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MACROPRUDENTIAL RESEARCH NETWORK

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by Fabio Fornari[2]

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In 2010 all ECB publications feature a motif taken from the €500 banknote.

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Macroprudential Research Network

This paper presents research conducted within the Macroprudential Research Network (MaRs). The network is composed of economists from the European System of Central Banks (ESCB), i.e. the 27 national central banks of the European Union (EU) and the European Central Bank. The objective of MaRs is to develop core conceptual frameworks, models and/or tools supporting macro-prudential supervision in the EU.

The research is carried out in three work streams:
1. Macro-financial models linking financial stability and the performance of the economy;
2. Early warning systems and systemic risk indicators;
3. Assessing contagion risks.

MaRs is chaired by Philipp Hartmann (ECB). Paolo Angelini (Banca d'Italia), Laurent Clerc (Banque de France), Carsten Detken (ECB) and Katerina Šmídková (Czech National Bank) are workstream coordinators. Xavier Freixas (Universitat Pompeu Fabra) acts as external consultant and Angela Maddaloni (ECB) as Secretary.

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The paper is released in order to make the research of MaRs generally available, in preliminary form, to encourage comments and suggestions prior to final publication. The views expressed in the paper are the ones of the author(s) and do not necessarily reflect those of the ECB or of the ESCB.

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Abstract

We forecast recession probabilities for the United States, Germany and Japan. The predictions are based on the widely-used probit approach, but the dynamics of regressors are endogenized using a VAR. The combined model is called a 'ProbVAR'. At any point in time, the ProbVAR allows to generate conditional recession probabilities for any sequence of forecast horizons. At the same time, the ProbVAR is as easy to implement as traditional probit regressions. The slope of the yield curve turns out to be a successful predictor, but forecasts can be markedly improved by adding other financial variables such as the short-term interest rate, stock returns or corporate bond spreads. The forecasting performance is very good for the United States: for the out-of-sample exercise (1995 to 2009), the best ProbVAR specification correctly identifies the ex-post classification of recessions and non-recessions 95% of the time for the one-quarter forecast horizon and 87% of the time for the four-quarter horizon. Moreover, the ProbVAR turns out to significantly improve upon survey forecasts. Relative to the good performance reached for the United States, the ProbVAR forecasts are slightly worse for Germany, but considerably inferior for Japan.

JEL classification: C25, C32, E32, E37.

Keywords: Recessions, forecasting, probit, VAR

Non-technical summary

The expected future level of economic activity is an essential piece of information for policy institutions, such as central banks or financial system surveillance authorities, but also for private-sector decision makers, such as commercial banks or investment funds. Accordingly, there is a vital interest in tools for forecasting the ups and downs of the business cycle. The literature on forecasting economic activity can be broadly divided into two strands: one dealing with forecasting the growth rates of GDP or industrial production, the other trying to predict recession and expansion phases or turning points.

This paper belongs to the second strand as it presents an approach to quantify recession probabilities. In the literature, so-called probit models are the major tool for computing the probability of a recession prevailing at a given point in the future. Among the choice of predictors, the slope of the yield curve has gained particular prominence: an inverted yield curve (long-term interest rate below short-term rate) in the current quarter implies a high probability of recession in the future, typically in one year's time. Using additional predictors (financial variables, commodity prices, surveys or macroeconomic variables) has been found to increase the forecasting power relative to slope-only probit models. The main characteristic of all such models, however, is that they are usually specified for one specific forecast horizon. Put differently, one needs to estimate a separate model for each forecast horizon of interest.

The model proposed in this paper, in contrast, can be employed to derive the complete set of recession probabilities for all coming quarters. That is, based on a set of observed predictors, it generates a complete 'term structure of recession probabilities'. This is accomplished by combining the standard probit model with a vector autoregressive model for the regressors. The combined model is called ProbVAR. Unlike similar models in the literature, it is easy to implement and does not rely on computationally-intensive estimation techniques.

Concerning the choice of predictors, we feed the ProbVAR with a number of financial variables. In addition to the term spread, we consider the level of the short-term interest rate, the corporate bond spread and stock returns. We constrain ourselves to financial variables as they are timely indicators that do not suffer from real-time problems.

We estimate the ProbVAR for the United States, Germany and Japan, using quarterly data from 1960Q1 to 2009Q4. The in-sample and out-of-sample fit are satisfactory for Germany and especially for the United States, but less so for Japan. In the out-of-sample exercise (1995 to 2009) for the United States, the best ProbVAR specification correctly identifies 95% of the quarters to be recessions or non-recessions for the one-quarter forecast horizon, and 87% for the four-quarter horizon; for Germany 91% and 76% of the examined quarters are correctly identified for the one- and four-quarter horizon, respectively; for Japan, the proportions of correctly identified quarters amount to 82% and 55%, respectively. The relatively poor out-of-sample results for Japan are possibly related to the particular pattern with which Japanese recessions occurred, i.e. with long durations and concentrated in the second half of the sample. For the United States, we find in addition that the ProbVAR forecasts are clearly superior to the forecasts of the Survey of Professional Forecasters.

For all countries considered, including additional regressors besides the slope of the yield curve improves the predictive ability over all horizons. For the United States, the best-performing models include the short-term interest rate and the international average of corporate bond spreads as additional predictors; for Germany, the short-term interest rate and the corporate bond spread enter besides the term spread; for Japan, the slope of the yield curve is accompanied by the short-term interest rate and the stock return.

We show that the ProbVAR is well-suited for providing a 'term structure of recession probabilities', i.e. the sequence of such probabilities from one to any future quarter ahead. This is not only helpful in gauging the length of a potentially forthcoming recession, but also for tracing the probability of exiting a recession in a future period, when a recession is already prevailing. Taking the US example, the ProbVAR anticipates the profile of the 2001 recession very well. Regarding the most recent recessionary period, associated with the financial turmoil that started in summer 2007, some model specifications correctly detected it, but they initially predicted this recession to be rather short-lived. This is related to the fact that the slope of the yield curve is still a prominent predictor in our ProbVAR models: due to the swift and strong monetary policy response to the crisis, the yield curve recorded a quick steepening thus leading the model to take a strong stance against recession, given the historical relationships between the term spread and business cycle turning points. In this occasion, the relevance of other financial variables (in this case the corporate bond spread), which could counteract this effect, became apparent.

Overall, the ProbVAR model appears to be a useful tool in various fields of applications. For instance, from a macro-prudential perspective, it can help to identify risks of recession in a consistent fashion across short- and medium-term horizons. The information needed to produce such recession probability forecasts is quickly updatable as regressors are financial variables, which are available in real time. For central banks and other policy institutions, the ProbVAR can likewise help to identify and predict periods of economic slack, but it also provides information on the expected remaining duration of a recession, once it has actually begun.

1 Introduction

Business cycle forecasting has attracted a great deal of applied econometric work. This comes as no surprise, as the expected future level of economic activity is an essential piece of information for policy institutions, such as central banks or financial system surveillance authorities, but also for private-sector decision makers, such as banks or investment funds.

The literature on forecasting economic activity can be broadly divided into two strands, one dealing with forecasting the growth rates of GDP or industrial production, the other trying to predict recession and expansion phases or turning points in the business cycle. Most approaches to forecasting growth rates are based on linear models (such as linear VARs or FAVARs), but several nonlinear alternatives (such as threshold or regime-switching models) have been also put forward in the literature. Econometric approaches aimed to forecast recessions are instead inherently nonlinear and usually employ discrete choice (e.g. probit) models: the probability of recession prevailing at a given future horizon is modeled as a linear combination of selected predictors. Among these, the slope of the yield curve (long-term minus short-term interest rate) has turned out to be particularly successful, see Estrella, Rodrigues, and Schich (2003), Estrella and Hardouvelis (1991), Estrella and Mishkin (1997), and Estrella (2007) among others.

Besides specifications based on the term spread alone, other variables have been found to be useful predictors in probit models of recessions, possibly accompanying the slope of the yield curve. As shown by King, Levin, and Perli (2007), the corporate bond spread also carries predictive power, especially when combined with the term spread. Dovern and Ziegler (2008) use a large set of variables (survey indicators, composite indices, measures of real economic activity and financial indicators) to forecast both economic growth and recession phases in the United States. They evidence that all these indicators improve the forecasting performance of growth rates at short horizons, although they do not necessarily improve the forecasts of recession probabilities. Similarly, Haltmaier (2008) finds that the oil price, a leading indicator of economic activity and a stock price index are significant predictors besides the term spread in a simple probit model for US recessions; the same (apart from the oil price) holds for Germany and Japan. However, false signals are a serious problem when considering out-of-sample predictions. Bellego and Ferrara (2009) draw principal components from a set of several indicators, ending up with three factors representing term spread variables, stock market variables and commodities, respectively, which they use to forecast euro-area recessions. Engemann, Kliesen, and Owyang (2010) find that oil prices have considerable predictive power for US recessions.[1] Finally, Wright (2006) shows that probit models that use both the level of the federal funds rate and the term spread predict US recessions and expansions better than models which are based on the term spread only. Besides domestic financial variables, regressors capturing the

[1] The latter two papers employ probit model with time-varying parameters.

international environment have been found to be useful predictors for economic activity as well. In a recent application based on probit models, Nyberg (2010) finds that stock market returns and the foreign term spread have additional predictive power to forecast recessions in the United States and Germany beyond the domestic term spread.

Discrete choice models for quantifying recession probabilities have been usually specified for one particular forecast horizon. Hence, if one sets out to predict recession probabilities over various, say H, horizons, one would need to estimate H different models. The main drawback of such an approach is that the H separate models do not account for the tight relationship between economic activity prevailing in the contiguous H forecast horizons and as such they possibly produce non-smooth swings in recession probabilities along the time horizon spanned by H. This is not a desirable model outcome, as it would stand in contrast to the evidence that recessionary months or quarters are clustered in time.

This paper presents an econometric approach that allows to predict recession probabilities for any set of horizons from one single model. At the same time, it is as straightforward to implement as the traditional fixed-forecast-horizon probit model. The model consists of two components. The first one is a standard probit relation linking a set of regressors to the recession probability in the subsequent quarter. With this component alone, we would be able to predict recession odds for the one-quarter-ahead horizon only. In order to allow for multi-period forecasts, the probit equation is linked to a VAR – the second component – which endogenizes the dynamics of the selected regressors. The combined model is referred to as a ProbVAR. At each point in time, the conditional probability of the economy being in recession h quarters ahead is obtained in closed-form from the ProbVAR. Forecasts for different horizons are consistent with one another, as they arise from one single model.

Concerning the choice of predictors, we feed the ProbVAR with a number of financial variables. Besides the term spread, we consider the level of the short-term interest rate, the corporate bond spread and stock returns. We constrain ourselves to financial variables as they are timely indicators that do not suffer from real-time problems.[2] Corporate bond spreads and stock returns belong either to the country for which recession probabilities are computed or they are averages over the G7 countries.

We estimate the ProbVAR for the United States, Germany and Japan, using quarterly data from 1960Q1 to 2009Q4. The in-sample and out-of-sample fit are satisfactory for Germany and especially for the United States, but less so for Japan. In the out-of-sample exercise (1995 to 2009) for the United States, the best ProbVAR specification correctly identifies 95% of the quarters to be recessions or non-recessions based on their

[2]Survey forecasts of economic activity would not be affected by real-term problems, either. However, they are available at a lower frequency than financial variables, and they may suffer from a publication lag.

ex-post classification for the one-quarter forecast horizon, and 87% for the four-quarter horizon; for Germany 91% and 76% of the quarters are correctly identified for the one- and four-quarter horizon, respectively; for Japan, the proportions of correctly identified quarters drop to 82% and 55%, respectively. The relatively poor out-of-sample results for Japan are possibly related to the particular pattern with which Japanese recessions occurred, i.e. with long durations and concentrated in the second half of the sample. For the United States we also find that the ProbVAR forecasts are clearly superior to the forecasts provided by the Survey of Professional Forecasters.

For the three countries considered, including additional regressors besides the slope of the yield curve improves the predictive ability over all horizons. For the United States, the best-performing models include the short-term interest rate and the international average of corporate bond spreads as additional predictors; for Germany, the short-term interest rate and the corporate bond spread are worth including beyond the term spread; for Japan the forecasting performance can be improved by using the short-term interest rate and the stock return as additional predictors besides the slope of the yield curve.

We show that the ProbVAR is well-suited for providing a 'term structure of recession probabilities', i.e. the sequence of such probabilities from one to any future quarter ahead. This is not only helpful in gauging the length of a potentially forthcoming recession, but also for tracing the probability of exiting a recession in a future period, when a recession has already begun. For the United States, the ProbVAR anticipates the profile of the 2001 recession very well. Regarding the most recent recessionary period, associated with the financial turmoil that started in summer 2007, some model specifications correctly detect it, but they initially predict this recession to be rather short-lived. This is related to the fact that the slope of the yield curve is still a prominent predictor in our ProbVAR models: due to the swift and strong monetary policy response to the crisis, the yield curve recorded a quick steepening thus leading the model to take a stance against recession, given the historical relationships between the term spread and business cycle turning points. In this occasion, the relevance of other financial variables (corporate spreads and stock returns) became apparent as they managed to counteract the signal stemming from the term spread.

Finally, we show that the ProbVAR is a useful device to derive impulse responses of recession probabilities to shocks in selected financial variables. As an illustration, we quantify the changes in recession probabilities following a shock to the term spread. Unlike with plain linear VARs, impulse responses in the ProbVAR depend on initial conditions and are not scaling proportionally with the size of shocks. For instance, for the United States, we find that a decrease in the slope of the yield curve by one percentage point leads to a rise of US recession probabilities: it peaks after four quarters, reaching a 25 percentage points increase over baseline. However, this effect is conditional on the assumption that the shock materializes in a situation where regressors are at their sample averages. Tracing the same shock for a situation where the initial yield curve is steeper leads to a much more subdued response.

In our ProbVAR model, we do not include information contained in past values of the recession indicator (the dychotomic 0/1 variables) as done by Kauppi and Saikkonen (2008)[3], or lagged values of the latent business cycle indicator as in the Qual-VAR model by Dueker (2005), which nests our ProbVAR specification. Hence, unlike these papers we are excluding direct feedback from the lagged business cycle variable itself, thereby omitting the information related to whether and since how long the economy has already been in recession. However, compared to the Dueker (2005) specification, ours comes with the advantage that estimation and forecasting is straightforward, not requiring any computationally-intensive methods, while the Qual-VAR rests on simulation-based inference. Whether the simpler model that we propose leads to an economically significant loss in forecasting ability is largely an empirical question. At least for the out-of-sample forecast of the 2001 recession in the United States, for which a direct comparison is feasible, the two approaches give rise to similar results.

The paper is organized as follows. The next section presents the ProbVar model, its estimation and the derivation of recession probabilities over arbitrary horizons. We also explain the calculation of nonlinear impulse responses. The third section presents an application of the model to forecasting recession probabilities in the three countries and evaluates both the in-sample and the out-of-sample forecasting performance; for the United States, we also compare the ProbVAR forecasting performance to that of survey forecasts; finally, we analyze how well the models capture the time profiles of the recessions in the two most recent episodes. The fourth section shows an application of ProbVAR-based impulse response analysis. The last section concludes.

2 ProbVAR: a VAR-augmented probit model

The standard probit specification to quantifying recession probabilities for a fixed forecast horizon of k periods is of the form

$$P(y_{t+k} = 1|X_t) = \Phi(\beta_0 + \beta' X_t), \tag{2.1}$$

where y_{t+k} is a binary variable equal to one if a recession prevails at time $t+k$ and zero otherwise; X_t is a vector containing predictors observed at t (i.e. possibly including variables lagged further); β_0 and β are a scalar and a vector of parameters, respectively; and $\Phi(\cdot)$ denotes the cumulative distribution function of a standard normal. Given estimated (β_0, β) and observed X_t, (2.1) delivers the conditional recession probability for time $t+k$. However, in order to obtain recession probabilities for other forecast horizons $h \neq k$, the above equation has to be re-estimated for the specific horizon h. Noteworthy, the esti-

[3] See also Nyberg (2010) for several variants of this model.

mated β_0 and β would vary freely across the different horizons, so there is no 'smoothness constraint' on the set of probability forecasts as a function of h.

In the following, we propose a model, that produces recession probabilities for an arbitrary set of forecast horizons with a single set of parameters. This is achieved by endogenizing the dynamics of the explanatory variables X_t using a VAR.

2.1 Model structure and estimation

The first ingredient of the model we propose is a VAR(p) for the dynamics of the regressors. Let x_t denote an $N \times 1$ vector of variables that are potentially useful in explaining the future recession probability. The evolution of x_t is assumed to follow a homoscedastic Gaussian VAR(p) with serially uncorrelated errors,

$$x_t = c_0 + A_1 x_{t-1} + \ldots A_p x_{t-p} + v_t, \quad v_t \sim N(0, S). \tag{2.2}$$

For the following, it is convenient to work with some arbitrary factorization of the residual covariance matrix: $v_t := \Sigma u_t$, with $\Sigma\Sigma' = S$ and $u_t \sim N(0, I)$. As usual, we can represent the VAR(p) (2.2) in companion form,

$$X_t^{(p-1)} = c + B X_{t-1}^{(p-1)} + R u_t, \tag{2.3}$$

where here and in the following $X_t^{(n)} \equiv (x_t', \ldots, x_{t-n}')'$ and with obvious definitions of c, B and R.

The second ingredient of the model is the standard probit relation, in which a latent variable y_t^* is specified as a linear function of explanatory variables:

$$y_t^* = \beta_0 + \beta' X_{t-k}^{(l)} + \epsilon_t, \quad \epsilon_t \sim N(0,1), \quad \text{and } \epsilon_t, u_s \text{ independent for all } s,\ t, \tag{2.4}$$

where ${X_{t-k}^{(l)}}' = (x_{t-k}', x_{t-k-1}', \ldots, x_{t-k-l}')$ with $k > 0, \quad l \geq 0$.

In (2.4), k defines at which point in the past the vector of predictors is positioned, and l determines the number of lags relative to k. For example, with $k = 2$ and $l = 2$, y_t^* will be dependent on x_{t-2}, x_{t-3} and x_{t-4}, while for $k = 2$ and $l = 0$, y_t^* will be dependent only on x_{t-2}.

Finally, a recession prevails at t if the binary indicator variable y_t equals unity, which is in turn the case if the latent y_t^* is positive:

$$y_t = 1, \text{ if } y_t^* > 0, \text{ and } y_t = 0, \text{ otherwise.} \tag{2.5}$$

The complete model comprises the VAR relation (2.3), the linear relation between the latent recession variable and the regressors (2.4), and the mapping (2.5) of y_t^* into the observable binary indicator y_t. The system will be referred to as a ProbVAR(p, k, l) model.

As each parameter of the ProbVAR system either appears only in (2.3) or only in (2.4) and the innovations ϵ and u are assumed to be independent, the parameters can be estimated consistently by separately estimating the VAR part (e.g. by OLS) and the probit part (by Maximum Likelihood).

2.2 Representing the model with a single state vector

The vector $X_t^{(p-1)}$ of the VAR in companion form can differ in length from $X_{t-k}^{(l)}$ used in the probit relation. For instance, it may be the case that there are no further lags beyond k in the probit relation, while the VAR comprises several lags, hence making $X_t^{(p-1)}$ longer than $X_{t-k}^{(l)}$. Likewise, there may be several lags required in (2.4) while in the VAR only one lag may suffice. However, for the derivation of recession probabilities over different horizons (see next subsection) it is convenient that $X_t^{(p-1)}$ and $X_{t-k}^{(l)}$ have the same length. We now describe how to represent both the probit regressors and their (Markovian) dynamics using the same state vector.

There are three cases to consider. If $l = p-1$, then $X_t^{(p-1)}$ in the VAR companion form and $X_{t-k}^{(l)}$ in the probit relation have the same length and the same relative lag structure.

If $p-1 > l$, then the X vector in the VAR companion form is larger. To represent the system in terms of this vector, the vector β in (2.4) has to be augmented with zeros. That is, we re-write (2.4) as

$$y_t^* = \beta_0 + \tilde{\beta}' X_{t-k}^{(p-1)} + \epsilon_t \tag{2.6}$$

where $\tilde{\beta}' = (\beta', 0_{N\cdot(p-1-l)})$.

If $p-1 < l$, the probit relation makes use of a richer lag structure than the VAR. In this case, a VAR(1) representation of this longer state vector will be employed. Denote the number of additional lags to be accommodated by $p^\Delta = l-(p-1)$, then the dynamics of $X_t^{(l)} = (X_t^{(p-1)}, X_{t-p}^{(p^\Delta-1)})$ are given by[4]

$$\begin{aligned} &\begin{pmatrix} X_t^{(p-1)} \\ X_{t-p}^{(p^\Delta-1)} \end{pmatrix} \\ = &\begin{pmatrix} c \\ 0_{(N\cdot p^\Delta)\times 1} \end{pmatrix} + \begin{pmatrix} [B \mid 0_{(N\cdot p)\times(N\cdot p^\Delta)}] \\ \left[0_{(N\cdot p^\Delta)\times(N\cdot(p-1))} \mid I_{N\cdot p^\Delta} \mid 0_{(N\cdot p^\Delta)\times N}\right] \end{pmatrix} \begin{pmatrix} X_{t-1}^{(p-1)} \\ X_{t-p-1}^{(p^\Delta-1)} \end{pmatrix} \\ + &\begin{pmatrix} R \\ 0_{(N\cdot p^\Delta)\times N} \end{pmatrix} u_t \end{aligned} \tag{2.7}$$

Unless otherwise noted, we will always work with the canonical state vector X_t (without superscript) defined as $X_t \equiv X_t^{(\max\{l,p-1\})}$, so that writing

$$X_t = c + BX_{t-1} + Ru_t, \tag{2.8}$$

[4] If $p = 1$, the sub-matrix $0_{(N\cdot p^\Delta)\times(N\cdot(p-1))}$ vanishes.

for the VAR dynamics of regressors, we won't distinguish explicitly, for instance, whether B is just the companion-form matrix of the VAR(p) in (2.3) or the augmented version appearing in (2.7).

2.3 Computing probabilities at the h-period horizon

Given the parameters $(c, B, R, \beta_0, \beta)$ of the ProbVAR, the objective is to compute probabilities of a recession occurring h periods ahead of a given time period t, i.e. $Pr(y_{t+h} = 1|X_t)$. For $h = k$, this is simply

$$Pr(y_{t+k} = 1|X_t) = \Phi(\beta_0 + \beta' X_t).$$

For $h > k$, the conditional probability is given by

$$Pr(y_{t+h} = 1|X_t) = Pr(y^*_{t+h} > 0|X_t) = Pr(\beta_0 + \beta' X_{t+h-k} > 0|X_t).$$

In order to compute this, the VAR dynamics has to be employed. Let $d \equiv h - k$. From (2.8) it follows that

$$X_{t+d} = B^d X_t + (I + B + \ldots + B^{d-1})c + \sum_{i=1}^{d} B^{d-i} R u_{t+i}$$

The distribution of X_{t+d} conditional on X_t is normal with conditional expectation

$$\mu_d(X_t) = B^d X_t + (I + B + \ldots + B^{d-1})c \tag{2.9}$$

and conditional variance-covariance matrix

$$V_d = \sum_{i=1}^{d} B^{d-i} R R' (B^{d-i})'. \tag{2.10}$$

Thus, the distribution of y^*_{t+h} conditional on X_t is also normal with conditional expectation

$$m_d(X_t) = \beta_0 + \beta' \mu_d(X_t)$$

and conditional variance

$$v_d^2 = \beta' V_d \beta + 1.$$

Accordingly, the probability of interest is given by[5]

$$\Pr(y_{t+h} = 1|X_t) = 1 - \Phi(0; m_d(X_t), v_d) = \Phi\left(\frac{m_d(X_t)}{v_d}\right). \tag{2.11}$$

Before examining the empirical performance of the ProbVAR model, it is worth noting that the 'Qual VAR' model by Dueker (2005) – briefly described in the introduction – is

[5]The function $\Phi(x)$ denotes the cdf of a standard normal evaluated at x, and the function $\Phi(x; a, b)$ is the cdf of $N(a, b^2)$ evaluated at x.

of a similar nature. However, it is more general than the ProbVAR, as it directly specifies the joint dynamics of y_t^* and X_t as a VAR.[6] Hence, compared to our recursive approach, where lagged values of y_t^* do not influence current y_t^* and X_t, it allows for richer dynamics as such feedback is allowed. As the latent variable enters the VAR, estimating the model and computing predictions has to rely on simulation-based filtering techniques, whereas both tasks are less computationally demanding for the ProbVAR model that we propose. Whether the additional complexity of the Dueker approach adds value for a given forecast exercise remains largely an empirical issue.

2.4 Impulse response analysis

Similar to a linear VAR, the ProbVAR can be used to trace the effect of an unexpected change in one state variable at time t on the time profile of subsequent recession probabilities. Formally, we can define an impulse response function as

$$IR(h; X_t, \delta) := \Pr(y_{t+h} = 1 | X_t + \delta) - \Pr(y_{t+h} = 1 | X_t). \tag{2.12}$$

As a crucial difference to standard VAR analysis, the sequence of probability responses will not only depend on the shock vector δ but also on the current state X_t. This is an immediate implication of the probabilities being a non-linear function of the state vector X_t.

In choosing δ we are faced with the same considerations as with shock identification in linear VAR analyses. In the empirical analysis below, we do not attempt to provide a structural shock identification and use the concept of generalized impulse responses as in Pesaran and Shin (1998). That is, we fix the size of the shock to the ith equation in (2.2), say δ_i, and set the complete shock vector as the conditional expectation of shocks, given the ith shock, taking into account the covariance matrix of the errors:

$$(\delta_1, \ldots, \delta_N)' = E(v_t | v_{i,t} = \delta_i).$$

In practice, the computation of the conditional expectation employs the normality assumption $v_t \sim N(0, S)$ in (2.2), where S is replaced by its OLS estimate $\hat{S}$.[7]

[6] Yet another alternative would consist of deriving recession probabilities directly from a VAR with observable GDP growth and other variables. Under the assumption of Gaussian innovations, an estimated linear VAR would also provide at each point in time the conditional probability of GDP growth being negative for at least two quarters from some time $t + h$ henceforth. To our knowledge, a comparison of such an approach and those based on latent business cycle indicators (Probit, Qual VAR, ProbVAR) has not yet been conducted in the literature.

[7] The remaining elements of δ are filled with zeros such that it complies with the overall length of the canonical state vector X_t in (2.8).

3 Recession probabilities for the United States, Germany and Japan

We apply ProbVAR models to estimate recession probabilities in the United States, Germany and Japan since the 1960s. We first describe the dependent variables, i.e. the binary recession indicators, and the pool of potential regressors. Thereafter, we report the in- and out-of-sample forecasting performance of selected specifications. For the United States, we also compare the recession forecasts generated by the ProbVAR to those from surveys. For each country, we also conduct an in-depth analysis of the most recent recession and the preceding one.

3.1 Data on recessions and explanatory variables

The data set consists of time series of recession indicators and financial variables, which serve as potential predictors. Data are quarterly and span the period from 1960Q1 to 2009Q4.

The business cycle dating for the United States is taken from the NBER, while the recession classifications for Germany and Japan are from the Economic Cycle Research Institute (ECRI).[8] A given quarter is considered as a recession if at least one month in that quarter was identified to be a recession.

We consider the following group of potential regressors: the slope of the yield curve (ten-year government bond yield minus short-term interest rate), sl, the short-term interest rate itself, i, the year-on-year log return of a broad composite stock price index, sr, and the corporate bond spread, cs. All these time series are taken from Global Financial Data. For the stock return we try both the domestic values of the variables and the average value of that variable for the G7 countries (indicated by the superscript $G7av$). Similarly, we compute the average of the corporate bond spread (also indicated by the superscript $G7av$), using the data series from Canada, Germany, the United Kingdom and the United States. The other countries are not used for constructing the average as either no series (France) or no reliable series (Italy, Japan) for corporate bond spreads were available. The international (G7 average) counterpart of the domestic stock returns and corporate bond spreads is taken into account because several papers evidence that commonalities are present in the business cycles of main economic areas. Therefore, to the aim of forecasting the probability of recession in one country, variables that point to the simultaneous occurrence of a brightening or deteriorating international environment potentially matter beyond domestic indicators.

[8] http://www.businesscycle.com.

3.2 Model specification

To the aim of forecasting recession probabilities in the three countries, we run the ProbVAR model with different combinations of regressors and different lag specifications. For comparison, we also estimate sets of simple probit models by employing equation (2.1) for each specific horizon. Recall that in contrast to the ProbVAR models, the simple probit models have to be estimated separately for each forecast horizon h.

For both model types (ProbVAR and simple probit), we consider specifications with one to three regressors. We always include the slope sl of the domestic yield curve since this variable has been shown to be key in forecasting recessions in the literature. Besides the slope-only specification, we consider specifications with one or two additional regressors from the following pool of five variables: $\{i,\ sr,\ sr^{G7av},\ cs,\ cs^{G7av}\}$. To restrict the total number of examined combinations we do not consider set-ups that contain sr together with sr^{G7av}, or cs together with cs^{G7av}, leaving us with a total set of 14 regressor combinations.

Concerning the lag structure of the ProbVAR, we fix the lag k in the probit relation (2.4) to one, in order to have one quarter as the shortest forecast horizon. As additional lags in the probit relation (2.4), we consider $l = 0$, i.e. no additional lags beyond one, and $l = 3$, i.e. the probit relation contains regressors lagged by one to four quarters. The lag structure of the VAR is chosen to be either one or four. Hence, using the terminology introduced in section 2.1, we consider four dynamic specifications for each set of regressors: a ProbVAR($p = 1$,$k = 1$,$l = 0$), a ProbVAR(1,1,3), a ProbVAR(4,1,0) and a ProbVAR(4,1,3). For example, the ProbVAR(1,1,3) features a VAR(1) for the regressors x_t while the lags x_{t-1}, x_{t-2}, x_{t-3} and x_{t-4} enter the probit relation.

When dealing with the simple probit models, the lag length of the regressor is determined by the forecast horizon. Thus, for instance, to the aim of forecasting recessions six quarters ahead via the slope of the yield curve, the simple probit model has to be estimated on data pairs $(y_t,\ sl_{t-6})$ possibly enhanced with additional lags, i.e. $(y_t,\ sl_{t-6}, sl_{t-6-1}, \ldots, sl_{t-6-l})$. In analogy to the ProbVAR case we consider the cases $l = 0$ and $l = 3$.

We compare models by looking at recession forecasts between one and six quarters ahead. We consider in-sample and out-of-sample forecasts. For the in-sample version, the parameters of the ProbVAR and simple probit models are estimated on data going from 1960Q1 to 2009Q4.[9] For the out-of-sample exercises, the models are estimated on

[9]More precisely, the first data tuple used for estimation always includes the recession indicator of 1962Q2. This is done in order to guarantee that all specifications will provide forecasts for the same sequence of quarters. For instance, for the simple probit model with $l = 0$ and a forecast horizon of six quarters, the probability of recession in 1961Q3 could be forecast with variables dated 1960Q1, but for the specification with $l = 3$ (richer lag structure of probit), the required regressors lie before the beginning of the data set. For the richest specifications (simple probit with $l = 3$, and ProbVAR with $l = 3$ and/or $p = 4$), 1962Q2 is the first date for which a recession probability can be forecast over the six-quarter

expanding windows of data. The first window ends in $T_0^* = 1994Q4$, meaning that the last y_t in this window is recorded at that quarter. Given the estimated parameters based on this window, the respective state vector is plugged into the estimated model and recession probability forecasts for times $T_0^* + h$, $h = 1, 2, 3, \ldots$ are computed and stored. Then the model is estimated on a window expanded by one quarter, and so forth.

3.3 Goodness of fit

Tables 1 to 12 show the fit of the 14 (identifying the regressor combination) times 4 (identifying the lag specifications) different ProbVAR models and the 14 times 2 simple probit models for the United States, Germany and Japan, respectively. For each country and model type, we present in-sample and out-of-sample results.[10]

The upper part of the tables shows the mean absolute forecast error (MAE), given by the mean absolute difference between the model-implied probabilities and the actual outcome of the binary recession variable, i.e.

$$MAE = \frac{1}{T}\sum_{t=1}^{T}\left(\left|y_t - \widehat{\Pr}(y_t = 1)\right|\right). \tag{3.13}$$

The bottom part of the tables reports the so-called R^2_{count} measure of fit, i.e. the weighted sum of fractions of correctly identified recession and non-recession periods,

$$R^2_{count} = \frac{n_{11}}{n_1}\cdot\frac{n_1}{n} + \frac{n_{00}}{n_0}\cdot\frac{n_0}{n} = \frac{n_{11}+n_{00}}{n}, \tag{3.14}$$

where n is the number of quarters for which forecasts have been computed, n_1 is the number of recessions, $n_0 = n - n_1$ is the number of non-recession quarters, n_{11} is the number of correctly identified recessions and n_{00} is the number of correctly identified non-recessions.

It is important to note that the R^2_{count} fit measure as well as the single 'hit ratios' of correctly identified recessions, $\frac{n_{11}}{n_1}$, or correctly identified non-recessions, $\frac{n_{00}}{n_0}$, are all relying on a conversion of the model-implied recession probabilities into a binary variable. That is, given an estimated probability of recession $\widehat{\Pr}(y_t = 1)$ for some time t, a mapping $D : [0,1] \mapsto \{0,1\}$, $\hat{y}_t = D\left(\widehat{\Pr}(y_t = 1)\right)$, has to be applied to convert probabilities into alleged recessions or non-recessions. Usually, such a decision rule depends on a threshold $\Pr^*$ such that $\hat{y}_t = 1$, if $\widehat{\Pr}(y_t = 1) > \Pr^*$ and $\hat{y}_t = 0$ otherwise. Hence, the choice of

horizon.

[10] For our model comparisons, we assume that the same model specification is used across time. Alternatively, the out-of-sample exercises could be based on recursively choosing the best model specifications (for instance, based on information criteria). Our approach of looking at the models separately, yields a more clear-cut result on the individual models' performances. However, it could in fact be the case that the best forecasting model changes over time, so that our best specification could be improved upon in terms of forecast accuracy. We leave such an analysis for future research.

the threshold $\Pr^*$ is key. Setting $\Pr^* = 0.5$ is often too conservative a criterion, especially when the overall fraction of recessions in the sample is relatively small.[11] A natural choice is to set $\Pr^*$ equal to the frequency of recessions measured over a long period. We set $\Pr^* = 0.20$ for the United States and Japan and $\Pr^* = 0.25$ for Germany.[12] While in the out-of-sample exercises, this threshold could be adjusted period by period, we treat it as constant so that results are independent of such time variation.

For the United States, variable specifications 6,7,11,14 could not be estimated for at least one lag combination (or at least one horizon for the simple probit models) and at least one sub-sample in the out-of-sample forecasting exercise. As is well-known in the standard probit literature, this problem arises when the regressor and lag structure is too rich, so that it would provide a perfect fit.[13] Therefore, the corresponding lines are left blank in the result tables for the United States[14] Finally, for Japan, there are blank lines for all specifications that include the domestic corporate bond spread, since this is not available for Japan as discussed in section 3.1.

3.3.1 United States

What should be considered the preferred specification for the United States? Looking at both the in- and the out-of-sample performance, at both the MAE and R^2_{count} measure of fit, and at both ProbVAR and simple probit models (Tables 1 - 4), the model specifications containing the term spread sl, the short-term interest rate i, and the international average of corporate bond spreads cs^{G7av} appears to dominate, but the specification including the term spread sl, the domestic stock return sr, and the international average of corporate bond spreads cs^{G7av} also performs very well. These specifications will therefore be used for the more in-depth analyses of the model performance in the remainder of the paper. Regarding the lag specification, the richer probit equation is preferred (i.e. four lags rather than one) while for the ProbVAR models, where also the VAR lags have to be determined, the more parsimonious model is selected. That is, we consider ProbVAR(1,1,3) specifications (1 lag in the VAR, probit relation including variables from $t-1$ to $t-4$.).

In the out-of-sample exercise, considering the average forecasting performance between one and six quarters ahead, the (sl, i, cs^{G7av})-ProbVAR(1,1,3) correctly predicts 83% of the forecast quarters to have been recessions or non-recessions, respectively. For the one-quarter horizon this fraction even amounts to 95%, while standing at 85% for the

[11] See the discussion in the chapter on discrete choice models in Greene (2003).

[12] Computed over the whole sample, the fraction of recessions amounts to 0.20 for the United States, to 0.28 for Germany, and to 0.21 for Japan.

[13] See the discussion in Greene (2003), p. 683-4.

[14] For reasons of consistency the corresponding lines are also left blank in the tables with in-sample results.

four-quarter horizon. This is markedly higher compared to the slope-only specification with the same lag structure, for which the R^2_{count} measures for forecast horizons of one and four quarters only amount to 71% and 69% respectively.

Comparing the best out-of-sample performances of the simple probit specifications with those of the ProbVAR models (table 4 vs. table 3), it turns out that there are no sizeable differences, a result that holds with respect to both fit measures, MAE and R^2_{count}. Hence, the ProbVAR specifications, for which forecasts for all horizons come from the same model, do neither appear superior nor inferior compared to the simple probit specifications, which have to be estimated separately for each forecast horizon. However, the ProbVAR comes with some other advantages as will be discussed below.

Figures 1 - 2 show the in-sample fit of the ProbVAR and the simple probit models, respectively. The figures contain the fit from the slope-only specification as well as the fit from the (sl, i, cs^{G7av})-specification. Apparently, the richer specification outperforms the slope-only counterpart because it assigns higher probability to recession periods as well as because it reduces the number of 'false signals', i.e. recessionary signals when in fact the respective quarter would not turn out to be a recession.

Figure 3 displays the results for the out-of-sample analysis with ProbVAR specifications including also the (sl, cs^{G7av}, sr)-specification. It turns out that the superiority of the (sl, i, cs^{G7av})-specification over the slope-only specification is mainly due to the fact that the former produces fewer false signals. The other specification, (sl, cs^{G7av}, sr), despite assigning even higher probabilities to recession periods tends to generate false probability spikes, on account of the relatively more volatile stock-return regressor that it features.

Concerning the most recent recession, none of the considered specifications is able to stand out. On the one hand, the (sl, cs^{G7av}, sr) and slope-only version have been falsely signaling high recession probabilities for more than one year before the start of the recession, while the (sl, i, cs^{G7av})-specification has not. The false signals are emitted mainly on account of the slope of the yield curve, which was flat or even negative during the period. The (sl, i, cs^{G7av}) specification, in contrast, manage to counteract this effect on account of the relatively low level of the short-term interest rate, which by itself gave a signal against recession.

On the other hand, the (sl, i, cs^{G7av}) and the slope-only specification do not manage to clearly recognize (in the sense of two- and four-quarter predictions) the latest recession episode when it had already started. The reason is that monetary policy reacted very quickly and strongly at the onset of the financial crisis, which led to a steep yield curve and a low short rate - both being signals against the occurrence of a recession based on historical data patterns. The (sl, sr, cs^{G7av})-specification fares better as it takes into consideration the strong stock market losses and the high corporate bond spreads, thus counteracting the signal coming from the slope of the yield curve. A more detailed analysis on how well the ProbVAR models manage to trace the time profile of the last two recessions is conducted in sub-section 3.5.

Turning to the simple-probit models, the out-of-sample results in figure 4 are broadly similar to the respective ProbVAR counterparts, although with some noteworthy differences. For instance, for the two-quarter horizon, the simple probit version with the (sl, sr, cs^{G7av})-specification features a major false probability spike in the late 1990s, which is less sharp in the corresponding ProbVAR version. By contrast, the latter model sends a wrong signal in 2003, which does not occur for the simple probit version. For the most recent recession, the probability assessments are roughly similar, but at the four-quarter horizon the ProbVAR (sl, sr, cs^{G7av})-specification appears to perform better than its simple-probit counterpart. Overall, while the quantitative measures of forecasting performance do not see a clear advantage in using the ProbVAR over the simple probit or the other way round, the graphical results do not evidence clear dominance of one of the two model types either.

3.3.2 Germany

Compared to the encouraging outcomes for the United States, the results of the German specifications are somewhat inferior across all forecast horizons, which holds for both in- and out-of-sample forecast exercises and for both ProbVAR and simple probit models. Nonetheless, the lowest average MAE and highest average R^2_{count} measures for the out-of-sample results from the ProbVAR, which stand at 0.27 and 0.75, respectively, are high enough to consider the behavior of the model as overall satisfactory. The best performing specification includes the slope of the yield curve, the level of the short-term interest rate and the corporate bond spread, (sl, i, cs). The specification that replaces the corporate spread by the stock return does only marginally worse.

Again, as already observed for the United States, the preferred model specification greatly improves over the standard slope-only specification. The latter reaches an MAE of 0.42 and an average R^2_{count} of 0.38 only. Looking also at the (sl, i)-specification, one observes that including the short-term interest rate in addition to the slope already improves the forecasting performance, but the additional inclusion of the corporate bond spread (or stock return) leads to a further improvement. For the in-sample results (table 5), the relative improvement over the slope-only specification is not as strong as in the out-of-sample case, but it is nonetheless remarkable.

The three panels of figure 5 show the in-sample fit of the ProbVAR models with the preferred (sl, i, cs) specification and the slope-only specification. The former is obviously better at discriminating between recession and non-recession periods, a result that holds for all of the chosen forecast horizons. The simple-probit versions of these specifications (figure 6) give a similar pattern.

Figure 7 shows the out-of-sample performance of the (sl, i, cs) and the slope-only specification. For each forecast horizon, the latter does not generate enough variability in the probability forecasts, so that it attributes very small probabilities of recessions to periods which were actually recessions. Furthermore it provides a fairly large number of 'wrong signals' (for the recession probability threshold of 0.25). The preferred (sl, i, cs)-specification reduces the number of false signals, which is the main reason for its better overall fit measures. However the model does not manage to predict the beginning of the two recessions during the period considered. A similar pattern emerges for the simple-probit version (figure 8).

As we did for the United States, we also include in figures 7 and 8 the out-of-sample performance of the (sl, cs^{G7av}, sr) specification. This specification does not include the short-term interest rate as regressor. Compared to the (sl, i, cs)-specification with the highest R^2_{count} and lowest MAE, the (sl, cs^{G7av}, sr)-model recognizes the two recessions earlier. It also manages to keep near-term recession probabilities in 2005 and 2006 lower than the slope-only version does, but overall produces more false signals than the preferred (sl, i, cs) specification. The graphical comparison between the ProbVAR models and their simple-probit counterparts shows similar patterns, which is in line with the fact that the goodness-of-fit measures also gave similar magnitudes.

3.3.3 Japan

Figure 9 highlights that the Japanese economy has exhibited a pattern of recession periods significantly different from the US and German experience. From the beginning of our sample in 1960 to 1991 only one recession has been recorded, namely between 1973Q4 and 1975Q1. From 1992 until the end of the sample, in contrast, Japan has experienced a recession for about 50% of the time. This odd distribution of recession periods obviously poses a challenge for any forecast model.[15]

According to the MAE and R^2_{count} measures, the in-sample performance of both the best ProbVAR and the best simple probit specifications for Japan (tables 9 and 10) do not differ much from the results obtained for Germany. However, when judging the out-of-sample performance of the model (tables 11 and 12) , there is a significant deterioration relative to the results obtained for the other two countries.

Based on both in-sample and out-of-sample results, the preferred specification contains the slope, the short-term interest rate and the stock return as regressors. Again, this improves upon the slope-only specification, but not as sizably as was the case for the best specifications of the other two countries. As for the US and Germany, we also include a

[15]Based on a sample from 1978 to 1997, Hirata and Ueda (1998) find that the term spread has some predictive power for Japanese recessions, but it is by far not as strong as in the case of the United States. They also find some predictive content of stock market data for longer forecast horizons, but this predictor produces fairly noisy signals.

third specification that does not contain the short-term interest rate, but which contains the international average of corporate bond spreads

In the out-of-sample forecast exercises (figures 11 and 12), neither of the model specifications manage to predict the beginnings of the first two recessions (1997Q1 and 2000Q3) for the considered time period. The preferred specification only recognizes the start of the 2000Q3 recession for the one-period forecast horizon (not shown in the picture). For the most current recession, however, the preferred (sl, i, sr)-specification manages to forecast it at the two-quarter horizon. The (sl, cs^{G7av}, sr)-specification is also able to do so and additionally manages to emit fewer false signals during the time between the last two recessions.

Finally, comparing the results from the ProbVAR specifications to the corresponding simple probit models, the forecasting quality appears to be similar.

3.4 Comparison to survey forecasts (US only)

For the United States we are also able to compare the forecasting performance of the ProbVAR to results from a survey. In the Survey of Professional Forecasters (SPF), participants are asked to state the probability of a decline in US GDP in each of the four subsequent quarters. For the one-quarter horizon, the resulting series of probabilities has been 'nicknamed' as the 'anxious' index. Although the survey question does not directly ask whether the economy will be in recession, we nevertheless treat the respective probability responses as comparable to subjective recession probabilities. Figure 13 compares the recession probabilities from the survey for the one- and four-quarter horizon with the out-of-sample predictions from the two ProbVAR(1,1,3) specifications, (sl, i, cs^{G7av}) and (sl, cs^{G7av}, sr), for the quarters between 1996 and 2009. It turns out that the range of recession probabilities announced during this time is smaller for the surveys than for the ProbVAR. In particular, for the four-quarter horizon, the survey-based probabilities range between 9% and 28%, while those from the models range from zero to around 100%. A visual inspection suggests that the two ProbVAR specifications are superior to the SPF forecasts. In fact, the upper panel of table 13 shows that the two considered ProbVAR models, but also almost all other ProbVAR specifications, give rise to smaller mean absolute forecast errors than the survey. Moreover, the difference in MAEs between the preferred (sl, i, cs^{G7av})- and (sl, cs^{G7av}, sr)-model-based probabilities and the probabilities stemming from the surveys is also statistically significant (see the lower panel of table 13), as is the case as well for a few other specfications. Notably, there are only two ProbVAR specifications ('slope only' and 'slope with corporate bond spread') that do significantly worse than the survey for the one-quarter horizon.

3.5 Zooming in on the last two recession periods

As argued above, the ProbVAR model is ideally suited to generate at a given point in time the whole sequence of recession probabilities for the coming quarters. Such a 'term structure of recession probabilities' can likewise be generated with a set of simple probit models. However, a separate set of parameters has to be estimated for each horizon, which lacks the consistency imposed by the ProbVAR. In the following subsections we discuss these model-implied time profiles of recession probabilities for the most recent recession and the previous one in the three countries.

3.5.1 United States

In order to see to what extent the ProbVAR model traces the recession profile of the US 2001 recession, we estimate the model parameters (both the VAR part and the probit part) using data up to and including 2000Q3, i.e. we use only information dated two quarters before the start of the recession. We then use the regressors and their required lags prevailing at that date to compute the recession probability for 2000Q4, 2001Q1, ..., 2003Q3. We employ the same model specifications as discussed in section 3.3.1: the slope sl as only regressor; the preferred specification with the slope, the short-term interest rate and the international corporate bonds spread (sl, i, cs^{G7av}) as regressors; and the specification with the slope, the average corporate bond spread, but the short-term interest rate replaced by the stock return, (sl, cs^{G7av}, sr).

The left panel of figure 14 plots the three term structures of recession probabilities between 2000Q4 and 2003Q3 based on 2000Q3 information. The slope-only and the (sl, i,cs^{G7av})-specification trace the time path of the recession very well. They initially assign a small recession probability (well below 20%, the chosen threshold as discussed in subsection 3.3) to the subsequent quarter (2000Q4), in which the recession in fact did not occur, to then fit the time profile of the unfolding recession. The slope-only specification produces probabilities of 35%, 43%, 51% and 41 %, for the four recession quarters, the preferred (sl, i, cs^{G7av})-specification provides somewhat lower probabilities of 21%, 44%, 40% and 26%, which nevertheless exceed the 20% threshold. Moreover, the preferred specification also manages to assign probabilities of below 20% for the quarters beyond the recession's end. In this sense, the model provides an accurate forecast of the length of the recession, when the econometrician stood two quarters before the beginning of the recession. The slope-only model also produces declining recession probabilities, but the decline takes place at a lower speed. The (sl, cs^{G7av}, sr)-specification manages to produce very high probabilities for the recession quarters (88%, 84%, 68% and 45%) as well as a quick decay of such probabilities thereafter. However, it wrongly predicts the recession to start one quarter earlier than it actually did.

We also include in this figure the QualVAR-based results of Dueker (2005). Recall that this model is more general as our ProbVAR since it also allows for feedback from the lagged latent business cycle indicator y^*_{t-l} to the current y^*_t and regressors X_t. In Dueker's study, X_t comprises real quarterly GDP growth, quarterly CPI inflation, the slope of the yield curve as well as the federal funds rate. He claims that "a recession probability above 50% for 2001:Q3 and 2001:Q4 is a rather strong signal of recession. This is especially true in light of the difficulties that professional forecasters and the leading indicators had in anticipating the 2001 recession." Against this background, the ProbVAR specifications do very well in this exercise and turn out to be similarly discriminative as the probabilities obtained by Dueker. As a difference, his recession probability for 2001Q1 is somewhat lower than all our ProbVAR specification. However, as the begin of the recession is dated as March 2001 (i.e. only the last third of 2001Q1 was in recession), it is not clear which of the results should be considered superior for this quarter. In the last recession quarter 2001Q4, in contrast, our probabilities are somewhat lower than Dueker's, but still indicative of a recession. For the subsequent out-of-recession quarters, our recession probabilities turn out to be superior as they show a quicker decay than Dueker's.

How do our ProbVAR results compare to the application of twelve (the number of horizons) simple probit models? The left panel of figure 15 shows that the corresponding sequences of probabilities do not match the recession profile as nicely as the different specifications of the ProbVAR do. The simple probit specifications do capture the beginning of the recession in a similar fashion as their ProbVAR counterparts. However, the recession probabilities stay high for too long (slope-only specification) or go down too early (the two richer specification). Moreover, using the information set dated 2000Q3, the simple probit specifications forecast the start of *another* recession in 2003Q1. This points to another advantage of the ProbVAR versions of the model, namely that recession probabilities tend to revert to their unconditional means when considering long forecast horizons.

Another exercise to assess a model's ability at fitting recession probabilities is to look at how well it predicts the exit from a recession given that the forecaster stands within the recession. With the same set of parameters as before, but using state variables dated 2001Q3, i.e. standing in the third recession quarter, we produced again the recession probabilities for the twelve subsequent quarters. As shown by the right panel of figure 14, the sl- and (sl, i, cs^{G7av})-ProbVAR models do a very good job in assigning a high probability of recession to the following quarter (which was the last recession quarter), while dropping below 10% thereafter (when recession in fact was over) to finally slowly converge towards the long-run recession frequency. The (sl, cs^{G7av}, sr)-specification essentially also shows the correct time profile but forecasts the length of the recession as one or two quarters longer than it actually turned out to be. The simple probit models performed similarly well in this experiment (right panel of figure 15), but produce an undesirable increase of recession probabilities towards the more distant forecast horizons.

Unfortunately, the models seem to have some problems in fitting the most recent recession, which the NBER dated to have started in December 2007. The out-of-sample forecasting exercise is based on parameters estimated on data up to 2007Q2 and on state variables up to 2007Q4. Through these ingredients we produce recession probabilities for the twelve quarters after 2007Q4. Figure 16, left panel, plots the results for our three ProbVAR model specifications. The slope-only specification and the (sl, i, cs^{G7av})-specification correctly assign a high recession probability for 2008Q1 but they foresee a swift decay of these probabilities thereafter. The (sl, cs^{G7av}, sr)-specification, in contrast, does not see a recession in 2008Q1, but implies high probabilities for the US economy to be in recession between 2008Q2 and 2009Q1.

The exercise is repeated with the forecaster standing in 2008Q3 (i.e. the quarter including the Lehman Brothers' collapse), but still keeping the parameters fixed at the values based on estimating the model up to 2007Q2. The slope-only specification does not recognize a recession for the following quarters at all. The preferred (sl, i, cs^{G7av})-specification assigns a high recession probability for 2008Q4 but predicts the recession to end thereafter. The (sl, cs^{G7av}, sr)-specification, in contrast, adequately implies very high recession probabilities for the four subsequent quarters (2008Q4 to 2009Q3), after which the probabilities quickly decline.

Why do the sl and (sl, i, cs^{G7av}) specifications fail to see a protracted recession, given the intensity of the financial turmoil? The reason most likely has to be found in the very strong reaction of US monetary policy, which led to a sudden rise of the slope of the yield curve as the short term interest rate was aggressively lowered since the start of the crisis. In fact, the slope of the yield curve turned from being negative in the first quarter of 2007, to a positive value of more than two percentage points by the end of 2008. Given the estimated historical regularities, the slope-only model could not avoid interpreting this as a strong signal against recession. Specification (sl, i, cs^{G7av}) counteracts the effect coming from the slope through the inclusion of the corporate bond spread, which widened during the crisis, thereby contributing to increasing the model-implied recession probability. However, the specification also includes the short-term interest rate, which reached historical lows during the crisis, thus contributing to decreasing the model-implied recession probability again. The successful third specification, in contrast, compensated the falsely comforting signal from the term spread by including both the corporate bond spread and stock returns.

Finally, the simple probit versions of the slope-only and the (sl, i, cs^{G7av})-specifications (figure 17) disappointed similarly to their ProbVAR counterparts during the most current recession. The (sl, cs^{G7av}, sr)- specification, that performed well in its ProbVAR version, produces markedly inferior results when the forecasts are based on separately estimated simple probit models.

3.5.2 Germany

Similarly to what occurred in the United States, the year 2001 marked the beginning of a recession also for the German economy. However, this lasted nearly three years, significantly longer than the US one. Analogously to what we did for the corresponding US exercise, we estimated the models until 2000Q3 and with this set of parameters as well as the regressors observed until this point in time, we produced forecasts for recession probabilities for up to twelve quarters ahead, i.e. for the period ranging between 2000Q4 and 2003Q3. Figure 18, left panel, shows the results for the slope-only specification, the (sl, i, cs) specification that was preferred on the basis of the goodness-of-fit measures in the out-of-sample forecasts, and the (sl, cs^{G7av}, sr) specification. Conditional on the information set dated 2000Q3, all specifications essentially fail to recognize the pattern of the coming recession. While the (sl, i, cs)- and (sl, cs^{G7av}, sr)-specifications produce too low conditional recession probabilities for the quarters ahead, the slope-only version implies probabilities of around 40 to 50% (left panel of figure 18), but, as already seen in the time series figures for the out-of-sample results , the latter specification tends to produce high recession probabilities for most of the quarters, i.e. also in no-recession periods. The simple-probit variants of these specifications share similar problems (left panel of figure 19), but the (sl, i, cs)-specification now at least implies recession probabilities exceeding 30% for all periods from 2001Q3 henceforth.

Standing now in 2001Q3, i.e. with other eight quarters of recession to follow, and keeping the model parameters at their estimates based on data up until 2000Q3 but using predictive variables observed until 2001Q3, the (sl, cs^{G7av}, sr)-specification produces very reasonable predictions (see the right panel of figure 18), implying recession probabilities exceeding 85% for the four quarters to come, which then go down monotonically to 30% at the end of the recession to eventually fade out thereafter. The slope-only and the (sl, i, cs)-specification, by contrast, do not perform satisfactorily, as the former implies a flat path of recession probabilities of around 35%, while the latter implies recession odds even below 10%. Using the same prediction exercise, the simple-probit versions (right panel of figure 19) of these specifications again do similarly badly, where here the (sl, i, cs)-specification even sees rising recession probabilities after the recession's end (forecast horizon of nine to twelve quarters).

Turning to the most recent recession, the parameters are estimated with data until 2007Q2. Using these parameters, the first set of forecasts is produced using predictors until 2007Q4, when it was still far from clear that the strains in financial markets would eventually have had such sizable adverse macroeconomic implications for Germany. The (sl, cs^{G7av}, sr)-ProbVAR model, left panel of figure 20, anticipates recession probabilities as of around 40% for the first half of 2008 to then drop considerably thereafter. The (sl, i, cs)-version envisages the highest recession probability at the three-quarter horizon, i.e. for

2008Q3, but lower probabilities before and after that time. The slope-only model shows a mild monotonous decrease from a probability of 55% to 40% over the twelve following quarters.

Constructing the forecast in 2008Q3, while keeping the parameters on their pre-recession estimates (i.e. as of 2007Q3), the slope-only specification produces the same forecast as in 2007Q4, (right panel of figure 20), i.e. failing again to come up with a discriminating path of recession probabilities. The richer specifications, by contrast, see high recession probabilities exceeding 95% for 2008Q4 and 2009Q1. For 2009Q2, the first quarter after the recession, they also imply such high probabilities, but for 2009Q3, the (sl, cs^{G7av}, sr)-model implied probabilities sharply drop to 50% and then moves swiftly below 20%. The time profile of the recession probabilities envisaged beyond-2009Q2 is somewhat inferior for the (sl, i, cs)-version, as they tend to revert less quickly than for the (sl, cs^{G7av}, sr)-specification.

When the forecast exercise is based on the same variable specifications but using simple-probit models (figure 21) there is no major improvement relative to the ProbVAR models. The main differences are observed for the (sl, i, cs)-specifications. Based on 2007Q4 information, the simple-probit version predicts higher recession probabilities than the ProbVAR model for the last two quarters of the recession, but it also assigns recession probabilities exceeding 50% for all periods after the end of the recession. Standing in 2008Q3, the simple probit version of the (sl, i, cs)-specification is similarly successful as its ProbVAR counterpart in predicting the recession to stay for the next two quarters, but it implies higher recession probabilities than the ProbVAR for the time beyond the recession.

3.5.3 Japan

The first Japanese recession that we use to check the model's forecasting ability starts in 2000Q3 and ends in 2003Q2. Besides its extreme length, another peculiarity is given by the fact that it started only four quarters after the end of the previous recession (itself having a remarkable length of 11 quarters). Hence it comes as no surprise that the ProbVAR as well as the simple probit models have a hard time in assigning high probabilities to such an event. Standing in 2000Q1, i.e. two quarters before the beginning of the recession, it is only the (sl, i, sr)-specification that emits a 'mild' recessionary signal with probabilities rising slightly above 40% (see the left panels of figures 22 and 23).

When the forecast is made standing within the recession, i.e. placing the forecaster at 2001Q1, the latter specification correctly foresees the recession to last for another two years (right panel of figure 22). The (sl, cs^{G7av}, sr)-model also recognizes high recession probabilities for the near future, but misinterprets the recession as being rather short-lived. The corresponding simple-probit versions of the forecast specifications give rise to similar results. Conditional on 2000Q1 information, the simple probit specifications with the (sl, i, sr) variables emits somewhat stronger recession signals than the corresponding ProbVAR (right panel of 23). However, conditioning on variables observed until 2001Q1, the model falsely implies rising recession probabilities for the time after the end of the recession, i.e. for forecast horizons between nine and twelve quarters.

The most recent recession started for Japan in 2008Q1. Again we use the same strategy as before, estimating the model's parameters using information until 2007Q2 and then making forecasts using 2007Q4 and 2008Q3 regressors, respectively. The (sl, i, sr)-ProbVAR model predicts a recession with a 65% probability for 2008Q1, but implies lower recession probabilities around 50% for the following quarters (left panel of figure 24). The (sl, cs^{G7av}, sr)-predictors give strong recession signals (80% and 55%) for the first two quarters of 2008, but see the recession as rather short-lived as the probabilities drop markedly for the periods thereafter. The slope-only specification is essentially non-discriminative as it shows a 30% recession probability throughout the forecast horizons.

Making the recession probability forecasts from 2008Q3 (and again freezing parameters on their 2007Q2 estimates), the (sl, cs^{G7av}, sr)-model performs particularly well (right panel of figure 24). It assigns probabilities exceeding 90% for the following quarters, which were in fact still a recession. For 2009Q2 the estimated probability is still high, although the economy was already out of the recession in that quarter, but for 2009Q3 the recession odds drop to below 50% to then converge quickly to the long-run average of 20%. The (sl, i, sr)-specification assigns similarly high probabilities to the recession periods, but the probabilities go down too slowly for the quarters subsequent to the recession's end. The simple-probit version of the models (figure 25) produces a similar picture, with the (sl, cs^{G7av}, sr)-specification being even better discriminating than in the ProbVAR framework, but the (sl, i, sr)-induced recession probability profile being somewhat inferior compared to the ProbVAR case.

4 Illustrating impulse response analysis

In section 2.4, we explained how the ProbVAR model can be employed to generate impulse responses, i.e the changes in recession probabilities in reaction to unexpected changes in the explanatory variables. We shed more light on this method empirically by showing how recession probabilities respond to an unexpected change in the slope of the yield curve, for the United States, Germany and Japan. This particular type of shock is chosen since the term spread is one of the most prominent variables used in previous studies on recession forecasting, and because it enters the preferred specification of all the countries considered. However, we do not analyze the effect of a shock to the term spread in the slope-only specification, but rather in the specifications with the preferred forecast properties as outlined before: (sl, i, cs^{G7av}) for the United States, (sl, i, cs) for Germany, and (sl, i, sr) for Japan.

Due to the nonlinearity of the ProbVAR specification, the impulse responses differ from those obtained from linear VARs in two important dimensions: first, changing the size of the impulse vector does not lead to a proportional change in the impulse responses; second, impulse responses will depend on the initial state vector. To illustrate these points we show for each country six versions of the impulse responses, which come from three different shock sizes, combined with two different initial conditions. All results are based on 'generalized' impulse responses in the sense of Pesaran and Shin (1998).[16]

For the United States, a decrease of the slope of the yield curve by one percentage point leads to an increase in subsequent recession probabilities as expected. For the scenario in which the slope shock hits in a situation when all variables are at their sample means, the recession probability shows its peak response in the fourth quarter after the shock with a magnitude of somewhat over 25 percentage points, see figure 26. Thereafter, it slowly peters out towards zero. If the same shock is taken to be of double size, the peak response occurs likewise after four quarters. However, the probability more than doubles compared to the half-sized shock and amounts to nearly 70 percentage points. As a third variant of the shock we consider a one-percentage-point *increase* in the slope. While this shock mirrors the flattening shock considered first, the two responses of recession probabilities are not at all mirror images of each other: the maximum response only reaches 10 percentage points in absolute magnitude; in addition, the peak response is recorded after five rather than four quarters.

The same three shocks are again considered for different initial conditions. This time, regressors are also set equal to their sample averages, with the exception of the initial slope of the yield curve which is increased by one sample standard deviation. Hence, we start from a situation characterized by lower recession probabilities for the quarters ahead. The results for the three shocks point out clearly the other effect of nonlinearity, namely that impulse responses are dependent on the initial condition. Here, the responses associated with an initially steeper yield curve are much more dampened than their counterparts based on the initial slope being at its sample mean.

Turning to the results for Germany, figure 27, the same asymmetry as for the United States is observable, but to a less extent. In particular, the dependence on initial conditions is less distinct. The response to a one-percentage-point shock of the slope is lower than in the US case (slightly below 20 percentage points), but the peak occurs already in the third quarter after the shock. The responses to the double-sized negative shocks are also smaller than their US counterparts, the reactions to the positive slope shocks are slightly more distinct and peak earlier.

Finally, for Japan the response to the term spread shock is very small, see figure 28.

[16] That is, the initial shock vector is given as the expectation of shocks to the driving variables in the respective specification (e.g., slope, short-term interest rate and average corporate bond spread for the US) conditional on the shock to the slope assuming the specified magnitude.

The maximum reaction to a flattening of the yield curve by one percentage point occurs after one quarter and is below 10 percentage points. The dependence on initial conditions is negligible.

5 Conclusion

In this paper, the traditional probit regression approach for forecasting recession probabilities is enhanced by endogenizing the dynamics of regressors using a VAR. With the resulting 'ProbVAR' model it is possible to generate a smooth 'term structure of recession probabilities' at each point in time. The model is straightforward to estimate: first, a traditional probit relation between recession probabilities and regressors (lagged by one quarter) is estimated; then the VAR for the regressors is estimated, e.g. by ordinary least squares. Given the probit and VAR parameters, the ProbVAR yields a closed-form expression for the recession probability at any future horizon, conditional on the current predictors and possibly their lags.

We apply the model to forecasting the recession phases of the United States, Germany and Japan from 1960 to 2009. As for the choice of regressors, we use the slope of the yield curve, but also other financial variables, such as the level of the short-term interest rate, the corporate bond spread and the return on a broad-based stock market index. The latter two variables have been employed both in their domestic form, i.e. by selecting each of the variables with reference only to the country under examination, or taking averages of their values over the G7 economies.

The in- and out-of-sample performance of the best ProbVAR specifications is very good for the United States, somewhat less satisfactory for Germany, and considerably inferior for Japan. The good forecast quality for the binary series 'US recession' stands somewhat in contrast to findings that US macroeconomic activity has become harder to forecast during the two decades ('great moderation') preceding the recent crisis, see, e.g., D'Agostino, Giannone, and Surico (2006). However, as pointed out in that study, nonlinear models may in principle be more successful, which seems to be confirmed to some extent by our results.

For all countries, the best specifications feature other financial variables beyond the slope of the yield curve, and the gain in forecasting precision compared to the slope-only specification is considerable. For the United States, we also compare the forecasting quality of the ProbVAR models to that of the Survey of Professional Forecasters. For our out-of-sample period, 1996 to 2009, the model significantly improves upon the survey-based forecasts for the one-quarter and the four-quarter horizons. This result is in line with the finding of Rudebusch (2008), which are, however, based on the term spread as the sole regressor.

As mentioned, in the ProbVAR framework, recession probabilities for all forecasting horizons are based on one single model. We compare the recession probabilities that originate from the ProbVar for horizons of one to six quarters to the comparable predictions stemming from standard probit models, which are separately estimated for each forecast horizon. Essentially, the comparison between the ProbVAR and simple probit models is analogous to the debate about 'direct' vs. 'iterated multi-step' forecasts in linear VAR models. If the model is correctly specified, the iterative forecast should be more efficient, while direct forecasts should be more robust to model mis-specification.[17] Hence, determining which forecast is superior is essentially an empirical question, and the literature does not appear to have found evidence for general superiority of one or the other approach across several applications. In our context, it turns out that the forecast quality is overall similar across the two approaches. However, the ProbVAR model tends to imply smoother and economically more plausible time profiles of recession probabilities, compared to the term structures of recession probabilities implied by separately estimated standard probit models.

The out-of-sample forecasting exercise has pointed out the importance of cross-checking various specifications. For instance, while in the 2001 US recession the model specification with term spread, short-term interest rate and corporate bond spread has been very successful, it failed to recognize the most current US recession. This is because monetary policy measures led short-term rates to historical lows, while at the same time the yield curve steepened considerably. Given the historical relationships, the latter constellation is interpreted by the econometric model as a strong hint against the occurrence of a recession. Hence, during that period, it was preferable to also consider such model specifications, in which other financial variables can compensate the expansion signal stemming from the term spread.

Finally, we have shown that the ProbVAR model can also be used to conduct impulse response analysis, which cannot be achieved with traditional fixed-horizon probit models. In a similar way as with traditional linear VARs, we can trace the effect of a shock to the financial variables on the recession probabilities in the following quarters. However, as the model is non-linear, there are two important differences to linear-VAR impulse responses. First, positive and negative shocks of the same absolute size have different absolute effects on recession probabilities. Second, the impact of the same shock on recession probabilities depends on the initial conditions, i.e. on the constellation of financial variables at the time at which the shock occurs.

Overall, the ProbVAR model appears to be a useful tool in various fields of applications. For instance, from a macro-prudential perspective, it can help to identify the risks of a forthcoming recession in a consistent fashion across short- and medium-term horizons. Forecasting information is quickly updatable as regressors are financial variables, which

[17] See, e.g., Marcellino, Stock, and Watson (2006).

are available in real time. For central banks and other policy institutions, the ProbVAR can likewise help to identify and predict periods of economic slack, but it also provides information on the expected remaining duration of a recession, once it has actually begun.

The results of this paper are encouraging and call for several extensions. For instance, the ProbVAR may be extended to a bi- or multivariate approach in the sense that it predicts the joint probability of recession for two or more countries. Second, while we employ averages of financial variables from various countries, it appears worthwhile to explore the forecasting power of financial factors drawn from a larger set of financial data. Third, in our approach, the same set of variables appeared in the VAR and in the probit relation. This one-to-one relation may be fruitfully relaxed: there may be variables in the VAR part that help forecasting, e.g., the term spread, but these variables may not need to show up in the probit relation.

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A Tables

	Specification	Forecast horizon: 1q				Forecast horizon: 4 q				Forecast horizon: 6 q				Av. over forec. hor. 1 to 6 q			
		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4	
		VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)
1	sl	0.30	0.30	0.22	0.22	0.30	0.30	0.25	0.25	0.30	0.30	0.27	0.27	0.30	0.30	0.25	0.25
2	sl, i	0.30	0.30	0.21	0.21	0.29	0.29	0.24	0.24	0.30	0.30	0.28	0.26	0.29	0.29	0.24	0.24
3	sl, cs	0.24	0.24	0.15	0.15	0.29	0.28	0.22	0.22	0.30	0.29	0.27	0.26	0.28	0.27	0.21	0.21
4	sl, sr	0.19	0.19	0.12	0.12	0.28	0.28	0.25	0.24	0.30	0.29	0.27	0.27	0.26	0.26	0.22	0.21
5	sl, i, cs	0.23	0.23	0.14	0.14	0.28	0.27	0.21	0.21	0.29	0.28	0.26	0.25	0.27	0.26	0.20	0.20
6	sl, i, sr																
7	sl, cs, sr																
8	sl, cs^{G7av}	0.20	0.20	0.13	0.13	0.28	0.28	0.21	0.21	0.31	0.30	0.27	0.26	0.26	0.26	0.20	0.20
9	sl, sr^{G7av}	0.21	0.21	0.14	0.14	0.29	0.27	0.25	0.23	0.30	0.28	0.27	0.25	0.27	0.25	0.22	0.21
10	sl, i, cs^{G7av}	0.18	0.18	0.10	0.10	0.27	0.25	0.19	0.19	0.30	0.29	0.26	0.24	0.25	0.24	0.18	0.17
11	sl, i, sr^{G7av}																
12	sl, cs, sr^{G7av}	0.19	0.19	0.10	0.10	0.29	0.27	0.23	0.21	0.31	0.28	0.27	0.25	0.26	0.25	0.20	0.19
13	sl, cs^{G7av}, sr	0.16	0.16	0.06	0.06	0.28	0.27	0.21	0.21	0.31	0.30	0.29	0.28	0.25	0.24	0.19	0.18
14	sl, cs^{G7av}, sr^{G7av}																

	Specification	Forecast horizon: 1q				Forecast horizon: 4 q				Forecast horizon: 6 q				Av. over forec. hor. 1 to 6 q			
		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4	
		VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)
1	sl	0.56	0.56	0.76	0.76	0.74	0.71	0.69	0.69	0.79	0.76	0.67	0.67	0.70	0.68	0.69	0.70
2	sl, i	0.71	0.71	0.78	0.78	0.78	0.79	0.71	0.71	0.71	0.74	0.67	0.68	0.75	0.75	0.72	0.72
3	sl, cs	0.74	0.74	0.82	0.82	0.72	0.73	0.78	0.77	0.71	0.72	0.69	0.69	0.72	0.73	0.77	0.77
4	sl, sr	0.82	0.82	0.89	0.89	0.68	0.65	0.73	0.73	0.67	0.64	0.69	0.68	0.72	0.70	0.76	0.76
5	sl, i, cs	0.79	0.79	0.85	0.85	0.69	0.72	0.80	0.80	0.65	0.72	0.71	0.71	0.72	0.74	0.79	0.79
6	sl, i, sr																
7	sl, cs, sr																
8	sl, cs^{G7av}	0.78	0.78	0.84	0.84	0.69	0.70	0.81	0.81	0.61	0.56	0.69	0.69	0.71	0.70	0.79	0.79
9	sl, sr^{G7av}	0.77	0.77	0.86	0.86	0.66	0.72	0.69	0.75	0.63	0.71	0.69	0.72	0.68	0.73	0.75	0.78
10	sl, i, cs^{G7av}	0.81	0.81	0.89	0.89	0.72	0.73	0.84	0.84	0.63	0.68	0.74	0.74	0.73	0.75	0.84	0.84
11	sl, i, sr^{G7av}																
12	sl, cs, sr^{G7av}	0.80	0.80	0.90	0.90	0.63	0.73	0.74	0.79	0.59	0.65	0.68	0.68	0.68	0.74	0.78	0.80
13	sl, cs^{G7av}, sr	0.87	0.87	0.94	0.94	0.72	0.68	0.76	0.77	0.60	0.59	0.61	0.61	0.73	0.72	0.78	0.78
14	sl, cs^{G7av}, sr^{G7av}																

Table 1: United States. In-sample forecasting performance (1960Q1 to 2009Q4) of several ProbVAR specifications: MAE (top panel) and R^2_{count} (bottom panel). sl: slope of the yield curve, i: short-term interest rate, cs: corporate bond spread, sr: year-on-year stock return, sr^{G7av}: average stock return over the G7 countries, cs^{G7av}: average corporate bond spread over Canada, Germany, the United Kingdom, and the United States. Shaded entries indicate the minimum MAE and the maximum R^2_{count} for the respective forecast horizon.

	Specification	Forecast horizon: 1q		Forecast horizon: 4 q		Forecast horizon: 6 q		Av. over forec. hor. 1 to 6 q	
		Additional probit lags:		Additional probit lags:		Additional probit lags:		Additional probit lags:	
		0	3	0	3	0	3	0	3
1	sl	0.30	0.22	0.25	0.24	0.25	0.25	0.27	0.24
2	sl, i	0.30	0.21	0.24	0.23	0.25	0.24	0.26	0.23
3	sl, cs	0.24	0.15	0.23	0.21	0.25	0.23	0.24	0.20
4	sl, sr	0.19	0.12	0.24	0.23	0.25	0.25	0.23	0.20
5	sl, i, cs	0.23	0.14	0.22	0.20	0.25	0.22	0.23	0.19
6	sl, i, sr								
7	sl, cs, sr								
8	sl, cs^{G7av}	0.20	0.13	0.20	0.19	0.25	0.23	0.21	0.18
9	sl, sr^{G7av}	0.21	0.14	0.25	0.21	0.23	0.22	0.24	0.19
10	sl, i, cs^{G7av}	0.18	0.10	0.19	0.19	0.24	0.22	0.20	0.17
11	sl, i, sr^{G7av}								
12	sl, cs, sr^{G7av}	0.19	0.10	0.23	0.19	0.23	0.21	0.22	0.17
13	sl, cs^{G7av}, sr	0.16	0.06	0.20	0.18	0.24	0.22	0.20	0.15
14	sl, cs^{G7av}, sr^{G7av}								

	Specification	Forecast horizon: 1q		Forecast horizon: 4 q		Forecast horizon: 6 q		Av. over forec. hor. 1 to 6 q	
		Additional probit lags:		Additional probit lags:		Additional probit lags:		Additional probit lags:	
		0	3	0	3	0	3	0	3
1	sl	0.56	0.76	0.71	0.71	0.70	0.71	0.66	0.72
2	sl, i	0.71	0.78	0.72	0.74	0.70	0.75	0.71	0.75
3	sl, cs	0.74	0.82	0.76	0.81	0.73	0.77	0.74	0.81
4	sl, sr	0.82	0.89	0.72	0.73	0.71	0.71	0.73	0.78
5	sl, i, cs	0.79	0.85	0.77	0.82	0.75	0.78	0.77	0.83
6	sl, i, sr								
7	sl, cs, sr								
8	sl, cs^{G7av}	0.78	0.84	0.81	0.82	0.75	0.77	0.79	0.81
9	sl, sr^{G7av}	0.77	0.86	0.70	0.78	0.73	0.74	0.73	0.79
10	sl, i, cs^{G7av}	0.81	0.89	0.84	0.83	0.78	0.78	0.82	0.85
11	sl, i, sr^{G7av}								
12	sl, cs, sr^{G7av}	0.80	0.90	0.76	0.82	0.78	0.76	0.77	0.83
13	sl, cs^{G7av}, sr	0.87	0.94	0.79	0.85	0.74	0.79	0.80	0.85
14	sl, cs^{G7av}, sr^{G7av}								

Table 2: United States. In-sample forecasting performance (1960Q1 to 2009Q4) of several simple probit specifications: MAE (top panel) and R^2_{count} (bottom panel). sl: slope of the yield curve, i: short-term interest rate, cs: corporate bond spread, sr: year-on-year stock return, sr^{G7av}: average stock return over the G7 countries, cs^{G7av}: average corporate bond spread over Canada, Germany, the United Kingdom, and the United States. Shaded entries indicate the minimum MAE and the maximum R^2_{count} for the respective forecast horizon.

	Specification	Forecast horizon: 1q				Forecast horizon: 4 q				Forecast horizon: 6 q				Av. over forec. hor. 1 to 6 q			
		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4	
		VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)
1	sl	0.32	0.32	0.25	0.25	0.31	0.32	0.27	0.27	0.32	0.32	0.29	0.30	0.32	0.32	0.27	0.28
2	sl, i	0.27	0.27	0.20	0.20	0.28	0.27	0.24	0.25	0.28	0.28	0.29	0.28	0.28	0.28	0.25	0.25
3	sl, cs	0.29	0.29	0.18	0.18	0.31	0.31	0.27	0.27	0.32	0.32	0.28	0.28	0.30	0.30	0.25	0.26
4	sl, sr	0.20	0.20	0.12	0.12	0.30	0.31	0.27	0.31	0.31	0.33	0.29	0.32	0.27	0.28	0.24	0.26
5	sl, i, cs	0.24	0.24	0.15	0.15	0.26	0.26	0.25	0.25	0.28	0.28	0.28	0.28	0.26	0.26	0.24	0.24
6	sl, i, sr																
7	sl, cs, sr																
8	sl, cs^{G7av}	0.21	0.21	0.14	0.14	0.27	0.27	0.23	0.23	0.31	0.31	0.30	0.29	0.26	0.26	0.22	0.21
9	sl, sr^{G7av}	0.21	0.21	0.16	0.16	0.30	0.29	0.28	0.30	0.32	0.31	0.29	0.30	0.28	0.27	0.25	0.26
10	sl, i, cs^{G7av}	0.16	0.16	0.12	0.12	0.23	0.22	0.23	0.23	0.27	0.27	0.29	0.28	0.22	0.21	0.21	0.21
11	sl, i, sr^{G7av}																
12	sl, cs, sr^{G7av}	0.21	0.21	0.15	0.15	0.30	0.29	0.28	0.30	0.32	0.31	0.29	0.30	0.28	0.27	0.25	0.26
13	sl, cs^{G7av}, sr	0.18	0.18	0.10	0.10	0.29	0.28	0.22	0.23	0.32	0.33	0.30	0.31	0.26	0.26	0.20	0.21
14	sl, cs^{G7av}, sr^{G7av}																

	Specification	Forecast horizon: 1q				Forecast horizon: 4 q				Forecast horizon: 6 q				Av. over forec. hor. 1 to 6 q			
		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4	
		VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)
1	sl	0.51	0.51	0.71	0.71	0.82	0.65	0.69	0.67	0.76	0.73	0.67	0.60	0.68	0.65	0.68	0.66
2	sl, i	0.76	0.76	0.85	0.85	0.76	0.76	0.82	0.82	0.76	0.76	0.67	0.69	0.76	0.76	0.77	0.78
3	sl, cs	0.64	0.64	0.80	0.80	0.82	0.71	0.69	0.69	0.75	0.71	0.71	0.64	0.71	0.68	0.72	0.70
4	sl, sr	0.84	0.84	0.93	0.93	0.67	0.64	0.75	0.65	0.69	0.60	0.76	0.58	0.73	0.70	0.79	0.70
5	sl, i, cs	0.82	0.82	0.89	0.89	0.78	0.78	0.80	0.78	0.76	0.76	0.80	0.71	0.79	0.79	0.80	0.78
6	sl, i, sr																
7	sl, cs, sr																
8	sl, cs^{G7av}	0.75	0.75	0.78	0.78	0.84	0.76	0.78	0.80	0.73	0.80	0.64	0.62	0.77	0.75	0.75	0.76
9	sl, sr^{G7av}	0.80	0.80	0.87	0.87	0.69	0.73	0.67	0.69	0.67	0.67	0.69	0.64	0.72	0.74	0.74	0.72
10	sl, i, cs^{G7av}	0.85	0.85	0.95	0.95	0.84	0.87	0.85	0.82	0.76	0.73	0.71	0.71	0.84	0.85	0.83	0.82
11	sl, i, sr^{G7av}																
12	sl, cs, sr^{G7av}	0.76	0.76	0.82	0.82	0.69	0.73	0.69	0.69	0.67	0.65	0.71	0.64	0.70	0.72	0.75	0.71
13	sl, cs^{G7av}, sr	0.84	0.84	0.87	0.87	0.73	0.67	0.80	0.71	0.62	0.62	0.75	0.64	0.73	0.70	0.79	0.72
14	sl, cs^{G7av}, sr^{G7av}																

Table 3: United States. Out-of-sample forecasting performance (1995Q1 to 2009Q4) of several ProbVAR specifications: MAE error (top panel) and R^2_{count} (bottom panel). sl: slope of the yield curve, i: short-term interest rate, cs: corporate bond spread, sr: year-on-year stock return, sr^{G7av}: average stock return over the G7 countries, cs^{G7av}: average corporate bond spread over Canada, Germany, the United Kingdom, and the United States. Shaded entries indicate the minimum MAE and the maximum R^2_{count} for the respective forecast horizon.

	Specification	Forecast horizon: 1q		Forecast horizon: 4 q		Forecast horizon: 6 q		Av. over forec. hor. 1 to 6 q	
		Additional probit lags:		Additional probit lags:		Additional probit lags:		Additional probit lags:	
		0	3	0	3	0	3	0	3
1	*sl*	0.32	0.25	0.28	0.27	0.28	0.30	0.29	0.27
2	*sl, i*	0.27	0.20	0.25	0.25	0.27	0.27	0.26	0.24
3	*sl, cs*	0.29	0.18	0.28	0.26	0.29	0.28	0.29	0.25
4	*sl, sr*	0.20	0.12	0.28	0.29	0.30	0.31	0.26	0.25
5	*sl, i, cs*	0.24	0.15	0.25	0.23	0.27	0.26	0.25	0.22
6	*sl, i, sr*								
7	*sl, cs, sr*								
8	sl, cs^{G7av}	0.21	0.14	0.24	0.26	0.29	0.26	0.24	0.22
9	sl, sr^{G7av}	0.21	0.16	0.29	0.27	0.28	0.27	0.27	0.25
10	sl, i, cs^{G7av}	0.16	0.12	0.24	0.25	0.27	0.25	0.22	0.21
11	sl, i, sr^{G7av}								
12	sl, cs, sr^{G7av}	0.21	0.15	0.29	0.25	0.28	0.24	0.26	0.24
13	sl, cs^{G7av}, sr	0.18	0.10	0.24	0.27	0.30	0.26	0.24	0.21
14	sl, cs^{G7av}, sr^{G7av}								

	Specification	Forecast horizon: 1q		Forecast horizon: 4 q		Forecast horizon: 6 q		Av. over forec. hor. 1 to 6 q	
		Additional probit lags:		Additional probit lags:		Additional probit lags:		Additional probit lags:	
		0	3	0	3	0	3	0	3
1	*sl*	0.51	0.71	0.69	0.71	0.69	0.62	0.63	0.69
2	*sl, i*	0.76	0.85	0.82	0.80	0.82	0.75	0.80	0.78
3	*sl, cs*	0.64	0.80	0.67	0.73	0.67	0.69	0.66	0.72
4	*sl, sr*	0.84	0.93	0.67	0.65	0.62	0.62	0.71	0.73
5	*sl, i, cs*	0.82	0.89	0.78	0.80	0.80	0.78	0.81	0.81
6	*sl, i, sr*								
7	*sl, cs, sr*								
8	sl, cs^{G7av}	0.75	0.78	0.78	0.73	0.69	0.75	0.75	0.75
9	sl, sr^{G7av}	0.80	0.87	0.67	0.73	0.73	0.73	0.72	0.75
10	sl, i, cs^{G7av}	0.85	0.95	0.84	0.75	0.78	0.80	0.84	0.82
11	sl, i, sr^{G7av}								
12	sl, cs, sr^{G7av}	0.76	0.82	0.67	0.76	0.75	0.75	0.72	0.76
13	sl, cs^{G7av}, sr	0.84	0.87	0.76	0.69	0.65	0.75	0.76	0.76
14	sl, cs^{G7av}, sr^{G7av}								

Table 4: United States. Out-of-sample forecasting performance (1995Q1 to 2009Q4) of several simple probit specifications: MAE error (top panel) and R^2_{count} (bottom panel). sl: slope of the yield curve, i: short-term interest rate, cs: corporate bond spread, sr: year-on-year stock return, sr^{G7av}: average stock return over the G7 countries, cs^{G7av}: average corporate bond spread over Canada, Germany, the United Kingdom, and the United States. Shaded entries indicate the minimum MAE and the maximum R^2_{count} for the respective forecast horizon.

	Specification	Forecast horizon: 1q				Forecast horizon: 4 q				Forecast horizon: 6 q				Av. over forec. hor. 1 to 6 q			
		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4	
		VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)
1	sl	0.35	0.35	0.34	0.34	0.37	0.37	0.36	0.36	0.39	0.39	0.38	0.38	0.37	0.37	0.36	0.36
2	sl, i	0.31	0.31	0.29	0.29	0.33	0.34	0.31	0.31	0.36	0.37	0.35	0.35	0.33	0.33	0.31	0.31
3	sl, cs	0.32	0.32	0.31	0.31	0.36	0.36	0.35	0.35	0.38	0.39	0.38	0.38	0.35	0.36	0.34	0.35
4	sl, sr	0.25	0.25	0.22	0.22	0.35	0.37	0.32	0.33	0.38	0.40	0.37	0.39	0.32	0.34	0.30	0.31
5	sl, i, cs	0.24	0.24	0.21	0.21	0.31	0.31	0.29	0.29	0.35	0.36	0.34	0.35	0.30	0.30	0.27	0.28
6	sl, i, sr	0.20	0.20	0.17	0.17	0.30	0.32	0.27	0.27	0.35	0.37	0.34	0.34	0.28	0.29	0.25	0.25
7	sl, cs, sr	0.25	0.25	0.21	0.21	0.35	0.36	0.32	0.33	0.38	0.40	0.37	0.39	0.32	0.34	0.30	0.31
8	sl, cs^{G7av}	0.33	0.33	0.32	0.32	0.36	0.36	0.36	0.36	0.38	0.38	0.38	0.38	0.36	0.36	0.35	0.35
9	sl, sr^{G7av}	0.26	0.26	0.24	0.24	0.35	0.36	0.34	0.35	0.38	0.39	0.38	0.39	0.33	0.34	0.32	0.32
10	sl, i, cs^{G7av}	0.27	0.27	0.26	0.26	0.31	0.31	0.31	0.31	0.35	0.35	0.35	0.35	0.31	0.31	0.30	0.30
11	sl, i, sr^{G7av}	0.22	0.22	0.19	0.19	0.31	0.31	0.29	0.29	0.35	0.36	0.35	0.35	0.29	0.29	0.27	0.27
12	sl, cs, sr^{G7av}	0.26	0.26	0.24	0.24	0.35	0.36	0.34	0.35	0.38	0.39	0.38	0.39	0.33	0.34	0.32	0.32
13	sl, cs^{G7av}, sr	0.25	0.25	0.20	0.20	0.35	0.36	0.31	0.32	0.38	0.40	0.37	0.38	0.32	0.34	0.29	0.30
14	sl, cs^{G7av}, sr^{G7av}	0.26	0.26	0.23	0.23	0.35	0.36	0.33	0.34	0.38	0.39	0.38	0.38	0.33	0.34	0.31	0.31

	Specification	Forecast horizon: 1q				Forecast horizon: 4 q				Forecast horizon: 6 q				Av. over forec. hor. 1 to 6 q			
		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4	
		VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)
1	sl	0.62	0.62	0.64	0.64	0.54	0.54	0.57	0.58	0.48	0.48	0.52	0.53	0.55	0.56	0.58	0.59
2	sl, i	0.67	0.67	0.69	0.69	0.65	0.64	0.67	0.68	0.59	0.61	0.59	0.61	0.65	0.66	0.66	0.67
3	sl, cs	0.72	0.72	0.72	0.72	0.60	0.59	0.63	0.63	0.51	0.51	0.56	0.54	0.61	0.61	0.64	0.64
4	sl, sr	0.76	0.76	0.82	0.82	0.63	0.58	0.67	0.64	0.57	0.47	0.56	0.53	0.65	0.61	0.69	0.66
5	sl, i, cs	0.78	0.78	0.80	0.80	0.67	0.69	0.71	0.72	0.59	0.64	0.60	0.63	0.69	0.70	0.71	0.73
6	sl, i, sr	0.82	0.82	0.86	0.86	0.69	0.70	0.72	0.72	0.59	0.58	0.61	0.67	0.72	0.71	0.73	0.75
7	sl, cs, sr	0.75	0.75	0.81	0.81	0.64	0.56	0.66	0.63	0.58	0.49	0.57	0.51	0.66	0.61	0.69	0.65
8	sl, cs^{G7av}	0.67	0.67	0.68	0.68	0.57	0.58	0.55	0.56	0.51	0.52	0.52	0.53	0.59	0.59	0.60	0.60
9	sl, sr^{G7av}	0.74	0.74	0.73	0.73	0.65	0.58	0.64	0.61	0.52	0.54	0.56	0.54	0.64	0.62	0.64	0.63
10	sl, i, cs^{G7av}	0.74	0.74	0.74	0.74	0.68	0.69	0.66	0.70	0.59	0.65	0.58	0.63	0.68	0.70	0.68	0.70
11	sl, i, sr^{G7av}	0.80	0.80	0.83	0.83	0.65	0.75	0.69	0.71	0.60	0.62	0.57	0.69	0.70	0.73	0.70	0.75
12	sl, cs, sr^{G7av}	0.74	0.74	0.75	0.75	0.64	0.59	0.64	0.61	0.56	0.55	0.58	0.56	0.65	0.64	0.65	0.64
13	sl, cs^{G7av}, sr	0.75	0.75	0.80	0.80	0.61	0.59	0.66	0.65	0.55	0.49	0.56	0.54	0.64	0.62	0.69	0.67
14	sl, cs^{G7av}, sr^{G7av}	0.74	0.74	0.75	0.75	0.62	0.57	0.59	0.62	0.54	0.55	0.54	0.48	0.64	0.62	0.64	0.62

Table 5: Germany. In-sample forecasting performance (1960Q1 to 2009Q4) of several ProbVAR specifications: MAE (top panel) and R^2_{count} (bottom panel). sl: slope of the yield curve, i: short-term interest rate, cs: corporate bond spread, sr: year-on-year stock return, sr^{G7av}: average stock return over the G7 countries, cs^{G7av}: average corporate bond spread over Canada, Germany, the United Kingdom, and the United States. Shaded entries indicate the minimum MAE and the maximum R^2_{count} for the respective forecast horizon.

	Specification	Forecast horizon: 1q		Forecast horizon: 4 q		Forecast horizon: 6 q		Av. over forec. hor. 1 to 6 q	
		Additional probit lags:		Additional probit lags:		Additional probit lags:		Additional probit lags:	
		0	3	0	3	0	3	0	3
1	sl	0.35	0.34	0.36	0.36	0.39	0.38	0.36	0.36
2	sl, i	0.31	0.29	0.31	0.28	0.37	0.31	0.32	0.29
3	sl, cs	0.32	0.31	0.35	0.35	0.38	0.38	0.35	0.34
4	sl, sr	0.25	0.22	0.33	0.32	0.38	0.37	0.31	0.30
5	sl, i, cs	0.24	0.21	0.29	0.25	0.35	0.30	0.28	0.25
6	sl, i, sr	0.20	0.17	0.29	0.25	0.36	0.31	0.27	0.24
7	sl, cs, sr	0.25	0.21	0.33	0.32	0.38	0.37	0.31	0.30
8	sl, cs^{G7av}	0.33	0.32	0.36	0.35	0.38	0.38	0.35	0.35
9	sl, sr^{G7av}	0.26	0.24	0.35	0.34	0.39	0.38	0.33	0.32
10	sl, i, cs^{G7av}	0.27	0.26	0.31	0.28	0.36	0.31	0.31	0.28
11	sl, i, sr^{G7av}	0.22	0.19	0.30	0.26	0.37	0.30	0.28	0.25
12	sl, cs, sr^{G7av}	0.26	0.24	0.34	0.34	0.38	0.37	0.32	0.31
13	sl, cs^{G7av}, sr	0.25	0.20	0.32	0.31	0.37	0.36	0.30	0.28
14	sl, cs^{G7av}, sr^{G7av}	0.26	0.23	0.34	0.33	0.38	0.37	0.32	0.30

	Specification	Forecast horizon: 1q		Forecast horizon: 4 q		Forecast horizon: 6 q		Av. over forec. hor. 1 to 6 q	
		Additional probit lags:		Additional probit lags:		Additional probit lags:		Additional probit lags:	
		0	3	0	3	0	3	0	3
1	sl	0.49	0.51	0.46	0.47	0.41	0.41	0.47	0.49
2	sl, i	0.60	0.65	0.59	0.73	0.52	0.72	0.59	0.70
3	sl, cs	0.62	0.63	0.55	0.58	0.48	0.47	0.56	0.58
4	sl, sr	0.72	0.79	0.58	0.60	0.46	0.45	0.61	0.63
5	sl, i, cs	0.76	0.78	0.65	0.78	0.53	0.75	0.66	0.77
6	sl, i, sr	0.78	0.82	0.66	0.73	0.51	0.71	0.67	0.76
7	sl, cs, sr	0.72	0.78	0.58	0.62	0.47	0.51	0.61	0.64
8	sl, cs^{G7av}	0.55	0.61	0.47	0.53	0.43	0.44	0.49	0.55
9	sl, sr^{G7av}	0.68	0.68	0.54	0.56	0.41	0.46	0.56	0.58
10	sl, i, cs^{G7av}	0.72	0.74	0.60	0.73	0.54	0.72	0.63	0.73
11	sl, i, sr^{G7av}	0.76	0.81	0.65	0.74	0.52	0.70	0.66	0.77
12	sl, cs, sr^{G7av}	0.69	0.70	0.54	0.57	0.48	0.52	0.58	0.60
13	sl, cs^{G7av}, sr	0.72	0.78	0.61	0.63	0.49	0.49	0.63	0.65
14	sl, cs^{G7av}, sr^{G7av}	0.67	0.69	0.54	0.56	0.44	0.48	0.57	0.60

Table 6: Germany. In-sample forecasting performance (1960Q1 to 2009Q4) of several simple probit specifications: MAE (top panel) and R^2_{count} (bottom panel). sl: slope of the yield curve, i: short-term interest rate, cs: corporate bond spread, sr: year-on-year stock return, sr^{G7av}: average stock return over the G7 countries, cs^{G7av}: average corporate bond spread over Canada, Germany, the United Kingdom, and the United States. Shaded entries indicate the minimum MAE and the maximum R^2_{count} for the respective forecast horizon.

	Specification	Forecast horizon: 1q				Forecast horizon: 4 q				Forecast horizon: 6 q				Av. over forec. hor. 1 to 6 q			
		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4	
		VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)
1	*sl*	0.43	0.43	0.42	0.42	0.42	0.42	0.42	0.42	0.41	0.42	0.41	0.42	0.42	0.42	0.42	0.42
2	*sl, i*	0.31	0.31	0.30	0.30	0.31	0.33	0.31	0.31	0.32	0.35	0.32	0.33	0.31	0.33	0.30	0.31
3	*sl, cs*	0.35	0.35	0.34	0.34	0.41	0.42	0.37	0.38	0.41	0.42	0.41	0.42	0.39	0.40	0.37	0.37
4	*sl, sr*	0.22	0.22	0.19	0.19	0.40	0.42	0.35	0.38	0.41	0.44	0.41	0.43	0.35	0.37	0.31	0.33
5	*sl, i, cs*	0.21	0.21	0.20	0.20	0.31	0.33	0.29	0.30	0.32	0.35	0.31	0.34	0.28	0.30	0.27	0.27
6	*sl, i, sr*	0.18	0.18	0.16	0.16	0.32	0.34	0.31	0.32	0.33	0.38	0.32	0.35	0.28	0.30	0.27	0.28
7	*sl, cs, sr*	0.21	0.21	0.18	0.18	0.38	0.39	0.33	0.34	0.41	0.44	0.40	0.43	0.34	0.35	0.30	0.31
8	sl, cs^{G7av}	0.36	0.36	0.38	0.38	0.42	0.42	0.47	0.47	0.42	0.42	0.44	0.44	0.40	0.40	0.43	0.43
9	sl, sr^{G7av}	0.25	0.25	0.22	0.22	0.41	0.42	0.38	0.39	0.42	0.43	0.41	0.42	0.36	0.37	0.34	0.35
10	sl, i, cs^{G7av}	0.26	0.26	0.27	0.27	0.33	0.35	0.32	0.35	0.33	0.35	0.34	0.36	0.31	0.32	0.31	0.33
11	sl, i, sr^{G7av}	0.20	0.20	0.18	0.18	0.31	0.34	0.31	0.32	0.32	0.36	0.31	0.34	0.28	0.30	0.27	0.29
12	sl, cs, sr^{G7av}	0.23	0.23	0.22	0.22	0.39	0.41	0.34	0.36	0.42	0.44	0.41	0.43	0.35	0.36	0.32	0.33
13	sl, cs^{G7av}, sr	0.19	0.19	0.17	0.17	0.41	0.42	0.39	0.40	0.42	0.45	0.43	0.45	0.35	0.36	0.32	0.34
14	sl, cs^{G7av}, sr^{G7av}	0.24	0.24	0.23	0.23	0.42	0.43	0.42	0.43	0.43	0.44	0.43	0.44	0.37	0.38	0.36	0.37

	Specification	Forecast horizon: 1q				Forecast horizon: 4 q				Forecast horizon: 6 q				Av. over forec. hor. 1 to 6 q			
		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4	
		VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)
1	*sl*	0.44	0.44	0.44	0.44	0.36	0.33	0.36	0.36	0.33	0.33	0.33	0.33	0.38	0.37	0.38	0.38
2	*sl, i*	0.69	0.69	0.73	0.73	0.71	0.69	0.71	0.69	0.69	0.75	0.71	0.75	0.71	0.71	0.71	0.72
3	*sl, cs*	0.47	0.47	0.51	0.51	0.47	0.44	0.51	0.51	0.38	0.36	0.45	0.42	0.45	0.43	0.50	0.49
4	*sl, sr*	0.75	0.75	0.85	0.85	0.47	0.49	0.58	0.60	0.44	0.36	0.45	0.44	0.55	0.52	0.65	0.62
5	*sl, i, cs*	0.80	0.80	0.82	0.82	0.73	0.76	0.73	0.75	0.71	0.67	0.69	0.62	0.75	0.75	0.75	0.75
6	*sl, i, sr*	0.91	0.91	0.87	0.87	0.64	0.62	0.71	0.71	0.65	0.58	0.64	0.58	0.74	0.71	0.74	0.72
7	*sl, cs, sr*	0.71	0.71	0.76	0.76	0.51	0.53	0.65	0.62	0.45	0.45	0.47	0.45	0.56	0.56	0.65	0.63
8	sl, cs^{G7av}	0.47	0.47	0.53	0.53	0.38	0.44	0.24	0.29	0.33	0.40	0.29	0.25	0.40	0.44	0.35	0.37
9	sl, sr^{G7av}	0.67	0.67	0.69	0.69	0.49	0.45	0.53	0.56	0.33	0.31	0.40	0.35	0.50	0.50	0.56	0.55
10	sl, i, cs^{G7av}	0.75	0.75	0.75	0.75	0.69	0.75	0.65	0.69	0.67	0.67	0.67	0.71	0.71	0.72	0.71	0.72
11	sl, i, sr^{G7av}	0.85	0.85	0.89	0.89	0.69	0.64	0.67	0.64	0.71	0.65	0.69	0.69	0.74	0.71	0.75	0.74
12	sl, cs, sr^{G7av}	0.65	0.65	0.71	0.71	0.51	0.51	0.60	0.60	0.40	0.33	0.45	0.49	0.53	0.51	0.61	0.61
13	sl, cs^{G7av}, sr	0.73	0.73	0.80	0.80	0.51	0.51	0.53	0.56	0.38	0.31	0.36	0.33	0.55	0.52	0.59	0.58
14	sl, cs^{G7av}, sr^{G7av}	0.71	0.71	0.75	0.75	0.49	0.42	0.44	0.40	0.29	0.27	0.25	0.35	0.49	0.48	0.50	0.50

Table 7: Germany. Out-of-sample forecasting performance (1995Q1 to 2009Q4) of several ProbVAR specifications: MAE error (top panel) and R^2_{count} (bottom panel). sl: slope of the yield curve, i: short-term interest rate, cs: corporate bond spread, sr: year-on-year stock return, sr^{G7av}: average stock return over the G7 countries, cs^{G7av}: average corporate bond spread over Canada, Germany, the United Kingdom, and the United States. Shaded entries indicate the minimum MAE and the maximum R^2_{count} for the respective forecast horizon.

	Specification	Forecast horizon: 1q		Forecast horizon: 4 q		Forecast horizon: 6 q		Av. over forec. hor. 1 to 6 q	
		Additional probit lags:		Additional probit lags:		Additional probit lags:		Additional probit lags:	
		0	3	0	3	0	3	0	3
1	sl	0.43	0.42	0.42	0.42	0.42	0.42	0.42	0.42
2	sl, i	0.31	0.30	0.31	0.32	0.35	0.35	0.31	0.32
3	sl, cs	0.35	0.34	0.38	0.38	0.41	0.44	0.37	0.38
4	sl, sr	0.22	0.19	0.37	0.38	0.41	0.42	0.32	0.32
5	sl, i, cs	0.21	0.20	0.30	0.29	0.36	0.36	0.28	0.28
6	sl, i, sr	0.18	0.16	0.31	0.33	0.36	0.35	0.28	0.28
7	sl, cs, sr	0.21	0.18	0.33	0.34	0.40	0.42	0.30	0.31
8	sl, cs^{G7av}	0.36	0.38	0.46	0.47	0.46	0.46	0.43	0.44
9	sl, sr^{G7av}	0.25	0.22	0.39	0.40	0.42	0.42	0.35	0.35
10	sl, i, cs^{G7av}	0.26	0.27	0.32	0.36	0.37	0.40	0.32	0.34
11	sl, i, sr^{G7av}	0.20	0.18	0.32	0.33	0.35	0.34	0.29	0.29
12	sl, cs, sr^{G7av}	0.23	0.22	0.35	0.36	0.41	0.43	0.32	0.33
13	sl, cs^{G7av}, sr	0.19	0.17	0.39	0.41	0.44	0.45	0.33	0.34
14	sl, cs^{G7av}, sr^{G7av}	0.24	0.23	0.42	0.44	0.45	0.46	0.36	0.37

	Specification	Forecast horizon: 1q		Forecast horizon: 4 q		Forecast horizon: 6 q		Av. over forec. hor. 1 to 6 q	
		Additional probit lags:		Additional probit lags:		Additional probit lags:		Additional probit lags:	
		0	3	0	3	0	3	0	3
1	sl	0.36	0.36	0.27	0.33	0.27	0.29	0.31	0.32
2	sl, i	0.67	0.71	0.69	0.65	0.69	0.60	0.69	0.65
3	sl, cs	0.45	0.49	0.51	0.47	0.44	0.35	0.47	0.45
4	sl, sr	0.69	0.82	0.55	0.51	0.38	0.38	0.57	0.58
5	sl, i, cs	0.80	0.84	0.75	0.78	0.58	0.67	0.73	0.76
6	sl, i, sr	0.91	0.87	0.69	0.64	0.56	0.62	0.74	0.72
7	sl, cs, sr	0.67	0.73	0.58	0.55	0.49	0.44	0.61	0.60
8	sl, cs^{G7av}	0.40	0.49	0.25	0.29	0.29	0.24	0.31	0.36
9	sl, sr^{G7av}	0.60	0.65	0.47	0.44	0.27	0.29	0.46	0.48
10	sl, i, cs^{G7av}	0.78	0.75	0.67	0.64	0.58	0.42	0.69	0.62
11	sl, i, sr^{G7av}	0.87	0.89	0.67	0.58	0.69	0.64	0.77	0.72
12	sl, cs, sr^{G7av}	0.58	0.62	0.49	0.53	0.44	0.36	0.52	0.53
13	sl, cs^{G7av}, sr	0.69	0.76	0.51	0.47	0.25	0.20	0.51	0.52
14	sl, cs^{G7av}, sr^{G7av}	0.64	0.64	0.35	0.38	0.27	0.33	0.44	0.45

Table 8: Germany. Out-of-sample forecasting performance (1995Q1 to 2009Q4) of several simple probit specifications: MAE error (top panel) and R^2_{count} (bottom panel). sl: slope of the yield curve, i: short-term interest rate, cs: corporate bond spread, sr: year-on-year stock return, sr^{G7av}: average stock return over the G7 countries, cs^{G7av}: average corporate bond spread over Canada, Germany, the United Kingdom, and the United States. Shaded entries indicate the minimum MAE and the maximum R^2_{count} for the respective forecast horizon.

	Specification	Forecast horizon: 1q				Forecast horizon: 4 q				Forecast horizon: 6 q				Av. over forec. hor. 1 to 6 q			
		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4	
		VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)
1	sl	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33
2	sl, i	0.28	0.28	0.28	0.28	0.29	0.29	0.28	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.28	0.29
3	sl, cs																
4	sl, sr	0.23	0.23	0.22	0.22	0.31	0.32	0.30	0.31	0.32	0.34	0.32	0.33	0.29	0.30	0.29	0.29
5	sl, i, cs																
6	sl, i, sr	0.19	0.19	0.19	0.19	0.27	0.28	0.27	0.27	0.29	0.30	0.29	0.29	0.25	0.26	0.25	0.26
7	sl, cs, sr																
8	sl, cs^{G7av}	0.30	0.30	0.28	0.28	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.32	0.32	0.32	0.32
9	sl, sr^{G7av}	0.29	0.29	0.28	0.28	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.32	0.32	0.32	0.32
10	sl, i, cs^{G7av}	0.26	0.26	0.24	0.24	0.29	0.29	0.29	0.29	0.29	0.30	0.29	0.29	0.28	0.28	0.28	0.28
11	sl, i, sr^{G7av}	0.24	0.24	0.23	0.23	0.28	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.27	0.27	0.27	0.27
12	sl, cs, sr^{G7av}																
13	sl, cs^{G7av}, sr	0.23	0.23	0.21	0.21	0.31	0.31	0.30	0.31	0.32	0.33	0.32	0.33	0.29	0.29	0.28	0.29
14	sl, cs^{G7av}, sr^{G7av}	0.29	0.29	0.27	0.27	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.32	0.32	0.31	0.31

	Specification	Forecast horizon: 1q				Forecast horizon: 4 q				Forecast horizon: 6 q				Av. over forec. hor. 1 to 6 q			
		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4	
		VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)
1	sl	0.54	0.54	0.54	0.54	0.53	0.52	0.54	0.54	0.51	0.51	0.53	0.52	0.53	0.53	0.53	0.53
2	sl, i	0.74	0.74	0.73	0.73	0.72	0.74	0.71	0.72	0.70	0.72	0.69	0.71	0.72	0.73	0.71	0.72
3	sl, cs																
4	sl, sr	0.80	0.80	0.82	0.82	0.67	0.61	0.68	0.64	0.60	0.53	0.61	0.52	0.69	0.65	0.70	0.67
5	sl, i, cs																
6	sl, i, sr	0.82	0.82	0.81	0.81	0.75	0.74	0.75	0.74	0.76	0.72	0.76	0.71	0.77	0.76	0.77	0.74
7	sl, cs, sr																
8	sl, cs^{G7av}	0.64	0.64	0.67	0.67	0.50	0.49	0.54	0.54	0.46	0.48	0.52	0.51	0.54	0.53	0.56	0.56
9	sl, sr^{G7av}	0.68	0.68	0.68	0.68	0.54	0.54	0.51	0.54	0.50	0.58	0.51	0.56	0.58	0.61	0.56	0.59
10	sl, i, cs^{G7av}	0.76	0.76	0.78	0.78	0.72	0.73	0.68	0.68	0.72	0.72	0.66	0.66	0.73	0.73	0.70	0.70
11	sl, i, sr^{G7av}	0.76	0.76	0.75	0.75	0.75	0.74	0.73	0.72	0.72	0.70	0.72	0.69	0.74	0.74	0.73	0.72
12	sl, cs, sr^{G7av}																
13	sl, cs^{G7av}, sr	0.82	0.82	0.82	0.82	0.65	0.62	0.67	0.64	0.59	0.53	0.59	0.52	0.69	0.66	0.69	0.66
14	sl, cs^{G7av}, sr^{G7av}	0.71	0.71	0.73	0.73	0.55	0.55	0.53	0.54	0.48	0.54	0.51	0.56	0.59	0.60	0.57	0.60

Table 9: Japan. In-sample forecasting performance (1960Q1 to 2009Q4) of several ProbVAR specifications: MAE (top panel) and R^2_{count} (bottom panel). sl: slope of the yield curve, i: short-term interest rate, cs: corporate bond spread, sr: year-on-year stock return, sr^{G7av}: average stock return over the G7 countries, cs^{G7av}: average corporate bond spread over Canada, Germany, the United Kingdom, and the United States. Shaded entries indicate the minimum MAE and the maximum R^2_{count} for the respective forecast horizon.

	Specification	Forecast horizon: 1q		Forecast horizon: 4 q		Forecast horizon: 6 q		Av. over forec. hor. 1 to 6 q	
		Additional probit lags:		Additional probit lags:		Additional probit lags:		Additional probit lags:	
		0	3	0	3	0	3	0	3
1	sl	0.33	0.33	0.33	0.32	0.33	0.32	0.33	0.32
2	sl, i	0.28	0.28	0.28	0.28	0.29	0.28	0.28	0.28
3	sl, cs								
4	sl, sr	0.23	0.22	0.31	0.30	0.32	0.32	0.29	0.28
5	sl, i, cs								
6	sl, i, sr	0.19	0.19	0.27	0.26	0.28	0.27	0.25	0.25
7	sl, cs, sr								
8	sl, cs^{G7av}	0.30	0.28	0.33	0.32	0.32	0.31	0.32	0.31
9	sl, sr^{G7av}	0.29	0.28	0.33	0.32	0.33	0.32	0.32	0.31
10	sl, i, cs^{G7av}	0.26	0.24	0.28	0.26	0.28	0.26	0.28	0.26
11	sl, i, sr^{G7av}	0.24	0.23	0.28	0.28	0.29	0.28	0.27	0.27
12	sl, cs, sr^{G7av}								
13	sl, cs^{G7av}, sr	0.23	0.21	0.30	0.29	0.32	0.31	0.29	0.27
14	sl, cs^{G7av}, sr^{G7av}	0.29	0.27	0.33	0.31	0.32	0.31	0.32	0.30

	Specification	Forecast horizon: 1q		Forecast horizon: 4 q		Forecast horizon: 6 q		Av. over forec. hor. 1 to 6 q	
		Additional probit lags:		Additional probit lags:		Additional probit lags:		Additional probit lags:	
		0	3	0	3	0	3	0	3
1	sl	0.54	0.54	0.54	0.53	0.54	0.53	0.54	0.53
2	sl, i	0.74	0.73	0.69	0.72	0.69	0.70	0.71	0.72
3	sl, cs								
4	sl, sr	0.80	0.82	0.63	0.65	0.54	0.53	0.67	0.67
5	sl, i, cs								
6	sl, i, sr	0.82	0.81	0.76	0.73	0.71	0.71	0.76	0.75
7	sl, cs, sr								
8	sl, cs^{G7av}	0.64	0.67	0.53	0.56	0.54	0.59	0.56	0.59
9	sl, sr^{G7av}	0.68	0.68	0.54	0.55	0.53	0.53	0.57	0.58
10	sl, i, cs^{G7av}	0.76	0.78	0.69	0.72	0.68	0.72	0.71	0.74
11	sl, i, sr^{G7av}	0.76	0.75	0.72	0.72	0.68	0.71	0.72	0.73
12	sl, cs, sr^{G7av}								
13	sl, cs^{G7av}, sr	0.82	0.82	0.63	0.67	0.58	0.62	0.68	0.70
14	sl, cs^{G7av}, sr^{G7av}	0.71	0.73	0.54	0.57	0.54	0.61	0.58	0.61

Table 10: Japan. In-sample forecasting performance (1960Q1 to 2009Q4) of several simple probit specifications: MAE (top panel) and R^2_{count} (bottom panel). sl: slope of the yield curve, i: short-term interest rate, cs: corporate bond spread, sr: year-on-year stock return, sr^{G7av}: average stock return over the G7 countries, cs^{G7av}: average corporate bond spread over Canada, Germany, the United Kingdom, and the United States. Shaded entries indicate the minimum MAE and the maximum R^2_{count} for the respective forecast horizon.

	Specification	Forecast horizon: 1q				Forecast horizon: 4 q				Forecast horizon: 6 q				Av. over forec. hor. 1 to 6 q			
		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4	
		VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)
1	sl	0.51	0.51	0.53	0.53	0.53	0.53	0.53	0.53	0.54	0.54	0.54	0.54	0.53	0.53	0.53	0.53
2	sl, i	0.50	0.50	0.52	0.52	0.55	0.55	0.54	0.55	0.56	0.56	0.56	0.57	0.54	0.54	0.54	0.55
3	sl, cs																
4	sl, sr	0.35	0.35	0.36	0.36	0.51	0.52	0.50	0.52	0.53	0.53	0.53	0.54	0.47	0.48	0.47	0.48
5	sl, i, cs																
6	sl, i, sr	0.28	0.28	0.28	0.28	0.51	0.52	0.50	0.51	0.55	0.56	0.55	0.56	0.45	0.46	0.45	0.46
7	sl, cs, sr																
8	sl, cs^{G7av}	0.45	0.45	0.41	0.41	0.55	0.55	0.56	0.56	0.56	0.56	0.55	0.55	0.53	0.53	0.51	0.51
9	sl, sr^{G7av}	0.45	0.45	0.45	0.45	0.54	0.53	0.55	0.55	0.54	0.53	0.55	0.54	0.52	0.51	0.53	0.52
10	sl, i, cs^{G7av}	0.41	0.41	0.37	0.37	0.54	0.55	0.56	0.56	0.58	0.59	0.57	0.57	0.51	0.52	0.50	0.50
11	sl, i, sr^{G7av}	0.39	0.39	0.41	0.41	0.56	0.56	0.57	0.57	0.57	0.58	0.58	0.59	0.51	0.52	0.53	0.53
12	sl, cs, sr^{G7av}																
13	sl, cs^{G7av}, sr	0.32	0.32	0.29	0.29	0.52	0.53	0.52	0.53	0.54	0.54	0.54	0.53	0.47	0.47	0.46	0.47
14	sl, cs^{G7av}, sr^{G7av}	0.42	0.42	0.40	0.40	0.55	0.55	0.56	0.57	0.56	0.55	0.55	0.55	0.52	0.52	0.52	0.51

	Specification	Forecast horizon: 1q				Forecast horizon: 4 q				Forecast horizon: 6 q				Av. over forec. hor. 1 to 6 q			
		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4		Probit Lags:1		Probit lags: 1 to 4	
		VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)	VAR(1)	VAR(4)
1	sl	0.38	0.38	0.36	0.36	0.33	0.33	0.35	0.35	0.29	0.29	0.27	0.27	0.34	0.34	0.32	0.32
2	sl, i	0.55	0.55	0.47	0.47	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.40	0.47	0.47	0.45	0.44
3	sl, cs																
4	sl, sr	0.80	0.80	0.78	0.78	0.49	0.47	0.49	0.51	0.36	0.31	0.36	0.36	0.58	0.55	0.55	0.54
5	sl, i, cs																
6	sl, i, sr	0.71	0.71	0.67	0.67	0.45	0.47	0.47	0.47	0.45	0.44	0.45	0.47	0.53	0.53	0.52	0.51
7	sl, cs, sr																
8	sl, cs^{G7av}	0.55	0.55	0.60	0.60	0.40	0.40	0.29	0.31	0.31	0.35	0.24	0.24	0.42	0.42	0.38	0.38
9	sl, sr^{G7av}	0.60	0.60	0.62	0.62	0.44	0.40	0.38	0.33	0.36	0.35	0.27	0.33	0.47	0.45	0.41	0.42
10	sl, i, cs^{G7av}	0.51	0.51	0.53	0.53	0.45	0.44	0.45	0.44	0.42	0.38	0.45	0.44	0.46	0.45	0.46	0.45
11	sl, i, sr^{G7av}	0.49	0.49	0.44	0.44	0.44	0.40	0.44	0.38	0.44	0.42	0.44	0.38	0.46	0.43	0.43	0.40
12	sl, cs, sr^{G7av}																
13	sl, cs^{G7av}, sr	0.78	0.78	0.82	0.82	0.55	0.47	0.53	0.47	0.36	0.27	0.35	0.31	0.58	0.53	0.58	0.55
14	sl, cs^{G7av}, sr^{G7av}	0.60	0.60	0.65	0.65	0.44	0.42	0.27	0.29	0.36	0.35	0.25	0.24	0.48	0.45	0.40	0.40

Table 11: Japan. Out-of-sample forecasting performance (1995Q1 to 2009Q4) of several ProbVAR specifications: MAE error (top panel) and R^2_{count} (bottom panel). sl: slope of the yield curve, i: short-term interest rate, cs: corporate bond spread, sr: year-on-year stock return, sr^{G7av}: average stock return over the G7 countries, cs^{G7av}: average corporate bond spread over Canada, Germany, the United Kingdom, and the United States. Shaded entries indicate the minimum MAE and the maximum R^2_{count} for the respective forecast horizon.

	Specification	Forecast horizon: 1q		Forecast horizon: 4 q		Forecast horizon: 6 q		Av. over forec. hor. 1 to 6 q	
		Additional probit lags:		Additional probit lags:		Additional probit lags:		Additional probit lags:	
		0	3	0	3	0	3	0	3
1	*sl*	0.51	0.53	0.53	0.55	0.55	0.55	0.53	0.55
2	*sl, i*	0.50	0.52	0.55	0.57	0.58	0.60	0.54	0.56
3	*sl, cs*								
4	*sl, sr*	0.35	0.36	0.52	0.54	0.55	0.56	0.48	0.49
5	*sl, i, cs*								
6	*sl, i, sr*	0.28	0.28	0.52	0.55	0.58	0.60	0.46	0.48
7	*sl, cs, sr*								
8	sl, cs^{G7av}	0.45	0.41	0.55	0.56	0.55	0.55	0.52	0.52
9	sl, sr^{G7av}	0.45	0.45	0.55	0.57	0.55	0.56	0.52	0.54
10	sl, i, cs^{G7av}	0.41	0.37	0.56	0.57	0.59	0.63	0.52	0.52
11	sl, i, sr^{G7av}	0.39	0.41	0.56	0.58	0.59	0.61	0.52	0.54
12	sl, cs, sr^{G7av}								
13	sl, cs^{G7av}, sr	0.32	0.29	0.53	0.54	0.54	0.56	0.47	0.48
14	sl, cs^{G7av}, sr^{G7av}	0.42	0.40	0.56	0.56	0.55	0.56	0.52	0.52

	Specification	Forecast horizon: 1q		Forecast horizon: 4 q		Forecast horizon: 6 q		Av. over forec. hor. 1 to 6 q	
		Additional probit lags:		Additional probit lags:		Additional probit lags:		Additional probit lags:	
		0	3	0	3	0	3	0	3
1	*sl*	0.38	0.36	0.35	0.31	0.31	0.33	0.35	0.32
2	*sl, i*	0.55	0.47	0.45	0.42	0.44	0.38	0.47	0.42
3	*sl, cs*								
4	*sl, sr*	0.80	0.78	0.47	0.49	0.38	0.27	0.56	0.52
5	*sl, i, cs*								
6	*sl, i, sr*	0.71	0.67	0.44	0.44	0.42	0.38	0.51	0.47
7	*sl, cs, sr*								
8	sl, cs^{G7av}	0.55	0.60	0.31	0.25	0.31	0.35	0.38	0.39
9	sl, sr^{G7av}	0.60	0.62	0.31	0.24	0.33	0.31	0.41	0.37
10	sl, i, cs^{G7av}	0.51	0.53	0.47	0.42	0.44	0.35	0.47	0.43
11	sl, i, sr^{G7av}	0.49	0.44	0.42	0.40	0.36	0.36	0.43	0.40
12	sl, cs, sr^{G7av}								
13	sl, cs^{G7av}, sr	0.78	0.82	0.51	0.38	0.29	0.25	0.54	0.50
14	sl, cs^{G7av}, sr^{G7av}	0.60	0.65	0.25	0.25	0.27	0.33	0.39	0.39

Table 12: Japan. Out-of-sample forecasting performance (1995Q1 to 2009Q4) of several simple probit specifications: MAE error (top panel) and R^2_{count} (bottom panel). sl: slope of the yield curve, i: short-term interest rate, cs: corporate bond spread, sr: year-on-year stock return, sr^{G7av}: average stock return over the G7 countries, cs^{G7av}: average corporate bond spread over Canada, Germany, the United Kingdom, and the United States. Shaded entries indicate the minimum MAE and the maximum R^2_{count} for the respective forecast horizon.

		ProbVAR(1,1,0)		ProbVAR(4,1,0)		ProbVAR(1,1,3)		ProbVAR(4,1,3)	
	forecast horizon →	1	4	1	4	1	4	1	4
1	*sl*	0.08	0.00	0.08	0.00	0.01	-0.05	0.01	-0.05
2	*sl, i*	0.03	-0.04	0.03	-0.04	-0.04	-0.07	-0.04	-0.07
3	*sl, cs*	0.05	-0.01	0.05	-0.01	-0.06	-0.05	-0.06	-0.05
4	*sl, sr*	-0.04	-0.02	-0.04	-0.01	-0.12	-0.05	-0.12	-0.01
5	*sl, i, cs*	0.00	-0.05	0.00	-0.06	-0.09	-0.07	-0.09	-0.06
6	*sl, i, sr*								
7	*sl, cs, sr*								
8	*sl, cs^{G7ave}*	-0.03	-0.04	-0.03	-0.05	-0.10	-0.09	-0.10	-0.09
9	*sl, sr^{G7ave}*	-0.03	-0.01	-0.03	-0.02	-0.08	-0.03	-0.08	-0.02
10	*sl, i, cs^{G7ave}*	-0.08	-0.09	-0.08	-0.10	-0.12	-0.09	-0.12	-0.09
11	*sl, i, sr^{G7ave}*								
12	*sl, cs, sr^{G7ave}*	-0.03	-0.01	-0.03	-0.02	-0.09	-0.04	-0.09	-0.02
13	*sl, cs^{G7ave}, sr*	-0.06	-0.03	-0.06	-0.04	-0.14	-0.10	-0.14	-0.09
14	*sl, cs^{G7ave}, sr^{G7ave}*								

		ProbVAR(1,1,0)		ProbVAR(4,1,0)		ProbVAR(1,1,3)		ProbVAR(4,1,3)	
	forecast horizon →	1	4	1	4	1	4	1	4
1	*sl*	2.92	-0.32	2.92	-0.17	0.20	-1.56	0.20	-1.54
2	*sl, i*	0.86	-2.03	0.86	-1.96	-0.90	-2.62	-0.90	-2.57
3	*sl, cs*	1.86	-0.72	1.86	-0.88	-1.26	-1.65	-1.26	-1.50
4	*sl, sr*	-1.31	-1.04	-1.31	-0.48	-3.91	-2.04	-3.91	-0.33
5	*sl, i, cs*	-0.04	-2.78	-0.04	-3.06	-2.75	-2.58	-2.75	-2.60
6	*sl, i, sr*								
7	*sl, cs, sr*								
8	*sl, cs^{G7ave}*	-0.67	-1.47	-0.67	-1.62	-2.23	-3.01	-2.23	-2.91
9	*sl, sr^{G7ave}*	-1.15	-0.71	-1.15	-1.52	-1.68	-1.13	-1.68	-0.44
10	*sl, i, cs^{G7ave}*	-2.88	-4.01	-2.88	-4.39	-4.00	-4.33	-4.00	-4.47
11	*sl, i, sr^{G7ave}*								
12	*sl, cs, sr^{G7ave}*	-0.56	-0.68	-0.56	-1.54	-1.43	-1.36	-1.43	-0.41
13	*sl, cs^{G7ave}, sr*	-1.12	-0.92	-1.12	-1.21	-3.00	-2.40	-3.00	-1.54
14	*sl, cs^{G7ave}, sr^{G7ave}*								

Table 13: United States. Out-of-sample recession probabilities based on expanding-window estimation implied by selected ProbVAR specifications vs. probability of GDP decline h quarters ahead as expressed by Survey of Professional Forecasters (SPF). Forecast horizons are 1 quarter and 4 quarters. Forecast periods are from 1996Q2 to 2009Q4. The top panel displays the difference between mean absolute forecast error (MAE) of the respective ProbVAR specification and the SPF. Cells with negative numbers, indicating superior ProbVAR forecasts, are shaded in green. The bottom panel displays t-statistics of Diebold-Mariano tests on forecast superiority, t-values smaller than -1.65 (ProbVAR beats SPF) shaded in green, larger than 1.65 (SPF beats ProbVAR) shaded in red. The t-statistics are based on regressing the difference of the MAEs between the two models on a constant. The t-statistic of the estimate of this constant uses the Newey-West correction with the number of lags corresponding to the forecast horizon.

B Figures

B.1 United States: In- and out-of-sample fit

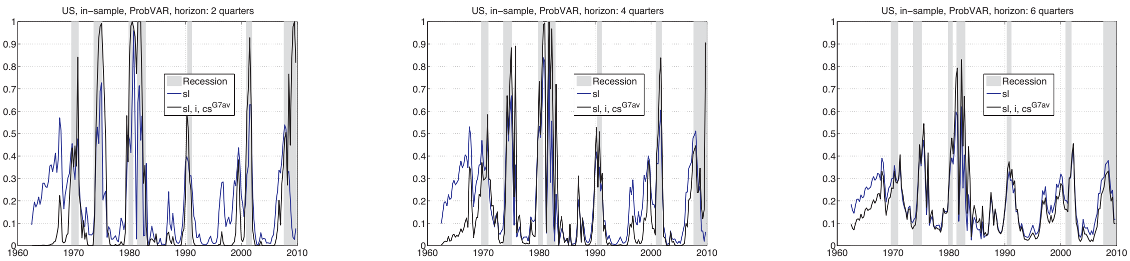


Figure 1: United States. In-sample fit (model-implied conditional probabilities vs. recessions) of selected ProbVAR specifications for prediction horizons of 2, 4 and 6 quarters.

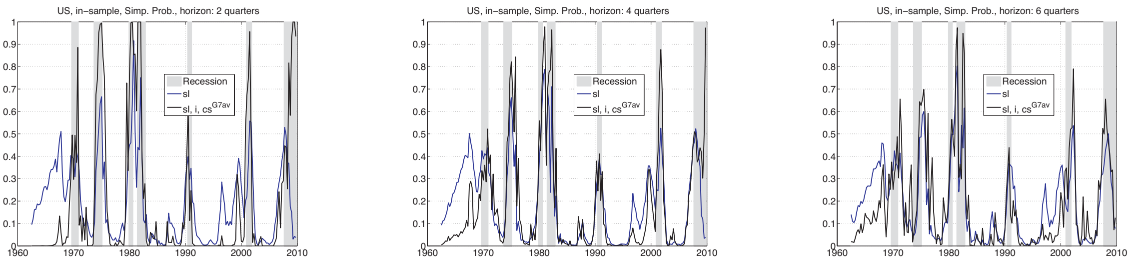


Figure 2: United States. In-sample fit (model-implied conditional probabilities vs. recessions) of selected simple probit specifications for prediction horizons of 2, 4 and 6 quarters.

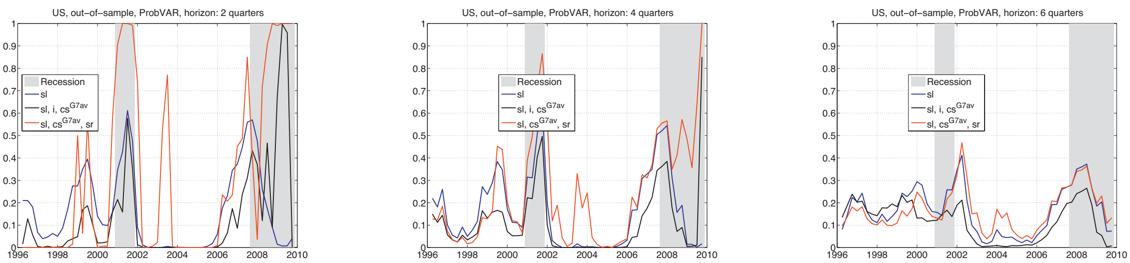


Figure 3: United States. Out-of-sample fit (model-implied conditional probabilities vs. recessions) of selected ProbVAR specifications for prediction horizons of 2, 4 and 6 quarters.

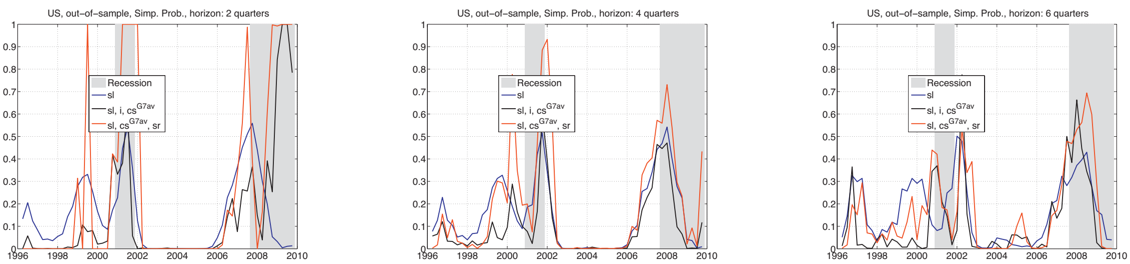


Figure 4: United States. Out-of-sample fit (model-implied conditional probabilities vs. recessions) of selected simple probit specifications for prediction horizons of 2, 4 and 6 quarters.

B.2 Germany: In- and out-of-sample fit

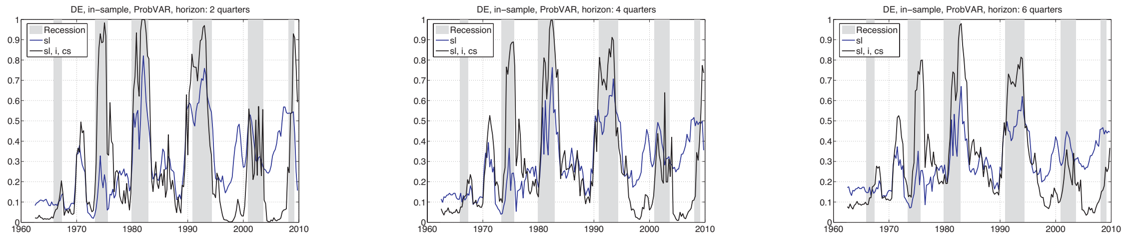


Figure 5: Germany. In-sample fit (model-implied conditional probabilities vs. recessions) of selected ProbVAR specifications for prediction horizons of 2, 4 and 6 quarters.

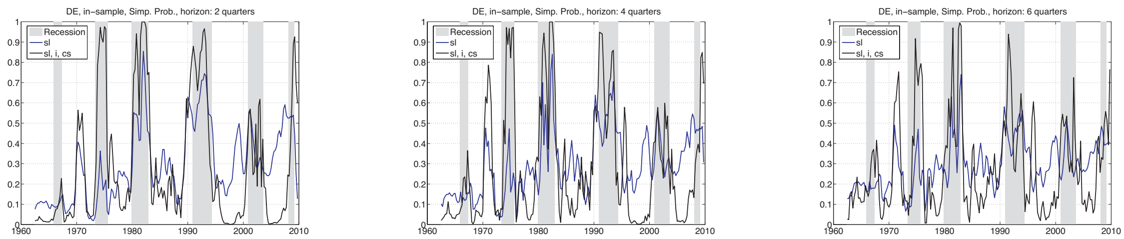


Figure 6: Germany. In-sample fit (model-implied conditional probabilities vs. recessions) of selected simple probit specifications for prediction horizons of 2, 4 and 6 quarters.

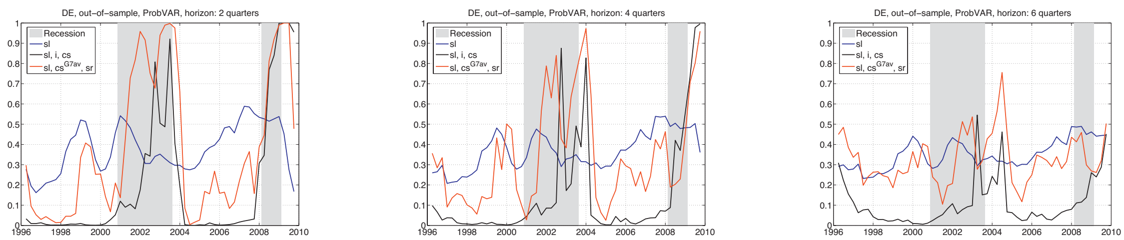


Figure 7: Germany. Out-of-sample fit (model-implied conditional probabilities vs. recessions) of selected ProbVAR specifications for prediction horizons of 2, 4 and 6 quarters.

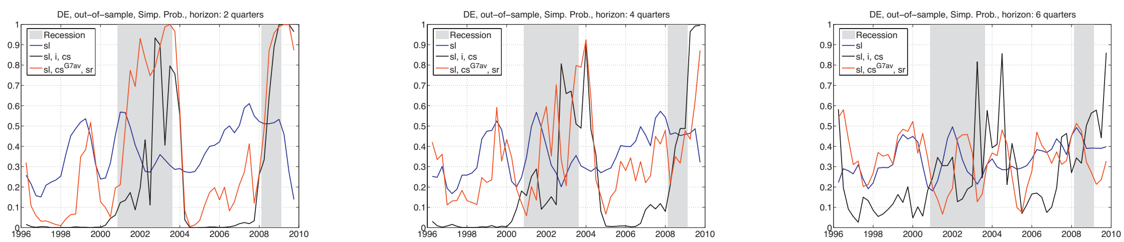


Figure 8: Germany. Out-of-sample fit (model-implied conditional probabilities vs. recessions) of selected simple probit specifications for prediction horizons of 2, 4 and 6 quarters.

B.3 Japan: In- and out-of-sample fit

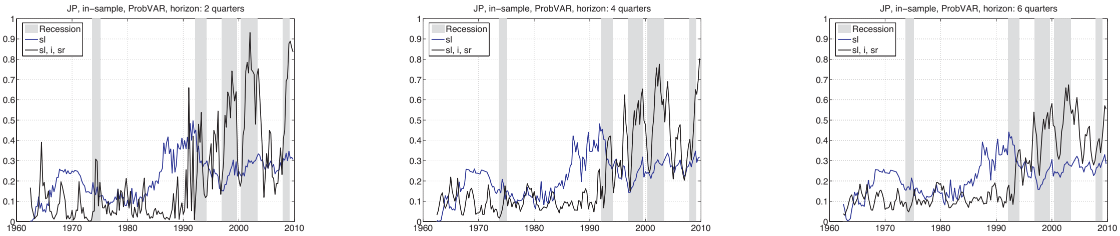


Figure 9: Japan. In-sample fit (model-implied conditional probabilities vs. recessions) of selected ProbVAR specifications for prediction horizons of 2, 4 and 6 quarters.

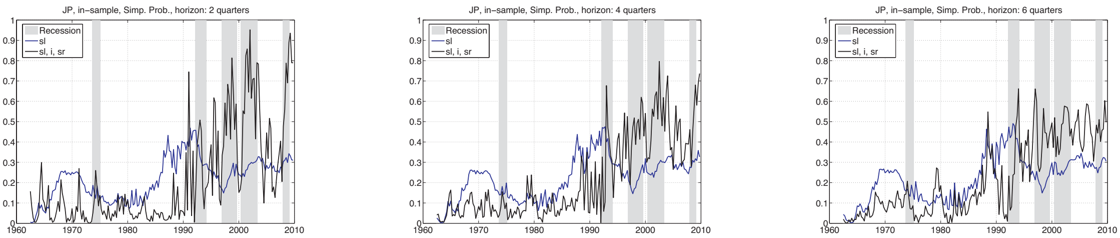


Figure 10: Japan. In-sample fit (model-implied conditional probabilities vs. recessions) of selected simple probit specifications for prediction horizons of 2, 4 and 6 quarters.

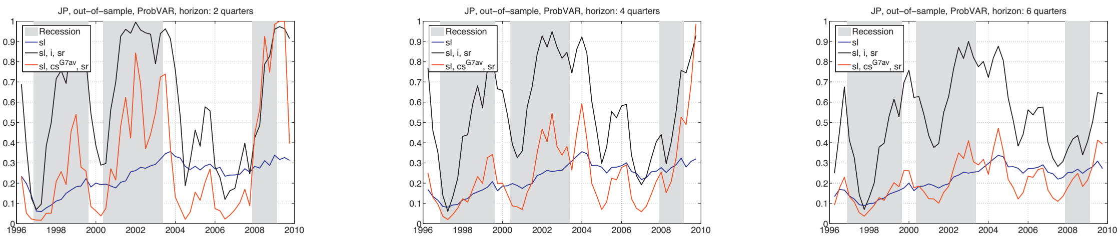


Figure 11: Japan. Out-of-sample fit (model-implied conditional probabilities vs. recessions) of selected ProbVAR specifications for prediction horizons of 2, 4 and 6 quarters.

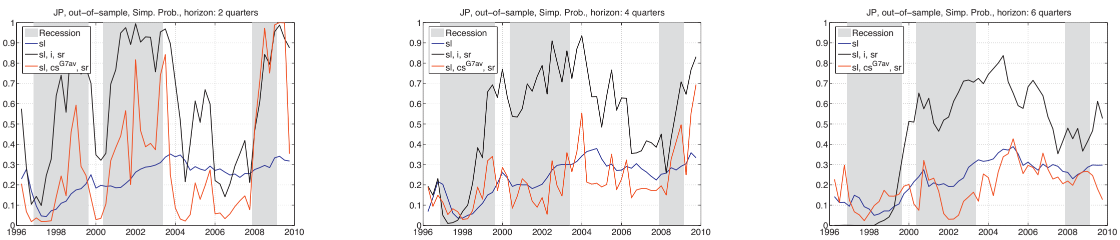
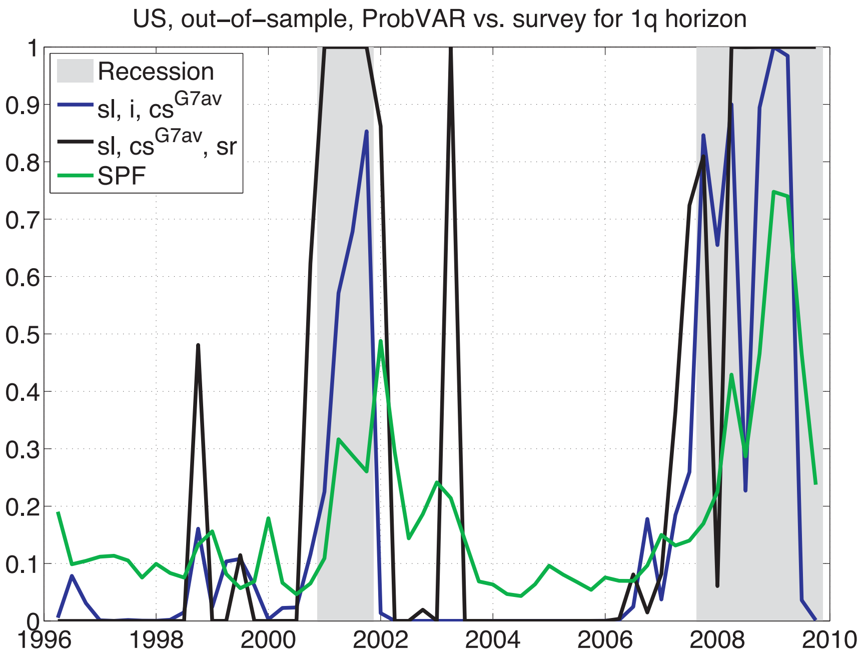


Figure 12: Japan. Out-of-sample fit (model-implied conditional probabilities vs. recessions) of selected simple probit specifications for prediction horizons of 2, 4 and 6 quarters.

B.4 United States: ProbVAR vs. Survey of Professional Forecasters



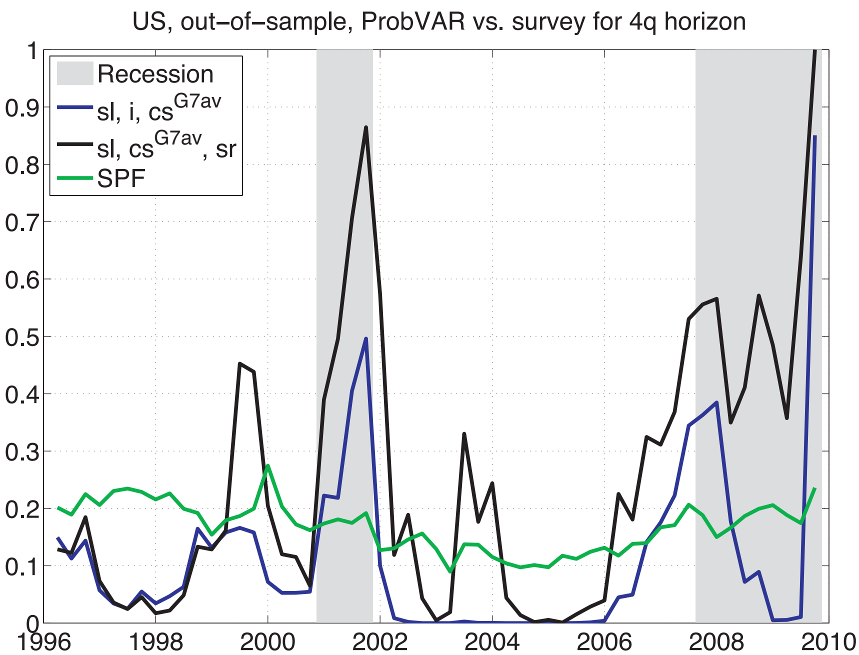


Figure 13: United States. Out-of-sample recession probabilities implied by selected ProbVAR(1,1,3) specifications vs. probability of GDP decline h quarters ahead as expressed by Survey of Professional Forecasters (SPF). Forecast horizons are 1 quarter (top panel) and 4 quarters (bottom panel). All recession probabilities refer to the probability of recession in the respective quarter, i.e. they are conditional on information dated h periods before.

B.5 United States: Zooming in on the recession that started in 2001

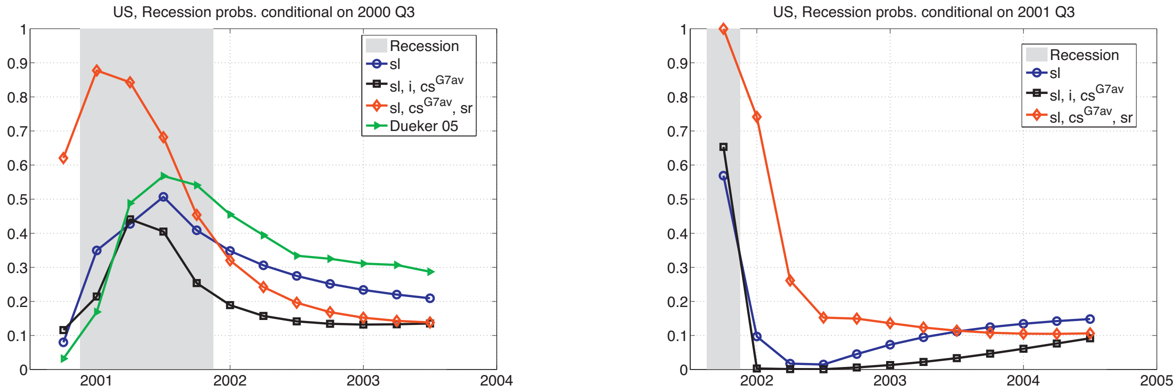


Figure 14: United States. Recession probabilities for 1, 2, ..., 12 quarters ahead as implied by selected ProbVAR specifications. Parameters estimated based on sample ending in 2000Q3. Prediction based on regressors observed in 2000Q3 (left) and 2001Q3 (right). The green line with triangles are the out-of-sample recession probabilities obtained by Dueker (2005), table 2, also based on 2000Q3 information.

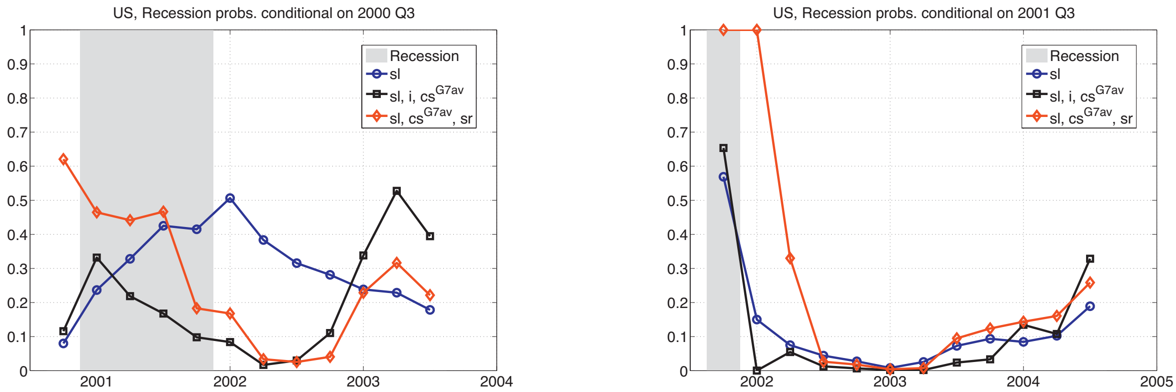


Figure 15: United States. Recession probabilities for 1, 2, ..., 12 quarters ahead as implied by selected simple probit specifications. Parameters estimated based on sample ending in 2000Q3. Prediction based on regressors observed in 2000Q3 (left) and 2001Q3 (right).

B.6 United States: Zooming in on the recession that started in 2007

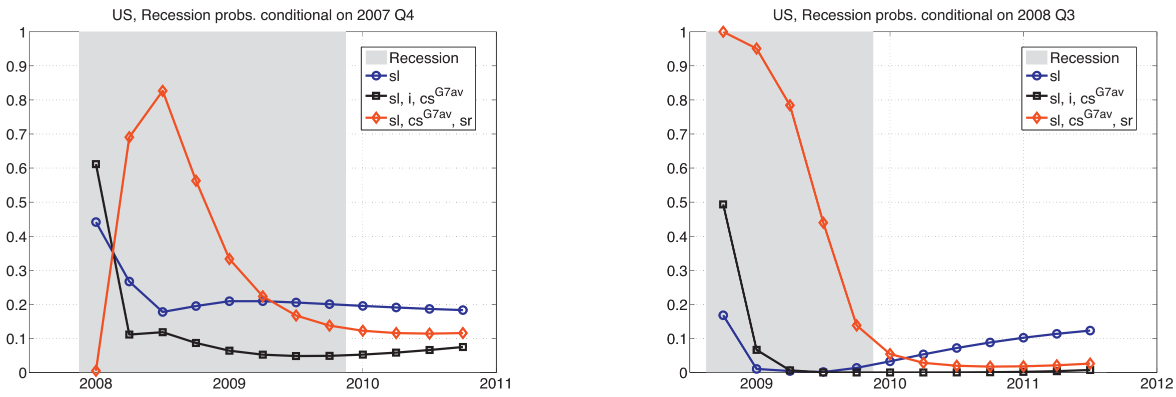


Figure 16: United States. Recession probabilities for 1, 2, ..., 12 quarters ahead as implied by selected ProbVAR specifications. Parameters estimated based on sample ending in 2007Q2. Prediction based on regressors observed in 2007Q4 (left) and 2008Q3 (right).

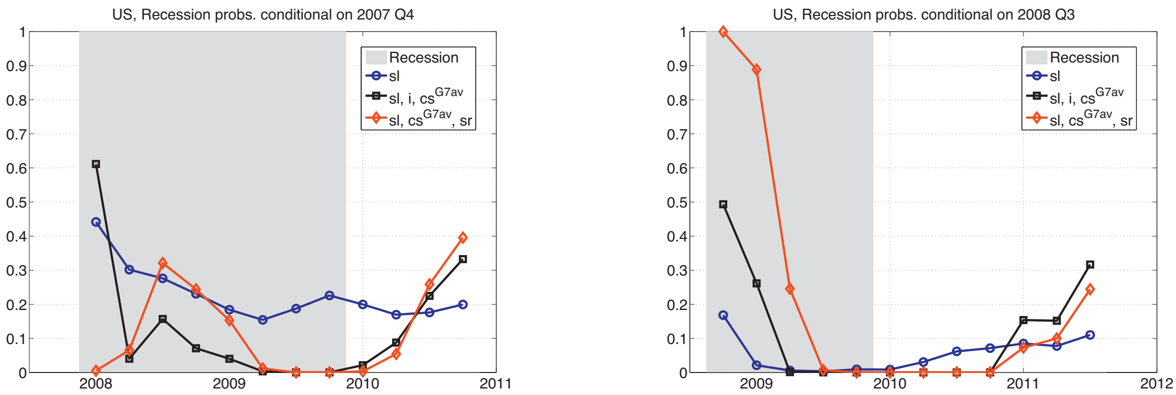


Figure 17: United States. Recession probabilities for 1, 2, ..., 12 quarters ahead as implied by selected simple probit specifications. Parameters estimated based on sample ending in 2007Q2. Prediction based on regressors observed in 2007Q4 (left) and 2008Q3 (right).

B.7 Germany: Zooming in on the recession that started in 2001

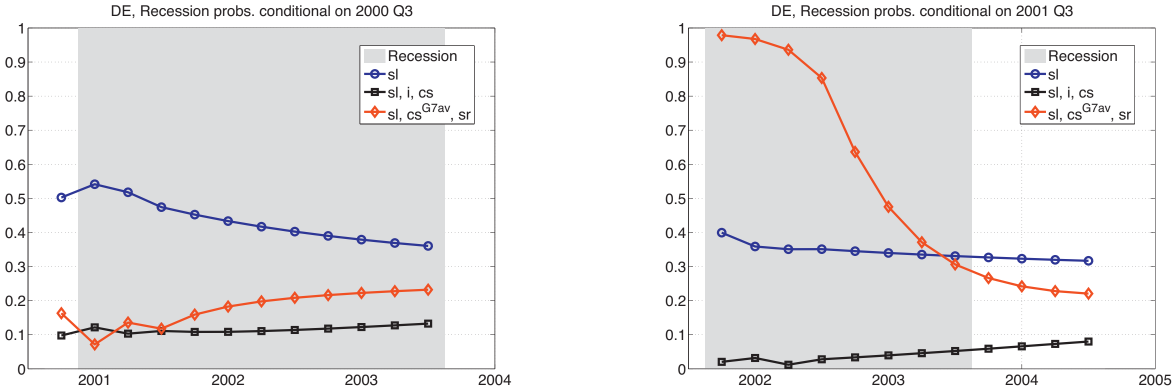


Figure 18: Germany. Recession probabilities for 1, 2, ..., 12 quarters ahead as implied by selected ProbVAR specifications. Parameters estimated based on sample ending in 2000Q3. Prediction based on regressors observed in 2000Q3 (left) and 2001Q3 (right).

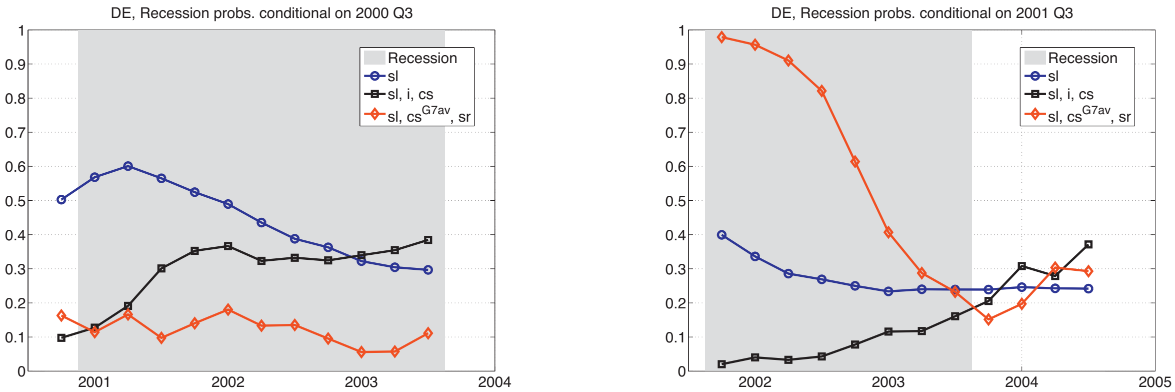


Figure 19: Germany. Recession probabilities for 1, 2, ..., 12 quarters ahead as implied by selected simple probit specifications. Parameters estimated based on sample ending in 2000Q3. Prediction based on regressors observed in 2000Q3 (left) and 2001Q3 (right).

B.8 Germany: Zooming in on the recession that started in 2008

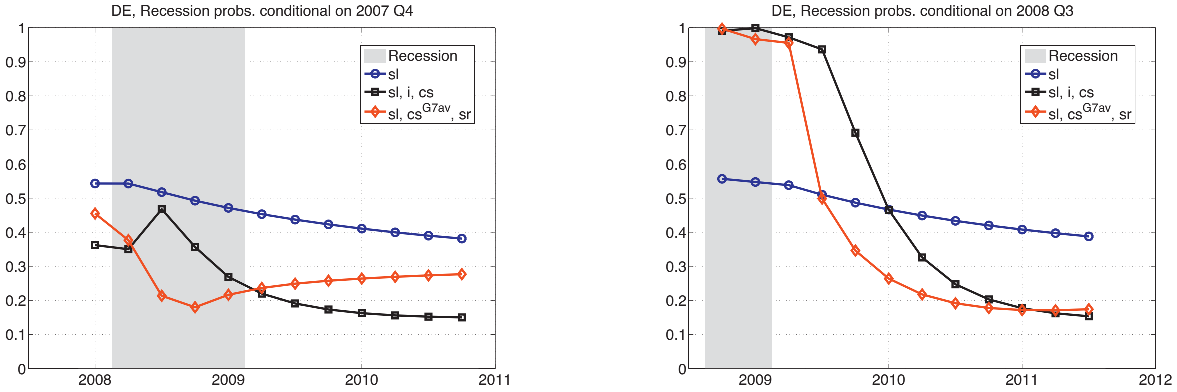


Figure 20: Germany. Recession probabilities for 1, 2, ..., 12 quarters ahead as implied by selected ProbVAR specifications. Parameters estimated based on sample ending in 2007Q2. Prediction based on regressors observed in 2007Q4 (left) and 2008Q3 (right).

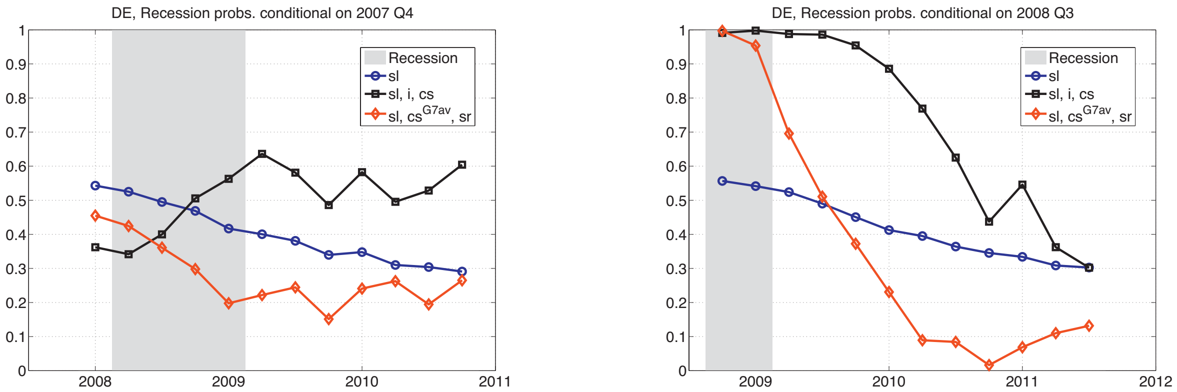


Figure 21: Germany. Recession probabilities for 1, 2, ..., 12 quarters ahead as implied by selected simple probit specifications. Parameters estimated based on sample ending in 2007Q2. Prediction based on regressors observed in 2007Q4 (left) and 2008Q3 (right).

B.9 Japan: Zooming in on the recession that started in 2000

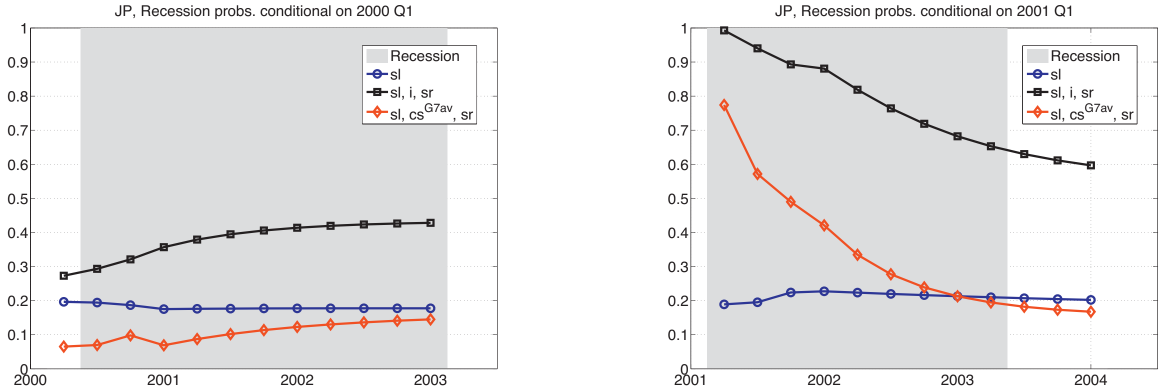


Figure 22: Japan. Recession probabilities for 1, 2, ..., 12 quarters ahead as implied by selected ProbVAR specifications. Parameters estimated based on sample ending in 2000Q1. Prediction based on regressors observed in 2000Q1 (left) and 2001Q1 (right).

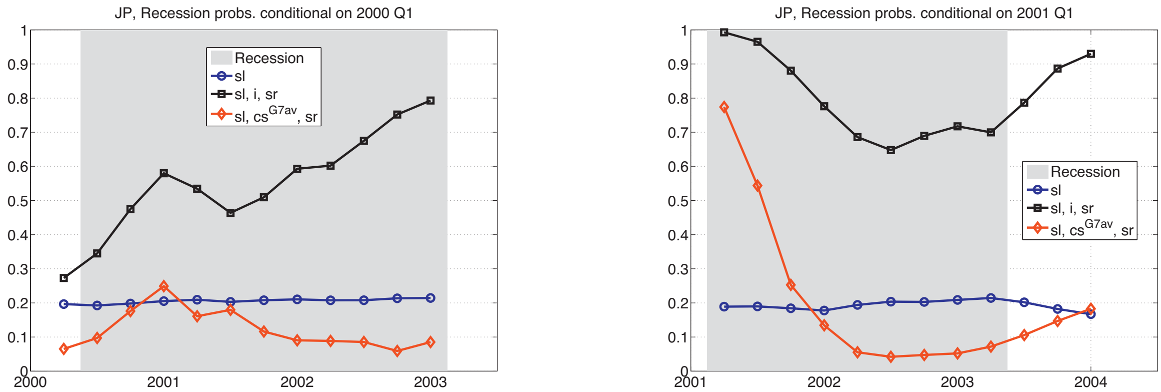


Figure 23: Japan. Recession probabilities for 1, 2, ..., 12 quarters ahead as implied by selected simple probit specifications. Parameters estimated based on sample ending in 2000Q1. Prediction based on regressors observed in 2000Q1 (left) and 2001Q1 (right).

B.10 Japan: Zooming in on the recession that started in 2008

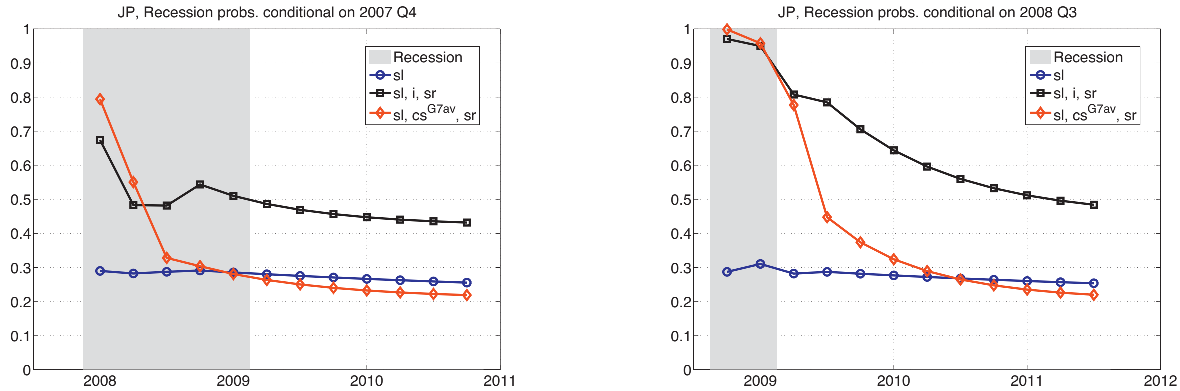


Figure 24: Japan. Recession probabilities for 1, 2, ..., 12 quarters ahead as implied by selected ProbVAR specifications. Parameters estimated based on sample ending in 2007Q2. Prediction based on regressors observed in 2007Q4 (left) and 2008Q3 (right).

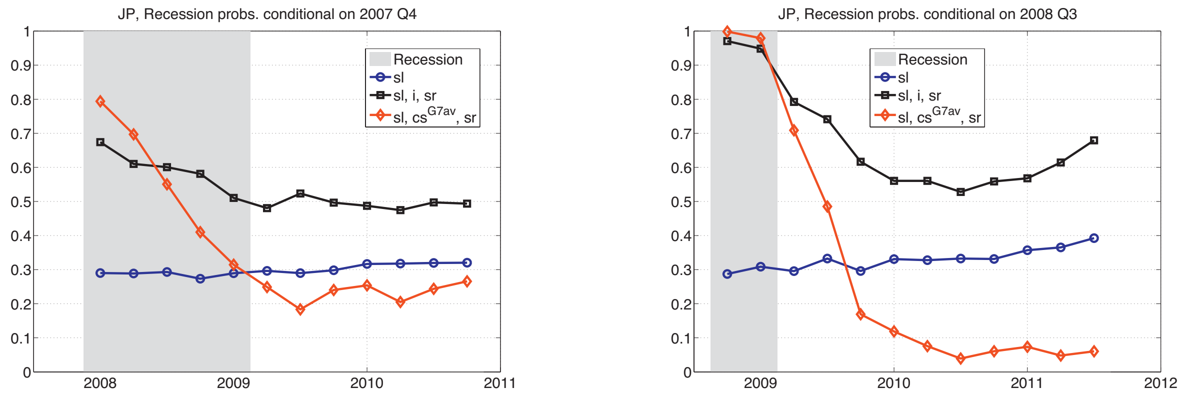


Figure 25: Japan. Recession probabilities for 1, 2, ..., 12 quarters ahead as implied by selected simple probit specifications. Parameters estimated based on sample ending in 2007Q2. Prediction based on regressors observed in 2007Q4 (left) and 2008Q3 (right).

B.11 Impulse response analysis

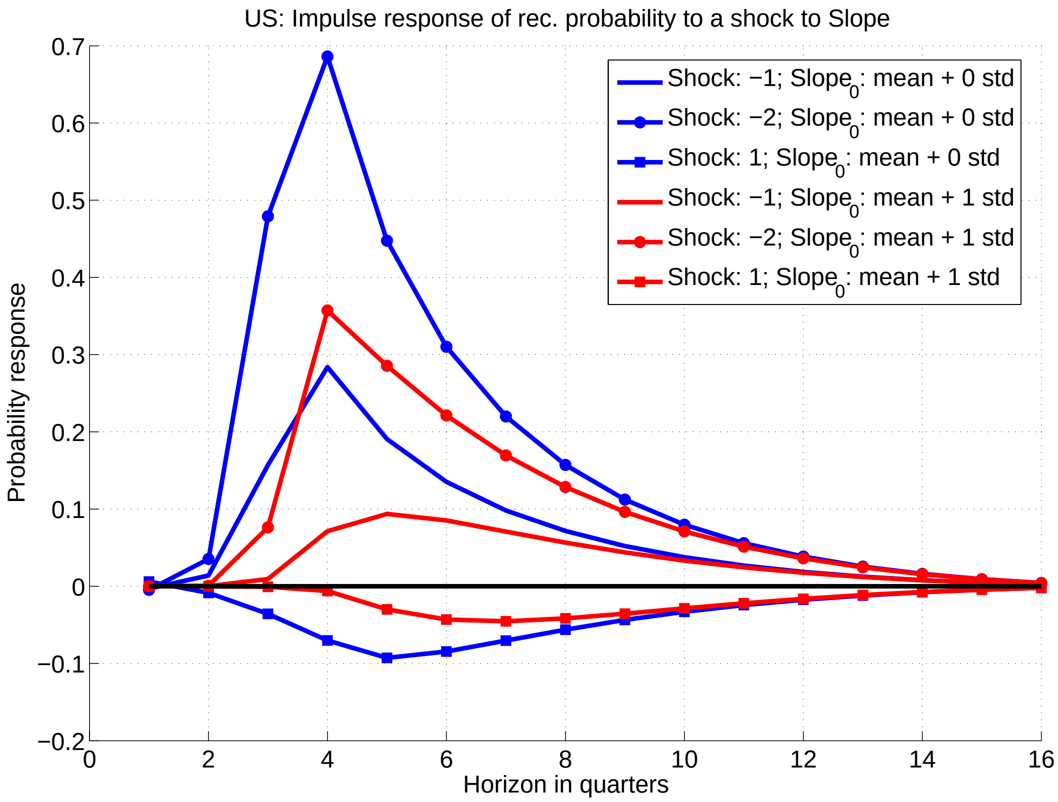


Figure 26: Impulse response of recession probabilities to a shock to the slope of the yield curve. – United States, ProbVAR(1,1,3) with slope, short-term interest rate and average corporate bond spread. Estimated using data until 2009Q4. Blue lines represent responses to a shock to the slope of the yield curve of -1 pp (plain line), -2 pp (line with circles), and +1 pp (line with squares). The initial regressors are at their sample averages. Red lines: the same, but for another initial condition of regressors: all variables and their lags are at their sample averages, but the slope equals the sample average plus one sample standard deviation.

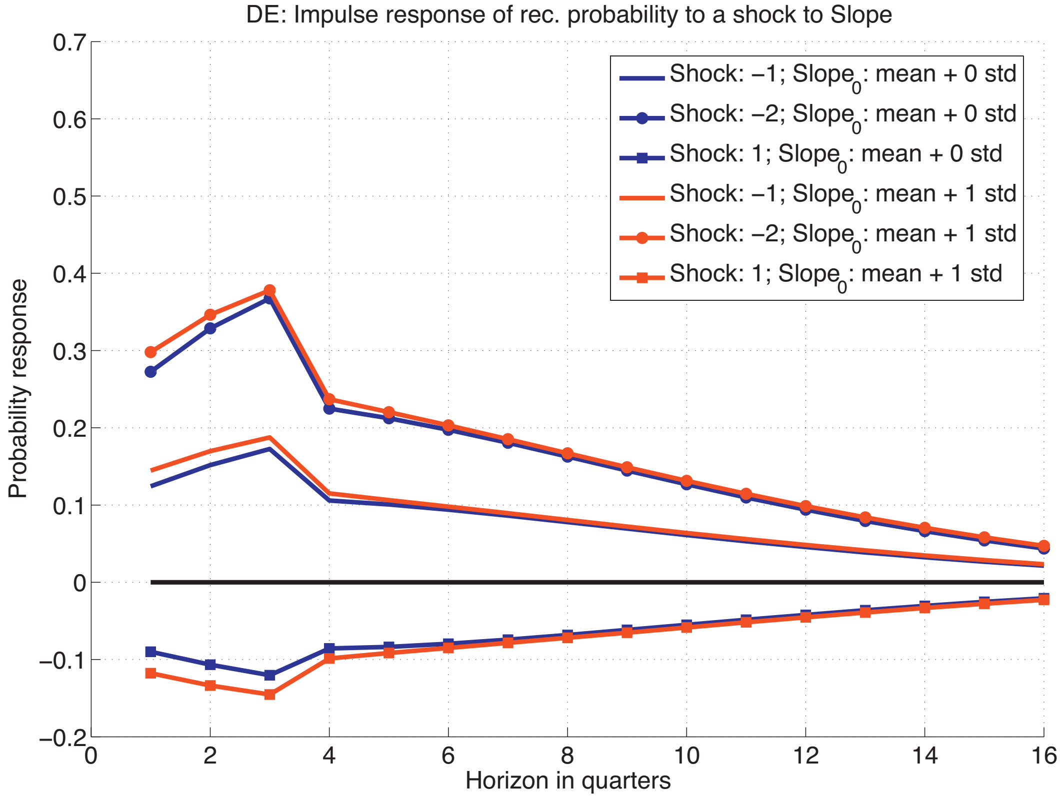


Figure 27: Impulse response of recession probabilities to a shock to the slope of the yield curve. – Germany, ProbVAR(1,1,3) with slope, short-term interest rate and average stock return. See figure 26.

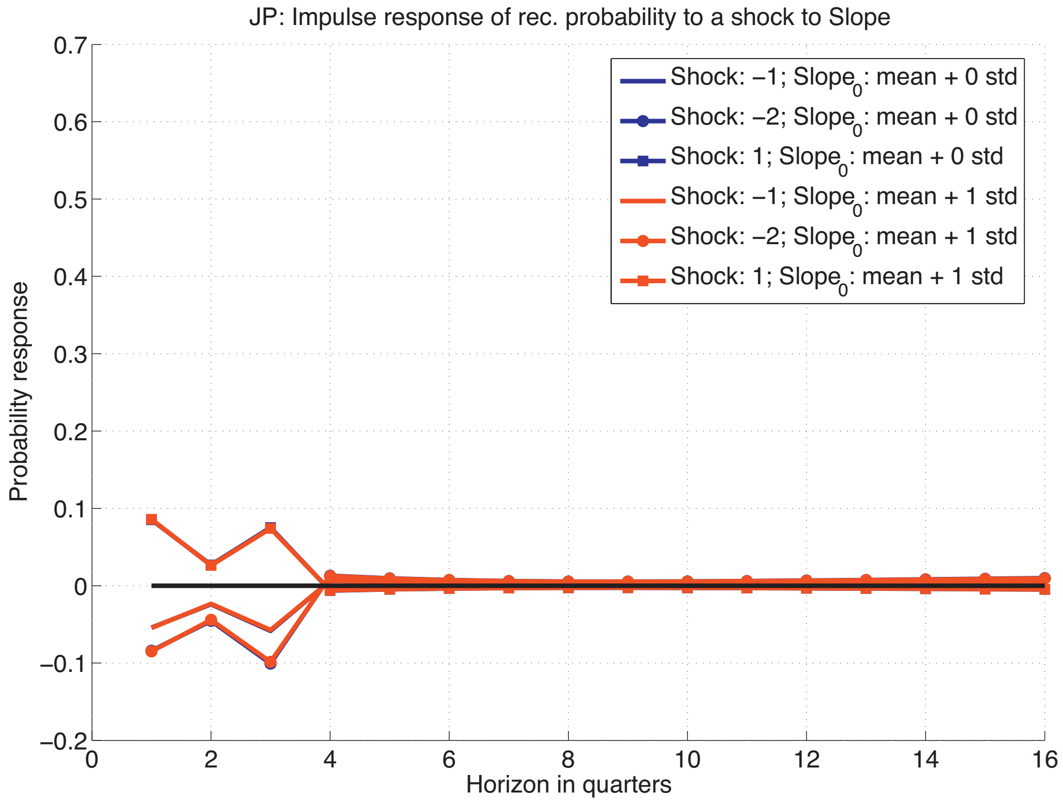


Figure 28: Impulse response of recession probabilities to a shock to the slope of the yield curve. – Germany, ProbVAR(1,1,3) with slope, short-term interest rate and average stock return. See figure 26.