can be written in matrix notation as follows, where capital letters denote a matrix and bold letters a vector:

(5) $\mathbf{V}_t = i_t \, \mathbf{J}_0 + M \, \mathrm{E}_t \mathbf{V}_{t+1} + L \, \boldsymbol{\eta}_t$

where

$$\mathbf{V}_t := \begin{pmatrix} x_t \\ \pi_t \end{pmatrix}; \quad \mathbf{J}_0 := \begin{pmatrix} -\varphi \\ -\lambda\varphi \end{pmatrix}; \; M := \begin{pmatrix} 1 & \varphi \\ \lambda & \beta + \lambda\varphi \end{pmatrix}; \; L := \begin{pmatrix} 1 & 0 & 0 \\ \lambda & 1 & 0 \end{pmatrix} \text{ and } \boldsymbol{\eta}_t := \begin{pmatrix} g_t \\ u_t \\ a_t \end{pmatrix}$$

The disturbance vector η here has a third component, the sunspot process *a,* which does not influence the Vector of endogenous variables **V (**note the third column of zeros in *L***)**.

Blanchard and Kahn (1980) have shown that the system

(6) $\mathrm{E}_t \mathbf{V}_{t+1} = M^{-1} \mathbf{V}_t + L^* \boldsymbol{\eta}_t$

has a unique bounded solution if and only if both eigenvalues of M^{-1} lie outside the unit circle. Our system (5) would thus have a unique bounded solution if and only if both eigenvalues of *M* lie inside the unit circle[22]. Equation (6) in the appendix computes the eigenvalues of *M.* It is easy to show that the first eigenvalue has modulus smaller 1 and the second greater than 1 for all possible parameter values. Equilibrium is thus indeterminate due to sunspot equilibria.

Having proven the existence of sunspot equilibria in our model, we pursue to construct a special sunspot equilibrium linking financial stability to price stability. We specify the Gaussian distribution of η to be

(7) $\mathrm{E}_t(\boldsymbol{\eta}_{t+1}) = K \cdot \boldsymbol{\eta}_t$ and $Var_t(\boldsymbol{\eta}_{t+1}) = S(i_t - i_{t-1})$ with

(8) $$K := \begin{pmatrix} \mu & 0 & 0 \\ 0 & \rho & 0 \\ 0 & 0 & \nu_2^{-1} \end{pmatrix} \quad \text{and} \quad S(i_t - i_{t-1}) := \begin{pmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & \xi(i_t - i_{t-1})^2 \end{pmatrix}$$

where ν_2 is the unstable eigenvalue of matrix *M*, exceeding 1 (see equation (6) of the appendix).
We then define matrix *T* and vector $\boldsymbol{J_n}$ as follows:

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[22] We assume reasonably that after solving the central bank's optimisation problem, i will depend only on the vector of disturbances and thus the solution of the system including optimal monetary policy will thus not affect the matrix M.

(9)

$$T \;:= \begin{pmatrix} \frac{(1-\mu\beta)}{(1-\mu(1+\beta+\lambda\varphi)+\beta\mu^2)} & \frac{\rho\varphi}{(1-\rho(1+\beta+\lambda\varphi)+\beta\rho^2)} & -\beta-\lambda\varphi+\nu_2 \\ \frac{\lambda}{(1-\mu(1+\beta+\lambda\varphi)+\beta\mu^2)} & \frac{(1-\rho)}{(1-\rho(1+\beta+\lambda\varphi)+\beta\rho^2)} & \lambda \end{pmatrix}$$

(10) $\mathbf{J}_{n+1} := M\,\mathbf{J}_n \qquad n > 0$

Theorem 1

The following expression is a formal solution to the system (5):

(11) $\mathbf{V}_t := T \cdot \boldsymbol{\eta}_t + \sum_{n=0}^{\infty} \mathbf{J}_n \, \mathrm{E}_t \, i_{t+n}$

The proof is shown in appendix 1.1.3. Matrix T reveals how the sunspot process *a* affects the vector of endogenous variables **V** for a given interest rate process. As the standard solution of CGG is a solution of (5), any other solution must differ from it by some solution of the associated homogenous equation. Any solution of the associated homogenous equation has the form of the M eigenvector associated to the eigenvalue larger than 1, multiplied by a martingale. The two first columns of the matrix T stem from the standard CGG problem (without sunspots). We add the third column to take into account the supplementary element a_t. The sunspot process is relevant despite the fact that there is no "real" influence of *a* on **V** as shown in equation (5), simply due to the fact that agents expect *a* to be important for **V**. We see that depending on the process *a* and for $\xi > 0$ there exists an infinite number of bounded solutions to our two-equation system.

The variance specification of η constitutes the decisive link to financial instability. The assumptions depicted in matrices *K* and *S* are that the sunspot process *a* follows a stable autoregressive process, uncorrelated to the inflation and output gap disturbances and that the variance of a_{t+1} is $\xi\,(i_t - i_{t-1})^2$. Thus the larger the change in interest rates, the higher the next period's variance of the sunspot process[23]. The crucial assumption is that large changes in interest rates always incorporate an element of surprise, which a) either conveys new information about the central bank's analysis of the state of the economy or b) inflicts actual losses on the intermediary system (which is most relevant for the transmission process of monetary policy). The selection of this specific sunspot specification is arbitrary and in this sense we exploit the indeterminacy provided by the forward-looking New Keynsian model. The fact that we chose this and not some other specification, though, relies on the

[23] Our definition of a sunspot process is consistent with the original definition by Azariades and Guesnerie (1986) in the sense that there is no underlying relationship between the sunspot variable and the endogenous variables except via self-fulfilling expectations. Obviously our sunspot process *a* differs from the original literal sunspots in the sense that it is influenced by past policy decisions, while the literal sunspots remain unaffected by any of the reactions they might have triggered on earth. In our case the "endogeneity" of the sunspot process is exogenously given and thus we confidently stick to the terminology of considering our case a sunspot equilibrium. In some sense our specification resembles the Froot and Obstfeld (1991) "intrinsic bubble" idea.

fact that we consider the belief that a smooth interest rate path is beneficial for macroeconomic stabilisation to be reasonable. We are fortunate enough not to rely on how well founded the link between financial stability and interest rate smoothing really is. Important is that the relevant market participants' perceptions that smooth interest rates are conducive to financial stability is not necessarily unreasonable and has some theoretical underpinning[24]. Consequently we consider our sunspot specification as a special case but not as unrealistic[25]. The sunspot environment allows us to remain rather unconcerned about the lack of micro-foundations of this part of the model. One could even side with the view that by definition, there cannot be a micro-foundation for sunspot beliefs.

Nevertheless we would like to motivate our special sunspot specification. For example, the 1994 series of interest rate hikes of the Fed is an example of the first channel. Bond rates peaked and the yield curve steepened, due to the new information that the Fed was concerned about inflationary pressure. In the same sense, the January 2001 Fed rate cut by 50 basis points let "The Economist" to speculate whether the Fed knew something about financial stability problems what the rest of the world did not. Thus in reality large interest rate changes can potentially lead to higher uncertainty and thus trigger all kinds of speculation about a possibly dismal state of the economy. Our variance specification assumes that the larger the interest rate change, the more likely it is that the central bank is perceived as being acting under the impression of very significant new information and the larger is the probability that it's actions are considered "behind the curve".

Loretan and English (2000) argue that large changes in interest rates will increase the volatility of financial market prices and correspondingly also the correlation of asset returns. According to standard risk/return portfolio management theory, this will lead to portfolio adjustments towards less risky positions and could potentially trigger large asset price movements. They interpret the first careful and later accelerating pace of interest rate increases by the FED in 1994 in this light. They also cite from the 1995 Humphrey-Hawkings report, which shows that the FED had been concerned about possibly large portfolio shifts as a reaction to larger interest rate moves. The 1994 example deals with an interest rate hike, but the portfolio shift argument applies equally to interest rate cuts, which would also increase asset return correlation. Potential asset price movements due to the above mentioned portfolio shifts leads to the second channel to justify our sunspot specification. This channel refers to a large (and surprising) change in interest rates leading to actual losses of the intermediary system, if the latter were wrongly positioned. The classical example here is of course the interest rate hike, which deteriorates the balance sheets of intermediaries involved in maturity transformation by converting

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[24] The following citation taken from the Financial Times, January 9, 2000, page 17 fits particularly well: "There is also the endless guessing game: does the market believe in the inflation-growth trade-off because there is one, or because it thinks that Mr. Greenspan does? ".

[25] Most likely an asymmetric specification (so that only interest rate hikes deteriorate the degree of financial stability) would be considered even more likely to qualify for a sunspot. We chose the symmetric case, as the mathematics were much easier and our general point is still likely to hold.

variable rate liabilities into fixed rate assets. But the potential for losses does not only exist for interest rate hikes (though the aggregate intermediary sector relevant for the transmission of monetary policy is still likely to be engaged in the classical maturity mismatch) but could also follow surprising interest rate cuts, when individual players positioned themselves wrongly. A large player's bankruptcy might trigger a systemic crisis. This is the reason why also the second channel can be used to motivate our symmetric variance specification.

Apart from these more intuitive channels, interest rate smoothing has also been discussed in the literature both as the result of monetary policy adhering to financial stability objectives as well as a possible manifestation of financial stability. Cukierman (1990), for example, shows that a central bank will smooth real short-term interest rates when it cares also for profits of the banking system besides inflation and the output gap. Goodfriend (1987) on the other hand expanding on Poole (1970) defines a financial stability objective by punishing a volatile interest rate development in the central bank loss function. McCallum (1994) argues that interest rate smoothing would be compatible with a monetarist lender-of-last resort policy, defined as lending to the markets via open market operations to avoid both a systemic crisis and lending to insolvent banks. And Miron (1989) reports that the main reason of the Fed's foundation in 1914 was the goal to smooth interest rates for financial stability reasons. Moreover, recently several observers have claimed that empirically excessively smooth policy rates might be due to financial stability considerations of central banks[26].

So far we have explained why a change in interest rates might lead to financial instability. In the model a change in interest rates increase the variance of the sunspot process, which ceteris paribus would increase the variance of inflation and output. It remains to be motivated why we believe that a financial crisis could have exactly these consequences. Here we refer to Goodhart and Huang (1999) who argue that a major bank's failure increases inflation variability due to the fact that the transmission mechanism becomes more uncertain (the money multiplier more variable) and thus renders the task of the central bank much more difficult. Or to cite Goodhart and Huang (1999, p. 5) "When failures occur, and people start to panic, their behaviour is likely to become far less predictable. Policy mistakes become much more likely." They also present empirical evidence for three selected crises where the money multiplier actually became less predictable[27].

Note that we allow the central bank to eliminate what we call a "sunspot financial crisis"[28], which potentially could trigger highly volatile inflation and output gap paths, by smoothing interest rates: Equation (8) reveals that the variance of the sunspot disturbance a (and thus the sunspot process a itself) can be reduced to zero, by a steady interest rate policy.

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[26] See Clarida, Gali, Gertler (1999), Goodhart (1999) and the discussion in Woodford (1999).
[27] See also Kaufmann (1998, p. 45).

3. The "non-Bellmanity" of the optimising problem

CGG (1999) solve the optimisation problem under discretion by minimising the loss function (3) given the constraint (2). The first order conditions are then used to derive the equilibrium relationship between x and π, which then is inserted in (1) to obtain the optimal interest rate reaction function, equation (4). Note that this is in essence a maximisation with respect to i_t for the current t, so with respect to a *scalar* (instead of, for example, a process, or a function). In the following we will concentrate on optimisation under discretion. This seems to be a natural choice when dealing with financial instability, because we think it is illusionary to assume a monetary policy authority could credibly commit to a rule in a crisis situation[29]. In CGG (1999) the optimisation is performed in correspondence with the Bellman principle (see Appendix 1.2.4 for a formal presentation). The Bellman principle basically states that an agent planning to start optimising tomorrow, can do no better today than to optimise taking future optimal plans as given.[30] The optimal strategy is time-consistent. CGG (1999, p. 1672) assume that future inflation and output are not affected by today's action and that the central bank cannot influence expectations about future inflation and output. As a consequence, the central bank disregards the future in solving its decision problem. Eventually, it is this separability between the present and the future, which, first of all, gives meaning to the Bellman concept of optimality and second, provides a solution, which is optimal with respect to this concept[31]. But in more general cases, this concept is not applicable. Generally speaking, the Bellman equation can be applied when the current value of the objective function only depends on the current value of the control variable and not on the reaction function as such.[32]. If this is the case, standard theory applies and the existence of an optimal control solution in the form of a reaction function or a feedback rule, has been proven. But if it is not the case, then the policy rule cannot be derived anymore in accordance with the Bellman principle despite dealing with optimisation under discretion. As we will point out, by its very structure, the decision problem of CGG – on which we focus throughout this paper – happens to fall outside of the formal pattern for the Bellman principle. In the standard CGG model the objective function relies on the reaction function as such, which is evident due to the use of the expectation operator, and relies on it in a way that cannot be transformed into a

[28] We always refer to sunspot financial crises or sunspot financial instability but obviously the sunspot equilibria considered here are a more general phenomenon. The sunspot component could be linked to any possible process uncorrelated with the macroeconomic fundamentals x and π.

[29] Blinder (1999) might disagree. Drawing on his experience as a central banker he mentions that, in practice, credibility is a very important element in the utility function of a central banker. Thus the benefit to build or defend credibility by sticking to previously announced policies despite the crisis situation could also justify considering commitment solutions. In game theoretic language Blinder is arguing that reputation effects in repeated games are really important as they make commitment credible. But we would still doubt that given a normal rate of time preference such credibility considerations would win the day during a severe financial crisis.

[30] See Obstfeld and Rogoff (1996, p. 719)

[31] See Currie and Levine (1993, p. 129).

formally well-specified Bellman problem (see Appendix §1.2.4). Note that this violation of "Bellmanity" does not depend on the fact that we have to deal with indeterminacy and thus does not rely on the sunspot property. To see this remember that on the one hand sunspots arise only due to the fact that the homogenous equation associated with our equation (5) has non-zero solutions. On the other hand, the "non-Bellmanity" of the model originates due to the particular appearance of the expectation operator E_t appears in the definition of the loss function (3). Obviously, this structure of the loss function does not depend on the spectrum of the matrix M. Assuming the variance of the sunspot process *a* to be influenced by past interest rate changes, is simply a way to ascertain that it would be definitely wrong to make the same assumptions as CGG, i.e. that the process of the control variable is independent of today's value of the control. In the case without sunspots (or possibly a different sunspot process), such an assumption might turn out to have no consequence for the optimal policy rule, although this would still need explicit proof.[33]

The following example is meant to illustrate this technical point that in case the pay-off depends on the optimal rule itself, the equilibrium looses the property of time-consistency[34]. This example is a two-player game. It is simple, abstract, and lacks economic content. Despite these drawbacks, it happens to exactly reproduces the technical difficulty that we encounter in our model (except in the degenerate sub-case without sunspots). This is why we invite the reader to examine it.

We search for a solution in form of a reaction function, so that the optimal rule is a function of the state variables. Starting from any given value of a state variable, one can compute a rule maximising the objective function. But nothing guarantees that two rules computed from two different starting points will actually coincide. Let us consider a game with one player in discrete time and a state space consisting only of two possible states of nature: X_1 and X_2. The player controls the state variable entirely. A decision rule provides the next state of nature as a function of the current one. There are exactly 4 possible decision rules, namely: $d(1) = (X_1\text{->}X_1, X_2\text{->}X_1)$, $d(2) = (X_1\text{->}X_1, X_2\text{->}X_2)$, $d(3) = (X_1\text{->}X_2, X_2\text{->}X_1)$ and $d(4) = (X_1\text{->}X_2, X_2\text{->}X_2)$. Let us assume that the objective function, O, takes the form of a discounted sum of some function h of the state variable and of the decision rule, d(z) with z = 1, 2, 3 or 4, thus $O = h(X, d(z))$. We suppose that $h(X_1, (X_1\text{->}X_2, X_2\text{->}X_1))=1$, that $h(X_2, (X_1\text{->}X_1, X_2\text{->}X_2))=1$, and that h is zero for the six remaining combinations of $X_{1,2}$ and d(z). We further

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32 The reader will find a more in depth and technical discussion of this point in the subsection 1.2.4 of the appendix.

33 This is the case for the standard rule derived by CGG. See theorem 3.

34 Note that Sargent (1999, p. 46) provides an example, in which dynamic programming does not lead to a time consistent solution. In his case it is also true that the objective function depends on the future values of the control variable. But the major difference is that in his case time consistency is lost due to the switch from a Cournot to a Stackelberg game. The result is thus not surprising as the Stackelberg solution corresponds to a commitment solution. In our case instead the reason for time inconsistency is the generalisation of the relationship between utility and the process of the control variable.

assume that the discount factor is strictly positive but small. The optimal rule computed in X_1 is to switch states permanently, i.e. (X_1->X_2,X_2->X_1) while the optimal rule computed in X_2 is to never change states, i.e. (X_1->X_1,X_2->X_2). Assuming in X_1 that the player will hold on to switching strategy (X_1->X_2,X_2->X_1) clearly contradicts the assumption of optimal behaviour.
The dependence of the objective function h on the rule d(z) is the origin of the "non-bellmanity" of our monetary policy problem. The very same sort of dependence in our monetary policy case is depicted in equation (38) of appendix 1.2.4.

We thus have to define a more general concept of optimality in order to make any statement about optimal monetary policy in the sunspot case. In brief, we define a policy rule (see appendix 2.1.4) as optimal, when a policy rule for any $t > t_0$, where t itself is a parameter in the t_0 decision problem, equals the optimal rule for t_0. A formal definition of our notion of optimality is found in definition 6 in appendix 2.1.4, where the term "self-reply" is introduced[35]. A reaction function is optimal when it maximizes the objective function over the set of all possible reaction functions, under the hypothesis that the market believes that the central bank will always follow this reaction function. Observe that such a definition incorporates the same circularity than a Nash equilibrium. Our equilibrium is a fixed point of the "self-reply" mapping in the same sense a Nash equilibrium is a fixed point of the best reply mapping. The analogy between optimality according to our definition and a Nash equilibrium is not only a formal one. One can understand the central bank's problem as a game between an "explicit" player, the central bank, and an implicit player, the market. One could make this hidden player explicit, by deriving its behaviour from some optimal program. In this (more complicated) reformulation of the very same game, the optimal strategy in the sense of the above definition would become the strategy prescribed by a Nash equilibrium.
To summarise the discussion on optimality, one should mention that:

- A generalisation of the standard central bank's problem as exposed in CGG exhibits an explicit dependency of the objective on the policy rule that makes it "non-Bellmanian".
- Thus a rigorous definition of optimality is a relevant issue, as the Bellman principle is not applicable.
- This forces us to write down a definition of optimality which is reasonable even when the objective function depends on the policy rule as a whole and not only on the value taken by the policy rule at the current state variable.
- Consequently, we decide to define optimal policy rules as policy rules that are fixed points of a so-called "self-reply" mapping (Definition 6 in the Appendix). This definition restores the time-consistency property.

[35] We chose the term "self-reply" because we are not in a game with two players equally and explicitly involved.

Theorem 2

Given a standard optimal control problem (as defined by equation (27) and objective function (28) of the appendix) then the optimal control in Bellman's sense is a closed loop control that is also optimal in the sense of Definition 6.

The proof is depicted in appendix § 1.2.5. Thus our definition encompasses the Bellman principle for those cases where the latter is correctly applicable.

4. A conflict between price stability and financial stability

In a standard New Keynsian model the central bank's optimal reaction function (under discretion) would typically look like equation (4). If a financial shock determines next periods inflation expectations[36] $E_t\pi_{t+1}$, it will reflect the fact that in order to minimise the intertemporal loss function it is not optimal to keep next period's inflation right at target (or close to the rate prescribed by an optimal output smoothing when $\alpha > 0$). A financial shock then leads to a deviation from the desired inflation rate at least for one period. The reason would be that over the infinite optimisation horizon it must pay to accept deviations from the inflation target in the short-term in order to minimise output and/or inflation variances over the whole period. It is not necessary that the financial shock shows up explicitly in the unconditional inflation expectations term, it could also be that financial instability changes inflation expectations by changing weights in the reaction function on other shocks of the system.

We will prove the existence of a conflict between price stability and financial stability with the following two conditions:

1. We will show that if there is an optimal policy rule, then this policy rule reacts to the occurrence of the sunspot crisis. We accomplish this by showing that the standard rule without sunspots cannot be optimal anymore in the sunspot case.
2. We prove that expected inflation for the next period under the optimal rule in the sunspot case is different from (actually it will be larger than) inflation expectations under the standard rule in the case without sunspots.

The first condition is necessary because if the standard rule would still be optimal in the sunspot case, the fact that inflation expectations are different under the sunspot and the standard case (condition 2), could simply mean that the occurrence of a crisis makes it impossible to achieve the same inflation

[36] $E_t\pi_{t+1}$ here is the true unconditional inflation expectations of the public, incorporating the reaction of the central bank.

rate, but as there is nothing monetary policy can do about it, it would be meaningless to call this a policy conflict. The second condition is necessary because if next period's inflation expectations would be the same for the standard and the sunspot case, it seemingly is no problem for monetary policy to offset the financial shock without any costs in terms of price stability under the new policy rule. Also then it would not make sense to call this a conflict between price and financial stability.

In order to prove conditions 1 and 2 above, we still need some further results.

Theorem 3

If ξ is equal to zero, then there exists only one optimal policy rule in the form of a linear function of the state variables. This policy rule is the standard rule.

The proof is found in appendix 2.2.1. Theorem 3 states that when we are in the case without sunspots and the optimal solution is restricted to be a linear function of the state variable vector $\boldsymbol{\eta}$, the standard rule as presented by Clarida, Gali and Gertler (1999) is the only linear solution also according to the self-reply notion of optimality. The optimal reaction function then is:

(12) $$\mathrm{i}_t := \boldsymbol{\omega} \cdot \boldsymbol{\eta}_t \quad \text{with}$$

(13) $$\boldsymbol{\omega} := \begin{pmatrix} \omega_x \\ \omega_\pi \\ 0 \end{pmatrix} \quad \text{and}$$

(14) $$\begin{pmatrix} \omega_x \\ \omega_\pi \end{pmatrix} = \begin{pmatrix} \dfrac{1}{\varphi} \\ \dfrac{\lambda - \lambda\rho + \alpha\rho\varphi}{(\alpha + \lambda^2 - \alpha\beta\rho)\varphi} \end{pmatrix}$$

Together with Theorem 2 we can thus rely on the standard rule to provide us with the first and second moments of inflation for the case without sunspots.

Theorem 4

If ξ is large enough, then the standard rule (12)-(14) is not optimal in the sense of Definition 6.

The formal proof is presented in appendix 2.2.2. The proof consists of inserting the standard rule (12) into the general solution of the system, equation (11). We then collect the terms through which the sunspot component can influence the decision on the optimal instrument in period t. When ξ is large enough these terms will dominate the loss function.[37] Minimising the relevant parts of the loss function with respect to i_t we obtain (the dominant part of) the optimal policy rule which also depends on i_{t-1} (see equation 61 in the appendix). We thus are able to show that with sunspot financial crises, the standard rule, which does not depend on i_{t-1}, cannot be the optimal rule anymore. This result is very intuitive, as smoothing interest rates is what dampens the financial crisis and thus its effects on inflation and the output gap. The central bank will thus have to adapt its optimal rule to the occurrence of a sunspot financial crisis, although we cannot derive the optimal rule itself. Thus our first condition for a conflict is fulfilled.

The second condition to establish a conflict is then to show that inflation expectations under the respective optimal rules are different in the sunspot and the no-sunspot cases. We will be able to prove this only for the limiting case of $\xi\to\infty$. Although by the principle of continuity this holds also for ξ large enough.

Assuming $\xi\to\infty$ the optimal rule[38] would have to be marginally close to $i_t = i_{t-1}$ in which case the sunspot component would be marginally close to zero as its variance is reduced to zero and we are asymptotically back in the standard model in terms of the shocks hitting the economy. This observation is important as we want to compare expected inflation on equal terms, i.e. inflation in the sunspot case should not be larger simply because there is an additional shock hitting the economy. Assume first that α is zero or close to it, i.e. output does not play a role in the central bank's loss function. In this case we know that expected inflation is zero in the case without sunspots, as the central bank is able to counteract all relative demand (g) and cost-push/mark-up (u) shocks to keep inflation exactly on target as long as it follows the standard optimal rule of equations (12)-(14).

Shifting the formal solution of our model as specified in equation (11) forward by one period and taking expectations at period t on both sides using equation (7), we obtain equation (15).

$$(15)\quad \mathrm{E}_t \mathbf{V}_{t+1} = T \cdot K \cdot \boldsymbol{\eta}_t + \sum_{n=0}^{\infty} \mathbf{J}_n \, \mathrm{E}_t \, i_{t+1+n}$$

Under the $i_t = i_{t-1}$ policy rule the second term on the right hand side of (15) will be zero, as no deviations from the long-run value will be observed. Thus we can compute the expected inflation in period t+1 under the asymptotically optimal policy rule for $\xi\to\infty$:

[37] To simply show that the loss with the standard rule is larger then with a constant interest rate rule does not prove anything yet about optimality according to definition 6. Although dominated, the standard rule outcome could still be a Nash equilibrium. The crucial part is what follows, i.e. that the optimal interest rate reaction function depends on the past interest rate, and thus cannot be a Nash equilibrium.

[38] When ξ is tending to infinity it is the only case when we know at least one of the (possibly several) optimal rules for the sunspot case.

(16) $$E_t\,\pi_{t+1} = \frac{\lambda\mu}{\left(1-\mu\left(1+\beta+\lambda\varphi\right)+\beta\mu^2\right)}g_t + \frac{(1-\rho)\rho}{\left(1-\rho\left(1+\beta+\lambda\varphi\right)+\beta\rho^2\right)}u_t$$

which is clearly different from zero as long as the economy is subject to either output gap or cost-push shocks.

If α is not equal to zero but tends to infinity expected inflation in the standard no sunspot case is given by equation (17).

(17) $$\lim_{\alpha\to\infty} E_t\,\pi_{t+1} = \frac{\rho}{1-\beta\rho}u_t$$

Even in the absence of relative demand shocks g, one can show that the second term on the right hand side of equation (16) always exceeds the right hand side of (17). Thus the result that expected inflation in the sunspot and no-sunspot cases differs (and is larger in the sunspot case) does not depend on α being zero.

The intuition for the conflict between short-term price stability and financial stability is that a strategy trying to keep the fixed-horizon (unconditional) inflation forecast equal to the inflation target will result in a sub-optimal outcome for overall price stability (defined like in a standard loss function as the present discounted value of $E[(\pi-\pi^*)^2]$, where π^* is the target rate of inflation). It is useful to recall the following identity:

(18) $$E_t[(\pi_{t+n}-\pi^*)^2] = \mathrm{Var}\,(\pi_{t+n}) + [E_t(\pi_{t+n})-\pi^*]^2$$

Thus when the central bank is able to directly influence the variance of inflation, it might not be optimal to only target the (squared) bias $E_t(\pi_{t+n})-\pi^*$ of inflation[39]. In our example the central bank is able to directly influence the variance of inflation through its impact on the variance of the sunspot process. In reality it is less clear whether such situations occur frequently.

5. Discussion

An immediate criticism of our example could be to question the empirical relevance of the degree of forword-lookingness in the baseline New Keynsian model employed here. This could be of concern because the possibility of sunspot equilibria might vanish with a sufficient degree of history dependence in the optimal policy rule.[40] Gali (2000) discusses the empirical evidence and shows that recent studies have found evidence in favour of the forward-looking Phillips curve as depicted in

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[39] See Broadbent and Walton (2000).
[40] See Svensson and Woodford (1999).

equation (2), so that the sunspot possibility cannot be dismissed simply for the lack of empirical relevance of the chosen model specification.

A more severe argument against the relevance of our example is the fact that Bernanke and Woodford (1997) as well as Svensson and Woodford (1999) show that the problem of indeterminacy can be solved by the central bank committing to the appropriate weights of a Taylor rule. A Taylor rule is generally found to provide a good approximation to optimal monetary policy[41]. Thus could not a central bank avoid the possibility of sunspot financial crises simply by committing to a well-specified instrument rule? We would tend to argue that a Taylor-rule like behaviour in a close-to-crisis situation might consecutively loose credibility as events unfold, exactly because an instrument rule is not the optimal policy and not time-consistent, once a crisis happens. The fact that the behaviour of major central banks' could ex-post decently be described by Taylor rules does not mean that there is a credible ex-ante commitment to such a rule. We thus do not find the argument convincing enough to discard the sunspot crisis possibility. In this context it would be interesting to analyse whether a loss function giving some weight to a reference value for money growth could avoid the indeterminacy by introducing some history dependence in the reaction function. The answer has to be left for future research.

Another argument against the empirical relevance of our case stems from the empirical literature on financial crises. Gorton (1988), Calomiris and Gorton (1991) and Goodhart (1995) present evidence that banking crises tend to be related to the business cycle. Bordo, Duecker and Wheelock (2000) claim periods of financial distress historically are caused by shocks to inflation or the price level. To reconcile these empirical findings with our model, one would need to argue that the likelihood of a sunspot crisis rises in a recession, which might very well be true but resembles an attempt to immunise the sunspot theory from falsification.[42] Thus it certainly is worthwhile to also adopt the real business cycle view of financial crises and check the possibility or likelihood of conflict cases. This also has to be left for future research. But the mere scarcity of findings of conflict in this field (derived from first principles), in our view is a sign of the general tenuousness of the conflict case between price stability and financial stability. The source of the tenuousness is that if the crisis happens in a recession, a more expansionary monetary policy is unlikely to create inflation but would rather tend to stabilise both inflation and the financial system. Our conflict case became possible exactly because the occurrence of the sunspot crisis is unrelated to the current state of inflation and the business cycle.

[41] See again Gali (2000, p. 3).

[42] Although one could argue that our case is something of a hybrid between a pure sunspot and the real business cycle view of financial crises. The variance of the sunspot process is uncorrelated to this period's output gap and cost-push disturbance, but not to last period's in case the central bank reacts to the latter disturbances in a standard way. Thus when the economy is changing speed the standard interest rate response of the central bank will increase the severity of possible sunspot financial crises. This is an observation, which is not at odds with the empirical evidence cited above.

An interesting extension of this paper would be to analyse whether our specific sunspot equilibrium is learnable in the sense of Honkapohja and Mitra (2001).
It is worthwhile to summarise the three major general assumptions, which drive our result.

1. crises encompass a self-fulfilling element, unrelated to economic fundamentals.
2. crises have significant effects on future inflation and output variability.
3. crises (or rather the self-fulfilling dynamics driving the crises) can actually be contained by central bank monetary policy.

All three conditions could each be questioned to cast some doubt on the relevance of our conflict result[43]. Or they could be used as a very general checklist to identify situations in which an explicit reaction to financial instability is warranted for purely monetary policy reasons.
An obvious and important caveat is that we have not considered the moral hazard issue, i.e. the question whether financial market participants' behaviour would not shift towards more (excessive) risk taking, once an explicit optimal monetary policy reaction for crises situations is known to the public.
Some obvious *policy conclusions* can be drawn from the analysis.
First of all, a conflict case stresses the importance of conducting monetary policy with a sufficiently long-horizon, which might imply deviating from the inflation target in the short-run.
Second, if a central bank expects financial stability to be affected by its monetary policy decisions - even if this reaction is only based on sunspot beliefs - and abstracting from moral hazard considerations, it will better take these beliefs into account for purely price stability oriented motives. Note that on first reading this apparently contradicts the recommendation by Bernanke and Woodford (1997) who argued that monetary policy should not be based on any variable influenced by market expectations (like consensus inflation forecasts or asset prices). In their instrument rule approach such behaviour exactly introduces the circularity leading to indeterminacy of equilibrium and is thus best avoided. In our optimal monetary policy setting, the indeterminacy is structurally unavoidable due to the high degree of "forward-lookingness" in the model. Trying to dampen the effects of the sunspot process is then the optimal reaction[44].
The specific link between macroeconomic stability and interest rate smoothing as specified in the model is not meant to be taken literally (although as argued above we believe it is not unrealistic). We rather intend to make a general point about the importance of self-fulfilling beliefs for monetary policy. In this sense the following citation from Woodford (1994, p. 324) fits particularly well: "The

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[43] Blanchard (2000, p. 5) agrees with (at least) assumptions number 1 and 3. "Bubbles by definition, are not based on fundamentals, but on animal spirits. And there are good reasons to believe that the stance of monetary policy can excite or dampen these spirits."
[44] Recently Svensson and Woodford (1999) and Evans and Honkapohja (2001) also argued in favour of incorporating private sector expectations in the central banks monetary policy rule.

idea is that a theoretical understanding of why a certain design for a ladder, for example, would be unstable, is of practical use even if the theory cannot be used to predict in which direction the ladder will fall on any given occasion".

Finally it should be emphasised that we consider the three basic assumptions mentioned previously as rather restrictive, so that in reality conflict cases will rather be the exception than the rule.

6. Summary

We have shown how the standard New Keynsian IS-AS model in the mode of Clarida, Gali and Gertler (1999) is subject to sunspot equilibria due to its forward-looking nature (see Svensson and Woodford (1999)). We link the sunspot equilibria to financial crises so that the central bank can dampen the economic consequences of a crisis, which can be arbitrarily large with a smooth interest rate policy. We suggest a more general Nash-type concept of optimality tackling the time inconsistency problem of the optimal rules obtained from dynamic programming, which enables us to show that with sunspot financial crises the standard optimal monetary policy rule is no longer optimal. We depict the possibility that the occurrence of a financial crisis triggers a conflict case between short-term price stability and financial stability. The results are derived within the current standard macroeconomic model, which is - except for the by definition arbitrary sunspot component - perfectly micro-founded. We derive the general conditions under which such a conflict is possible and combine these with the available empirical evidence to conclude that - abstracting from moral hazard issues – policy relevant conflict cases cannot be excluded but are likely to be rather rare events.

Appendix

This mathematical appendix summarizes the mathematical formalization and results of the paper.

1 The model

The presentation of the model is divided into two sections. The first section focuses on the dynamics of the model. Our presentation of those dynamics allow us to consider not only the standard solution, but also other solutions that involve sunspot equilibria. The second section focuses on the decision problem faced by the central bank within this model. In this second section, we give the notion of "optimal policy" of the central bank a rigorous meaning, and this rigorous meaning remains valid under circumstances general enough to allow us to handle the case with sunspots.

1.1 The dynamical system

In this section, we focus on the dynamical aspects of our problem. After recalling what was the original formulation of the dynamics, we rewrite the system by making use of a vector formulation, which we will use from then on. This vector formulation encompasses the original formulation to be found in Clarida, Gali and Gertler (1999), however it is slightly more general. This is because it should express, not only the original situation described in Clarida, Gali and Gertler, but also variants relying on sunspot equilibria. We end this section by using the vector formulation of the dynamical system to present the solution to the system.

1.1.1 Original formulation of the dynamic system

We start with the equations given in Clarida, Gali and Gertler (1999, 1665), i.e. their equations (2.1) and (2.2).

$$\begin{aligned} x_t &= -\varphi\left[i_t - \mathrm{E}_t\ \pi_{t+1}\right] + \mathrm{E}_t\ x_{t+1} + g_t \\ \pi_t &= \lambda\, x_t + \beta\ \mathrm{E}_t\ \pi_{t+1} + u_t \end{aligned} \tag{1}$$

Time t is discret. Those equations involve:

- the output gap x_t and the inflation π_t , which are random variables,
- the disturbance terms g_t , u_t , which are random variables,
- the policy rate i_t that can be interpreted as a general random variable,
- and the constant parameters φ, λ and β, whose economic interpretations are given in the paper (φ is an intertemporal elasticity of substitution, λ the elasticity of the Philips curve and β a "natural" rate of discount).

The symbol E_t stands for the expectations operator given information in period t which we will label "knowing t" from here on. Similarly, we will later use the symbol Var_t for the variance knowing t.

To denote expectation by making explicit reference to the law P of the process $X_\bullet$ knowing t, we will use the notation E^{P} and the term "expectation under the probability P".

To denote expectation knowing t and knowing that some given process $X_\bullet$ takes at time t the value x, we will use the notation $\mathrm{E}[\ |X_t=x]$ and the expression "expectation at time t knowing that $X_t=x$".

1.1.2 Vector formulation of the dynamic system

We regroup the output gap x_t and the inflation π_t into one vector $\mathbf{V}_t$.

$$\mathbf{V}_t := \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} \tag{2}$$

We regroup the disturbance terms g_t , u_t and a third one a_t to be used later on into one vector $\boldsymbol{\eta}_t$.

$$\boldsymbol{\eta}_t := \begin{pmatrix} g_t \\ u_t \\ a_t \end{pmatrix} \tag{3}$$

From the constant parameters of the model, we build the following matrices:

$$L := \begin{pmatrix} 1 & 0 & 0 \\ \lambda & 1 & 0 \end{pmatrix} \tag{4}$$

$$M := \begin{pmatrix} 1 & \varphi \\ \lambda & \beta + \lambda\varphi \end{pmatrix} \tag{5}$$

We denote with ν_1 and ν_2 the eigenvalues of M, ordering them such that $\nu_1 < \nu_2$:

$$\nu_1 = \frac{1+\beta+\lambda\varphi - \sqrt{(1+\beta+\lambda\varphi)^2 - 4\beta}}{2}$$
$$\nu_2 = \frac{1+\beta+\lambda\varphi + \sqrt{(1+\beta+\lambda\varphi)^2 - 4\beta}}{2} \qquad (6)$$

Finally, we build a sequence of vectors J_n, n>0 by giving:

$$\mathbf{J}_0 := \begin{pmatrix} -\varphi \\ -\lambda\varphi \end{pmatrix} \qquad (7)$$

and the recurrence relationship:

$$\mathbf{J}_{n+1} := M\,\mathbf{J}_n \qquad n > 0 \qquad (8)$$

Then we have:

Lemma 1:
The equations (1) can be rewritten as:

$$\mathbf{V}_t = i_t\,\mathbf{J}_0 + M\ \mathrm{E}_t\mathbf{V}_{t+1} + L\ \boldsymbol{\eta}_t \qquad (9)$$

Proof:
The proof consists of standard algebraic computations.

1.1.3 Formal solution of the dynamical system

We intend to find a solution for (9). For the moment, we are only interested in a formal solution, so:
- issues like convergence of sums or of limits are left aside,
- also, the issue of the full generality of the solution is left aside.

But we make two requirements:
- We want our formal solution to have an explicit form,
- also we want our formal solution to encompass the case provided by section 2 of Clarida, Gali and Gertler (1999), and to encompass another case that corresponds to a sunspot equilibrium.

Even if we remain on the formal level, we need to specify the law of the process $\boldsymbol{\eta}_t$ of disturbances. We will assume from now on three things, namely that:
- the distribution of $\boldsymbol{\eta}_t$ knowing t is normal,
- the average of this conditional distribution is a deterministic linear function of $\boldsymbol{\eta}_t$,
- the variance of this conditional distribution is a deterministic function of $\boldsymbol{\eta}_t$, i_{t-1} and i_t ; which deterministic function has a specific form.

Let us give an exact formulation to those three assumptions. From the constant parameters of the model, we build the following constant matrix:

$$K := \begin{pmatrix} \mu & 0 & 0 \\ 0 & \rho & 0 \\ 0 & 0 & \nu_2^{-1} \end{pmatrix} \qquad (10)$$

and from other constant parameters S_{11}, S_{12}, S_{22} and ξ satisfying $S_{11}\geq 0$, $S_{22}\geq 0$, $S_{12}^2 \leq S_{11} . S_{22}$ and $\xi \geq 0$, we build the following deterministic matrix:

$$S(i_t - i_{t-1}) := \begin{pmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & \xi\,(i_t - i_{t-1})^2 \end{pmatrix} \qquad (11)$$

The constraints satisfied by the parameters S_{11}, S_{12}, S_{22} and ξ ensure that $S=S(i_t - i_{t-1})^2$ is always positive. Then our assumption regarding the law of the process $\boldsymbol{\eta}_t$ is :

Assumption 1:
The distribution of $\boldsymbol{\eta}_t$ knowing t is a Gaussian distribution such that:

$$\mathrm{E}_t\left(\boldsymbol{\eta}_{t+1}\right) = K \cdot \boldsymbol{\eta}_t \tag{12}$$

$$Var_t\left(\boldsymbol{\eta}_{t+1}\right) = S\left(i_t - i_{t-1}\right)^2 \tag{13}$$

The parameter ξ measures the intensity of the sunspot effect, there is a sunspot if $\xi>0$, there is no sunspot if $\xi=0$. Some particular cases are of interest:

- In the case $\xi=0$, $\boldsymbol{\eta}_t$ follows a simple Gaussian autoregressive process, and its third component a_t is constantly zero.
- If moreover $S_{12}=0$, the two first components g_t and u_t of $\boldsymbol{\eta}_t$ follow independent simple Gaussian autoregressive processes, and we are in the case of Clarida, Gali and Gertler (1999), with S_{11} being called σ_g^2 and S_{22} being called σ_u^2.

Having specified the law of the process $\boldsymbol{\eta}_t$ of disturbances, we can find formal solutions of equation (9). In order to do so, we build, from the constant parameters of the model, a fourth matrix:

$$T := \begin{pmatrix} \dfrac{\beta(1-\mu\beta)}{(\beta\mu - v_1)(\beta\mu - v_2)} & \dfrac{\beta\rho\varphi}{(\beta\rho - v_1)(\beta\rho - v_2)} & -\beta - \lambda\varphi + v_2 \\ \dfrac{\beta\lambda}{(\beta\mu - v_1)(\beta\mu - v_2)} & \dfrac{\beta(1-\rho)}{(\beta\rho - v_1)(\beta\rho - v_2)} & \lambda \end{pmatrix}$$

$$= \begin{pmatrix} \dfrac{(1-\mu\beta)}{\left(1-\mu(1+\beta+\lambda\varphi)+\beta\mu^2\right)} & \dfrac{\rho\varphi}{\left(1-\rho(1+\beta+\lambda\varphi)+\beta\rho^2\right)} & -\beta - \lambda\varphi + v_2 \\ \dfrac{\lambda}{\left(1-\mu(1+\beta+\lambda\varphi)+\beta\mu^2\right)} & \dfrac{(1-\rho)}{\left(1-\rho(1+\beta+\lambda\varphi)+\beta\rho^2\right)} & \lambda \end{pmatrix} \tag{14}$$

We then have:

Theorem 1:

The process defined by:

$$\mathbf{V}_t := T \cdot \boldsymbol{\eta}_t + \sum_{n=0}^{\infty} \mathbf{J}_n \, \mathrm{E}_t \, i_{t+n} \tag{15}$$

is a formal solution of (9).

Proof:

Applying formula (15) to $\mathbf{V}_{t+1}$ one gets:

$$\mathbf{V}_{t+1} = T \cdot \boldsymbol{\eta}_{t+1} + \sum_{n=0}^{\infty} \mathbf{J}_n \, \mathrm{E}_{t+1} \, i_{t+1+n} \tag{16}$$

One takes expectations knowing t of both sides of (16). By using $\mathrm{E}_t\mathrm{E}_{t+1} = \mathrm{E}_t$ and the hypothesis (12) one gets:

$$\mathrm{E}_t \mathbf{V}_{t+1} = T \cdot K \cdot \boldsymbol{\eta}_t + \sum_{n=0}^{\infty} \mathbf{J}_n \, \mathrm{E}_t \, i_{t+1+n} \tag{17}$$

One left-multiplies both sides of (17) by M. By using the recurrence relationship (8) and replacing the dummy n+1 by n one gets:

$$M \cdot \mathrm{E}_t \mathbf{V}_{t+1} = M \cdot T \cdot K \cdot \boldsymbol{\eta}_t + \sum_{n=0}^{\infty} \mathbf{J}_{n+1} \, \mathrm{E}_t \, i_{t+1+n} = M \cdot T \cdot K \cdot \boldsymbol{\eta}_t + \sum_{n=1}^{\infty} \mathbf{J}_n \, \mathrm{E}_t \, i_{t+n} \tag{18}$$

Standard algebraic computations show that the matrices *K, L, M, T* satisfy the relationship:

$$L = T - M \cdot T \cdot K \tag{19}$$

One subtracts (18) from (15), then subtracts $M\,\mathrm{E}_t\mathbf{V}_{t+1}$ from both sides and applies (19). One obtains the equation (9), which completes the proof.

1.2 The optimization problem

In this section, we will give a rigorous formulation to the optimization problem faced by the central bank. Our first attempt will be to write this problem according to the standard optimal control theory, centered around Bellman's principle. In order to do so, we first specify some reasonable features of the formulation. Then we determine the forms of the state variable and the controlled motion. Then we determine the shape of the objective function. Arrived at this point, we observe that the form of the objective function is an obstacle to a formulation of the problem in terms of the standard optimal control theory. This leads us to propose a more general definition of optimality, which is able to handle this problem.

1.2.1 Reasonable features of an optimal control formulation of the model

We will attempt to set the model in the formal frame of the optimal control theory. The role of the control should evidently be attributed to the policy rate i_t. Besides, one needs to define:

- a state variable F_t, belonging to a state space Ω,
- a controlled equation giving the motion of state variable F_t as a function of state variable F_t itself and i_t, and
- a criterion to maximize, having the form of the expectation of some well-behaved functional of the future path of F_t.

The economics of the problem determine that the criterion to maximize must be the expectation a functional of the future path of $\mathbf{V}_t$. Because of the form (15) of $\mathbf{V}_t$, the state variable F_t should encompass at least the disturbance term $\boldsymbol{\eta}_t$. In other words, the optimal control formulation sets that the vector $\boldsymbol{\eta}_t$ is given by a deterministic function of F_t.
Moreover F_t should be driven by a controlled equation of the form:

$$\mathbf{F}_{t+1} = \mathbf{A}(\mathbf{F}_t , i_t) + \mathrm{B}(\mathbf{F}_t , i_t)\Delta\mathbf{W}_t \tag{20}$$

where i_t is the control, the symbol Δ denotes the first difference

$$\Delta X_t := X_{t+1} - X_t \tag{21}$$

and $\Delta\mathbf{W_t}$ is a sequence of i.i.d. normal centered Gaussian vectors, so that $\mathbf{W_t}$ is a discrete-time Wiener process. The assumptions made above constrain the form of B() in a way which depends of the relationship between $\boldsymbol{\eta}_t$ and F_t. Here are two examples:

- If for example the state variable can be identified to $\boldsymbol{\eta}_t$ itself, then K is a diagonal matrix of eigenvalues μ, ρ, and $1/\nu_2$.
- If the state variable can be decomposed into $\boldsymbol{\eta}_t$ (first three components) and i_{t-1} (fourth component), then K is a diagonal matrix of eigenvalues μ, ρ, $1/\nu_2$ and 0.

We will focus on the second one.

1.2.2 Choice of the state variable and of the controlled motion

We formalize this choice into the:

Assumption 2:

$$F_t := (\boldsymbol{\eta}_t , i_{t-1}) \tag{22}$$

Lemma 2:

Under assumptions 1 and 2, the vector A() and the matrix B() of equation (20) take the specific forms:

$$\mathbf{A}(\mathbf{F}_t , i_t) := \begin{pmatrix} \mu & 0 & 0 & 0 \\ 0 & \rho & 0 & 0 \\ 0 & 0 & {\nu_2}^{-1} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \mathbf{F}_t + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \cdot i_t = \begin{pmatrix} & [K] & & 0 \\ & & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \mathbf{F}_t + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \cdot i_t \tag{23}$$

$$B(\mathbf{F}_t , i_t) := B(i_t) = \begin{pmatrix} S_{11} & S_{12} & 0 & 0 \\ S_{12} & S_{22} & 0 & 0 \\ 0 & 0 & \xi\,(i_t - i_{t-1})^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} & [S(i_t - i_{t-1})] & & 0 \\ & & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{24}$$

Proof:
Straightforward.

1.2.3 Choice of the objective function

For the sake of consistency with Clarida, Gali and Gertler (1999), the objective function must take the form:

$$\Lambda := -\mathrm{E}_t \sum_{n=0}^{\infty} \beta^n \, \mathbf{V}^{'}_{t+n} Q \mathbf{V}_{t+n} \tag{25}$$

where the superscript ' indicates transposition and where Q is the constant, strictly positive matrix, defined as:

$$Q := \begin{pmatrix} \alpha & 0 \\ 0 & 1 \end{pmatrix} \tag{26}$$

The parameter α is assumed to be positive. Then formula (25) coincides with the objective function equation (2.7) in Clarida, Gali and Gertler (1999), (cf p. 1668).

1.2.4 Difference between this problem and the standard optimal control problem

Having set the dynamics of the system and the objective function of the problem, we should be in a position to complete the formulation of the problem as an optimal control theory problem. Let us first recall by a representative example what are the usual notions and the standard results of the optimal control theory.

- A state variable X summarizes the relevant information about the state of the system. X is taken in a state space Ω, which is typically a finite-dimension space.
- A control variable b represents the decision to be taken at each step. B is taken in a space Ω_1. Typically Ω_1 is the tangent space of Ω or a convex part of it.
- One has a dynamic controlled equation of the same form of (20)

$$X_{t+1} = \mathbf{A}(X_t, b_t) + \mathrm{B}(X_t, b_t)\Delta \mathbf{W}_t \tag{27}$$

(where $X_\bullet$ is the process of the state variable, $b_\bullet$ is the process of the control, and thus X_t is the state variable at time t and b_t is the control at time t.)

- One has an objective function Λ. At decision time t_0 the state variable has value x, and the objective function Λ takes the form:

$$\Lambda = \Lambda(x, b_\bullet) := -\mathrm{E}\left[\sum_{t=t_0}^{\infty} \beta^{t-t_0} \, h(X_t, b_t) \mid X_{t_0} = x\right] \tag{28}$$

where the expectation is taken at time t_0 under the weak solution of (28) that starts in x at time t_0.
(In this expository example, we have assumed no explicit dependency upon time for the controlled motion (27) or for the objective function (28). This is only for the sake of simplicity of notations that we assume this property of time-homogeneity, although the theory is of course able to handle such dependencies. Note however that the problem we are studying also shares this property of time-homogeneity.)

According to the standard optimal control theory, one would get, formally at least, the following results:

- First, there is a process $b^*_\bullet$ maximizing the objective function (28) for all initial state variable x.

$$\forall b_\bullet, \Lambda(x, b^*_\bullet) \geq \Lambda(x, b_\bullet) \tag{29}$$

- Second, the value function U=U(x), defined as the maximum of the objective function

$$U(x) := \max_{b_\bullet} \Lambda(x, b_\bullet) = \Lambda(x, b^*_\bullet) \tag{30}$$

is solution of the Bellman equation:

$$U(x) = \max_{b} \left(h(x, b) + \beta \mathrm{E}\left[U(X_{t+1}) \mid X_t = x\right]\right) \tag{31}$$

where E[$|X_t=x$] denotes expectation at time t knowing that $X_t=x$. (Notice that the maximum in (31) is taken on b, the *current value* of the control, while the maximum in (30) was taken on $b_\bullet$, the *process* of the control.)

- Third, the optimal control $b^*_\bullet$ takes the particular form of a reaction function:

$$b^*_t \equiv b^*(X_t) \tag{32}$$

Of course, getting those results in a rigorous way involves many technicalities. For example:

- one must assume some regularity of the function h,
- one may have to define in a weakened sense the solutions of the Bellman equation (31) (like, in the time-continuous case, the so-called “viscosity solutions”, needed if the variance is not uniformly bounded from below),

- one may have to replace “for all initial x” by “for nearly all initial x”, so up to a set of zero measures,
- or one may have to understand the optimal control as a L^{∞}_{loc}, therefore defined up to a set of zero measures.

But the *meaning*, the *existence* and the *closed-loop form* of the optimal control are not problematic.

To exemplify what we understand under “standard optimal control theory”, we would like to provide a reference. The book “Stochastic control by functional analysis methodes”, by A. Bensoussan (1992) is in our opinion well suited (although it deals with the continuous time case instead of the discrete time case). The reader may observe how the possible strategies of the agent are first represented with a stochastic process (this is Chap. IV, § 1.2 p. 140), as we did with our control process $b_\bullet$ starting from (27) and (28). Optimality is a priori defined via a maximization over those processes, (this is formula (1.20) p.142) and thus the optimal control corresponds to our process $b^*_\bullet$ rather than to a reaction function that would correspond to our optimal policy rule. But then, the optimal control is proven to take the particular form of a reaction function (this is the proof of theorem 3.1, (Chap. IV, § 3 pp. 151-153)) This fact, that corresponds here to (32), is, in the book, emphasized in a remark (this is remark 3.1 p. 153). This fact is key: Should the optimal control actually take the form of a reaction function, then fairly obviously any problems of time-inconsistency are precluded. But again we want to stress: This fact cannot be taken for granted a priori, it does deserve to be proven, and the proof usually relies on a specific form of the decision problem (in essence our equation (27) and (28)). Thus when one cannot bring a decision problem into this form, it is not legitimate to postulate that the optimal control is a reaction function (or in other words a feedback, or a policy rule).

Let us then check whether both the dynamics and the objective function of the problem (20)-(26) have the required form. For what concerns dynamics, as we already noticed, (20) and (27) have the same form, so no difficulty arises from the side of the dynamics of the problem. We then consider the objective function.
The question is then to know whether this objective function (25) can be brought under that form (28). Let us consider the following transformation of objective functions:

$$\Phi(\Lambda) := \mathrm{E}_{t+1}\left(\Lambda(X_t, b_\bullet) - \beta\, \Lambda(X_{t+1}, b_\bullet)\right) \tag{33}$$

We observe that a function Λ of the form (28) necessarily fulfills

$$\Phi(\Lambda) = h(X_t, b_t) \tag{34}$$

where the right side depends only on the value b_t of the control at time t. By contrast, applying the same Φ to the function Λ defined in (25) one finds

$$\Phi(\Lambda) = \mathbf{V}_t' Q \mathbf{V}_t \tag{35}$$

but this, due to (15) can be written as:

$$\Phi(\Lambda) = -\mathrm{E}_t \left(T \cdot \boldsymbol{\eta}_t + \sum_{m=0}^{\infty} \mathbf{J}_m \,\mathrm{E}_t\, i_{t+m} \right)' Q \left(T \cdot \boldsymbol{\eta}_t + \sum_{m=0}^{\infty} \mathbf{J}_m \,\mathrm{E}_t\, i_{t+m} \right) \tag{36}$$

This evidently does not only depend on the value i_t of the control at time t, but involves the law of the process $i_\bullet$ as a whole. Consequently the objective function (25) cannot be brought under the form (28), and the standard optimal control theory does not apply here.
However the objective function (25) can be brought under a more general form, namely:

$$\Lambda = \Lambda(x, b_\bullet) := -\mathrm{E}\left[\sum_{t=t_0}^{\infty} \beta^{t-t_0}\, h(X_\bullet, b_\bullet \mid t) \mid X_{t_0} = x \right] \tag{37}$$

where $X_\bullet$, $b_\bullet \mid t$ denotes the law of the (joint) process ($X_\bullet$, $b_\bullet$) knowing t.
It is important to observe two things:

- This law $X_\bullet$, $b_\bullet \mid t$ is not a number, but an infinite-dimensional object, and consequently h is a functional and not a function).
- This law, which is knowing t, is itself a random object at time t_0 since t_0 is before t. The external expectation in (37) is knowing t_0. Consequently (37) refers to expectations knowing t_0 of functional of laws knowing t.

To see that the objective function (25) can indeed be written under form (37), it is enough to replace in (25) the terms V by their expression (15). One gets:

$$\Lambda := -\mathrm{E}_t \sum_{n=0}^{\infty} \beta^n \left(T \cdot \boldsymbol{\eta}_{t+n} + \sum_{m=0}^{\infty} \mathbf{J}_m \,\mathrm{E}_{t+n}\, i_{t+n+m} \right)' Q \left(T \cdot \boldsymbol{\eta}_{t+n} + \sum_{m=0}^{\infty} \mathbf{J}_m \,\mathrm{E}_{t+n}\, i_{t+n+m} \right) \tag{38}$$

which has clearly of the form (37).

Since it is straightforward that the form (28) is a particular case of the form (37), one could think about building a more general framework, suited to those objective functions that have form (37). This is what the next subsection is about.

1.2.5 Formulation within an enlarged framework

As we said, the standard optimal control theory does not apply here. Still, it is possible to define an optimal control process like in (29) and a value function like in (30), by relying on mere topological arguments. But it is not possible any more to ensure a priori that the optimal control will take the particular form of a reaction function, like in (32), or that the value function will obey a Bellman equation like (31).
We wish however that the optimal control, in a new sense that we will have to define, could take the particular form of a reaction function. Only then could it have the economic interpretation of a policy rule.
We will focus on the case of optimization under discretion. In this discretionary case, the central bank chooses, at each step, the interest rate without being constrained by anything. The optimal choice might or might not take the form of a function of the current value of the state variable. If it does, however, the expectations about the future behavior of the central bank must reflect that the central banks obey that particular reaction function (or, in other words, follows that particular policy rule). Given those expectations, the optimal choice of the central bank must actually coincide with the prescription of the reaction function for the current value of the state variable.
This is a fixed point problem. The definition we have to introduce is formally analogue to the definition of a Nash equilibrium, which is the fixed point of the so called "best reply" mapping. It can also be understood as the Nash equilibrium of a game between an explicit player, the central bank, and an implicit player, the "market".
Let us formalize the definition of an optimal policy rule. We set:

Definition 1:
A closed loop control *b() is a measurable deterministic function (i.e. a* L^{∞}_{loc} *function) of the state variable x, valued in* Ω_1*, giving the control at time t* b_t *as* $b_t = b(X_t)$*.*

Definition 2:
We call the law of that stochastic process, which the state variable X_t *and the control* b_t *jointly follow, the* dynamic law.

Now remember that objective functions of the form (37) refer to a probability. This probability is the law of a stochastic process jointly followed by the state variable and the control and consequently corresponds to the definition of a dynamic law. (Notice that (37) involves this probability in a complex way, since it mentions the law knowing t_0 of the laws of the process ($X_\bullet$, $b_\bullet$) knowing various $t \geq t_0$. However this complexity is in itself not material.) This enables us to set:

Definition 3:
If an objective function has form (37), we say that the probability appearing in formula (37) is the underlying dynamic law *of this objective function.*

The dynamic law will be of the form given by (27). Replacing in (27) b_t by $b(X_t)$ one gets:

$$X_{t+1} = \mathbf{A}(X_t, b(X_t)) + \mathrm{B}(X_t, b(X_t))\Delta\mathbf{W} \tag{39}$$

Definition 4:
We say that the dynamic law is subject to *the closed loop control b() if this dynamic law is the weak solution of (39). We say that the dynamic law is subject to the closed loop control b()* from *time T if this dynamic law is the weak solution of (39) for all t>=T.*

Definition 5:
We say that the dynamic law is (b,x,t)-first-free subject to the closed loop control b() *iff:*

- $\boldsymbol{X}_t = \boldsymbol{x}$,
- $b_t = b$,
- *and this dynamic law is subject to the closed loop control b() from t+1.*

Having set those five definitions, we are now in position to formalize the definition of an optimal closed loop control.

Definition 6:
A closed loop control is optimal *iff it associates to each state variable x the value which maximizes the objective function over b whose underlying dynamic law is (b,**x**,t)-first-free subject to this closed loop control.*

A first thing to inspect is whether this definition of optimality actually generalizes the definition of the standard optimal control theory.

Theorem 2:
Consider the optimal control problem, given by the dynamic controlled equation (27) and the objective function (28). Then the optimal control in Bellman's sense is a closed loop control that is also optimal in the sense of Definition 6.

Proof:
Choose a time t in N, a state variable X_t in Ω, and a control b_t in Ω_1 . By combining (29) and (32), one gets:

$$\forall b_\bullet, \Lambda\left(X_t, b^*\left(X_\bullet\right)\right) \geq \Lambda\left(X_t, b_\bullet\right) \tag{40}$$

where $b^*()$ is the optimal control in Bellman's sense written as a reaction function (this can be done because of (32)).
The combined equation holds for all control processes $b_\bullet$, so it holds for that particular control process, which is (b, X_t, t)-first-free subject to $b^*()$. Since this is valid for any t, X_t and b_t, it follows that $b^*()$ is optimal in sense of Definition 6.

The Bellman formalism applies in situations where (27) gives the dynamics and (28) gives the objective function. Definition 6 defines a notion of optimality that applies in situations where (27) gives the dynamics and (37) gives the objective function. Since (37) is more general than (28), this is a enlargement of the Bellman notion of optimality. But it is interesting only if optimal control in the sense of Definition 6 actually exists in situations where optimal control in the sense of Bellman is not defined. The general existence theorem for a decision problem with controlled dynamic (27) and objective function (37) is still an unsolved problem.

2 The conflict

The remaining part of the appendix formalizes the potential conflict arising between financial stability and price stability in the case of sunspots. We start with a rigorous formulation of this decision problem, based on the assumptions and definitions made so far. We then turn to the policy rule introduced in the Clarida-Gali-Gertler paper in the discretionary case. We show that, in the case without sunspots, this policy rule is optimal in the sense that we introduced above. The presentation of those solutions is divided into two sections. We show that it looses it optimality when the sunspot linking price stability to smoothness of policy rates is present and is large enough.

2.1 Summary of the decision problem

To fully specify the problem, one needs to indicate the variables, the dynamics, the objective function and the definition of optimality, in case this definition is not standard. We define those four items, and are then able to define a "self-reply" mapping of the problem. An optimal policy rule, in the sense that we choose to consider, is a fixed point of that well-defined self-reply mapping.

2.1.1 State variable and control variable.

By assumption 2, the state variable $\mathbf{F}_t$ is defined as $(\boldsymbol{\eta}_t, i_{t-1})$, which means that the state space Ω is R^4. The control variable is i_t , which means that Ω_1 is the real axis. In this case, we refer to the closed-loop controls as to the policy rules. So the formal definition of a policy rule i() is a L^∞_{loc} function from Ω to the real axis.

2.1.2 Dynamics of the problem

Assumptions 1 and 2 ensure via lemma 2 that the state variable $\mathbf{F}_t := (\boldsymbol{\eta}_t, i_{t-1})$ follows the dynamic controlled equation (20) (with A and B specified by (23) and (24)); this equation (20) is of the form of (27).

2.1.3 Objective function of the problem

The objective function is the one defined in (25), it can be rewritten as (38) and is therefore of the form of (37).

2.1.4 Notion of optimality

Since the dynamics can be brought under form (27) and since the objective function can be brought under form (37), it is possible to define optimal policy rules according to Definition 6. This is the definition we choose.

2.1.5 Formal definition of the self reply.

The self reply is a mapping defined on and valued in the space of policy rules, which is the space of L^{∞}_{loc} functions defined on the state space $\Omega = R^4$ and valued in the real axis. We will denote this mapping with the letter Φ. When applied to a policy rule i(), it gives another policy rule policy rule j()=Φ (i()). Consequently, to define the self reply is nothing else but to specify the value of j(**F**)=Φ (i())(**F**) for an arbitrary state variable **F**. This is what we are going to do.
Let us consider a policy rule i(). Let us replace in (20) i_t by i($\mathbf{F}_t$). We get:

$$\mathbf{F}_{t+1} = \mathbf{A}(\mathbf{F}_t, \mathrm{i}(\mathbf{F}_t)) + \mathrm{B}(\mathbf{F}_t, \mathrm{i}(\mathbf{F}_t))\Delta\mathbf{W}_t \tag{41}$$

According to Definition 4, the weak solutions of (41) that start at some initial time t_0 from some given value **F** of the state variable are subject to the closed loop control i(). The processes that start at t_0 from state x and that are weak solutions of (41) for $t \geq t_0+1$ are subject to the closed loop control i() from t_0+1. Among those processes, those who start at t_0 from state **F**, satisfy at time t_0, for some given value i of the policy rate:

$$\mathbf{F}_{t_0+1} = \mathbf{A}(\mathbf{F}_{t_0}, i) + \mathrm{B}(\mathbf{F}_{t_0}, i)\Delta\mathbf{W}_{t_0} \tag{42}$$

and then are weak solutions of (41) for $t \geq t_0+1$, are (i,**F**,t_0)-first-free subject to i(), according to Definition 5.
The formal definition of Φ thus becomes a simple matter of notation.

- We denote with P(i,**F**,t_0,i()) the law of the process which is (i,**F**,t_0)-first-free subject to i() .
- We denote with I(i,**F**,t_0,i())• the process which takes value i at time t_0 and i($\mathbf{F}_t$) at time $t \geq t_0+1$.
- Recall also that we denote the expectation under some probability P with the notation E^P.

Observe that those notations allow us to express the weak solution of (41) that starts in **F** at time t_0: This solution can be written as P(i(**F**),**F**,t_0,i()).
Then we can specify the value of j(**F**)=Φ (i())(**F**) through the formula:

$$\Phi(\mathrm{i}())(\mathbf{F}) \in \arg\max_{i} \ \Lambda(\mathbf{F}, \mathrm{I}(i, \mathbf{F}, t_0, \mathrm{i}())_{\bullet}) \tag{43}$$

Observe that t_0 becomes a dummy in (43) and could be replaced by any other character. Also, another thing is that the claimed measurability of j()=Φ(i()) is not readily apparent on (43). In all generality, the arg max is a correspondence or multivalued function. The possibility to choose j(**F**) among the possibly several elements of the arg max - in such a way that i depends measurably of **F** - is not a priori granted. We leave this technical issue aside.

2.2 The standard optimal rule

According to whether there are sunspots or not, the decision problem takes a different aspect. This last subsection shows that the presence of a sunspot linking financial stability to price stability destroys the optimality of the standard rule. We consider subsequently the cases without sunspot and with sunspot.

2.2.1 Case without sunspots: The solution of Clarida-Gali-Gertler

We need to introduce the rule of Clarida-Gali-Gertler in our system of notation. This rule is described in Clarida-Gali-Gertler (1999, 1672) equations (3.5) and (3.6). It takes the form of a linear function of g_t and u_t only (the two first components of $\mathbf{F}_t$.). In our notation, we just need to introduce the vector **o**, defined from the parameters of the model as:

$$\mathbf{o} := \begin{pmatrix} \dfrac{1}{\varphi} \\ \dfrac{\lambda - \lambda\rho + \alpha\rho\varphi}{(\alpha + \lambda^2 - \alpha\beta\rho)\varphi} \\ 0 \end{pmatrix} \tag{44}$$

Then the rule takes the form:

$$\mathrm{i}(\mathbf{F}_t) := \mathbf{o} \cdot \boldsymbol{\eta}_t \tag{45}$$

Definition 7:
We call standard rule *the rule (45) of Clarida-Gali-Gertler (1999) under the discretionary case.*

Then we have the:

Theorem 3:
If ξ is equal to zero, then there exists only one optimal policy rule in the form of a linear function of the state variables. This policy rule is the standard rule.

Proof:
We assume ξ=0. We will show that if a policy rule takes the form

$$\mathsf{i}\left(\mathbf{F}_t\right) := \boldsymbol{\omega} \cdot \boldsymbol{\eta}_t \tag{46}$$

where **ω** is a vector whose third component is zero,

$$\boldsymbol{\omega} := \begin{pmatrix} \omega_x \\ \omega_\pi \\ 0 \end{pmatrix} \tag{47}$$

and if this policy rule (46) is optimal in sense of Definition 6, then the vector **ω** is equal to the vector **o** defined in formula (44).
Let us denote the rule (46) with **ω**·. We have to determine Φ(**ω**·) and so to compute Φ(**ω**·)($\mathbf{F}_t$) for an arbitrary value of the state variable $\mathbf{F}_t$.
Let us consider some arbitrary value ι of the policy rate. Because ξ=0, the dynamic law subject to the standard rule (45) and the dynamic law (ι, $\mathbf{F}_t$, t)-first-free-subject to the standard rule are indeed equal, so in other words they are one and the same probability. We denote this probability with the symbol P.
We compute the value of Λ corresponding to the choice of the policy rate equal to ι at time t and following the standard rule (45) at times t+n, n>0. To compute this value, we start with expression (38). In this expression we replace i_t by ι and i_{t+m+n}, m+n>0, by $\boldsymbol{\omega}\cdot\boldsymbol{\eta}_{t+m+n}$. We get the expression:

$$\Lambda = \mathrm{E}^{\mathsf{P}} \left\{ \left(T \cdot \boldsymbol{\eta}_t + \sum_{n=1}^{\infty} \mathbf{J}_n \left(\boldsymbol{\omega} \cdot \boldsymbol{\eta}_{t+n}\right) + \mathbf{J}_0\, \iota \right)' Q \left(T \cdot \boldsymbol{\eta}_t + \sum_{n=1}^{\infty} \mathbf{J}_n \left(\boldsymbol{\omega} \cdot \boldsymbol{\eta}_{t+n}\right) + \mathbf{J}_0\, \iota \right) + \ldots \right\} \tag{48}$$

in which the subsequent terms denoted with "…" depend only on $\boldsymbol{\omega}\cdot\boldsymbol{\eta}_{t+m+n}$ but not on ι.
It is convenient to introduce the vectors **U** and $\mathbf{R}_t$. The vector U is given by:

$$\mathbf{U} := \begin{pmatrix} \alpha \\ \lambda \end{pmatrix} \tag{49}$$

The vector $\mathbf{R}_t$ is defined by:

$$\mathbf{R}_t := \mathrm{E}^{\mathsf{P}} \left\{ T \cdot \boldsymbol{\eta}_t + \sum_{n=1}^{\infty} \mathbf{J}_n \left(\boldsymbol{\omega} \cdot \boldsymbol{\eta}_{t+n}\right) \right\} = T \cdot \boldsymbol{\eta}_t + \sum_{n=1}^{\infty} \mathbf{J}_n \left(\boldsymbol{\omega} \cdot \mathrm{E}^{\mathsf{P}}\left(\boldsymbol{\eta}_{t+n}\right)\right) \tag{50}$$

Because of assumption 1, this is equal to:

$$\mathbf{R}_t = T \cdot \boldsymbol{\eta}_t + \sum_{n=1}^{\infty} \mathbf{J}_n \left(\boldsymbol{\omega} \cdot K^{\mathsf{n}} \cdot \boldsymbol{\eta}_t\right) \tag{51}$$

By formula (43), ι should maximize (48). We derive (48) with respect to ι. By setting this derivative equal to zero, we get for ι the following explicit expression:

$$\iota = \frac{\mathbf{U} \cdot \mathbf{R}_t}{\left(\alpha + \lambda^2\right)\varphi} \tag{52}$$

Formulae (51) and (52) prove that ι is also a linear function of $\boldsymbol{\eta}_t$. It can therefore be written under the form:

$$\iota := \boldsymbol{\omega}_2 \cdot \boldsymbol{\eta}_t \tag{53}$$

where $\boldsymbol{\omega}_2$ is a vector. The strategy (46) is equal to its own self-reply if and only if (46) and (53) are equal with probability 1. Given that ξ=0, the process **a**• remains identically equal to zero. Therefore (46) and (53) are equal with probability 1 if and only if the two first components of $\boldsymbol{\omega}$ and $\boldsymbol{\omega}_2$ are equal. So we have to find an explicit expression for $\boldsymbol{\omega}_2$ and to examine in which cases the two first components of $\boldsymbol{\omega}$ and $\boldsymbol{\omega}_2$ are equal.

The recurrence relationship (8) allows to show the identity:

$$\frac{\mathbf{U}\cdot\mathbf{J}_n}{(\alpha+\lambda^2)\varphi} \equiv \frac{\alpha\beta}{\alpha+\lambda^2}\frac{\nu_1^n-\nu_2^n}{\nu_1-\nu_2} - \frac{\nu_1^{n+1}-\nu_2^{n+1}}{\nu_1-\nu_2} \tag{54}$$

The right-hand terms of (54) are the n[th] terms of the power series expansion for the function f given by:

$$f(X) := 1 - \frac{(\alpha+\lambda^2)-\alpha\beta X}{(\alpha+\lambda^2)(\beta X^2-(1+\beta+\lambda\varphi)X+1)} \tag{55}$$

Therefore $\boldsymbol{\omega}_2$ is given by:

$$\boldsymbol{\omega}_2 = (T)'\,\mathbf{U} + \begin{pmatrix} f(\mu) & 0 & 0 \\ 0 & f(\rho) & 0 \\ 0 & 0 & 0 \end{pmatrix}\boldsymbol{\omega} \tag{56}$$

and what we have to solve is:

$$\begin{pmatrix}\omega_x\\ \omega_\pi\end{pmatrix} = \begin{pmatrix} \frac{(1-\mu\beta)}{(1-\mu(1+\beta+\lambda\varphi)+\mu^2)} & \frac{\lambda}{(1-\mu(1+\beta+\lambda\varphi)+\beta\mu^2)} \\ \frac{\rho\varphi}{(1-\rho(1+\beta+\lambda\varphi)+\beta\rho^2)} & \frac{(1-\rho)}{(1-\rho(1+\beta+\lambda\varphi)+\beta\rho^2)} \end{pmatrix}\begin{pmatrix}\alpha\\ \lambda\end{pmatrix} + \begin{pmatrix} f(\mu) & 0 \\ 0 & f(\rho)\end{pmatrix}\begin{pmatrix}\omega_x\\ \omega_\pi\end{pmatrix} \tag{57}$$

Standard algebra shows that the only solution is:

$$\begin{pmatrix}\omega_x\\ \omega_\pi\end{pmatrix} = \begin{pmatrix} \frac{1}{\varphi} \\ \frac{\lambda-\lambda\rho+\alpha\rho\varphi}{(\alpha+\lambda^2-\alpha\beta\rho)\varphi} \end{pmatrix} \tag{58}$$

This completes the proof.

2.2.2 Case with sunspots: The standard rule looses its optimality

We intend to prove that:

Theorem 4:
If ξ is large enough, then the standard rule (45) is not optimal in the sense of Definition 6.

Proof:
Let $\mathbf{F}_t=(\boldsymbol{\eta}_t, i_{t-1})$ the current value of the state variable. Let us call P_1 the dynamic law subject to the standard rule (45). For a given policy rate ι, let us call P_2 the dynamic law (ι, $\mathbf{F}_t$, t)-first-free-subject to the standard rule. We will use the sort-handed notation $\Lambda(\mathsf{P})$ to indicate the objective function computed under probability P. We will show that for ξ large enough, there is a policy rate ι distinct from $\mathbf{o}\cdot\boldsymbol{\eta}_t$ such that $\Lambda(\mathsf{P}_1) < \Lambda(\mathsf{P}_2)$.
Consider probability P_2 , so that all expectations are understood under this probabilityP_2 . We start from the expression (16) of $\mathbf{V}_{t+1}$, in which we replace the terms i_{t+1+n} by the values according to the standard rule (45). We get:

$$\mathbf{V}_{t+1} = T\cdot\boldsymbol{\eta}_{t+1} + \sum_{n=0}^{\infty}\mathbf{J}_n\,\mathrm{E}_{t+1}(\mathbf{o}\cdot\boldsymbol{\eta}_{t+1+n}) \tag{59}$$

But the third component of vector $\mathbf{o}$ is 0, so that the process $a_\bullet$ plays a role only via the term $T\cdot\boldsymbol{\eta}_{t+1}$. Moreover, the third diagonal term of the 3 x 3 matrix $T'\cdot Q\cdot T$ is strictly positive. We give this diagonal term the name Θ. An elementary calculation shows that it is given by :

$$\Theta := \left(T' \cdot Q \cdot T\right)_{3\,3} = \lambda^2 + \frac{\alpha}{4}\left(-1+\beta+\lambda\varphi-\sqrt{-4\beta+\left(1+\beta+\lambda\varphi\right)^2}\right)^2 \tag{60}$$

and is strictly positive. Hence, the P_2-expectation knowing t of $\mathbf{V}_{t+1}'\cdot Q\cdot \mathbf{V}_{t+1}$ is dominated by $\xi\,\Theta\,(\iota\text{-}i_{t-1})^2$
(Notice that the expectation knowing t of $\mathbf{V}_{t+1}'\cdot Q\cdot \mathbf{V}_{t+1}$ can be made arbitrary large by choosing a ξ large enough. As a consequence, for ξ large enough, Λ of the standard rule () is smaller that Λ of the rule of constant policy rate, so is dominated by this constant rate rule. However this, in itself, does not prove that the standard rule is not optimal in the sense of Definition 6.
Similarly, the P_2-expectation knowing t of $\mathbf{V}_{t+2}'\cdot Q\cdot \mathbf{V}_{t+2}$ is dominated by $\xi\,\Theta\,(\iota\text{-}i_{t+1})^2= \xi\,\Theta\;E_t(\iota\text{-}\mathbf{o}\cdot\boldsymbol{\eta}_{t+1})^2$ (given that P_2 is (ι, $\mathbf{F}_t$, t)-first-free-subject to the standard rule). Any other terms $\mathbf{V}_{t+n}'\cdot Q\cdot \mathbf{V}_{t+n}$, n>2, do not depend on ι.
Therefore by taking ξ large enough, one can make the ι that maximizes $\Lambda(P_2)$ arbitrarily close to the ι that maximizes $(\iota\text{-}i_{t-1})^2 + \beta\, E_t(\iota\text{-}\mathbf{o}\cdot\boldsymbol{\eta}_{t+1})^2$. An elementary calculation shows that this one is given by:

$$\iota = \frac{i_{t-1} + \beta\,\mathbf{o}'K\boldsymbol{\eta}_t}{\left(1+\beta\right)} \tag{61}$$

and therefore depends on i_{t-1} , reason for which it cannot generally coincide with $\mathbf{o}\cdot\boldsymbol{\eta}_t$, the policy rate prescribed by the standard rule. This completes the proof.

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