EUROPEAN CENTRAL BANK

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NMCM - WITH BOUNDEDLY RATIONAL LEARNING EXPECTATIONS

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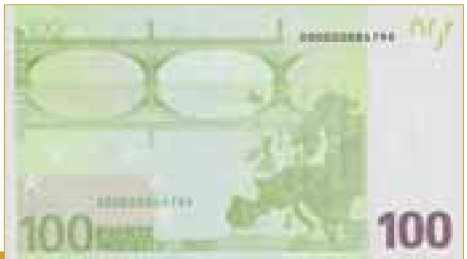

In 2011 all ECB publications feature a motif taken from the €100 banknote.

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CONTENTS

Abstract

Rational expectations has been the dominant way to model expectations, but the literature has quickly moved to a more realistic assumption of boundedly rational learning where agents are assumed to use only a limited set of information to form their expectations. A standard assumption is that agents form expectations by using the correctly specified reduced form model of the economy, the minimal state variable solution (MSV), but they do not know the parameters. However, with medium-sized and large models the closed-form MSV solutions are difficult to attain given the large number of variables that could be included. Therefore, agents base expectations on a misspecified MSV solution. In contrast, we assume agents know the deep parameters of their own optimising frameworks. However, they are not assumed to know the structure nor the parameterisation of the rest of the economy, nor do they know the stochastic processes generating shocks hitting the economy. In addition, agents are assumed to know that the changes (or the growth rates) of fundament variables can be modelled as stationary ARMA(p,q) processes, the exact form of which is not, however, known by agents. This approach avoids the complexities of dealing with a potential vast multitude of alternative mis-specified MSVs.

Using a new Multi-country Euro area Model with Boundedly Estimated Rationality we show this approach is compatible with the same limited information assumption that was used in deriving and estimating the behavioral equations of different optimizing agents. We find that there are strong differences in the adjustment path to the shocks to the economy when agent form expectations using our learning approach compared to expectations formed under the assumption of strong rationality. Furthermore, we find that some variation in expansionary fiscal policy in periods of downturns compared to boom periods.

Non technical summary

The dominant way to model expectations has been via model-consistent rational expectations (strong rationality). Whilst rational expectations (RE) can be taken as a theoretically well founded polar case, resorting to only them is not unproblematic. Therefore, given the extreme nature of the assumptions underlying rational expectations the literature has moved to a more realistic assumption of boundedly rational learning where agents are assumed to use only a limited set of information to form their expectations and do not know the complete structure of the model. The learning literature has pointed out that often a particular learning specification will produce a unique solution. However it is important to note that if the specific form of the learning process produces different solutions then the choice between these solutions and implicitly the corresponding Rational expectation equilibria (REE) is being made on the basis of a largely arbitrary decision. This motivates much of the argument presented in this paper that the choice of the form of the learning rule itself can be crucially important.

In the adaptive learning literature a standard assumption is that agents form expectations by using the correct model of the economy, but do not know the parameters i.e. agents have perfect knowledge about the structure of the economy and hence know the correct specification of the REE minimal state variable solution (MSV). However, in contrast to the RE solution, they have imperfect knowledge about the true values of the structural parameters and the implied parameter values of the true MSV solution. An alternative strand in the recent learning literature has been that, instead of using a correctly specified MSV, agents base their projections on a mis-specified MSV solution. This approach in effect drops the assumption of common information set of rational agents that fully understand the world, and therefore is more in line with the literature of heterogeneous agents with incomplete knowledge and expectations. Although agents fail to recognize the full set of correlations, agents are still rational in the sense that they avoid systematic mistakes by being willing to learn from past mistakes and change their behaviour.

Our approach deviates from all the aforementioned approaches. The basic principle of our approach is that it is compatible with the same limited information assumption that was used in deriving and estimating the behavioral equations of different optimizing agents. Hence, agents know the deep parameters of their own optimising frameworks, however, they are not assumed to know the structure nor the parameterisation of the rest of the economy. Neither do they know the stochastic processes generating shocks hitting the economy. Therefore, instead of using the correctly specified full model MSV, agents are assumed to use single equation MSV where the fundament variable, although endogenous in the whole model, is treated as predetermined for the optimising agent. Our approach is theoretically consistent as agents' local optimising decisions and future expectations are based on the same information set. It also avoids the complexities of dealing with a potential vast multitude of alternative misspecified MSVs.

In this paper we formalise this approach and apply it to a New Multi-Country Model (NMCM) see Dieppe et al (2011). The model can be characterised as an optimising agent - new keynesian model, but in contrast to standard DSGE models, we assume limited-information and it is more of a bottom-up approach as opposed to the standard DSGE top down approach where agents have full knowledge. In line with these limitations in the information base, all forward looking equations of the NMCM are estimated by the single equation instrumental variable method of GMM that requires rationality only under limited information.

In this paper we argue more formally that a single equation approach is more in line with the kind of bounded rationality assumed in the NMCM in defining the relevant information base for learning expectations than the approach based on the correctly specified reduced form of the whole model. Once we have presented the framework for the model and the implied estimation of the learning expectation equations, we study how the properties of the model differ from those obtained assuming perfect foresight (or model consistent) rational expectations. We further illustrate the implications by studying the impact of a fiscal policy expansion under learning expectations and

under rational expectations, when the policy change is credibly announced or alternatively its exact nature is unannounced (or uncredibly announced). We find that there are strong differences in the adjustment path to the shocks to the economy when agent form expectations using our learning approach compared to expectations formed under the assumption of strong rationality. Furthermore we show how, under our framework, the behaviour of the economy varies depending on the state of the economy and agents perceptions of the future.

Overall, we have aimed to outline a coherent and realistic framework for learning in limited information medium-scale models.

1 Introduction

The dominant way to model expectations has been via model-consistent rational expectations (strong rationality). Whilst rational expectations (RE) can be taken as a theoretically well founded polar case, resorting to only them is not unproblematic. It is well known that rational expectations can give rise to a multiplicity of solutions, sometimes terminal or transversality conditions may be enough to produce a unique solution but these conditions are always somewhat arbitrary. Rational expectations has also been criticised as it assumes too much information on the part of agents. Furthermore, it is well known that there have been difficulties in using large models that incorporate RE for forecasting, although there has recently been significant advances in using DSGE models for forecasting at policy-making institutions (e.g., Riksbank, Norges Bank, Bank of Finland, Czech National Bank and the ECB).

While rational expectation has been the dominant way to model expectation over the last forty years the literature on learning goes back almost as far as the rational expectations literature. Early work on learning includes Friedman (1975), Townsend (1978, 1983), Frydman (1982), Bray (1983) and Bray and Kreps (1984). This work focused almost exclusively on the stability properties of very small models, usually only one market. These models investigated a situation often referred to as 'rational learning' as it is assumed that agents know the true structure of the model being investigated but simply have to learn the parameter values. Given the extreme nature of the rational learning assumption the literature quickly moved to a more realist assumption of boundedly rational learning where agents are assumed to use only a limited set of information to form their expectations and do not know the complete structure of the model. Some early example from this literature include DeCanio (1979), Radner (1982) and Bray and Savin (1986), these examples focused on a case where the learning rule was the full reduced form of the model. Later papers began to use a learning rule which contained only a subset of the full set of reduced form variables and to define the idea of E-stability, when the parameters of the learning process converge to a fixed point (Evans 1989, Evans and Honkapohja 1994, 1995, 2001). Marcet and Sargent (1988, 1989) make the important link between E-stability and a conventional rational expectation equilibria (REE): when E-stability is attained the model has also reached a REE.

The learning literature (Evans 1986, Woodford 1990) has pointed out that often a particular learning specification will produce a unique solution and of course given the association of an E-equilibria with a REE, this implies that a particular REE is being chosen without recourse to these arbitrary transversality conditions. This illustrates that learning can bring positive advantages from an analytical standpoint. However it is important to note that if the specific form of the learning process produces different solutions then the choice between these solutions (and implicitly the corresponding REE) is still being made on the basis of a largely arbitrary decision. This motivates much of the argument presented in this paper that the choice of the form of the learning rule itself can be crucially important.

In the adaptive learning literature a standard assumption is that agents form expectations by using the correct model of the economy, but do not know the parameters (Evans and Honkapohja 2001), i.e. agents have perfect knowledge about the structure of the economy and hence know the correct specification of the REE minimal state variable solution (MSV), see McCallum (1983). However, in contrast to the RE solution, they have imperfect knowledge about the true values of the structural parameters and the implied parameter values of the true MSV solution. Hence, although correctly specified, the perceived law of motion (PLM) that agents use in updating their expectations deviates from the true MSV solution. Instead, the adaptive learning literature assumes that agents act like econometricians by continuously re-estimating and updating the parameters of the PLM taking into account observed expectation errors and all new information. Under these assumptions the actual law of motion (ALM) gradually converges to the model consistent RE solution. However, as discussed e.g. by Slobodyan and Wouters (2009), the short- and medium-run dynamics of the model may crucially depend on how much the initial estimates of the parameters deviate from those of the RE solution and, hence, may introduce non-voluntary arbitrariness into the dynamics of the estimated model.

Much of the learning literature has focussed on small, linear models where typically there is only one homogeneous / representative agent with a common information set, Milani (2007, 2009, 2010) and, hence, the correctly specified MSV solution for the whole model can be easily derived. With large and medium-sized models closed-form MSV solutions are difficult to attain given the large number of variables that could be included. Indeed, for non-linear models closed-form MSV solutions do not necessarily exist. Therefore, an alternative strand in the recent learning literature has been that, instead of basing their PLM on the correctly specified MSV, agents base it on a misspecified MSV solution. This approach in effect drops the assumption of common information set of rational agents that fully understand the world, and therefore is more in line with the literature of heterogeneous agents with incomplete knowledge and expectations. Although agents fail to recognize the full set of correlations, agents are still rational in the sense that they avoid systematic mistakes by being willing to learn from past mistakes and change their behaviour. Small model examples include Evans and Honkapohja (2003), Dennis and Ravenna (2008). The larger the model the larger the set of options among which to select a PLM. For example including only a subset of variables or including additional variables compared to the correctly specified MSV solution, then the selected PLM specification could be either under or over-parameterized. A key question then becomes of how to select from the various PLM when an obvious choice is not available. There has been a number of approaches to this, including choosing the explanatory variables that minimise the standard error of the regression, or ranking correlations according to their standard deviations, or identifying principle components and selecting the variables that mostly closely move with them (Beeby, Hall and Henry 2001). More recently De Grauwe (2010) used a model in which agents use simple rules (heuristics) to forecast the future, but these rules are then subjected to selection mechanism, so agents endogenously select the forecasting rules that have delivered the greatest fitness in the past. Finally, an alternative approach is to do Bayesian averaging over a variety of PLM.

Our approach deviates from all the aforementioned approaches. The basic principle of our approach is that it is compatible with the same limited information assumption that was used in deriving and estimating the behavioral equations of different optimizing agents. Hence, agents know the deep parameters of their own optimising frameworks, however, they are not assumed to know the structure nor the parameterisation of the rest of the economy. Neither do they know the stochastic processes generating shocks hitting the economy. Therefore, instead of basing their PLM on the correctly specified full model MSV, agents are assumed to base it on the single equation MSV where the fundament variable, although endogenous in the whole model, is treated as predetermined for the optimising agent. In addition, in line with the fact that most economic time series are I(1) variables, agents are assumed to know that the changes (or the growth rates) of fundament variables can be modelled as stationary ARMA(p,q) processes, the exact form of which is not, however, assumed to be known by agents. This suggests some form of heterogeneity of expectations, which could be due to cognitive limitations faced by agents in understanding the world, or that the observability of economic variables can be different across agents or that the costs of having full information are too large. This is also compatible with survey evidence showing clearly that expectations of aggregate economic variables differ across different sectors/agents, e.g. consumers and firms.

This approach implies that we avoid the conceptual difficulties encountered by DSGE models with adaptive learning based on the misspecified MSV, i.e. whilst the underlying specification and estimation of the typical DSGE model is effectively based on the optimisation of a single representative agent or a central planer who knows the structure of the whole model, the information set regarding the formation of learning is much more limited. Our approach, instead, is theoretically consistent as agents' local optimising decisions and future expectations are based on the same information set. It also avoids the complexities of dealing with a potential vast multitude of alternative misspecified PLMs.

In this paper we formalise this approach and apply it to a New Multi-Country Model (NMCM) see Dieppe et al (2011). The model can be characterised as a optimising agent - new keynesian model, but in contrast to standard DSGE models, we assume limited-information and, as in De Grauwe (2010), it is more of a bottom-up approach as opposed to the standard DSGE top down approach where agents have full knowledge. In line with these limitations in the information base, all forward looking equations of the NMCM are estimated by the single equation instrumental variable method of GMM that requires rationality only under limited information.

In the following section we argue more formally that a single equation approach is more in line with the kind of bounded rationality assumed in the NMCM in defining the relevant information base for learning expectations than the approach based on the correctly specified reduced form of the whole model. Once we have presented the framework for the model and the implied estimation of the learning expectation equations, we study how the properties of the model differ from those obtained assuming perfect foresight (or model consistent) rational expectations. We further illustrate the implications by studying the impact of a fiscal policy expansion under learning expectations and under rational expectations, when the policy change is credibly announced or alternatively its exact nature is unannounced (or uncredibly announced). We find that the departure of learning expectations solution from the rational expectations solution has a strong impact on the short-run properties of the model while in the longer run solution paths converge. Finally, we consider how the model properties change depending on the state of the economy.

As in Dieppe et al (2011), the model covers the 5 biggest euro area countries (Germany, France, Italy, Spain and the Netherlands), can be used either on a single country basis or as a linked euro area multi-country model, and it has been developed as macro model with both economic coherence, that matches the key characteristics of the data and is useful to analyse policy issues as well as being the main country macroeconomic tool used at the ECB in the context of the Eurosystem Macroeconomic Projection exercises (see ECB 2001).

2 Learning setup

In this section we formalise the limited information expectations formation approach. We start by a brief description of the adaptive learning framework under the assumption that all agents of the economy fully know the structure of the economy that is the standard case in Evans and Honkapohja (2001) and adopted e.g. by Milani (2007 and 2010) and Slobodyan and Wouters (2009) in empirical applications. Thereafter, we loosen this extreme information assumption to account for boundedly rational learning under imperfect information on the exogenous driving processes.

2.1 Updating beliefs under full information on model structure

Assume a stochastic linear or linearized structural model,

$$A_0 x_{t-1} + A_1 x_t + A_2 E_t y_{t+1} + \epsilon_t = 0 \tag{1}$$

where $x_t = [y_t', z_t']'$ is the vector of all model variables and $\epsilon_t = [v_t', e_t']'$ is the vector of white noise disturbances. Both of them are $(n+m) \times 1$ vectors. The vector y_t contains the n endogenous variables of the model and the vector z_t the other m variables driven by exogenous processes e.g.,

$$z_t = \rho z_{t-1} + e_t \tag{2}$$

where ρ is a $m \times m$ diagonal matrix of coefficients. Now the REE minimal state variable (MSV) solution of the model (1) with the driving processes (2) is of the form (McCallum 1983),

$$y_t = a + b y_{t-1} + c z_t + v_t \tag{3}$$

implying that the expected values of the endogenous variables are,

$$E_t y_{t+1} = a + b y_t + \rho c z_t \tag{4}$$

It is clear that in order to be able to apply (3), when forming their expectations about future outcomes, all agents must know exactly the structural model and its parameters, i.e. the model (1), as well as the stochastic structure that in our simple example is determined by (2). Endowing the real world agents with so much knowledge can hardly be considered realistic. More realistic would be to assume that the agents form their expectations under imperfect knowledge. For instance, the standard adaptive learning approach, as applied in Evans and Honkapohja (2001), assumes that agents know exactly the exogenous driving processes and the correct specification of the MSV solution but not its parameter values. Hence, they estimate the parameters $(\alpha_t, \beta_t, \gamma_t)$ of the MSV specification using the data available up to the point in time t and get the following Perceived Law of Motion (PLM),

$$y_t = \alpha_t + \boldsymbol{\beta}_t y_{t-1} + \gamma_t z_t + \delta v_t \tag{5}$$

Whenever new data becomes available (5) will be re-estimated and its parameters updated using, for instance, constant gain (perpetual learning) recursive least squares or Kalman filter algorithms. This procedure implies the following forecasting equation,

$$E_t y_{t+1} = \alpha_t + \boldsymbol{\beta}_t y_t + \rho \gamma_t z_t \tag{6}$$

and the Actual Law of Motion (ALM) is obtained after inserting (6) into (1).

As shown by Evans and Honkapohja (2001) the parameters of (6) converge to those of (4). Hence, the adaptive learning equilibrium converges to the REE. Now an interesting and, from the monetary policy point of view, a very important question is how much the replacement of rational expectations with adaptive learning expectations changes the adjustment dynamics of the model. This issue was studied in a medium size DSGE model by Slobodyan and Wouters (2009). Their finding was that learning expectations did not change the dynamics of the model much from that implied by fully rational expectations, at least, if the initial parameter values of (6) did not deviate too much from the true REE values of (4). This is intuitively easy to understand, because initial parameter values of (6) close to the true values of (4) imply that there does not exist much to learn and ALM utilizing (6) cannot deviate much from that utilizing (4). Hence, to obtain more marked differences between the adjustment dynamics based on rational expectations and adaptive learning expectations requires more constrained information base than what is the case, when the specification of the PLM corresponds to that of the correctly specified MSV solution. Constraining the information base is by itself no problem, because learning based on the misspecified MSV quite generally converges to an equilibrium, although not necessarily to REE, see Evans and Honkapohja (2001). The problem is rather how to constrain the information base in an intelligent and unambiguous way among innumerably possible ways to do it. In the following we show that in the NMCM framework, where the equations of the model are derived and estimated under bounded rationality, there is an unambiguous and theoretically well-founded way to define the information base for learning expectations.

2.2 Updating beliefs under limited information on model structure

In the NMCM the central DSGE assumption that all optimizing agents know fully the whole structure of the model including stochastic processes generating exogenous shocks is replaced by bounded rationality. Each agent knows only the parameters related to her optimization problem but need not know the rest of the model nor the stochastic exogenous processes driving the model. As the structure of the whole model is largely unknown to the agents, they quite often do not know how shocks that hit the economy are transmitted to the expected developments of the variables which are key determinants of their optimized choices. This is the case although shocks may quite often be easily observable. In line with these limitations in the information base all forward looking equations of the NMCM are estimated by the single equation instrumental variable method of GMM that requires rationality only under limited information. Hence, under these assumptions it is clear that the information content of the correctly specified MSV solution is much larger than what was required for the agents, when the equations of the NMCM were derived and estimated. In the following we argue that a single equation approach is much more in line with the kind of bounded rationality assumed in the NMCM in defining the relevant information base for learning expectations than the approach based on the correctly specified reduced form of the whole model.

Corresponding to the form of some single equation of the model (1) take a following example equation,

$$y_t = \beta E_t y_{t+1} + \delta y_{t-1} + y_t^* + \nu_t \tag{7}$$

where ν_t is a white noise shock and y_t^* is the fundament variable (possibly a complicated function of several variables endogenous in the whole model), the development of which is outside the control of the optimising agent in question. Hence, although endogenous in the full model, the development of y_t^* is predetermined for the agent just like truly exogenous variables of the model. As no uncertainty concerning the deep parameters of the underlying optimization framework was assumed, the decision making agent also knows the correct parameterization of equation (7).

For our purposes it is useful to express the structural parameters of (7) in terms of its roots. The roots of the homogenous part (7) are $\lambda_1 = \frac{1}{2\beta}\left(1 - \sqrt{1-4\beta\delta}\right)$ and $\lambda_2 = \frac{1}{2\beta}\left(1 + \sqrt{1-4\beta\delta}\right)$. The saddle path stability requires $\beta + \delta < 1$ that implies that $|\lambda_1| < 1$ and $|\lambda_2| > 1$. Now (7) can be rewritten as,

$$(1 - \lambda_1 L) E_t y_{t+1} = -\frac{\lambda_1 + \lambda_2}{(1 - \lambda_2 L)} E_t (y_t^* + \nu_t) \tag{8}$$

where L with $Lx_t = x_{t-1}$refers to lag operator. Now (8) implies the following solution for the following period expectations (see e.g. Sargent, 1979),

$$\begin{aligned} E_t y_{t+1} &= \lambda_1 y_t + \frac{\lambda_1 + \lambda_2}{\lambda_2} \sum_{i=0}^{\infty} \left(\frac{1}{\lambda_2}\right)^i E_t \left(y_{t+1+i}^* + \nu_{t+1+i}\right) && (9) \\ &= \lambda_1 E_t y_t + \left(\frac{\lambda_1 + \lambda_2}{\lambda_2 - 1}\right) y_t^* + E_t \Delta pv_{t+1} && (10) \\ &= \lambda_1^2 y_{t-1} + \left(\frac{\lambda_1 + \lambda_2}{\lambda_2 - 1}\right) (1 - \lambda_1) y_t^* + \left(\frac{\lambda_1 + \lambda_2}{\lambda_2}\right) E_t \Delta pv_{t+1} && (11) \end{aligned}$$

where

$$E_t \Delta pv_{t+1} = \left(\frac{\lambda_1 + \lambda_2}{\lambda_2 - 1}\right) \sum_{i=0}^{\infty} \left(\frac{1}{\lambda_2}\right)^i E_t \Delta y_{t+1+i}^*. \tag{12}$$

Equation (10) defines $E_t y_{t+1}$ in terms of information concerning current period realization on the dependent and fundament variables, on one hand, and the present value of expected changes of the fundament variable. Conventional assumption is that $E_t y_t = y_t$. which implies the stochastic disturbance ν_t in (7) is known at the point of time when expectations are formed. This need not, however, be true and, therefore, equation (11) presents the solution under the assumption that the lagged endogenous and the current fundament are observable at the moment of expectation formation. We return to this information issue later in this section.

Equation (9) implies the following current period solution,

$$y_t = \underbrace{\lambda_1 y_{t-1} + \left(\frac{\lambda_1+\lambda_2}{\lambda_2-1}\right) y_t^*}_{f_t} + \frac{1}{\lambda_2} E_t \Delta pv_{t+1} + \left(\frac{\lambda_1+\lambda_2}{\lambda_2}\right)\nu_t \tag{13}$$

This form of writing equation (7) depicts well the information available to the agent. In line with the information assumptions in underlying optimization the agent knows the f_t term but not how current and (possibly) lagged shocks hitting the economy are transmitted via the future changes of the fundament variable Δy_{t+i}^* to the expected present value term $E_t \Delta pv_{t+1}$. Econometricians know that most economic time series are I(1) variables, which implies that one can find a stationary ARMA(p,q) time series representation for their changes as, e.g.,

$$\Delta y_t^* = \mu + \phi(L)\left(\Delta y_{t-1}^* - \mu\right) + \psi(L)\, e_t \tag{14}$$

This gives the following Limited Information Minimal State Variable (LIMSV) presentation of equation (13),

$$\begin{aligned} y_t \quad = \quad & \underbrace{\lambda_1 y_{t-1} + \left(\frac{\lambda_1+\lambda_2}{\lambda_2-1}\right) y_t^*}_{f_t} + \frac{(\lambda_1+\lambda_2)}{(\lambda_2-1)^2}\mu + \Phi(L)\left(\Delta y_{t-1}^* - \mu\right) + \Psi(L)\, e_t \\ & + \left(\frac{\lambda_1+\lambda_2}{\lambda_2}\right)\nu_t \end{aligned} \tag{15}$$

If the forecasting agent knew all, what the econometrician knows, then she could use the LIMSV solution of (15) for forecasting $E_t y_{t+1}$ in (7). However, we do not assume that the agent knows the correctly specified ARMA(p,q) process and, hence, polynomials $\Phi(L)$ and $\Psi(L)$. In addition, also the drift term μ may be regime dependent, for instance, with respect to the growth of technical progress and/or population as well as the inflation target of the central bank. The underlying information assumptions of the NMCM, however, implies that the component f_t is known. It coincides with the LIMSV solution only if the ARMA(p,q) collapsed to $\Delta y_t^* = \mu + e_t$. However, the stationarity of (14) implies that $E_t \Delta pv_{t+1}$ term converges to a constant μ and hence, $f_t + \mu$ asymptotically converges towards the LIMSV solution. Hence we specify the following PLM relation,

$$y_t = \alpha_{0t} + \alpha_{1t}\left[\lambda_1 y_{t-1} + \left(\frac{\lambda_1+\lambda_2}{\lambda_2-1}\right) y_t^*\right] + u_t \tag{16}$$

$$\alpha_{0t} = \alpha_{0t-1} + \varepsilon_{0t} \tag{17}$$

$$\alpha_{1t} = \kappa + (1-\kappa)\cdot \alpha_{1t-1} + \varepsilon_{1t} \tag{18}$$

with $u_t \sim iid\left(0, \sigma_u^2\right)$ and $\varepsilon_{it} \sim iid\left(0, \sigma_i^2\right)$ and $\alpha_{it} \geq 0$. Parameter α_{0t} accounts for the drift and the adjustment dynamics of the unknown ARMA. The deviations of parameter α_{1t} from unity are related to the fact the f_{t-1} does not coincide exactly with correctly specified LIMSV. However, in line with the asymptotic properties of (14) parameter α_{1t} in equation (18) is specified to converge to unity with the speed determined by the size of parameter κ getting values in the interval $[0,1]$.

Now the PLM equation (16) implies the following forecasting equation

$$E_t y_{t+1} = \alpha_{0t} + \alpha_{1t}\left[\lambda_1 E_t y_t + \left(\frac{\lambda_1+\lambda_2}{\lambda_2 - 1}\right) y_t^*\right] \tag{19}$$

where the most straight-forward would be to assume that $E_t y_t = y_t$. This causes, however, simultaneity that is worsened by the fact that parameters α_{0t} and α_{1t} are also solved simultaneously with y_t. However, as discussed by Evans and Honkapohja (2001, Chapter 8.6) this problem can be circumvented by assuming that y_t is not included in the information set when forming expectations. This assumption is frequently used in the literature on indeterminacy (e.g. Milani 2007, 2010). Now, on the basis of equations (10) and (11) we re-define the square bracket term of (19) and end up with the following forecast equation for updating learning expectations,

$$E_t y_{t+1} = \alpha_{0t} + \alpha_{1t}\left[\underbrace{\lambda_1^2}_{\delta_1} y_{t-1} + \underbrace{\left(\frac{\lambda_1+\lambda_2}{\lambda_2 - 1}\right)(1+\lambda_1)}_{\delta_2} y_t^*\right] \tag{20}$$

Equation (20) together with (7) defines the ALM.

3 Kalman filter estimation of learning equations

Equations (16)-(18) form a state-space model where (16) is the measurement and (17)-(18) are the transition equations and can be estimated by a Kalman filter recursion. In matrix form it can be presented as follows,

$$y_t = X_t \boldsymbol{\alpha}_t + u_t \tag{21}$$

$$\boldsymbol{\alpha}_t = T\boldsymbol{\alpha}_{t-1} + \boldsymbol{\kappa} + R\boldsymbol{\varepsilon}_t \tag{22}$$

where $X_t = [1 \;\; (\delta_1 y_{t-1} + \delta_2 y_t^*)]$, $\boldsymbol{\alpha}_t = [\alpha_{0t} \;\; \alpha_{1t}]'$, $T = \begin{pmatrix} 1 & 0 \\ 0 & 1-\kappa \end{pmatrix}$, $\boldsymbol{\kappa} = [0 \;\; \kappa]'$ $R = I$ and $\boldsymbol{\varepsilon}_t \sim N(0, Q_t)$.

Following Harvey (1992) and Rockinger and Urga (2000), the variance-covariance matrix associated to $\hat{\alpha}_t$ is: $P_t = E_t[(\alpha_t - \hat{\alpha}_t)(\alpha_t - \hat{\alpha}_t)']$. The best estimates of P_t conditional on information at $t-1$ is

$$P_{t|t-1} = TP_{t-1}T' + RQ_tR' \tag{23}$$

Denote the variance of the residual u_t of the measurement equation by $H = \sigma_u^2$. Now the Kalman updating equations become,

$$\hat{\alpha}_t = \hat{\alpha}_{t|t-1} + \frac{P_{t|t-1}X_t'(y_{t-1} - X_t'\hat{\alpha}_{t|t-1})}{X_t P_{t|t-1} X_t' + H} \tag{24}$$

$$P_t = (I_m - \frac{P_{t|t-1}X_t'X_t}{X_t P_{t|t-1} X_t' + H})P_{t|t-1} \tag{25}$$

Equations (24)-(25) are the standard updating equations of the Kalman filter. In (24) $(y_{t-1} - X_t'\hat{\alpha}_{t|t-1})$ is called innovation and $\frac{P_{t|t-1}X_t'}{X_t P_{t|t-1} X_t' + H}$ the Kalman gain.

3.1 Optimising parameter estimation

The Kalman filter approach has the advantage of being very general, and can capture alternative forms of learning. The likelihood function can be concentrated so that only the ratio of the variance of the state equations to the measurement equations has to be estimated. Under the normalisation of $H = 1$, then Q_t becomes the signal to noise ratio. One learning algorithm used in the literature, is recursive least squares estimation which corresponds in the Kalman filter set-up to setting $Q_t = 0$; $H = 1$. In this case agents have infinite memory, with each observation being given equal weight. As Q_t rises agents effectively discount past observations more rapidly and the Kalman filter becomes equivalent to the constant gain algorithm used in the literature where agents give more weight to more recent observations. The Q matrix is therefore a measure of the rate at which agents are willing to update their forecasts. A higher Q means agents are more willing to learn and it also reflects their sensitivity to new information. However, there is a trade-off from discounting past observations in that there is a larger variance in the learning parameters, α_t. This willingness to learn is a further source of heterogeneity between economies. An alternative way to think about this is in terms of structural change: when agents give more weight to recent observations, it suggests agents are concerned about possible structural breaks in the past economic relationships. Indeed, the possibility of structural change suggests using a value of Q_t greater than zero.

As Milani (2005) shows, estimates vary strongly over a range of possible gain coefficients. Therefore, we follow Branch and Evans (2006) in estimating the degree of learning for each sector in each country. We do this by calculating the mean square forecast errors:

$$MSE(y_i) = \frac{1}{T}\sum_{t=0}^{T}(y_{i,t} - \hat{y}_{i,t})^2 \qquad (26)$$

where $\hat{y}_{i,t}$ is the forecast of the i-th component based on t-1 information and P_0 is set to a diagonal infinity matrix, and then computing the Q matrix that minimises the in-sample MSE by doing a grid search[1]. The value of Q which matters is its relative value compared with H, which is normalized to be 1. As both are normal distributions a relative value of Q twice as large as H will provide very rapid discounting with a half life of around 5 quarters. Higher values of Q would provide very volatile estimates and hence our search of Q is restricted to the range 0.0 to 1.9. As one of the aims of the model is to be used for forecasting, this method should provide the optimal forecasts given the model specification.

One key aspect when setting-up the Kalman filter updating mechanism concerns the initial values (priors) for $\hat{\alpha}$ and P_0. In some cases it has been found that the dynamics of learning models are sensitive to these choices, which adds a sense of arbitrariness that isn't present in the rational expectation solution, e.g. Slobodyan and Wouters (2009) find a crucial role of the initial beliefs to explain the improved fit in their DSGE model.

In our case, we use starting values as their expected long-run values i.e. α_0 =0 and α_1=1.[2] This is consistent with our framework and it thus is a reasonable starting point for the estimation. We also set P_0 equal to a diagonal infinity matrix, i.e. we assign a large uncertainty to such initial beliefs estimates.

[1] Alternatively we could have maximised the likelihood function, but as the likelihod function is basically made up of the squared prediction errors this should give virtually the same result.

[2] Instead of estimating learning forecast equations for the expected levels of dependent variables we estimate them for the expected changes. When the level form specification would imply the convergence of α_{0t} towards a small constant, the difference form specification implies convergence towards zero.

4 The Multi-Country Model Overview

The model used here is a new Multi-Country Euro Area Model (currently covering the 5 biggest euro area countries) with Boundedly Estimated Rationality, see Dieppe, Gonzalez Pandiella and Willman (2011). It was developed at the European Central Bank with the aim to match the key characteristics of the economies as well as to be used to produce the country projections and to provide policy analysis. The model has firm micro-economic foundations with the theoretic core of the model containing one exportable domestic good and one imported good. All central behavioural relations are based on the optimisation behaviour of three private sector decision making units (i.e. households, labour unions and firms) and the reaction functions of the government sector and the central bank. Expectation formation is treated explicitly and the model can be characterised as a limited-information - optimising agent - New Keynesian model.

As the available data does not disaggregate government into a separate institutional sector, the theoretical core of the model assumes a single domestic good produced by aggregated production function with total employment and total capital stock as inputs. Hence, the optimisation framework derives "true" behavioural relations for total employment, investment, private consumption and corresponding deflators and factor prices. For forecasting purposes, however, the accounting framework of the model is markedly more disaggregated, although feedback effects on the longer-run adjustment dynamics are specified via aggregate variables only.

The real world data which we have to confront stands in stark contrast to the predictions of many simple macroeconomic models. In particular we would point out that a simple model with a balanced growth path (BGP), as adopted e.g. by DSGE and other models related to the real business cycle (RBC) paradigm, would predict that the GDP-shares of labour and total factor income as well as the capital-output ratio are stationary. In the real data for our five countries this is clearly not the case. Therefore, as discussed by McAdam and Willman (2008), Solow (2000) and Blanchard (1997) we adopt a medium-run view regarding the underlying "trend" developments of our data in the sample period. Accordingly, the medium run developments, towards which the short-run dynamics converges, are allowed to deviate from the BGP. However, this view does not exclude the possibility that many processes, which from the medium run perspective may be advisable to treat as exogenous, are from the very long-run perspective endogenous and drive the medium run development eventually to converge to the BGP. Acemoglu (2002, 2003) gives an excellent example by showing that while technical progress is necessarily labour-augmented along the BGP, it may become capital-augmented in periods of transition reflecting the interplay of innovation activities, factor intensities and profitability. Given a below-unitary substitution elasticity this pattern promotes the asymptotic stability of income shares while precisely allowing them to fluctuate in the medium run. Accordingly, we allow non-unitary elasticity of substitution, non-constant augmenting technical progress and heterogeneous sectors with differentiated price and income elasticities of demand across sectors. We achieve this by following McAdam and Willman (2007 and 2008).

In addition to the relaxations concerning the medium-run development, the optimisation frameworks of agents contain a lot of frictional elements which are needed for explaining realistically the observed stylised short-run features. Labour is indivisible with important implications for the behaviour of all optimising agents. Regarding households' utility maximisation problem the indivisibility assumption simplifies the analysis, because the labour supply adjusts to the demand for labour conditional on the wage contract set by unions maximising either the utility of member households or targeting the warranted wage rate consistent with a desired employment rate. The basic framework in household's utility maximisation is Blanchard's (1985) overlapping generation framework that, however, is supplemented in many ways. Firstly to incorporate income uncertainty in a tractable way into the utility maximization framework we assume a two-stage approach in utility maximisation, Willman (2007). In the first stage, the consumer evaluates her risk-adjusted non-human and human wealth conditional on uncertain lifespan and labour income. Thereafter, in the second stage, conditional on the risk adjusted life-time resource constraint, the consumer is assumed to determine her optimal planned path of consumption.

In the profit maximising framework of the firm the assumption of indivisible labour, adjustment costs with respect to number of workers and convex costs with respect to work intensity introduce the discrepancy between paid hours and efficient hours. This explains the observed pro-cyclicality in labour productivity, when labour input is measured in heads or paid hours. It also introduces the ratio of efficient hours (per worker) to normal hours into optimal price setting on the top of the conventionally defined marginal cost of labour. The price setting of firms and the wage setting of unions are staggered with three-valued Calvo-signal, McAdam and Willman (2007). Part of firms (unions) keep prices (wages) fixed, another part changes prices (wages) following a backward-looking rule and the rest set them optimally. To capture the observed inertia in capital formation, capital stock and its rate of change are coupled with adjustment costs. Regarding the stock formation firms minimise quadratic losses induced by the deviations of inventories, on the one hand, and production, on the other hand, from their respective target levels related to the level of production implied by the production function, when existing inputs are utilised at their normal (cost minimising) rates.

All euro area countries are open economies and, therefore, also in our theoretic single domestic good framework a part of output is exported. However, firms face separate demand functions in domestic and export market leading to the pricing to market behaviour. This effectively separates the optimal price setting of exports from the rest of the firm's optimisation problem. We assume that the volume of exports is determined by the almost ideal demand system (AIDS) function. The advantage of this functional form compared to the conventional iso-elastic form is that now, compatibly with empirical evidence, the foreign competitors' price affect optimal export-price setting. The export demand and the optimal price setting form a two-equation system with cross-equation parameter constraints. This allows a model consistent way to estimate the price elasticity of export demand. Import side of the model is conventional being determined by domestic demand and the relative price of imports to domestic good.

The steady state form of the first-order conditions of profit maximising firms and the utility of member households maximising unions imply the 5-equation medium-run supply system that allows a consistent two-step estimation of the underlying deep parameters of the model. As the supply system contains cross-equation parameter constrains it is estimated with the method of non-linear SUR that León-Ledesma et al. (2010) have proven to be a very efficient estimation approach outperforming all single-equation methods. This system defines all parameters related to technology, production function and the mark-up allowing to define optimal frictionless prices, wages, labour demand and marginal cost and product concepts needed in estimating in the second stage the dynamic first order optimisation conditions of firms and unions. As the underlying framework is one of limited information, all dynamic equations containing the expectations of variables are estimated by the generalised method of moment (GMM).

In order to close the model the following additional relationships are required: a monetary policy rule, a fiscal policy rule and an exchange rate UIP rule. The model may be operated either with or without these rules, typically in a forecast the rules would be turned off while in policy simulations they are typically used. The fiscal policy rule is based on a reaction of taxes to the Government's debt to GDP ratio. The fiscal rule determines in the first place the path of the personal income tax rate. Transfers to households are modelled as a function of the unemployment rate. The monetary policy rule follows a simple Taylor rule specification in which the short term interest rate is determined by the inflation gap (where the inflation gap measures the distance between the actual inflation rate and its target) and the output gap along with the lagged interest effect. The exchange rate follows a standard forward-looking UIP equation. The key equations and parameters are in the appendix; see our sister paper Dieppe, Gonzalez Pandiella, and Willman (2011) for more details.

5 Estimated forecast equations for learning expectations

We now proceed to present the learning estimation results following the approach outlined in section (2.2) and (3). We apply it to the six main forward looking stochastic equations which are: consumption, price (GPP deflator) and wage inflation, investment, employment and inventories[3]. The uncovered interest rate parity condition for the real exchange rate contains also the expected forward exchange rate. However to retain the intrinsically forward looking nature of financial markets, this expectation is treated as rational.

The tables below present the key parameters for the learning equations[4] estimated since 1993Q1. For the time-varying parameters, α_{0t} and α_{1t} the table shows the end point of the parameter estimate (i.e. 2007Q2)[5]. We report δ_1, δ_2 and for the wage and price equations which include an additional lag, δ_3, which as in (20) are expressed in terms of the roots of the estimated forward looking equation. We also report the hyper-parameter for the Q matrix, obtained by minimizing the in-sample MSE[6]. The higher this parameter, the higher the variability in α_t and provides a reflection of structural change in those estimations. Finally, we report the in-sample R^2, which gives an indication of fit of the equations. The speed of convergence parameter, κ, in (22) was selected to be 0.03.

5.1 Employment expectations

Labour demand has a one lagged and a long-run (desired) number of workers, which is derived from the inverted production function.

$$\Delta E_t n_{t+1} = \alpha_{0t} + \alpha_{1t} \underbrace{\Delta(\delta_1 n_{t-1} + \delta_2 n_t^*)}_{\text{information}} + \varepsilon_{nt} \tag{27}$$

where $n_t = \log$ (number employed), and $n_t^* = \log(N_t^*)$ (inverted production function – see equation 34).

Learning Employment Estimation

	DE	FR	IT	ES	NL
α_0	0.0003	0.0002	0.0005	0.0009	0.0012
α_1	0.8481	0.9748	0.8106	0.9211	0.8049
δ_1	0.6803	0.7475	0.6292	0.4767	0.7475
δ_2	0.3197	0.2525	0.3708	0.5233	0.2525
Q	0.2000	0.0000	0.0150	0.0035	0.1000
$\mathbf{R^2}$	0.7380	0.5171	0.5968	0.8562	0.9690

5.2 Investment expectations

Capital accumulation reflects time to build considerations see section A.3.2 .

$$\Delta E_t k_{t+1} = \alpha_{0t} + \alpha_{1t} \underbrace{\left(\delta_1 \Delta k_{t-1} + \delta_2 \Delta k_t^*\right)}_{information} + \varepsilon_{ksrt} \tag{28}$$

[3] Expected HICP excluding energy is formed by a weighted average of the GDP deflator imports deflator excluding energy, p_t^{MN} and indirect energy prices.

[4] A number of expected variables are derived from these key estimated expectations - e.g. expected consumer prices are a weighted average of expected GDP factor prices and import prices.

[5] Although clearly they continue to evolve as new information comes into place.

[6] For Wages in Spain and Employment in France we started the estimation in 1997q1. This implied that the Q parameter was zero for these variables and was done to avoid undesirable roots. This approach is a minimal restriction compared to the alternative approach of projection facility Evans and Honakpohja (2001) which in effect restricts beliefs to a smaller neighbourhood.

where k_t is log of capital stock, $\Delta k_t^* = MPK_t - (1-\eta)\,UC_t + (1-d)\,(1-\delta)\,[\phi - (1-\eta)]$, MPK is marginal product of capital and UC is the real user cost of capital.

Learning Investment Estimation

	DE	FR	IT	ES	NL
α_0	-0.0003	0.0013	-0.0002	0.0021	0.0002
α_1	0.9805	1.0000	1.0116	0.8928	0.9232
δ_1	0.4830	0.6207	0.6078	0.6138	0.3928
δ_2	1.6950	1.7879	1.7796	1.7834	1.6267
Q	2.0000	2.0000	0.2000	0.3000	0.5000
R^2	0.9125	0.9033	0.3304	0.8600	0.8400

5.3 Consumption expectations

Household consumption follows an optimized framework with overlapping generations – see section (A.4). The expectation equation consistent with this equation is:

$$E_t C_{t+1} = \alpha_{0t} + \alpha_{1t} \log \underbrace{(\delta^1 C_{t-1} + \delta^2 C_t^*)}_{\text{information}} + \varepsilon_{ct} \tag{29}$$

where $C_t^* = (\gamma^y Y_t + \gamma^v V_t)$, Y_t is real labour income, and V_t is private sector total real wealth.

Note that in equilibrium with $\alpha_{1t} = 1$ terms $\frac{\delta_2}{1-\delta_1}\gamma^y$ and $\frac{\delta_2}{1-\delta_1}\gamma^v$equals the quarterly marginal propensities to consume out of labour income and wealth, respectively.

Learning Consumption Estimation

	DE	FR	IT	ES	NL
α_0	0.0042	0.0017	0.0008	0.0048	0.0195
α_1	0.9998	1.0001	0.9997	1.0002	0.9988
δ_1	0.7428	0.8464	0.8718	0.7286	0.8071
δ_2	14.4081	17.1154	7.6845	10.3162	8.8344
γ^y	0.0140	0.0079	0.0110	0.0221	0.0155
γ^v	0.0003	0.0001	0.0003	0.0004	0.0003
Q	0.0005	0.0040	0.0015	0.0750	0.0050
R^2	0.9337	0.9947	0.9790	0.9982	0.9939

5.4 Price expectations

Price equations follow the three-valued Calvo-signal, see equation (47), so that the expectation equation becomes:

$$\Delta E_t p_{t+1} = \alpha_{0t} + \alpha_{1t} \underbrace{\Delta(\delta_1 p_{t-1} - \delta_2 p_{t-2} + \delta_3 p_t^*)}_{\text{information}} + \varepsilon_{pt} \tag{30}$$

where p_t = gdp factor cost prices (log); $p_t^* = w_t - mpn_t + a_h\,(n_t^* - n_t) + \mu_t$ = frictionless equilibrium price level (log); w_t = compensation per worker (log); mpn_t= marginal product of labour (log); n_t^* = optimal number of workers (log) and n_t = actual employment (log);

Learning Price Estimation

	DE	FR	IT	ES	NL
α_0	0.0003	0.0021	0.0011	0.0022	0.0059
α_1	0.7967	0.9220	0.7525	0.7993	0.6055
δ_1	0.5871	0.4987	0.4186	0.2588	0.3151
δ_2	0.2592	0.2127	0.1723	0.0976	0.1230
δ_3	0.6721	0.7140	0.7538	0.8388	0.8079
Q	0.0375	0.0000	0.0175	0.0125	0.0000
R^2	0.6998	0.5832	0.7432	0.3780	0.3214

5.5 Wage expectations

As with prices, wages follow the three-valued Calvo-signal, with expectations based on frictionless equilibrium price level:

$$\Delta E_t w_{t+1} = \alpha_{0t} + \alpha_{1t} \underbrace{\Delta(\delta_1 w_{t-1} - \delta_2 w_{t-2} + \delta_3 w_t^*)}_{\text{information}} + \varepsilon_{wt} \tag{31}$$

where $w_t^* = (p_t^c + c_t) - n_t^f + \log\left(\sigma - 1 + \frac{MPN_t}{Y_t/N_t}\right) - h(time) + b_h\left(n_t^* - n_t^f\right)$,$w_t$=log of compensation per worker; MPN_t= marginal product of labour; n_t^* = optimal (desired) number of workers (log); n_t^F = labour force (log).

Learning Wage Estimation

	DE	FR	IT	ES	NL
α_0	0.0005	0.0006	0.0030	0.0026	0.0030
α_1	0.8474	0.8888	0.6543	0.9438	0.7087
δ_1	0.7790	0.4615	0.4371	0.5057	0.3598
δ_2	0.3666	0.1937	0.1815	0.2163	0.1439
δ_3	0.5876	0.7323	0.7444	0.7106	0.7841
Q	0.0150	0.0200	0.0075	0.0000	0.0050
R^2	0.8468	0.7108	0.6306	0.8992	0.5565

5.6 Inventories expectations

The desired equilibrium inventory stock KII* is based on the estimated CES production function and the dynamic equation reflects adjustment costs see equation (57) so the learning equation becomes:

$$\Delta E_t KII_{t+1} = \alpha_{0t} + \alpha_{1t} \underbrace{\Delta(\delta_1 KII_{t-1} + \delta_2 KII_t^*)}_{\text{information}} + \varepsilon_{nt} \tag{32}$$

where KII_t = inventory stock.

Learning Inventories Estimation*

	DE	FR	IT	ES	NL
α_0	-391.7	564.9	220.0	487.3	245.3
α_1	1.1662	0.8751	1.0979	1.0118	1.0749
δ_1	0.1516	0.4986	0.1678	0.1153	0.2139
δ_2	0.8714	3.9621	0.9615	0.6781	1.2353
Q	0.0000	0.0000	0.0000	0.0000	0.0000
R^2	0.5233	0.2378	0.2642	0.1546	0.2528

5.7 Learning Estimation observations

In general, the α_{1t} coefficient at 2007Q2 are close to 1, supporting the choice of using this form of the learning expectation rule. The main exceptions are the price equations where the α_{1t} parameters are lower (wages for the Netherlands is another exception).

There is quite some variability in the Q hyper-parameter across countries and equations, with estimates suggest a forgetting half-life of between 5 and 70 or more quarters. Looking at the variables, investment has a high Q hyper-parameter suggesting firms respond quickly to news to update their estimates whereas for consumption there is a longer half-life.

In general there is only small changes in the α_{1t} parameters over time with larger changes in the α_{ot} parameter. However, the variation in the parameters has an impact on the simulation results (see later). Overall, the estimation by Kalman filter provided a very good fit of data. This can be seen as being attributable to two factors, firstly the movement in parameters provides an additional degree of freedom, which captures variation, and secondly, the equations are optimised by minimising the squared error. The worse fits tend to be for the investment and inventory equations, with some R^2 of less than 0.8. Comparing the overall fit of the structural model equations under rational versus learning expectations we find little differences, suggesting the learning approach is a valid representation.

6 Scenario analysis

Above we have presented our learning framework and estimates under the assumption that agents have limited information. It is interesting to see how the interactions between the agents occur in the context of the full model and hence informative to do some shock/scenario - analysis. However, in undertaking scenario analysis it is important to consider exactly what underlying assumptions are required. Namely, it is important to consider what information set agents have, i.e. do households and firms have the same information set as Central Banks and Governments, or are there information asymmetries; is the shock anticipated or unanticipated, and finally it is important to distinguish between transitory and permanent shocks.

We will consider in this section, two alternative approaches to expectation formation. The first approach is the bounded rationality approach outlined above, where agents learn about the shocks and how the economy responds to those shocks. Under this approach we assume the Central Bank follows a Taylor rule, where it adopts the private agents' expectations for inflation rather than assuming the Central Bank has more information than private sector agents, or even full information[7]. However, we assume the real exchange rate follows a forward-looking, model-consistent UIP[8].

The second approach, for comparative purposes is the case of perfect foresight (model consistent) rationality where all agents have the same information set and know the duration of the shock and how the Central Bank and indeed the economy will react to the shock. In this approach, agents adjust their behaviour as soon as a change is anticipated, we call this 'announced and credible' shock. In this case where expectations are assumed to be fully rational with perfect foresight, the future expected value is simply replaced by the model future realisations and the model solved iteratively, such that the expectations are fully model consistent, i.e. expected outcomes are replaced with model outcomes:

$$E_t y_{t+1} = y_{t+1} \tag{33}$$

[7] Preston (2005) argues in favour of policy rules based on the bank's own forecast as he showed that if central bank adopt the private agents' expectations for its decisions without considering how they are formed, it may result in a self-fulfilling expectation problem and macroeconomic instabllity. The implications of replacing the expected inflation in the Talyor rule with either another rule, or the model-consistent rational expectations outcome are left for future work.

[8] The learning model can currently be simulated using a learning based UIP rule, but this clearly has different properties to the rational, model-consistent UIP. Using the latter could be interpreted as assuming rationality in the asset market.

For each country we have created a steady state baseline over a sufficiently long time horizon to enable us to perform standard shock simulations and to study the model properties. The comparative simulation analysis is undertaken over a horizon of 250 years[9].

We consider 5 different shocks covering a monetary policy shock, a demand shock and a supply shock, all occurring at time t:

1. *Short-term interest rate shock* - defined as a 50 b.p. increase in the short-term interest rate for 1 period followed by an interest rate rule.
2. *Permanent Government expenditure shock* – specified as a 0.5% of GDP increase in public consumption over the entire simulation horizon.
3. *Permanent Technology (TFP) shock* such that potential output is up by 1% over the entire horizon.
4. *Permanent appreciation of the euro nominal exchange rate* by 5% against all foreign currencies and
5. *Permanent world demand shock* of 1 per cent increase in extra euro area import demand.

We do the scenarios in single country mode, where there are no cross-country trade linkages and monetary policy and exchange rates react to single economy domestic developments. The simulations are presented in the figures at the end of the paper, where the results are presented as deviations from baseline, (real variables and employment are expressed as percentages deviations; prices as differences in year-on-year inflation rates and for interest rates, savings ratio, fiscal deficit, and unemployment, they are expressed as absolute deviations, either in percentage points or percent of GDP). We present results for the 5 estimated countries and consider the mechanisms and implications for the 5 shocks in turn.

6.1 One period shock to Interest Rates

We start by considering the reaction of the economy to a 1 period shock to the short-term interest rate followed by the Taylor rule. We consider the credibly announced rational expectations case where agents know that it is only a one period shock followed by a Taylor rule reaction. We compare it with the model simulated under learning. Figure 1 shows the response of the economy to this shock. In both cases, both demand and prices initially fall, but then start to return back to base as increased competitiveness aided by lower interest rates boost the economy. Under the rational / model consistent expectations scenario, although demand reacts quicker the responses are qualitative similar. However, the initial price responses are much larger than the learning and less persistent. This is because the learning model to some extent adds more frictions dampening the initial effects, so the model become more like a backward-looking model. However, while the initial effects are different, the adjustment paths are similar. This finding is similar to Slobodyan and Wouters (2009), who find the main differences under learning compared with the rational expectations are on the price side in their DSGE model.

6.2 Permanent government consumption shock

As noted by Van Brusselen (2009), there is no consensus on the response of the economy to a fiscal policy expansion. Indeed estimates in the literature vary widely depending on the model used. As

[9] Under rational expectations long simulation horizons ensures that the early part of the simulation path is (at most) marginally affected by the choice of terminal conditions. See Dieppe et al (2011) for more details.

the fiscal block is quite extensive we can consider expansionary fiscal policy through alternative instruments, but in this section we focus purely on an expansion in government expenditure. Figure 2 shows the impact of a permanent shock to Government consumption of 0.5% of GDP under learning and under perfect foresight. The fiscal rule, which is crucial from the standpoint of the sustainable development of government debt is operational, so government debt will return to previous levels, financed by higher taxes. In both cases, the impact of increasing government consumption by 0.5% of GDP is to initially increase GDP by around 0.5% of GDP, but subsequently there is strong crowding out due to higher interest rates, and hence higher cost of capital which reduces both investment and consumption as well as helps to reduce the surge in inflation. Furthermore, consistent with the uncovered interest rate parity condition, the shock triggers a depreciation of the domestic currency, thus boosting export demand. The initial depreciation of the currency reflects the fact that in spite of the initial rise of the interest rate it later decrease below the baseline and forces the real exchange rate to appreciate back to the baseline.

There are cross-country differences, both in the initial impact as well as the dynamics, see Figure 4. The differences are due to a variety of factors including openness of the economy, responsiveness to monetary policy, degree of financing constraints, perceived response of the Central Bank and perceived state of the economy. However, in general, cross-country differences are broadly similar between the rational and the learning versions.

In comparing the learning and the rational expectations approach, we notice that the learning model is stable and converging to the rational expectations solution. However, as also previously shown, the model with the learning process exhibits very different properties from a model with rational expectations with clear differences in the adjustment path to equilibrium. Indeed, the model seems to support the work of Orphanides and Williams (2004), who showed in a small model for inflation that policies which are efficient under rational expectations are not when agents use a learning process. We also see that the crowding out effects are much quicker and stronger in the anticipated rational expectations approach than under learning. Furthermore, monetary policy takes time to react, as the Central Bank learns about the duration of the shock. In the literature on learning it has become increasingly obvious that learning as opposed to rational expectations can impose a different set of constraints on how policy should be formulated. For recent surveys of this literature see Evans and Honkapohja(2008) and Bullard(2006). Indeed, it is also possible that some monetary policy rules can give rise to multiple REE and that the economy may not settle on the most desirable one, see Carlstrom and Fuerst (2004) and Evans and McGough (2005). The key insight of this literature is that monetary policy can have, in effect, a new role; to facilitate the learning process itself. One area of further investigation is to study the optimal monetary policy rule under learning as opposed to rational expectations in this model.

6.3 Other simulations

In addition to the two shocks above we also report the simulation results of an appreciation of the euro, a permanent world demand shock and a supply shock in the form of two technology shocks. As we have a CES production function, we consider both a shock to labour augmenting technical progress and a shock to capital augmenting, such that ceteris paribus potential output is permanently higher by 0.1% – i.e. the economy can produce more output for a given labour and capital. As with the previous simulations, after these shocks, there is an adjustment to a new equilibrium which is the same in the rational and the learning. However the key differences remain - more sluggish adjustment, particularly on the nominal side, which is a similar finding to Slobodyan and Wouters (2009).

6.4 Fiscal policy expansion under alternative expectations

Using our framework, we are able to illustrate the impact of different expectation assumptions. In doing this it is important to distinguish between transitory and permanent shocks, as clearly the response of the economy will depend on the duration of the shock hitting the economy. In this section we explore this in more detail.

A shock to the economy is by definition always unanticipated. However, in the case of a change of policy, consideration is needed as to whether the changes in policy are announced or unannounced and whether they are credible or uncredible. So far in the rational expectations perfect foresight case we have considered an announcement at point t where the future path of policy is fully credible and hence anticipated. We call this credibly announced rationality as agents know the future path of shocks and change their behaviour in advance. Whilst there are rigidities, the adjustments are therefore quick.

An alternative approach is of an unannounced or uncredible policy change. In this case, the information set is the same for all agents, but is one of protracted surprises, i.e. expectations will repeatedly not be fulfilled and agents will be surprised every period as they do not know the duration of the shock (for example a government continuously surprising firms and households). Therefore agents re-optimise each period as new information becomes available. This may to some extent be considered at odds with the rational expectations approach that does not allow systematic recurrent errors, but is a quite common approach in DSGE modelling.

The rationality assumption used in these two approaches are in some ways extreme: either there is full credibility or zero credibility and agents are continuously surprised. An alternative, perhaps more plausible case is that credibility is acquired, which is the learning approach. Indeed, in some ways, the learning approach is between these two approaches, in that agents are surprised, but are not continuously surprised and gradually learn about the shocks (e.g. if it is permanent or temporary) and adjust their behaviour accordingly. Therefore, we also compare learning to other extreme where agents know the model, but they don't know the future path of the shock. The learning approach would be expected to converge to the anticipated rationality solution.

We consider a temporary shock to government expenditure of 0.5% of GDP for 3 years after which government expenditure returns to back to base.[10] In all cases, the initial shock is unanticipated. Under the credibly announced scenario, both households, firms the central bank and the government know the duration of the shock. They therefore anticipate that debt-financed expenditures will lead to higher taxes. The second case we consider is where the change in government policy is unannounced or uncredible. In this case agents expect the government to return expenditure to their previous level of government spending, and agents are continuously surprised by the government, i.e. sequentially surprised. In this case, we also assume that the central bank has the same information set as the private agents and is also continuously surprised, and monetary policy reactions reflects this. Finally, the third case considered is the learning, bounded rationality, where agents don't know the duration of the shock, but learn about it, and subsequently are not continuously surprised. Figure 9 shows reactions under the three approaches.

The dynamics are quite different across the three different cases. In the first case, of credibly announced change in government spending, under model consistent expectations, agents react immediately to the shock and adjust their behaviour according, with quick crowding out, particularly for investment, higher interest rates, but also crowding out by households due to negative wealth effect on private consumption in anticipation of higher (future) tax burdens. Furthermore there are strong inflationary pressures in the first year. In the second case, where the duration of increased government spending is unannounced or uncredible the impact on the economy is different, with initially less inflationary pressures, and less crowding out in the second and third years. In the learning expectations

[10] An alternative assumption is that agents assume there is some persistent in Government spending, so instead of an immediate return to some baseline level, agents could assume government expenditure folows an AR(1) process.

case, where agents learn about the duration of the shock, the inflationary reaction is initially small - and similar to unnannounced / uncredible scenario, but as agents realise the shock is more persistent prices start to react quicker - more in line with the credible scenario. Private demand, particularly consumption, however reacts less and consequently after 3 years the boost in demand from increased government consumption is larger.

6.5 Time-variation in scenarios

As well as differences in the treatment of expectations, the impact of an expansionary fiscal policy potentially depends on the state of the economy, and the perceived (expected) state of the economy. In this respect, using the bounded-rationality framework, the simulations in figures 10 to 14 show the impact of starting the shock in 1999Q1, through to starting the shock in 2009Q4. The reported time frame is 8 quarters. Figures 10 and 11 show the response to an Interest rate shock of 50 basis points with exogenous exchange rates. The simulation shows that since 1999, inflation sensitivity to an interest rate shock has increased, which maybe interpreted as an increase in Central Bank credibility. However it is notable that the real effects remain basically the same over this period due to no essential change in real wage responses. This variation over time in the inflation process is due to the learning mechanism as agents adjust their expectations in response to the shocks hitting the economy, i.e. different parameters in the expectation equations. Figures 12 and 13 show the impact of a permanent increase in government expenditure, with monetary accommodation (i.e. exogenous interest rates and exchange rates). Price flexibility seems to have increased over the last 12 years in response to a demand shock, which results in faster crowding out of real effects. Secondly, the responsiveness of the economy to fiscal stimulus seems to vary over time, increasing during downturns (e.g. 2001), which is particularly noticeable during the recent period (2009). This is because the economy has some slack, i.e. is not being fully utilized and agents adjust their expectations to reflect this.

The scenarios analysis presented here are illustrative, and as our fiscal block is quite extensive we can do a range of alternative scenarios, e.g. a reduction in income tax, consumption tax or firm's social security payments or an increase in government transfers, and in a subsequent paper we consider the implications of coordinated versus uncoordinated fiscal expansion in the linked version of the model.

7 Conclusions

The learning approach to expectations is often criticised for being arbitrary. In this paper we have presented a limited information learning approach where each group of agents knows only the parameters related to their optimization problem but need not know the rest of the model nor the stochastic exogenous processes driving the model. Furthermore, as Milani (2005), has shown the learning estimates can vary strongly over different gain coefficients, therefore we optimised the key gain parameter, with the implicit assumption that agents choose the 'best performing rule'.

We have applied this learning framework to a new multi-country model consisting of three private sector decision making units, i.e. firms, trade unions and households. These agents are optimising but based on a limited information. Indeed, the core of the model is estimated with GMM that implicitly assumes boundedly rational expectations and thus consistent with the learning approach. It is for this reason that it is our preferred approach as the main specification for the model.

We have shown that under these assumptions, expectations based on learning (weak rationality), converges to the perfect foresight, (strong rationality) solution. However, there are strong differences in the adjustment path of the economy to an expansionary fiscal policy. Furthermore, under our framework, the behaviour of the economy varies depending on the state of the economy and agents perceptions of the future.

Overall, we have aimed to outline a coherent and realistic framework for learning in limited information medium-scale models. However, further research is needed to better to understand the importance of the information assumptions in the model.

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A Appendix: The Model Framework

The theoretical core of the model consists of three optimising private sector decision making units, i.e. utility maximising households, profit maximising firms and trade unions, which minimise the quadratic loss function under the staggered wage adjustment assumption. Monopolistically competing firms set prices, inventories and factor demands under the assumptions of indivisible labour. Output is in the short run demand-determined. Monopoly unions set wages and overlapping generation households make consumption/saving decisions. In the rest of this section we present the model equations. The detailed theoretical framework is in Dieppe et al (2011).

A.1 The Normalised CES production function

Our technology assumption is the "normalized" CES function allowing for time-varying factor-augmenting technical progress:

$$\frac{Y_t}{Y_0} = \left\{ \pi_0 \left[\Gamma_K(t,t_0) \frac{K_t}{K_0} \right]^{\frac{\sigma-1}{\sigma}} + (1-\pi_0) \left[\Gamma_N(t,t_0) \frac{h_t N_t}{h_0 N_0} \right]^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}} \tag{34}$$

where σ is the elasticity of substitution between capital and labour, π_0 distribution parameter equalling the capital share evaluated at the normalization point (subscript 0) and $\Gamma_i(t,t_0)$ define the (indexed) level of technical progress associated to factor i (with $\Gamma_i(t_0,t_0) = 1$). Technical progress follows a Box and Cox, (1964) functional form :

$$\log\left[\Gamma_i(t,t_0,\gamma_i,\lambda_i)\right] = \frac{\gamma_i t_0}{\lambda_i}\left[\left(\frac{t}{t_0}\right)^{\lambda_i} - 1\right] \tag{35}$$

where $i = N, K.$ and the log level of technical progress, $\Gamma_i(\bullet)$ is, therefore, a function of time, t (around its normalization point, t_0), a curvature parameter, λ_i, and has a growth rate of γ_i at the representative point of normalization[11].

A.2 The supply system

The behaviour of profit maximizing firms and the member households 'utility maximizing trade unions determine the long-run supply of the model as defined by the following 5-equation suppy system 36 - 40:

$$\log\left(\frac{P_t^y Y_t}{w_t N_t + q_t K_t}\right) - \log(1+\mu(t)) = 0 \tag{36}$$

[11] Note we scaled the Box-Cox specification by t0 to interpret γ_N and γ_K as the rates of labour- and capital-augmenting technical change at the fixed (i.e., representative) point.

$$\log\left(\frac{w_t N_t}{P_t^y Y_t}\right) - \log(1-\bar{\pi}) - \frac{1-\sigma}{\sigma}\left[\log\left(\frac{Y_t/\bar{Y}}{N_t/\bar{N}}\right) - \log\xi - \frac{\bar{t}\gamma_N}{\lambda_N}\left(\left(\frac{t}{\bar{t}}\right)^{\lambda_N} - 1\right)\right] + \log(1+\mu(t)) = 0 \quad (37)$$

$$\log\left(\frac{q_t K_t}{P_t^y Y_t}\right) - \log(\bar{\pi}) - \frac{1-\sigma}{\sigma}\left[\log\left(\frac{Y_t/\bar{Y}}{K_t/\bar{K}}\right) - \log\xi - \frac{\bar{t}\gamma_K}{\lambda_K}\left(\left(\frac{t}{\bar{t}}\right)^{\lambda_K} - 1\right)\right] + \log(1+\mu(t)) = 0 \quad (38)$$

$$\log\left(\frac{Y_t/\bar{Y}}{N_t/\bar{N}}\right) - \log(\xi) - \frac{\bar{t}\gamma_N}{\lambda_N}\left(\left(\frac{t}{\bar{t}}\right)^{\lambda_N} - 1\right) + \frac{\sigma}{1-\sigma}\log\left[\bar{\pi} e^{\frac{1-\sigma}{\sigma}\left[\frac{\bar{t}\gamma_N}{\lambda_N}\left(\left(\frac{t}{\bar{t}}\right)^{\lambda_N}-1\right) - \frac{\bar{t}\gamma_K}{\lambda_K}\left(\left(\frac{t}{\bar{t}}\right)^{\lambda_K}-1\right)\right]}\left(\frac{K_t/\bar{K}}{N_t/\bar{N}}\right)^{\frac{\sigma-1}{\sigma}} + (1-\bar{\pi})\right] = 0 \quad (39)$$

$$\log\left(\frac{N_t^F w_t}{P_t^c C_t}\right) + \log\left\{\sigma - 1 + (1-\bar{\pi})\left[\frac{Y_t/\left(\xi\cdot\bar{Y}\right)}{\frac{N_t}{\bar{N}} e^{\frac{\bar{t}\gamma_N}{\lambda_N}\left(\left(\frac{t}{\bar{t}}\right)^{\lambda}-1\right)}}\right]^{\frac{\sigma-1}{\sigma}}\right\} - \log\left(\frac{\sigma\kappa}{\bar{h}_t}\right) = 0 \quad (40)$$

where Y, N and K refer to output, employment and capital. P^y, w and q are their respective prices; C, P^c and N^F are consumption, consumption deflator and labour force. Bars above the variables refer to the sample averages. Normalised production function implies that $\vec{\pi} = \frac{\bar{q}\bar{K}}{\bar{w}\bar{N}+\bar{q}\bar{K}}$ is the capital share evaluated at the fixed point (sample mean).

Production Function Estimates

	FR	DE	IT	ES	NL
Elasticity of Substitution	0.532	0.614	0.614	0.55	0.575

All dynamic equations containing the leads of variables are estimated by the generalised method of moment (GMM) that is compatible with the assumption of bounded rationality[12].

A.3 Firms

A.3.1 Labour Demand

The desired (optimal) number of workers, N_t^* is derived from the inverted production function equation 34 such that:

$$N_t^* = \frac{\bar{N}(1-\bar{\pi})^{\frac{\sigma}{\sigma-1}}}{\Gamma_N(t)}\left[\left(\frac{Y_t}{\xi\bar{Y}}\right)^{\frac{\sigma-1}{\sigma}} - \bar{\pi}\left(\frac{K_{t-1}}{\bar{K}}\right)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{\sigma}{\sigma-1}} \quad (41)$$

Total wage costs can be presented as a convex function of the deviation of effective hours, h_t from normal hours, $\bar{h} = 1$:

[12] The instruments used in estimation are lags of dependent, driving and other relevant variables.

$$W_t = \overline{W}_t \left[h_t + \frac{a_h}{2} \cdot (h_t - 1)^2 \right] \quad (42)$$

Changes in employment are coupled with adjustment costs, $A_N(N_t, N_{t-1})$:

$$A_N(N_t, N_{t-1}) \;=\; \frac{a_N}{2} \cdot \Delta N_t \Delta n_t \quad (43)$$

where n=log(N).
Now the dynamic system of first order conditions imply the following labour demand:

$$n_t = \frac{D_t}{(1 + D_t + a_h/a_N)} n_{t+1} + \frac{1}{(1 + D_t + a_h/a_N)} n_{t-1} + \frac{a_h/a_N}{(1 + D_t + a_h/a_N)} n_t^* \quad (44)$$

$n_t^* = \log\left(N_t^*\right)$ (inverted production function)

$D_t = \frac{(1+(w_{t+1}-w_t))}{(1+r_t)_t} \cdot \frac{\left(1+(n_t^*-n_t)+a_h(n_t^*-n_t)^2\right)}{\left(1+\left(n_{t+1}^*-n_{t+1}\right)+a_h\left(n_{t+1}^*-n_{t+1}\right)^2\right)} \cdot$ = discounting factor (≈ 1)

A.3.2 Investment formation

Capital accumulation reflects time-to-build considerations. As with employment, we define the adjustment cost function $A(K_t, K_{t-1}, K_{t-2})$, as follows:

$$A(K_t, K_{t-1}, K_{t-2}) \;=\; \frac{a_K}{2} \cdot \Delta K_t \Delta k_t + \frac{a_K b_K^2}{2} \cdot \Delta K_{t-1} \Delta k_{t-1} - a_K b_K \cdot \Delta K_t \Delta k_{t-1} \quad (45)$$

where $k = \log K$ and $b_K \in [0,\, 1]$.
Now the dynamic system of first order conditions implies the investment equation:

$$\frac{\left(1-\Lambda^B\right)^2 b_K}{(1+r_t)\,(1+r_{t+1})} \Delta k_{t+2} - -\left(\frac{\left(1-\Lambda^B\right)^2 b_K^2}{(1+r_t)\,(1+r_{t+1})} + \frac{\left(1-\Lambda^B\right)(1+b_K)}{(1+r_t)} \right) \Delta k_{t+1}$$
$$+ \left(\frac{\left(1-\Lambda^B\right) b_K (1+b_K)}{(1+r_t)} + 1 \right) \Delta k_t - b_K \Delta k_{t-1}$$
$$= \frac{1}{a_K} \left(\frac{P_t}{(1+\mu_t)\, P_t^I} MPK_t - \left\{ \left(1-\Lambda^I\right) UC_t + \Lambda^B (1-\delta) \left[\alpha - \left(1-\Lambda^I\right)\right] \right\} \right) \quad (46)$$

Λ^B = LG-multiplier related to the borrowing constraint; α= the debt to capital stock ceiling ratio $0 \leq \alpha \leq 1$; MPK = marginal product of capital; UC = real user cost of capital; Λ^I= LG-multiplier related to the irreversibility of investment ; and a_K and b_K are adjustment cost parameters.

A.3.3 Price formation

Price and wage setting are staggered with three-valued Calvo-signal resulting in a conventional hybrid New Keynesian Phillips curve as in Gali and Gertler (1999):

$$\begin{aligned}\{\theta_p + \omega_p [1 - \theta_p (1-\beta)]\} \Delta p_t - \omega_p \Delta p_{t-1} - \beta\theta_p \Delta p_{t+1} \\ - \ (1-\omega_p)(1-\theta_p)(1-\beta\theta_p)(p_t^* - p_t) = 0 \quad (47)\end{aligned}$$

where p_t = log of gdp deflator at factor costs; $p_t^* = w_t - mpn_t + a_h (n_t^* - n_t) + \mu_t$ = log of the frictionless equilibrium price level; w_t = log of compensation per worker; mpn_t= log of the marginal product of labour (<= production function); n_t^* = optimal number of workers (log), n_t = actual employment (log); a_h is the overtime premium parameter determined by (42) and aggregate mark-up $\mu(t)$ is determined by the system 36-40. θ_p is the probability that firms don't change their prices, and ω_p is the probability prices are changed following a backward-looking rule. In estimation we assumed the four per cent annual discount rate, which in quarterly data implies $\beta = 0.99$.

Disaggregated Price Equations The post-tax HICP deflator is defined as:

$$p_t^{HX} = \frac{1 - tcir}{1 - tci_t} p_t^{HXT} \quad (48)$$

where p_t^{HXT} is the pre-tax HICP excluding energy, tci is the current implicit tax rate and tcir is the tax rate in the base year of price indices. We model the seasonal adjusted version of p_t^{HXT}, so-called p_t^{HXST} where the seasonal factors are estimated using a time-varying airline estimation procedure and kept fixed over the forecast horizon. We retain the Calvo price framework and parameters from above:

$$\begin{aligned}\{\theta_p + \omega_p [1 - \theta_p (1-\beta)]\} \Delta p_t^{HXST} - \omega_p \Delta p_{t-1}^{HXST} - \beta\theta_p \Delta p_{t+1}^{HXST} \\ - \ (1-\omega_p)(1-\theta_p)(1-\beta\theta_p)\left(p_t^{HXST*} - p_t^{HXST}\right) = 0 \quad (49)\end{aligned}$$

where p_t^{HXST} is the seasonal adjusted version of p_t^{HXT}, and p_t^{HXST*} is the long-run optimal non-energy HICP and is weighted average of the optimal GDP deflator p_t^* including indirect energy prices and imports deflator excluding energy, p_t^{MN} where the weights ϕ_1 are estimated by OLS and ϱ is set to 0.015 based on input-output tables. In addition, as with the Calvo price equation, we include a labour adjustment factor:

$$p_t^{HXST*} = \phi_1 p_t^{MN} + (1-\phi_1)((1-\varrho)p_t^* + \varrho p_t^{EI}) + a_h (n_t^* - n_t) \quad (50)$$

HICP energy (p_t^{HE}) is modelled as a mark-up of energy prices (or oil) and GDP deflator (p_t):

$$p_t^{HE} = \delta_1 p_t^{EI} + (1-\delta_1)(p_t) \quad (51)$$

The overall HICP p_t^H,then becomes a weighted average of HICP non-energy (post-tax), p_t^{HX} and HICP energy, p_t^{HE} where w_{et} is the weight of HICP energy in the overall HICP.

$$p_t^H \ = \ w_{et} \cdot p_t^{HE} + \ (1 - w_{et}) \cdot p_t^{HX} \quad (52)$$

The consumption deflator is linked via a simple bridge equation to seasonally adjusted HICP. All other domestic deflators (e.g. investment deflator) are specified as quasi-identities, i.e. modelled as weighted averages of domestic costs (measured by the value-added deflator defined above) and import prices (measured by the import deflator).

A.3.4 Wage Setting

Wages are also set via a staggered with three-valued Calvo-signal where part of unions keep wages fixed, θ_w, another part changes wages following backward-looking rule, ω_w, and the rest set them optimally:

$$\begin{aligned}\{\theta_w + \omega_w [1 - \theta_w (1 - \beta)]\} \Delta w_t = \omega_w \Delta w_{t-1} + \beta \theta_w E_t w_{t+1} \\ + \ (1 - \omega_w)(1 - \theta_w)(1 - \beta\theta_w)\{w_t^* - w_t\}\end{aligned} \tag{53}$$

where w_t = log of compensation per worker, and β, the discount factor, = 0.99. For the optimal frictionless wage rate, w_t^* we assume that part of the unions are utilitarian, a_{wu}, whist the rest are non-utilitarian:

$$\begin{aligned}w_t^* = a_{wu} \left[\left(p_t^C + c_t - n_t^F\right) - \log\left(\sigma - 1 + \frac{F_N^{CES}}{Y_t/N_t}\right) + \log \frac{\sigma\kappa}{\bar{h}(time)} \right] \\ + (1 - a_{wu}) \left[p_t + \log\left(\frac{F_N^{CES}}{1+\mu}\right) + \chi \log\left(\frac{F^{-1}(K_t, Y_t)}{\varpi \cdot N_t^F}\right)\right]\end{aligned} \tag{54}$$

$$F_N^{CES} = (1 - \pi_0)\left(\frac{Y_0}{N_0}\Gamma_N(t, t_0)\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Y_t}{N_t}\right)^{\frac{1}{\sigma}} \tag{55}$$

where c_t = consumption (log), p_t^C= consumption deflator (log) ; $F^{-1}(\cdot)$ = inverted CES production function (desired number of workers); N_t^F = labour force and the gap between optimal labour demand and supply measures the wage drift effect. [13].

A.3.5 Inventory investment

The desired equilibrium inventory stock KII* is based on the estimated CES production function:

$$KII = a + bF(K, N, t) \tag{56}$$

and inventories from the dynamic equation:

$$\begin{aligned}(1 - r \cdot A)\Delta KII_t = (1 - 2A)\Delta KII_t^* \\ - A[\Delta S_t - \Delta KII_t - \Delta F(\cdot)] + (1 - r) A [\Delta S_{t+1} - \Delta KII_{t+1} - \Delta F(\cdot, t+1)]\end{aligned} \tag{57}$$

S= Sales (Private consumption + exports)

A.4 Households

A forward looking aggregate consumption function with strong backward-looking frictional elements:

$$\begin{aligned}E_t \left\{ 1 + \frac{\gamma}{1.01}\left[(1-\pi)^2 aR_t - (0.01 + \pi)\left(1 - \frac{(1-\pi)a}{1.01}\right)\right]\right\} \frac{C_t}{Y_t} \\ - (1-\pi)\gamma \frac{R_t C_{t+1}}{Y_t} - a(1-\pi)\frac{C_{t-1}}{Y_t} - \left(\frac{1.01 - (1-\pi)a}{1.01}\right)\left(\frac{0.01+\pi}{1.01}\right) \\ \left\{\left(\frac{1}{1-\pi} - \gamma\right)\left(\frac{V_{t-1}}{Y_t} + 1\right) + \left(\frac{\Lambda(1.01 - (1-\pi)\gamma)}{0.01+\pi} - \frac{1}{1-\pi}\right)\right\} z_t = 0\end{aligned} \tag{58}$$

[13] For the simulations in the paper we assume all unions to be non-utilitarian.

where C_t is consumption, Y_t labour income net of payroll taxes minus transfers, V_{t-1}total wealth in the beginning of period. Parameter π= death probability; γ= forward information parameter; a= habit persistence parameter; Λ= income risk parameter and z_t refers to the set of instruments.

Total wealth, V_{t-1} at the beginning of period is :

$$V_{t-1} = \frac{P_I}{P_C}\left[\begin{array}{c} \left(\frac{P_S}{P_I}\right)^{b0}(1-s_H)\left(KSR_{t-1}-KGR_{t-1}\right)+ \\ \left(\frac{P_H}{P_I}\right)^{b1}s_H\left(KSR_{t-1}-KGR_{t-1}\right) \end{array}\right] + \frac{GDN_{t-1}+NFA_{t-1}}{P_C} \tag{59}$$

where P_I, P_C , P_S and P_H are investment deflator, consumption deflator, stock prices and the market price of housing, respectively. KSR is total capital stock, KGR is government sector capital stock and s_H is the share of housing stock of total private capital stock, and $b0$ and $b1$ are the elasticity parameters

A.5 Trade formation

2-equation system for the export volume and export price:

$$\left(\frac{P_X X}{P_{cx} MF}\right) = a + b \cdot f(time) - (\phi - 1)\left(p_X - p_{CX}\right) \tag{60}$$

$$p_X = a + \frac{1+(a+b\cdot f(time))/(\phi-1)}{2+(a+b\cdot f(time))/(\phi-1)}\left((1-a_x)(w-mpn)+a_x p_M\right) + \frac{1}{2+(a+b\cdot f(time))/(\phi-1)}p_{CX} \tag{61}$$

Where P_X = Export deflator (lower case refers to log); X = Export volume; P_{CX}= the external competitor export prices (lower case refers to log); MF = the world demand for exports; w = compensation per worker (log); mpn = marginal product of labour (log); p_M = import deflator (log), a = point market share; $\phi > 1$ is the representative point price elasticity of exports; b if different from zero measures the deviation of income elasticity of export demand from unity; and a_X = import content of exports (input-output estimate).

The dynamic export volume and export price equation follow conventional error correction equations.

The long-run equilibrium aggregate demand for imports:

$$m^* - e_{MR} = k(p_{MD} - p_{MN}) + b(x - e_{MR}) \tag{62}$$

where e_{MR} is the demand indicator for imports (import content weighted index of domestic demand); p_{MD} is domestic prices (gdp deflator); and p_{MN} is import prices excluding energy; (all logs). The dynamic equation follows a standard EC specification.

The import deflator excluding energy p_{MN} depends on the GDP deflator net of indirect taxes (p) and the competitors' import price (p_{CM})

$$p_{MN} = \phi_1(p + \log(1 - TX_1)) + (1 - \phi_1)p_{CM} \tag{63}$$

The dynamic equation follows a standard EC specification. The trade balance and net factor income equal the current account balance, which in turn is cumulated to give the stock of net foreign assets.

A.6 Parameter Estimates

		FR	DE	IT	ES	NL
Employment	a_h/a_N	0.0225	0.0388	0.0559	0.1414	0.0396
Investment	$1-\Lambda^B$	0.5087	0.4517	0.5067	0.5876	0.376
	b_K	0.7879	0.695	0.7796	0.7834	0.6267
	$1/a_K$	0.0126	0.0122	0.0129	0.0139	0.0163
	Λ^I	0.25	0.26	0.2	0.25	0.33
	a	0.8	0.8	0.855	0.8	0.75
NKPC	θ_p	0.7455	0.7646	0.7251	0.6705	0.6926
	ω_p	0.3531	0.3831	0.3235	0.2543	0.2807
	a_h	0.7515	0.399	0.1797	0.4212	0.4109
HICP energy/non energy	ϕ_1	0.21	0.15	0.1	0.033	0.17
	δ_1	0.33	0.21	0.15	0.21	0.35
NKPC Wage	θ_w	0.7364	0.7982	0.7301	0.7471	0.7077
	ω_w	0.3396	0.4412	0.3306	0.3555	0.2999
	χ	0.1	0.2	0.1	0.15	0.11
	a_{wu}	0.3	0.15	0.5366	0.2234	0.1745
Inventory	A	0.4705	0.3374	0.3501	0.3038	0.3802
Consumption	π	0.005	0.007	0.0053	0.0053	0.0053
	γ	0.8138	0.7923	0.6144	0.7192	0.6665
	a	0.9303	0.8715	0.9467	0.864	0.9098
	Λ	0.9031	0.8002	0.6711	0.8522	0.7196
	b0,b1	-	0.2828	0.2036	0.2388	0.336
2-equation export system	a	1.08	1.131	1.051	1.05	1.025
	ø	1.021	1.056	1.220	1.314	1.345
	a_X	0.385	0.167	0.400	0.465	0.672
Dynamic export	MF	0.846	0.745	0.663	0.68	0.926
	P_{CX}/P_X	-0.288	-0.239	-0.573	-0.399	-0.285
	Δx^*_{t-1}		-		0.271	
	$EC-term$	-0.152	-0.42	-0.157	-0.165	-0.088
Dynamic export price	ΔP^{x*}		0.795	0.529	0.726	0.93
	ΔP^{x*}_{t-1}		0.177			
	ΔP^{x}_{t-1}			0.2661		
	$\Delta P^x_{t-2}-\Delta P^x_{t-3}$	0.1333				
	$EC-term$	-0.328	-0.097	-0.171	-0.548	-0.11
Import	k	-0.782	-0.711	-1.001	-0.979	-0.576
	b	0.312	0.284	-	0.293	0.072
Dynamic Import	Δm^*	-	0.813	0.568	0.561	0.722
	Δm_{t-2}	-	-	0.132	-	-
	Δe_{MR}	0.642	-	-	-	-
	$EC-term$	-0.235	-0.121	-0.117	0.164	-0.229
Import price	ϕ_1	0.333	0.416	0.225	0.312	0.667
Dynamic Import price	Δp^*_{MN}	0.498	0.673	0.427	0.669	0.74
	$\Delta p^*_{MN,t-1}$	0.344	0.237	-	-	-
	$\Delta p^*_{MN,t-2}$	0.179	-	-	-	-
	$\Delta p_{MN,t-1}$	-	0.189	0.158	-	0.26
	$EC-term$	-0.118	-0.148	-0.235	-0.417	-0.16

A.7 Governments and Monetary Authority

Government receipts are split into a number of components: direct taxes on households earned income (T_f); direct taxes on firms (T_o), and indirect taxes (T_I), and other public income (OI_G) and (net) transfers (TR_F). Transfers as a proportion of nominal GDP (tr) are modelled as:

$$TR_F = tr.PY - \varkappa W(N - N_{bas}) \tag{64}$$

where $\varkappa$ is calibrated to 0.7. In addition, the fiscal authority has net interest payments on government debt (IN_G) and different types of primary expenditure categories, namely, government consumption (G_N) and government investment (I_{NG}) which are exogenous in real terms. The government consumption deflator follows both the price of home produced goods with a weight of δ^G and of imported goods with a weight of (1-δ^G) .The public deficit (D) each period is then the difference between the recipts and expenditures:

$$D = T_F + T_o + T_I + OI_G - TF_F - IN_G - G_N - I_{NG} \tag{65}$$

The fiscal authority's is faced by a budget constraint which says that public debt B_t is the cumulative sum of past public deficits (D) i.e.

$$B_t = B_{t-1} + D_t \tag{66}$$

The fiscal policy rule is based on a reaction of personal income taxes to the deviation of the government's debt to GDP ratio from its predetermined target and which contributes to adjustment towards the stock-flow equilibrium in the long-run.

$$\Delta\tau_t = \varphi_1(b_{t-1} - \bar{b}) + \varphi_2\Delta b_{t-1} \tag{67}$$

where τ_t is the personal income tax rate (T_F/Y), and b_t is the government debt to GDP ratio (B/Y), and $\bar{b}$ is the target. The parameters $\varphi_1$and φ_2 are set at 0.003 and 0.03 respectively.

The monetary policy rule follows a simple Taylor rule specification in which the short term interest rate i_t,is determined by the inflation gap, where this target level of inflation ($\Delta\hat{\bar{p}}_t$) is set to 2 per cent per annum and the output gap ($y_t - \bar{y}_t$) along with the lagged interest effect where ε_t is a serially uncorrelated shock to the interest rate:

$$i_t = (1 - 0.25) * i_{t-1} + 0.25 * (4 * 1.5 * (\Delta p_{t+1} - \Delta\hat{\bar{p}}_t) + 0.5 * (y_t - \bar{y}_t)) + \varepsilon_t \tag{68}$$

A.7.1 Financial Markets

The specification of the long-term interest rate equation is:

$$l_t = 0.7 * l_{t-1} + 0.3 * i_t \tag{69}$$

The exchange rate follows a standard real UIP equation.

$$e_t = e_{t+1} + (r_t^f - r_t)/400 \tag{70}$$

where e is the (log of) the real exchange rate and r_t^f.is the foreign real interest rate.

B Figures

Figure 1: Shock to short-term interest Rates (50bp)

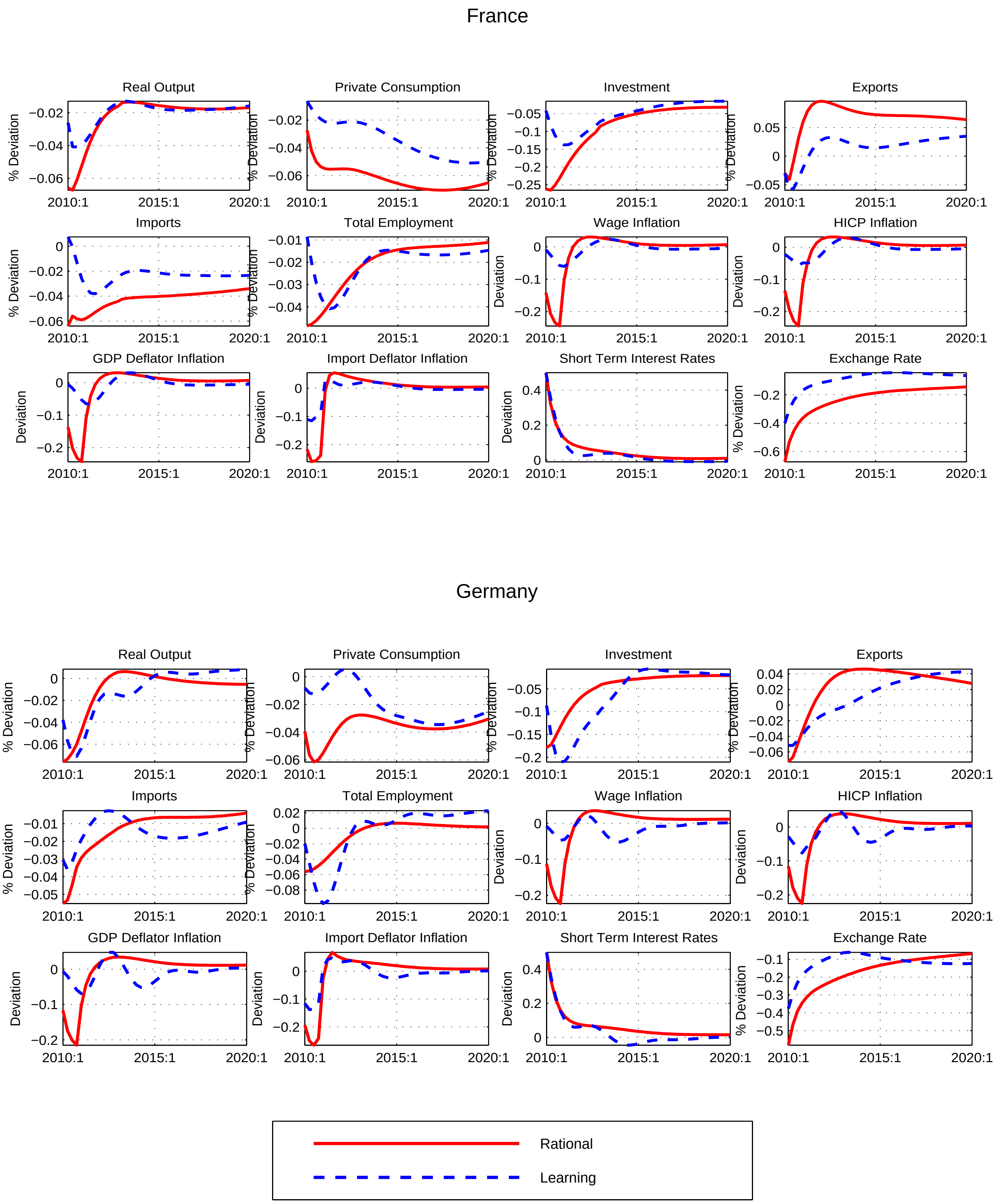


Figure 1: Shock to short-term interest Rates (50bp)

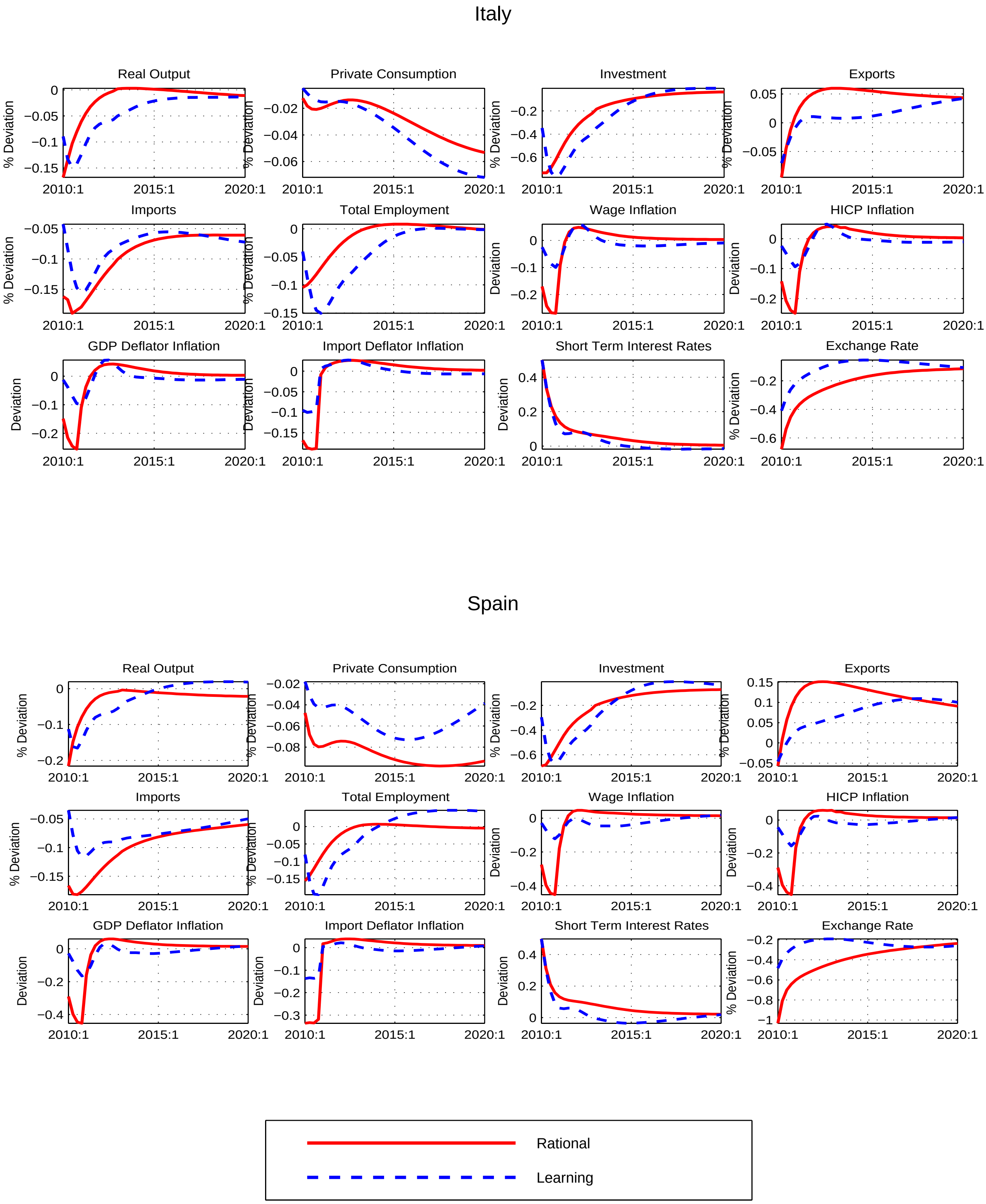
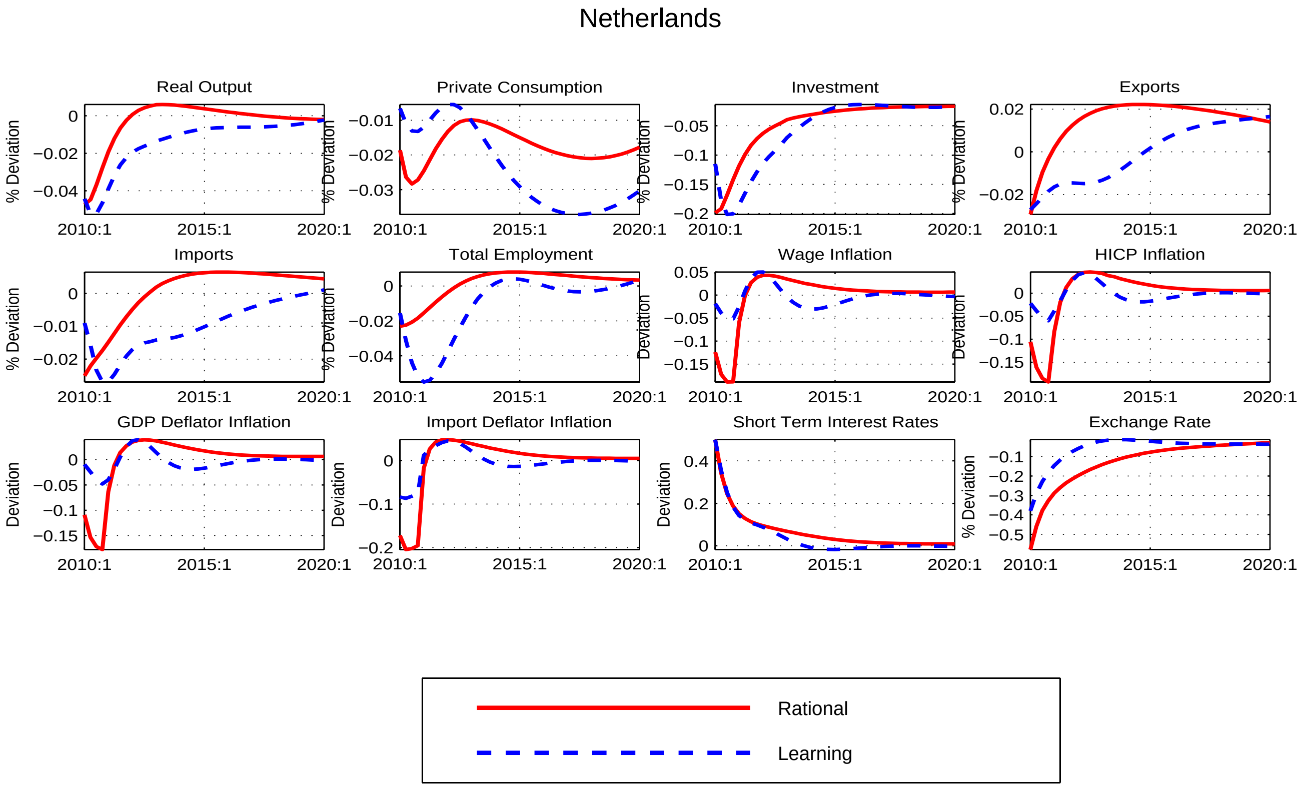


Figure 1: Shock to short-term interest Rates (50bp)



Real variables, employment and exchange rate are presented as percentage deviations from baseline, all other variables are in difference from baseline (either percentage points or percent of GDP). Inflation is computed as year-on-year rate of change.

Figure 2: Permament Government consumption

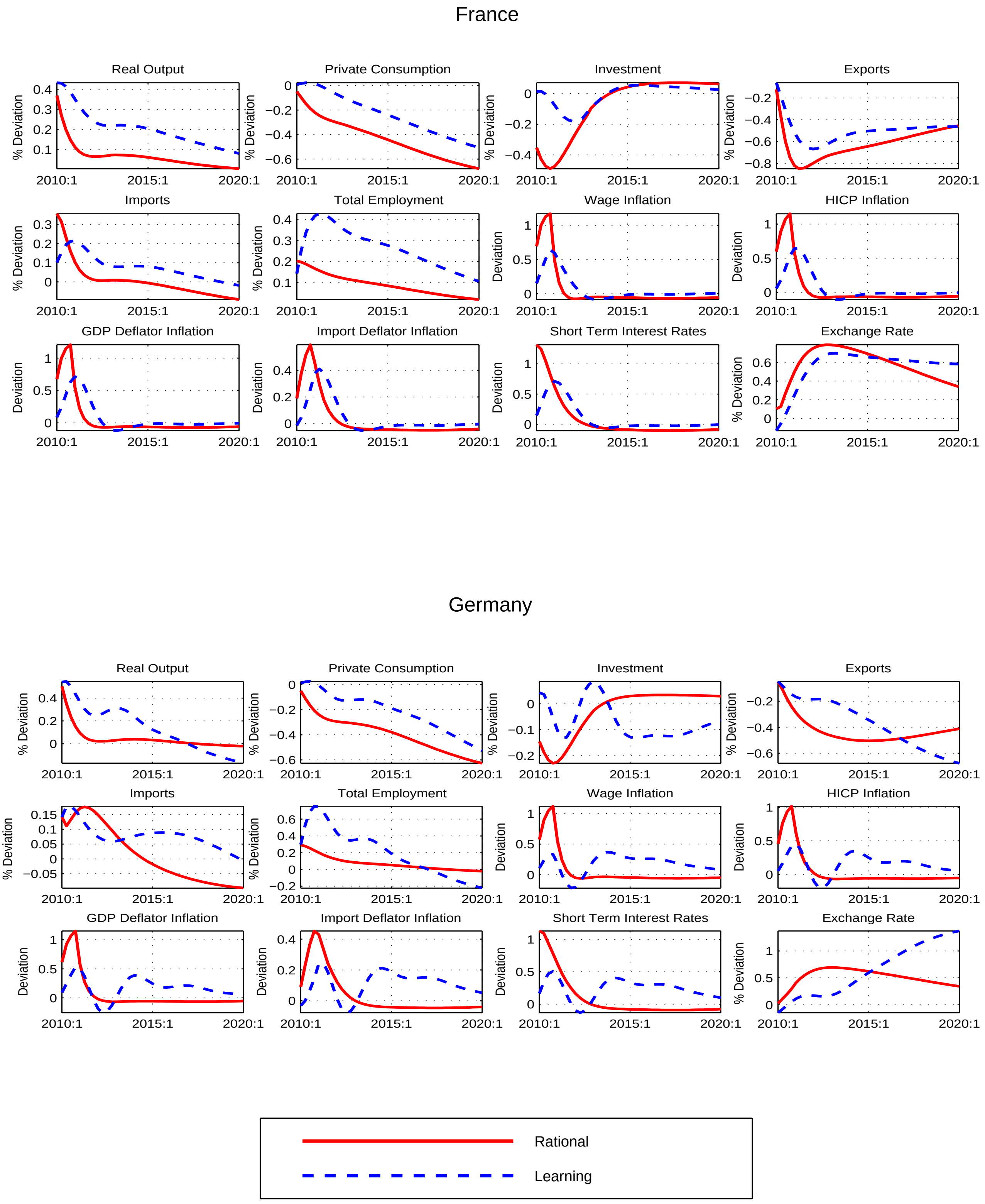
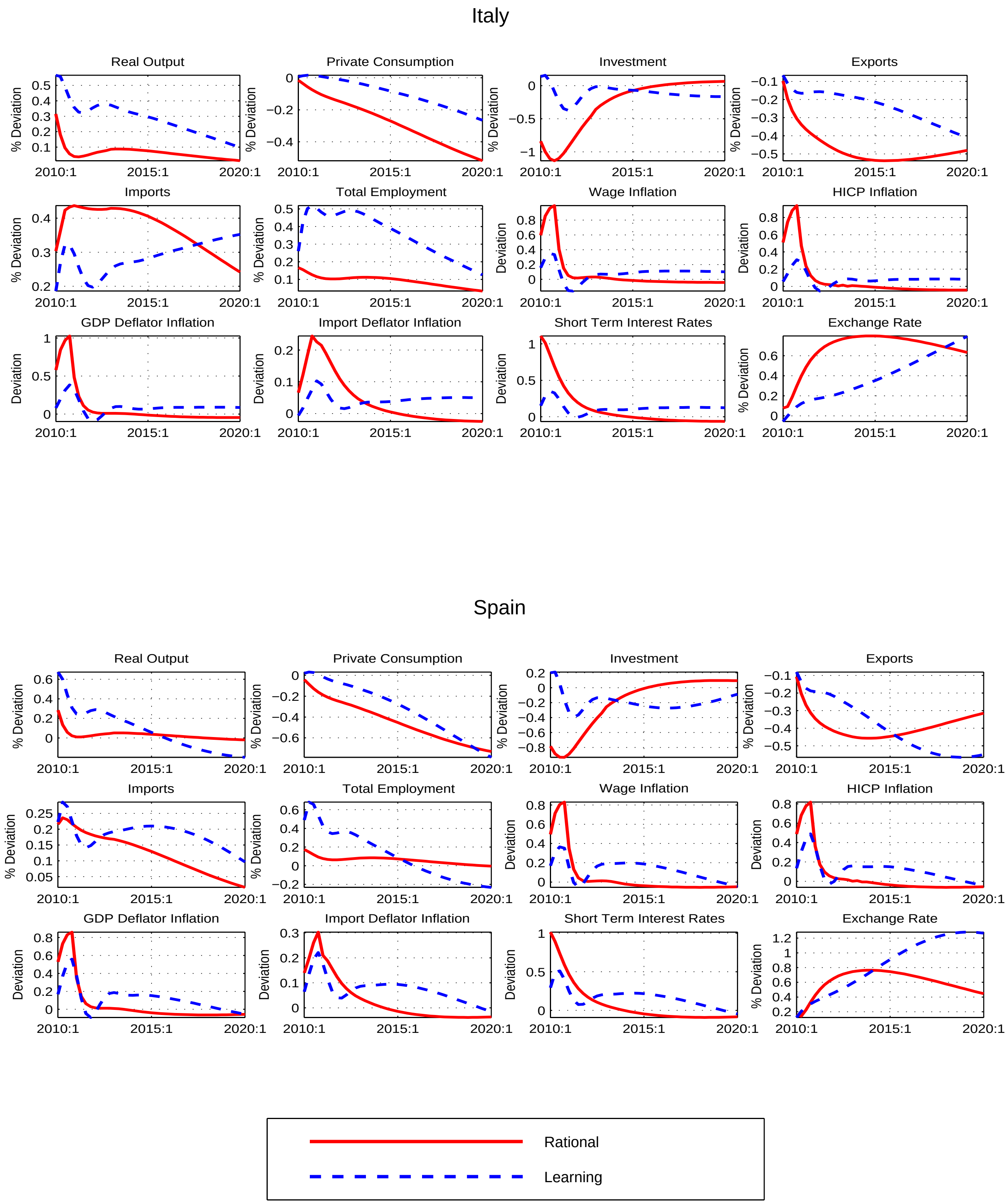


Figure 2: Permanent Government consumption



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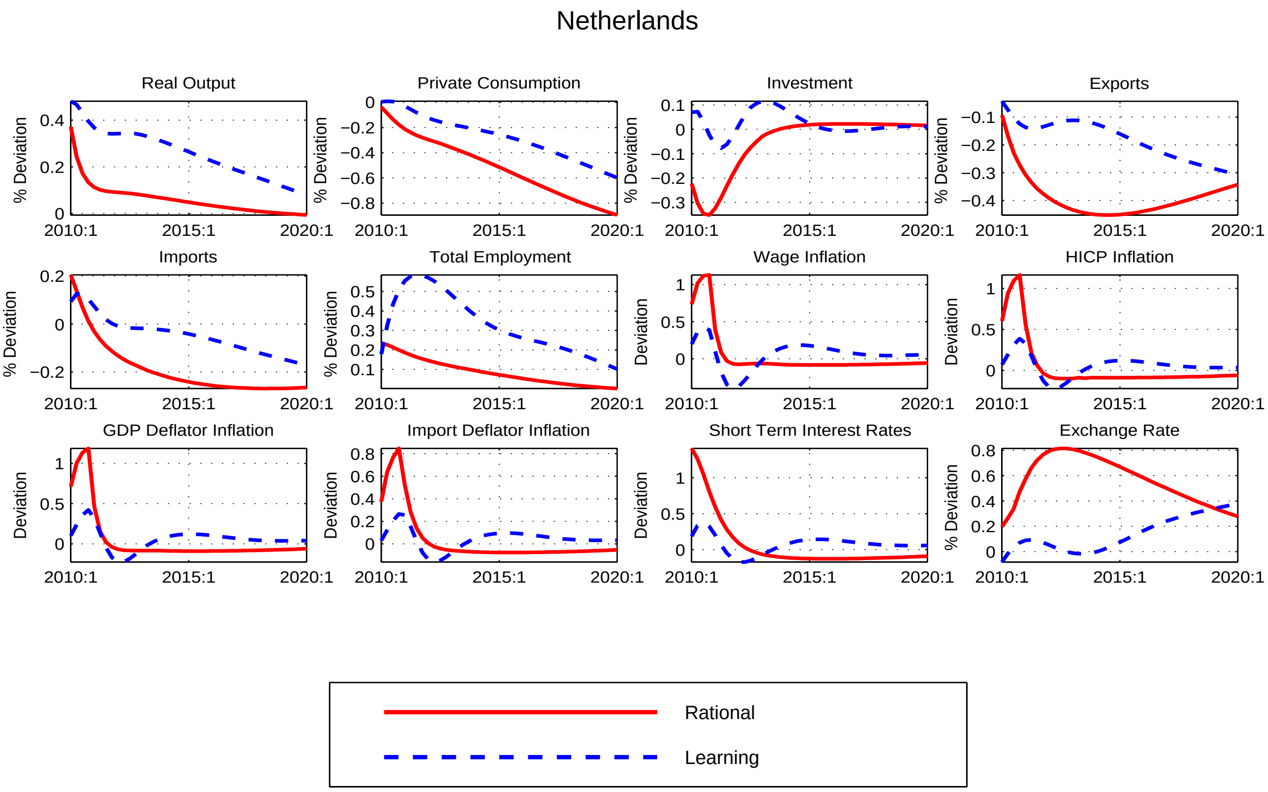
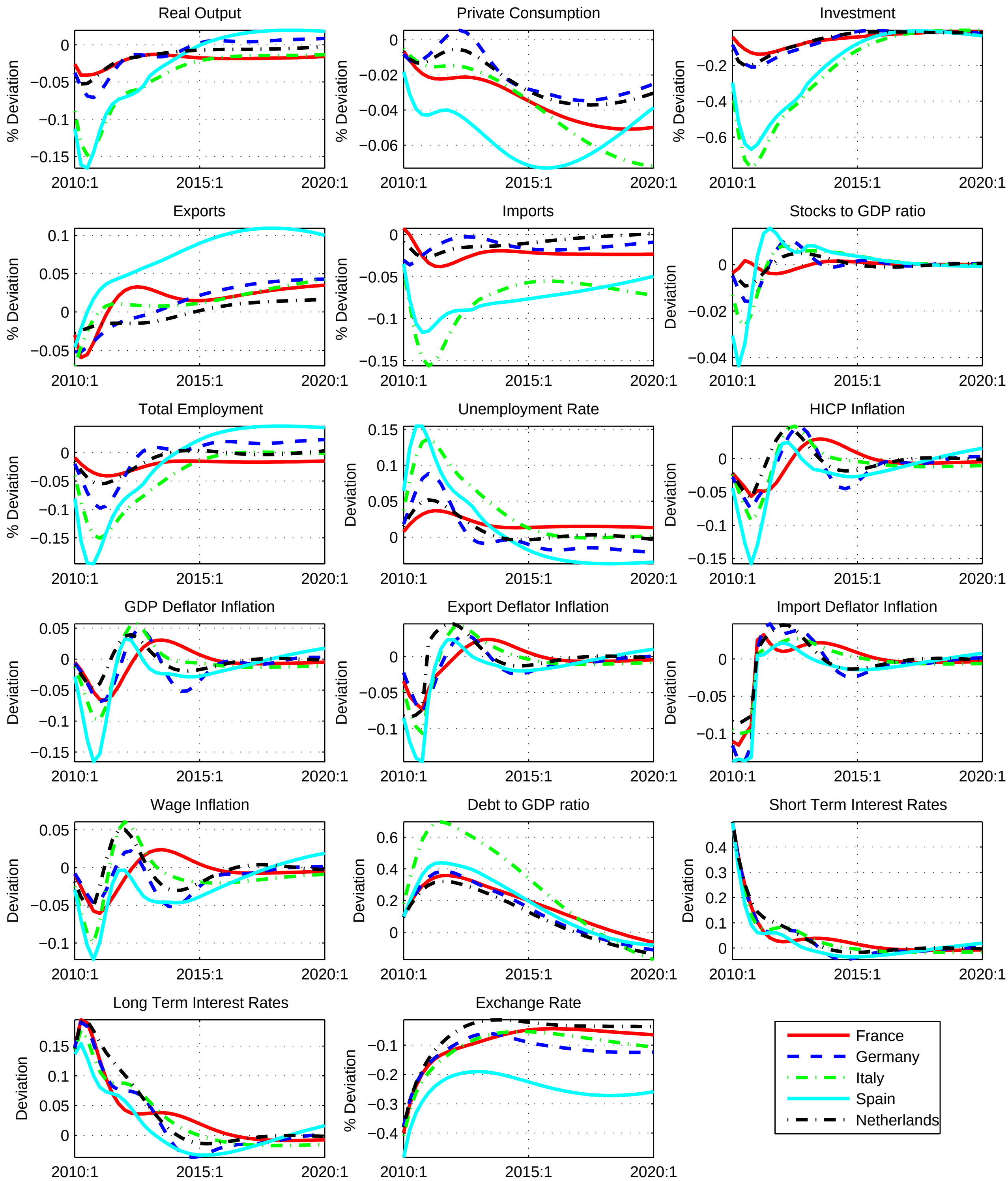


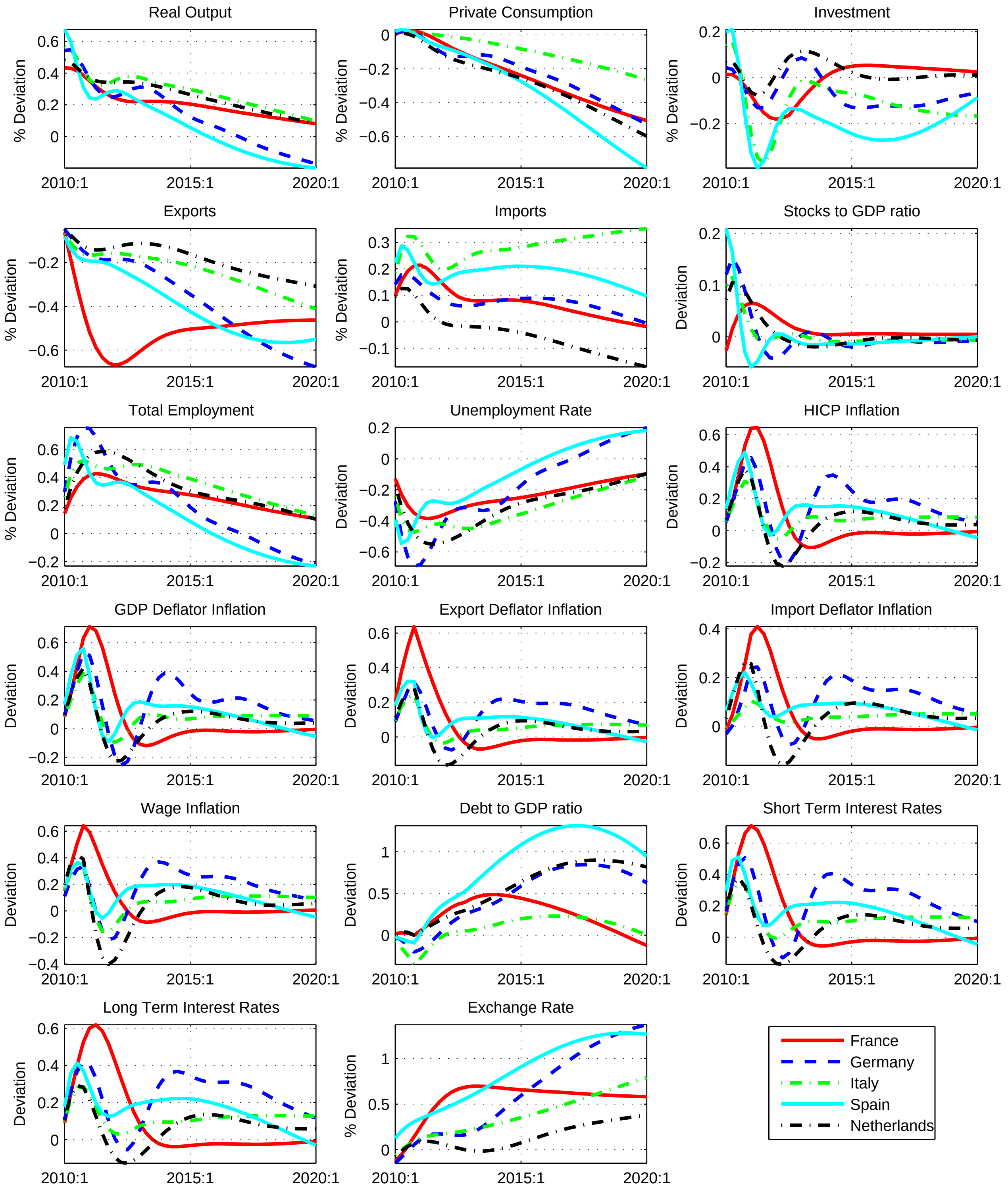
Real variables, employment and exchange rate are presented as percentage deviations from baseline, all other variables are in difference from baseline (either percentage points or percent of GDP). Inflation is computed as year-on-year rate of change.

Figure 3: Short−term Interest rate shock (50bp) − Learning



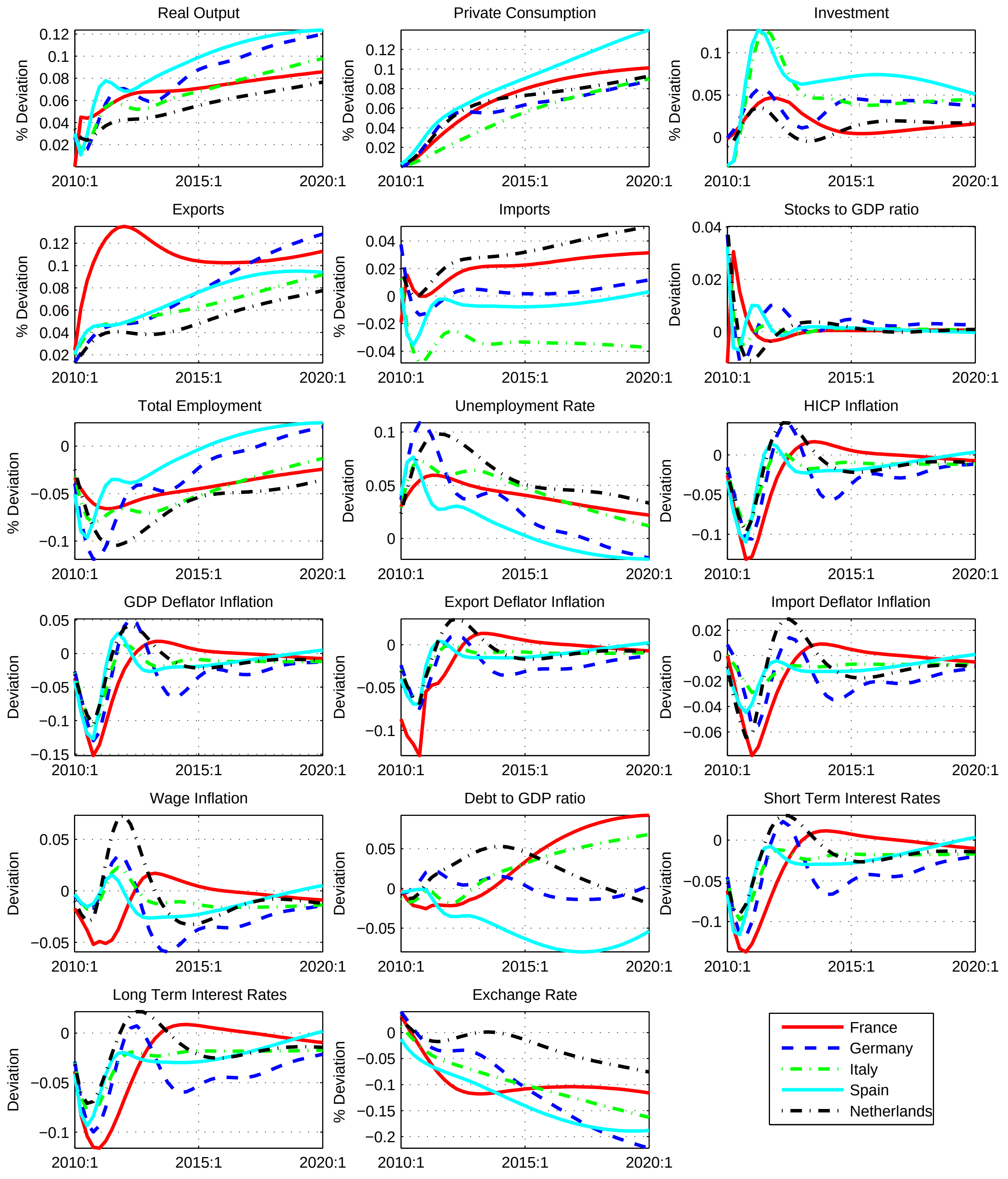
Real variables, employment and exchange rate are presented as percentage deviations from baseline, all other variables are in difference from baseline (either percentage points or percent of GDP). Inflation is computed as year-on-year rate of change.

Figure 4: Permanent Government Consumption (0.5% GDP) − Learning



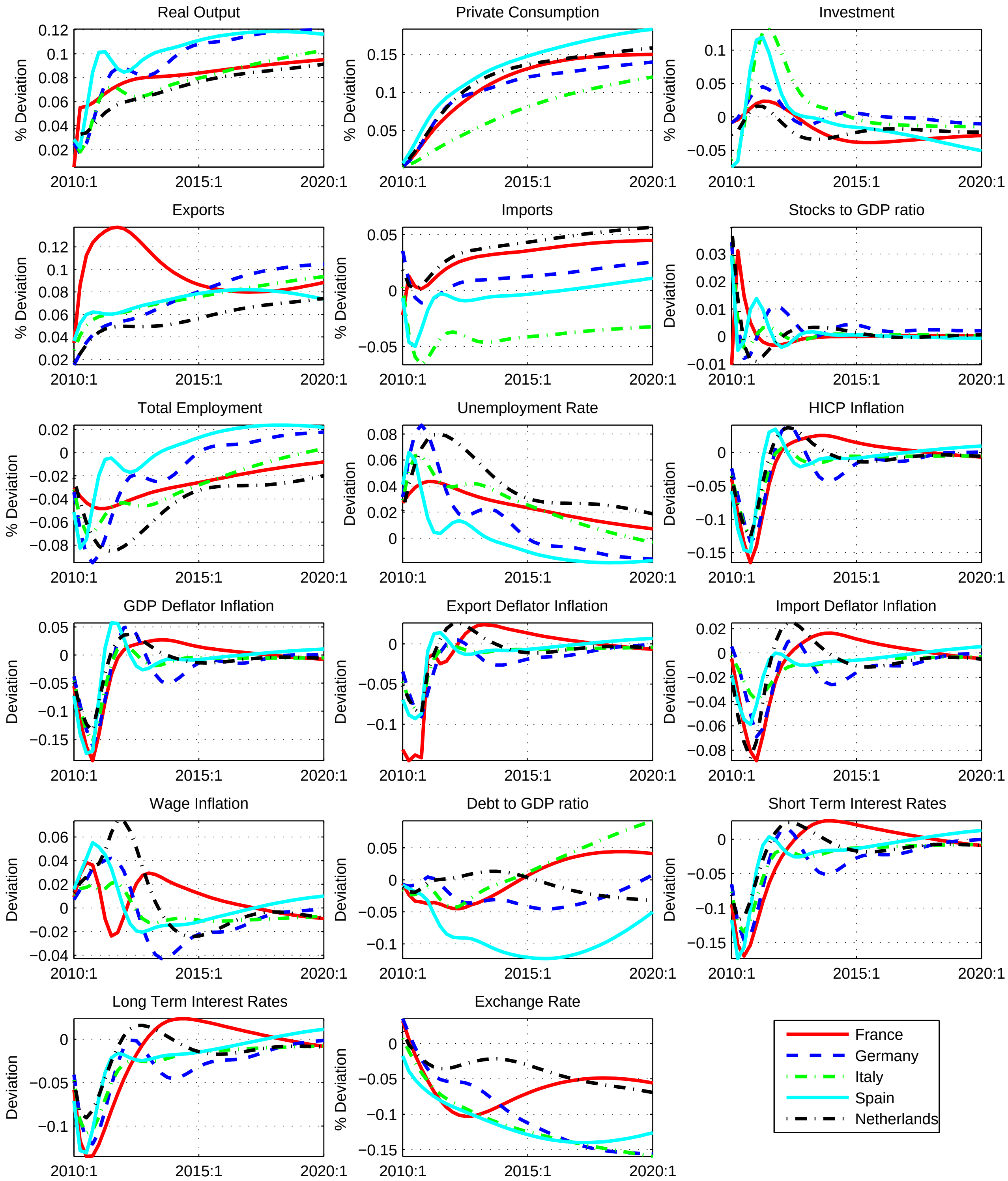
Real variables, employment and exchange rate are presented as percentage deviations from baseline, all other variables are in difference from baseline (either percentage points or percent of GDP). Inflation is computed as year-on-year rate of change.

Figure 5: Labour Augmenting Technology shock (0.1% GDP) − Learning



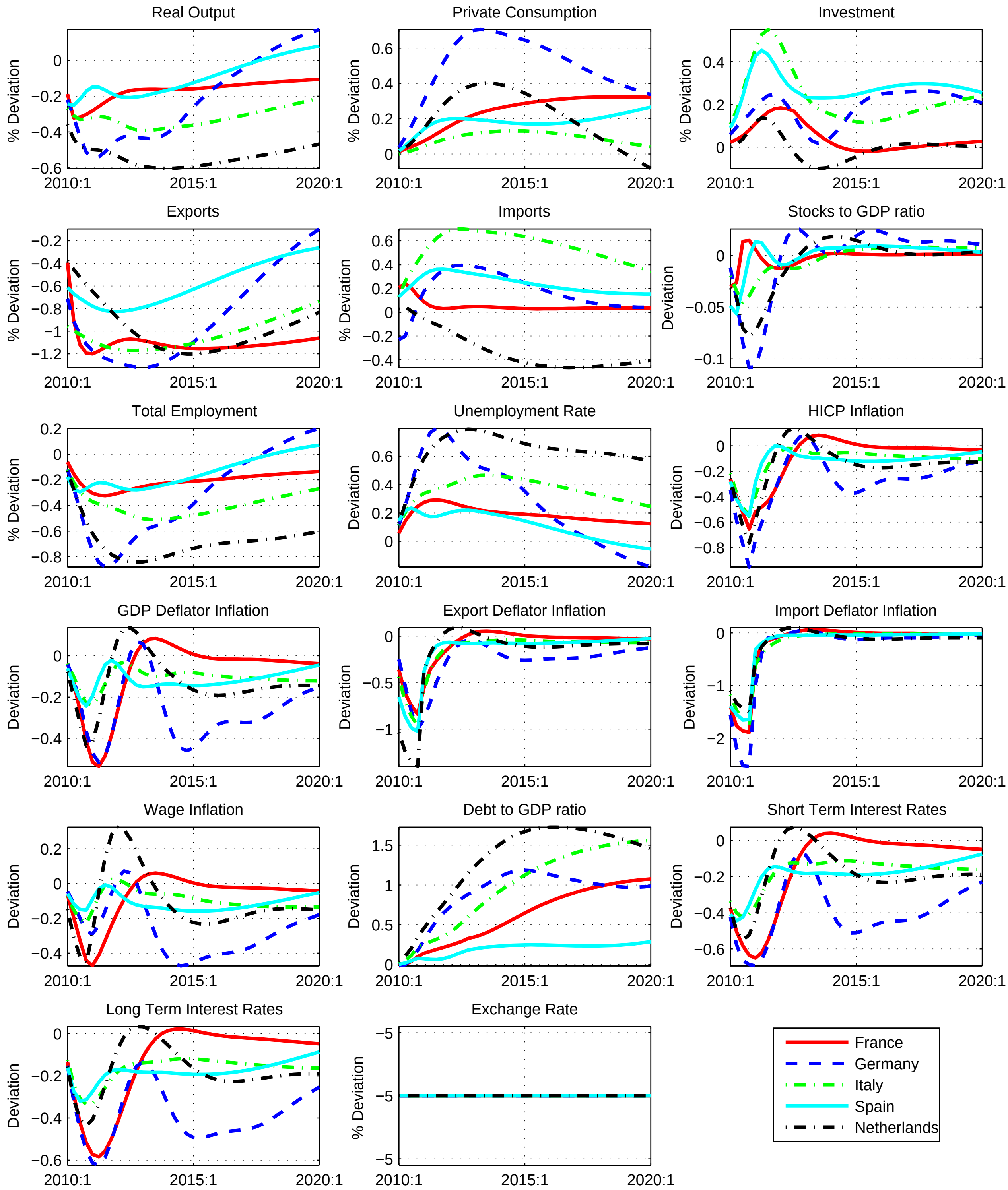
Real variables, employment and exchange rate are presented as percentage deviations from baseline, all other variables are in difference from baseline (either percentage points or percent of GDP). Inflation is computed as year-on-year rate of change.

Figure 6: Capital Augmenting Technology shock (0.1% GDP) − Learning



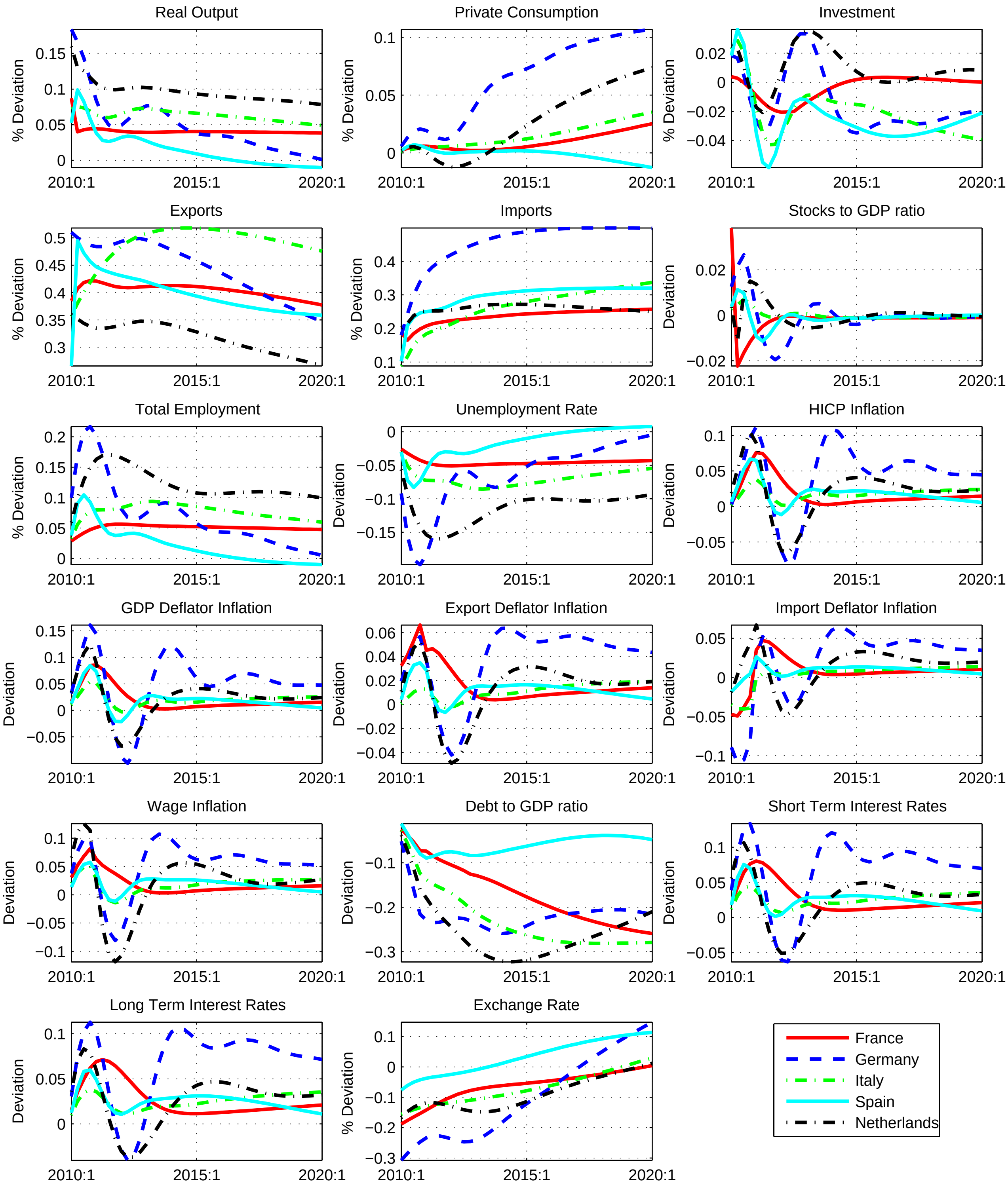
Real variables, employment and exchange rate are presented as percentage deviations from baseline, all other variables are in difference from baseline (either percentage points or percent of GDP). Inflation is computed as year-on-year rate of change.

Figure 7: Permanent Exchange rate shock (5%) − Learning



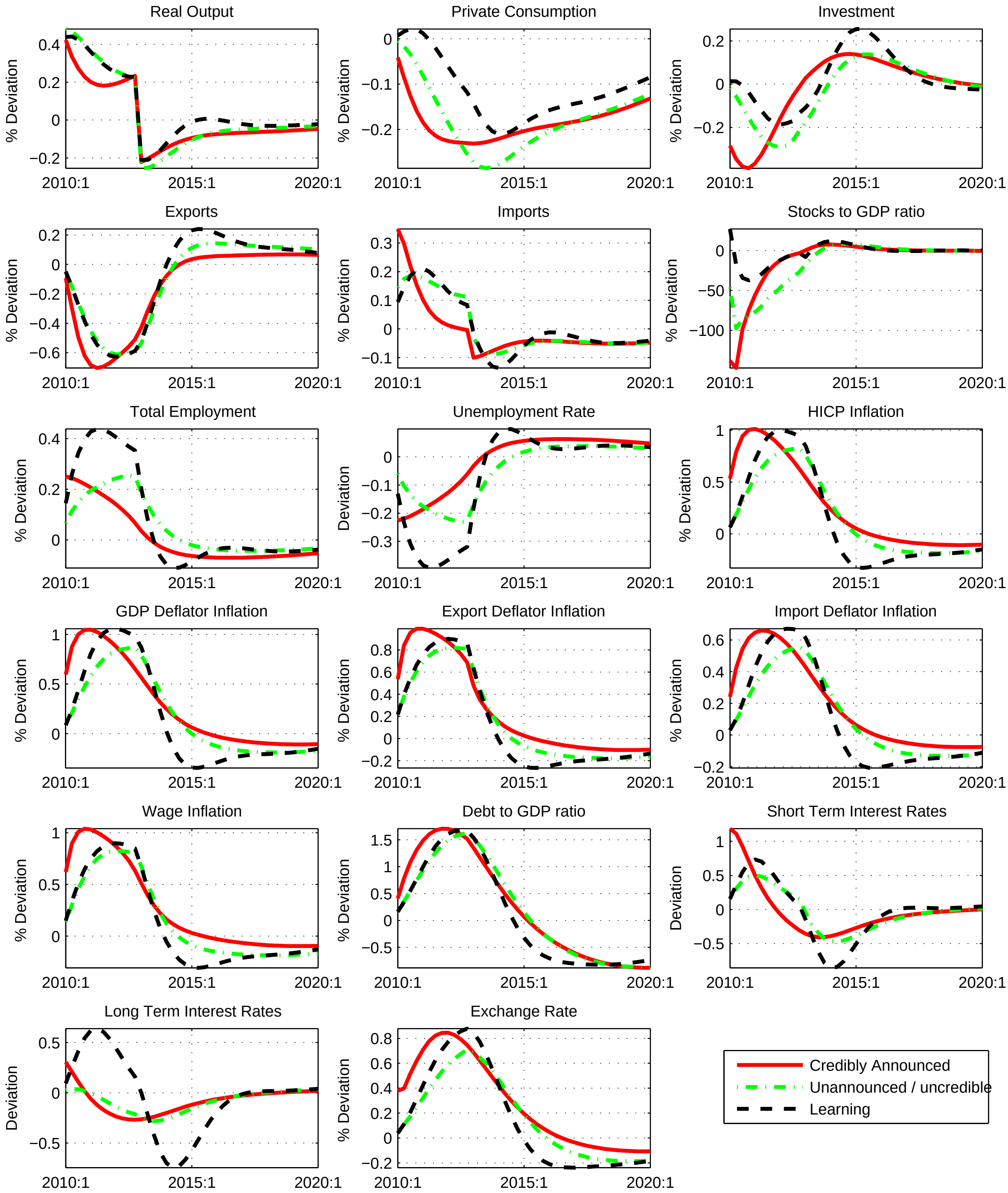
Real variables, employment and exchange rate are presented as percentage deviations from baseline, all other variables are in difference from baseline (either percentage points or percent of GDP). Inflation is computed as year-on-year rate of change.

Figure 8: Permanent World Demand shock (1%) − Learning



Real variables, employment and exchange rate are presented as percentage deviations from baseline, all other variables are in difference from baseline (either percentage points or percent of GDP). Inflation is computed as year-on-year rate of change.

Figure 9: Announced, unannounced and learning
Government Consumption (0.5% of GDP) for 3 years − France



Real variables, employment and exchange rate are presented as percentage deviations from baseline, all other variables are in difference from baseline (either percentage points or percent of GDP). Inflation is computed as year-on-year rate of change.

Figure 10: Shock to Interest rates (50 basis points):
Exogenous exchange rates; Impact on Real Output

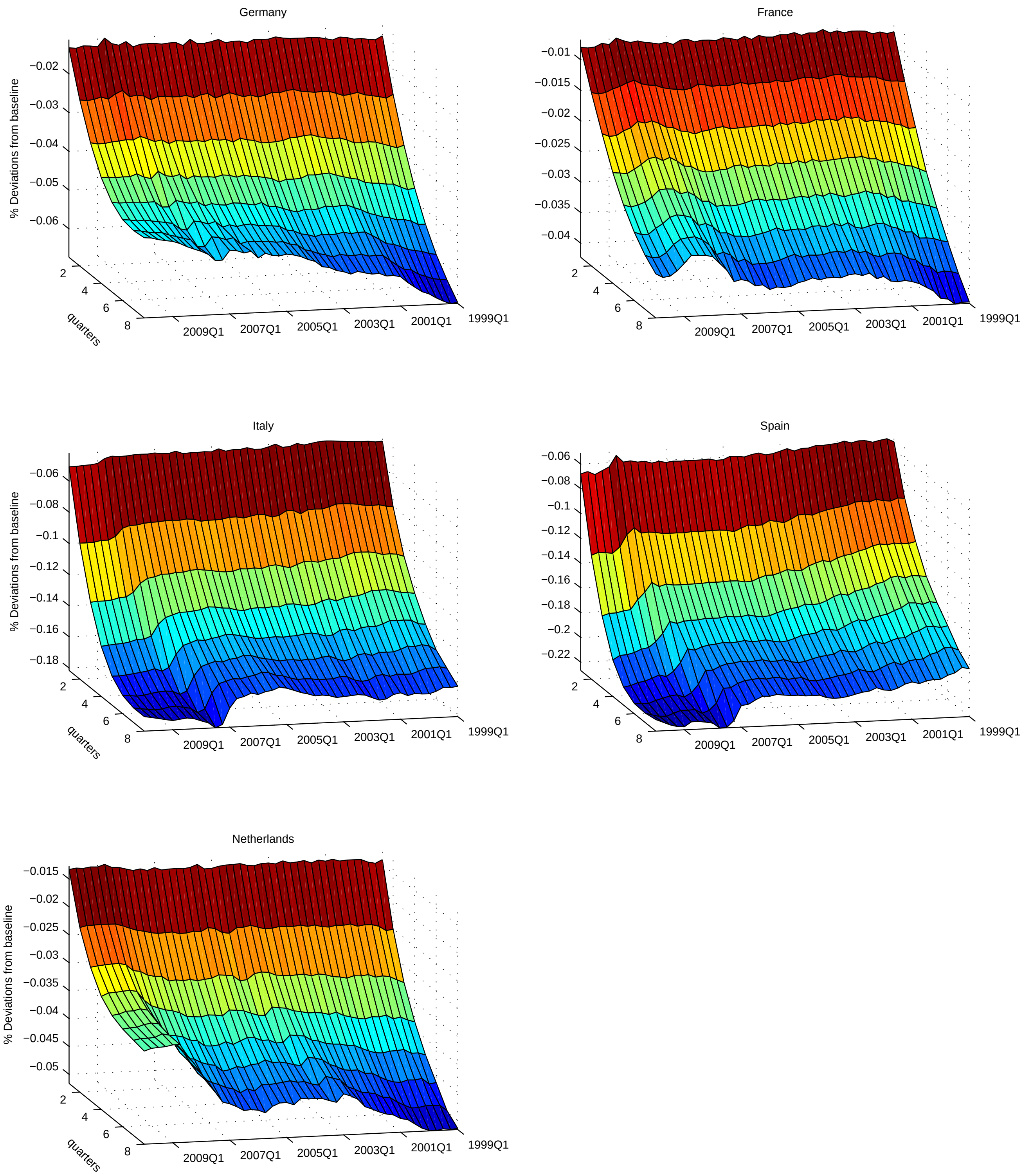


Figure 11: Shock to Interest rates (50 basis points):
Exogenous exchange rates; Impact on HICP Inflation

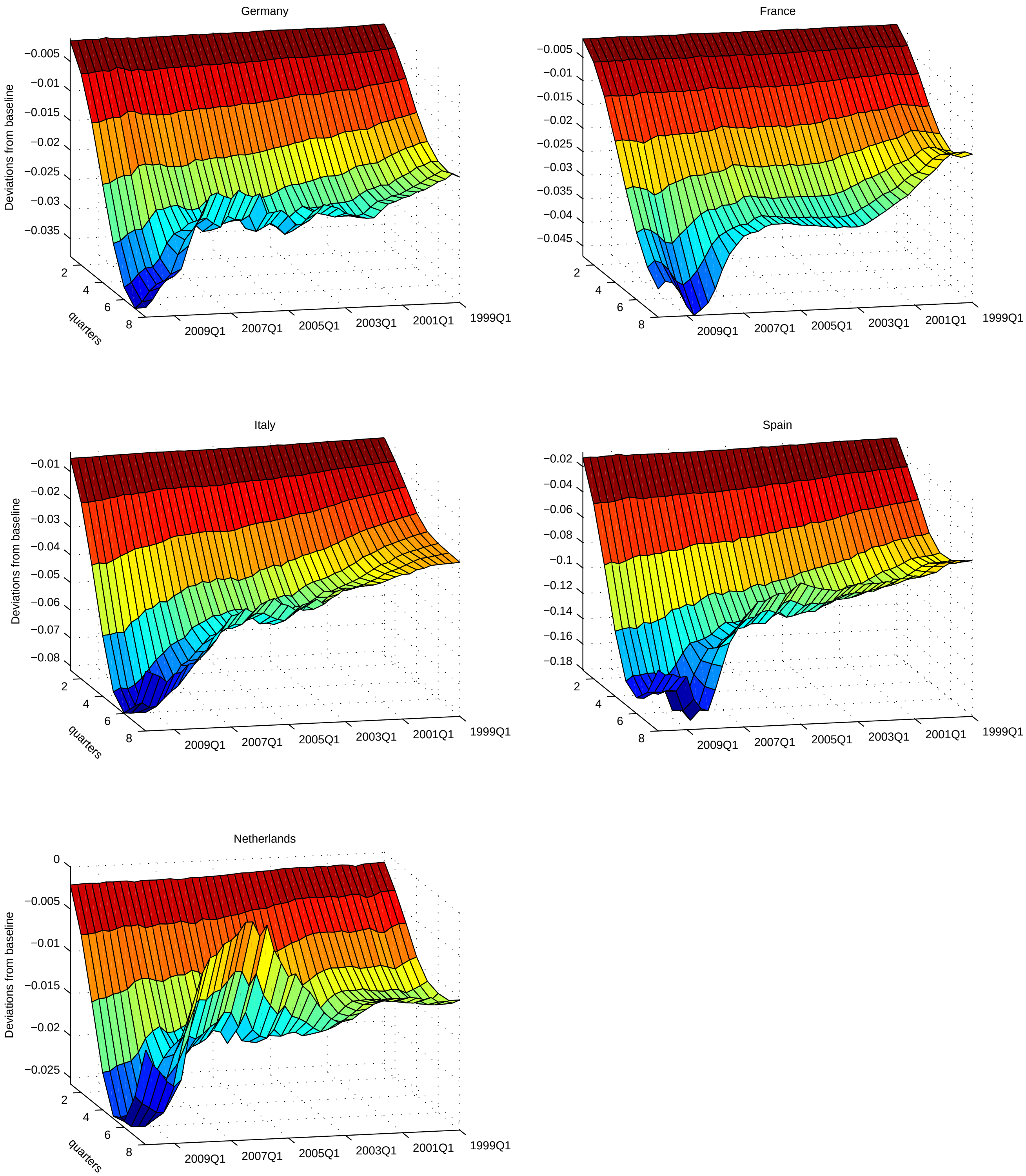


Figure 12: Permanent shock to government expenditure (0.5 % GDP): Exogenous interest rates; Impact on Real Output

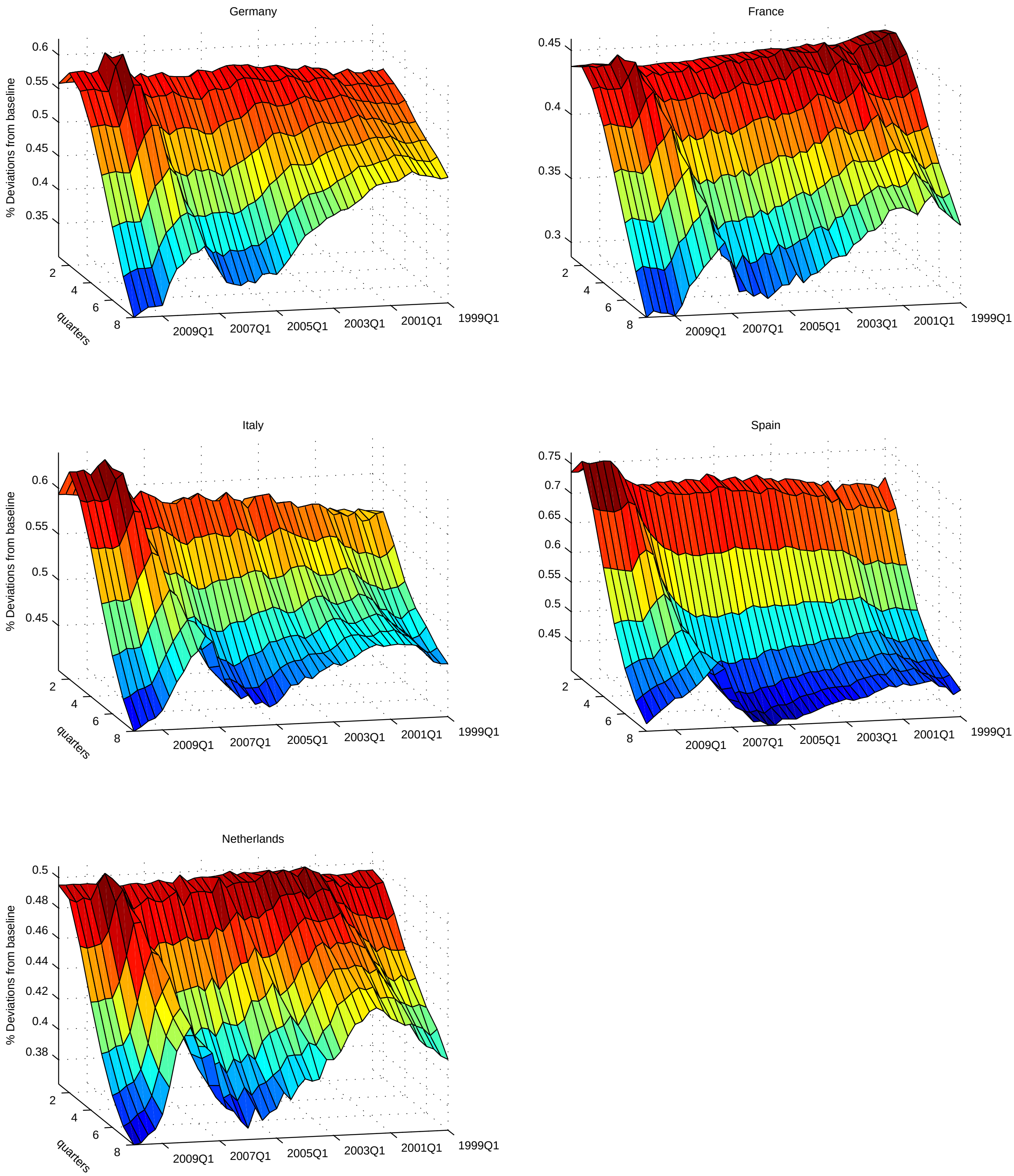


Figure 13: Permanent shock to government expenditure (0.5 % GDP):
Exogenous interest rates; Impact on HICP Inflation