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HOUSEHOLD LEVERAGE

by Stefano Corradin

In 2012 all ECB publications feature a motif taken from the €50 banknote.

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Abstract

I propose a life-cycle model where a finitely lived risk averse agent finances her housing investment choosing to provide a down payment. After signing the mortgage contract, the agent may strategically default and move into the rental market. Risk neutral lenders efficiently price mortgages charging a default premium to compensate themselves for expected losses due to default on a mortgage. As a result, mortgage value and amount of leverage are closely linked. An alternative is for the agent to rent the same house, paying a rent fully adjustable to house prices. The rent risk premium is set such that the agent is indifferent *ex ante* between owning with a mortgage and renting. Three main results arise. First, the optimal down payment and the house price volatility are positively related. The higher the house price volatility, the higher the down payment the agent provides to decrease the volatility of the equity share in the house. Second, in the presence of borrowing constraints, a higher risk of unemployment persistence and/or a substantial drop in labor income decreases the leveraged position the agent takes. Third, ruling out the effect of taking costly leverage on owning a house significantly biases the results in favor of owning over renting.

JEL Classification: G21, E21
Keywords: Default premium, rent risk premium, loan to value ratio, loan to income ratio and negative home equity.

1 Non-technical summary

Households leverage played a key role in the financial crisis of $2007-09$. According to Mian and Sufi (2010), after remaining relatively constant from 1998 to 2002, total mortgage debt grew by 34% from 2002 to 2006, while the total debt to income ratio increased by 0.8 from 2002 to 2006, which is more than one half standard deviation of the 1997 level. The bursting of the housing bubble in combination with a rapid expansion in the use of leverage in the years leading up to the crisis was a primary driver of the recession that began in December 2007. The widespread increase in default rates and resulting losses in mortgage backed securities was the result of falling house prices and severe falls in labor income. These events illustrate the need to understand how much leverage households should take to account for the house price and labor income risks. The goal of this paper is to study the leverage decision for first time home buyers modeling a rational utility-maximizing household.

I propose a life-cycle model where a finitely lived risk averse household head finances her housing investment by means of a fixed rate mortgage contract, whose face value depends on the down payment she chooses to provide. The agent is exposed to two sources of uncertainty: house prices and labor income. After signing the mortgage contract, while increasing her equity share in the house through amortization, the agent optimally determines the time at which defaulting and moving into the rental market provides a greater expected continuation utility than continuing the mortgage payments. Negative equity is necessary but on its own not sufficient to declare default. The agent may strategically default even when she is still able to make mortgage payments. Risk neutral lenders efficiently price mortgages charging a default premium to compensate themselves for expected losses due to default on a mortgage. As a consequence, the default premium mainly depends on the down payment the agent chooses to provide, the agent's propensity to default and the dead weight losses the lender faces in case of default. Understanding the consequences of taking costly leverage is also key to understanding the choice of owning a house. I determine the cost of ownership, such that, in the first period the agent is indifferent *ex ante* between owning with a mortgage and renting the same house. As a result, the cost of renting incorporates a rent risk premium that depends on the main parameters of the model. Then, I unveil some interesting implications calibrating the model with household level data on labor income and housing values from the Survey of Income and Program Participation (SIPP) of the U.S. Census Bureau from 1997 to 2005.

Existing literature has emphasized when households should default (see Kau and Keenan (1995), Downing, Stanton, and Wallace (2005), Campbell and Cocco (2003) and Campbell and Cocco (2011)), but has been silent on how much leverage a household should take, having the option of default on the mortgage itself. The innovation of my paper is to make the mortgage value and the amount of leverage interlinked. Because the agent is allowed to choose her optimal down payment in the first period, her optimal amount of leverage is a function of house price, income expectations, bankruptcy costs and other parameters. As a consequence, the mortgage value (and therefore the mortgage spread) cannot be determined without knowing the amount of leverage the agent takes. At the same time, the amount of leverage (and therefore the down payment) cannot be optimized without knowing the effect of leverage on mortgage value. The analysis addresses the following questions: (i) What is the optimal amount of leverage, and how does this depend on house price risk and labor income risk?; (ii) How does default premium depend on leverage, house price risk and labor income risk?; and (iii) How does default probability depend on leverage, house price risk and labor income risk?

Three main results arise. First, the optimal down payment and the house price volatility are positively related. The higher the house price volatility, the higher the down payment the agent provides to decrease the volatility of the equity share in the house. Second, in the presence of borrowing constraints, a higher risk of unemployment persistence and/or a substantial drop in labor income decreases the leveraged position the agent takes. Third, ruling out the effect of taking costly leverage on owning a house significantly biases the results in favor of owning over renting.

2 Introduction

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From a life-cycle point of view, an agent is willing to smooth housing consumption over time taking on a mortgage when her income and her available liquid assets are insufficient to afford the desired house; at the same time, she has an income and expects to be able to pay for the house over time. The long term durability of housing makes its purchase price relatively high, with the result being that a relevant mortgage coupon has to be paid over the course of several years. The use of such leverage to purchase the house magnifies the risk assumed by the buyer. From the lenders' prospective, the mortgage contract necessarily has a risk of default which must be priced in the coupon. The more leverage the household takes by increasing the mortgage, the greater the risk that an adverse event will lead to default. As a result, the cost of a mortgage has a major impact on the amount of leverage the agent takes on.

I propose a life-cycle model where a finitely lived risk averse household head finances her housing investment by means of a fixed rate mortgage contract, whose face value depends on the down payment she chooses to provide. The agent is exposed to two sources of uncertainty: house prices and labor income. After signing the mortgage contract, while increasing her equity share in the house through amortization, the agent optimally determines the time at which defaulting and moving into the rental market provides a greater expected continuation utility than continuing the mortgage payments. Negative equity is necessary but on its own not sufficient to declare default.

The agent may strategically default even when she is still able to make mortgage payments. Risk neutral lenders efficiently price mortgages charging a default premium to compensate themselves for expected losses due to default on a mortgage. As a consequence, the default premium mainly depends on the down payment the agent chooses to provide, the agent's propensity to default and the dead weight losses the lender faces in case of default. Understanding the consequences of taking costly leverage is also key to understanding the choice of owning a house. I determine the cost of ownership, such that, in the first period the agent is indifferent *ex ante* between owning with a mortgage and renting the same house. As a result, the cost of renting incorporates a rent risk premium that depends on the main parameters of the model. Then, I unveil some interesting implications calibrating the model with household level data on labor income and housing values from the Survey of Income and Program Participation (SIPP) of the U.S. Census Bureau from 1997 to 2005.[1]

Existing literature has emphasized when households should default (see Kau and Keenan (1995), Downing, Stanton, and Wallace (2005), Campbell and Cocco (2003) and Campbell and Cocco (2011)), but has been silent on how much leverage a household should take, having the option of default on the mortgage itself. The innovation of my paper is to make the mortgage value and the amount of leverage interlinked. Because the agent is allowed to choose her optimal down payment in the first period, her optimal amount of leverage is a function of house price, income expectations, bankruptcy costs and other parameters. As a consequence, the mortgage value (and therefore the mortgage spread) cannot be determined without knowing the amount of leverage the agent takes. At the same time, the amount of leverage (and therefore the down payment) cannot be optimized without knowing the effect of leverage on mortgage value. The analysis addresses the following questions: (i) What is the optimal amount of leverage, and how does this depend on house price risk and labor income risk?; (ii) How does default premium depend on leverage, house price risk and labor income risk?; and (iii) How does default probability depend on leverage, house price risk and labor income risk?

Three main contributions and results arise from this paper. First, the down payment and the house price volatility are positively related. As the house price volatility increases, the agent

[1] The SIPP collects income, asset and demographic information from a sample of approximately $20,000-30,000$ households. The main advantages of the SIPP are its large sample size and detailed information about covariates as well as its complete housing history.

faces the choice between paying a higher default premium in the mortgage payment, due to higher chances of default and a higher rent risk premium in the rental contract, due to the more volatile rental payments. The model's prediction is a natural consequence of the fact that the agent is taking leverage to finance the housing purchase. On one hand, a higher down payment decreases the volatility of the equity share in the house and hence decreases the default premium making the cost of leverage less onerous. On the other hand, the higher the down payment the higher the rent risk premium has to be to make the agent *ex ante* indifferent between owning with a mortgage and renting the same house. Although an increase in the house price volatility contributes to a decrease in leverage, the probability of default might be higher than the one obtained with a lower house price volatility.

Second, the riskiness of labor income and borrowing constraints directly affect the leveraged position the agent takes. In my model, the source of income risk is unemployment, or the probability of a large decline in labor income. The presence of borrowing constraints exacerbate the negative labor income shock making the default option more valuable and inducing the agent with large mortgage payments relative to labor income to face the choice between severe consumption cutback and default followed by a move into the rental market with a higher buffer of savings. A higher risk of unemployment persistence increases the opportunity to default because the expected duration of being unemployed increases. *Ceteris paribus*, the increasing opportunity to default increases the default premium. Consequently, the agent provides a higher down payment. Furthermore, a higher risk of unemployment persistence also uniformly increases the probability of default over time. Then, the expectation of a substantial drop in labor income substantially decreases the leveraged position the agent takes. Due to the drop in labor income, the probability of default rises quickly during the first years of the mortgage life, because borrowing constraints are more binding at the beginning, when the mortgage payment to labor income ratio is higher.

As highlighted by Campbell and Cocco (2003) and more recently by Campbell and Cocco (2011), the limit of current mortgage pricing literature is not to recognize the role played by labor income and borrowing constrains in determining default. In fact, the default occurrence is mainly driven by the house price process, and triggered by a negative equity position in a house. Mortgage termination is treated as the optimal response of a rational owner who chooses when optimal to default, but the implicit assumption is that the agent could borrow immediately and costlessly from other sources if labor income falls and is not sufficient to make the required mortgage payments.[2],[3]

Third, the model sheds light on the choice between owning and renting. Sinai and Souleles (2005) recently identified a hedging benefit of owning with respect to renting if the agent has a long horizon and is exposed to volatile rents. Their claim that owning is a natural hedge against rent fluctuations is intuitive, but is built on the assumption that the agent does not need to take a mortgage, because she is endowed with enough wealth to pay the full purchase price. In my model, the default premium measures the risk associated with cost of owning a house and reduces the amount of leverage the agent is willing to take, and hence the demand for owner-occupied housing. Because increasing leverage makes owning a house less attractive, the rent risk premium must decrease sufficiently to keep the owner indifferent between owning and renting. Thereby, ruling out the effect of taking costly leverage on owning a house significantly biases the results in favor of owning over renting.

The paper is organized as follows. Section 3 describes the agent's optimization problem. Subsection 3.1 derives the renter's problem. Subsection 3.2.1 derives the owner's problem in the presence of a mortgage, describing how default is endogenously determined. Subsection 3.2.2 provides the mortgage valuation. Section 4 describes the calibration of the model parameters. Section 5 presents the numerical results of the model. Finally, Section 6 gives the conclusions.

3 Statement of the problem

Consider an agent whose preferences are represented by a constant relative risk aversion utility function (CRRA), and depend on housing services H and other goods $C(t)$:

$$u\left(C(t), H\right) = \frac{(C^{\beta}(t)H^{1-\beta})^{1-\gamma}}{1-\gamma}, \tag{1}$$

[2]Generally, mortgage pricing literature considers two sources of uncertainty: interest rates and house prices. Under this approach, standard contingent claims techniques are used to calculate the optimal exercise policy (see Kau and Keenan (1995)).

[3]Empirical literature provided evidence that there is substantial heterogeneity in default behavior across homeowners even controlling for mortgage and property characteristics. Two possible explanations are advocated. One is that transaction costs associated with defaulting differ across homeowners (see Downing, Stanton, and Wallace (2005)). The second one is that trigger events, such as divorce, loss of a job, or accidents affect default behavior (see Vandell (1995), Deng, Quigley, and Van Order (2000) and Foote, Gerardi, and Willen (2008)).

where β measures the relative importance of numeraire consumption versus housing services, $\beta \in (0, 1)$, and γ is the curvature parameter and coefficient of relative risk aversion, $\gamma > 0$.

The agent is unconstrained in respect of owning versus renting choice. The agent is exposed to two sources of uncertainty: (i) the stochastic flow of labor income, L; and (ii) the stochastic house price per square foot, P. She chooses the status that maximizes her expected utility in the first period.

Housing stock H provides a constant flow of housing services to the agent. I make the assumption that (i) the housing stock H does not depreciate; (ii) the flow of housing services is δH, where $\delta = 1$ for convenience. The housing size H in the first period is given and the agent does not adjust the level of housing services, moving to a larger or smaller house, regardless of the path of house price, income and wealth.[4] The agent chooses consumption of all goods other than housing, $C(t)$, at any time t.

The agent has a finite horizon and discounts the utility function at the constant rate of time preference ρ:

$$\int_0^T e^{-\rho t} u\left(C(t), H\right) \mathrm{d}t, \tag{2}$$

where T is the terminal date. The agent also derives utility from terminal wealth $W(T)$, which can be interpreted as bequest motive:

$$V(W, T) = \frac{W(T)^{1-\gamma}}{1-\gamma}. \tag{3}$$

To examine the impact of labor uncertainty on the individual consumption path and on the choice to rent or own, I assume that the labor income $L(t) \geq 0$ can only take two values at any time t: $L^h(t)$ and $L^l(t)$ with $L^h(t) > L^l(t) > 0$ (h - high, l - low). The level of income in the high-regime, $L^h(t)$, grows at a time dependent rate

$$\mathrm{d}L^h(t) = f(t, e, D(t))L^h(t)\mathrm{d}t, \tag{4}$$

where $f(t, e, D(t))$ is a deterministic function of age t, the education level e and other individual demographic characteristics $D(t)$. The high regime can be interpreted as the phase in which the agent has an occupation. Thus, the deterministic component is calibrated to capture the hump

[4]This assumption makes the model numerically tractable and I will discuss it later.

shape of earnings over the life-cycle. When a high to low regime switching occurs, I assume that labor income drops at the level $L^l(t) = \phi L^h(t)$, where ϕ is a constant replacement rate. The negative shock can be due to unemployment, divorce, sickness or other reasons. In addition, I presume that $L(t)$ is observable and that its transition probability follows a Poisson law, such that $L(t)$ is a two-regime Markov chain. Let $\theta^{h\to l}$ denote the rate of leaving regime h for regime l. In fact, there is a probability $\theta^{h\to l}\Delta t$ that the value of the labor income $L(t)$ changes from $L^h(t)$ to $L^l(t)$ during a time interval Δt. In addition, the expected duration of regime h is $1/\theta^{h\to l}$. This set of assumptions captures the idea that labor income may randomly change.

The house price is given and is governed by a geometric Brownian motion:

$$\mathrm{d}P = \mu_P P \mathrm{d}t + \sigma_P P \mathrm{d}Z, \tag{5}$$

where μ_P is the rate of real home price appreciation, σ_P is the volatility parameter and Z is a Wiener process.

The wealth of the agent at time zero, $W(0)$, corresponds to the dollar amount in the risk free asset, $B(0)$, and does not take into account the present value of the future uncertain income stream. The risk free asset pays a constant interest rate r and its dynamic is governed by

$$\mathrm{d}B = rB\mathrm{d}t. \tag{6}$$

If the agent chooses to rent, she continuously pays a rent that is a constant fraction α, the rental cost, of the current house value, $H \times P(t)$, exposing herself to the rent fluctuations. The rental cost α is defined as the sum of the user cost, the cost of consuming the house, and the constant rent risk premium λ. I set the rent risk premium such that the agent is indifferent *ex ante* as to owning with a mortgage or renting in the first period.

If the agent chooses to own, she can finance the housing purchase in the first period, taking a mortgage $F(0)$. By assumption, the agent signs a mortgage contract with a level of income $L^h(0)$ in the first period. The face value of the mortgage in the first period is $F(0) = (1-\omega)H \times P(0)$, where ω is the down payment or the initial equity share in the house, $\omega \in (0, 1]$. Given house value $H \times P(0)$, the agent chooses the optimal down payment ω^* and therefore the optimal amount of leverage to take on.

The mortgage is a fully amortizing fixed rate contract,

$$F(0) = \int_0^T N e^{-ct} \mathrm{d}t, \tag{7}$$

where N is the mortgage coupon and c is the mortgage coupon rate. As a result, the total payout is

$$N = \frac{cF(0)}{1 - e^{-cT}}. \tag{8}$$

Because the mortgage is a fully amortizing contract, the agent gradually increases her equity share in the house due to amortization. Therefore, the mortgage balance at time T is null. By assumption, the interest component of the coupon is not tax deductible.[5] Thus, after signing the mortgage contract, the agent's wealth at any time t is $W(t) = B(t) + H \times P(t) - F(t)$, while at time T it is $W(T) = B(T) + H \times P(T)$.

The occurrence of default is triggered endogenously at the stopping time τ. As in Campbell and Cocco (2003) and Campbell and Cocco (2011), the agent optimally determines at which point to default based on whether defaulting and moving into the rental market or continuing to pay mortgage coupons provides her the greater expected utility. Since the dynamic of the labor income is regime dependent, so is the optimal default policy.

Default is assumed to be costly for the agent and the lender. The agent loses the house, incurring a transaction cost $\epsilon H \times P(\tau)$, is forced into the rental market, loses access to the mortgage market, and stops paying the mortgage coupons N. The optimal default threshold in each regime accounts for the possibility, for the agent, to default in the either regime. Therefore, the agent has to determine a default policy in each regime, while taking into account the optimal default policy in the other regime. Most of the time, the agent defaults with a negative equity position in the house in both regimes. The lender incurs a loss proportional to the housing value at default $\psi H \times P(\tau)$, due to missing coupon payments, delay and legal costs associated with repossession

[5] I disregard tax considerations as they are too country-specific. In the U.S., the homeownership is substantially subsidized. A owner can deduct the mortgage interest payment from the income tax decreasing the after-tax cost of owning with a mortgage. Then, the owner can deduct the property taxes and the points charged when the mortgage contract is signed. Moreover, the most fundamental subsidy is that homeowners are not taxed on the implicit rent they receive from their housing investment. Jaffee and Quigley (2006) provide a review and taxonomy of U.S. federal housing programs, including direct public expenditures on housing and indirect expenditures through the tax system.

of the collateral. The default premium κ compensates the lenders for the potential consequences of default. Campbell, Giglio, and Pathak (2011) used data on houses transactions in the state of Massachusetts over the period 1987 to March 2008 and found that houses in foreclosure are sold on average at a 28 percent discount.

I assume that the default decision is not affected by legal considerations and I assume that there is no recourse in the case of bankruptcy.[6] Ghent and Kudlyak (2011) quantified the effect of recourse on default and found that recourse affects default by lowering the borrower's sensitivity to negative equity. However, I abstract on how bankruptcy laws affect mortgage default (see Li, White, and Zhu (2011)) and how non-pecuniary factors, such as views about fairness and morality, affect the default incentives (see Guiso, Sapienza, and Zingales (2011)).

The default timing is endogenously determined and is affected by the default premium and the rent risk premium. In Campbell and Cocco (2003) and Campbell and Cocco (2011), a lower (higher) rent premium increases (decreases) the probability of default, but the exogenous default premium on the mortgage does not adjust accordingly.[7] Instead, in my model, the lenders take the optimal homeowner default policy as given. Hence, the default premium κ is set such that the lenders are indifferent as to the return r on the risk free asset B or the return c on the risky mortgage F.

In the next sections, I separately analyze the renter's and the owner's problem (Subsection 3.1 and 3.2.1 respectively) and I discuss the mortgage valuation in Subsection 3.2.2. The goal is to identify the optimal down payment ω^*, the default premium κ and the rent risk premium λ such that (i) the agent is indifferent *ex ante* as to owning with a mortgage or renting the house $H \times P(0)$ when labor income is in high regime; (ii) the mortgage fair value equals the mortgage face value in the first period. The optimal down payment and the two premia depend on the level of wealth $W(0)$, the labor income $L^h(0)$ and the house price $P(0)$.

In order to make the model numerically tractable, I make three simplifying assumptions. The first is that, as in Campbell and Cocco (2003) and Campbell and Cocco (2011), the owner can change her status, from owning to renting by defaulting on the mortgage, while the renter can never change her status.

[6] In recourse states, lenders may be able to obtain a deficiency judgement and to collect on debt not covered by proceedings from a foreclosure sale.

[7] This limitation is acknowledged by the two authors.

The second major assumption is that while the default and rent premia are determined endogenously at the mortgage's origination and rent's signature, they remain constant throughout the agent's horizon.[8] This precludes the possibility that, after signing the mortgage contract, the agent or the lender might renegotiate the mortgage coupon by varying the default premium. Increase in house prices would generate a motive for prepayment because the homeowner is paying a higher price for default risk than is fairly justified, since mortgage payments are based on the default probability appropriate at the origination date. It also precludes the possibility that, after signing a rental contract, a different rental cost might be charged by varying the rent risk premium. While the assumption is restrictive, it may be realistic if the fixed costs of negotiating contracts are as high as to limit the potential to renegotiate. This assumption mainly precludes the refinancing decision. Mortgage refinancing plays an important role in consumption smoothing (see Chen, Michaux, and Roussanov (2011)) and potentially in leverage decision.[9] In this latter case, the agent would be able to dynamically re-balance her leveraged position instead of statically maximizing it in the first period as in my model. Unfortunately, this extension would make the model intractable adding an additional state variable and making the mortgage pricing more complicated. I would need to identify the amount the agent would borrow and the associated default premium lenders would charge accounting for the house value, the labor income and the cash on hand at every time. Campbell and Cocco (2011) also do not account for the decision to refinance the mortgage contract. They solve the model under alternative assumptions regarding what households are allowed to do when they have accumulated positive home equity. For example, they allow them to sell and terminate the mortgage contract with different assumed transactions costs. Such alternative assumptions have little effect on default decision in states of house price declines which is also the focus of this paper.

Third, I abstract from prepayment decision not accounting for uncertainty in risk free rates. This is a simplification because homeowners have the option to prepay mortgages at their book values, possibly subject to prepayment penalty. They prepay and refinance when interest rates

[8] Campbell and Cocco (2003) and Campbell and Cocco (2011) assume exogenous constant default and rent premia. Yao and Zhang (2005) and Van Hemert (2008) assume an exogenous constant rent risk premium and a zero default premium.

[9] Mayer, Piskorski, and Tchistyi (2008) explore the practice of mortgage refinancing in a competitive dynamic lending model. The borrower's creditworthiness evolves stochastically over time. Borrowers who are hit by a series of negative wealth shocks will default, while borrowers who experience a series of positive wealth shocks will refinance to obtain a mortgage with a lower default premium. The model predicts that the equilibrium mortgage default premium increases as credit quality falls. Because they impose that house value is constant, the equilibrium default premium exclusively depends on the labor income evolution.

drop, as the present value of the current mortgage payments would exceed the mortgage's book value. The interaction between default option and prepayment option has previously been investigated in literature (see Kau et al. (1992), Kau and Keenan (1995), and Downing, Stanton, and Wallace (2005)). In my model this feature is absent as I assume interest rates to be constant. The prepayment option would definitely affect the leverage decision because the agent could refinance at lower rates potentially increasing the leverage she takes facing a lower mortgage yield. Campbell and Cocco (2003) allow costly refinancing to occur only if the agent has positive home equity and they show that the refinancing activity mostly takes place within the first half of the mortgage life.

The nature and properties of the solution in the presence of borrowing constraints need to be numerically investigated. This is also the case for the saving-consumption problem studied in the macroeconomics literature of buffer-stock models (see Zeldes (1989), Deaton (1991), Carroll (1997) and Gourinchas and Parker (2002)) and in Campbell and Cocco (2003) and Campbell and Cocco (2011). Appendix A provides an overview of the numerical schemes implemented to solve the agent's problem and the mortgage valuation.

3.1 Renting

I assume that the agent rents a house of size H, and she continuously pays a fraction α of the current house value $H \times P(t)$, the rental cost. The agent's wealth $W(t)$ corresponds to the risk free asset $B(t)$ at any time t. Following Deaton (1991), Carroll (1997) and Gourinchas and Parker (2002), the cash on hand $A(t)$ has the following dynamic:

$$\mathrm{d}A = \mathrm{d}B + \big((1-g)L^i - C^i - \alpha H \times P\big)\,\mathrm{d}t, \quad i = l, h, \tag{9}$$

where g is the tax rate on labor income. Given $A(0) \geq 0$, the renter's problem is to choose the numeraire consumption, $C^i(t)$, to solve:

$$V^r(A(0), P(0), 0, i) = \sup_{C^i(t)} \mathrm{E}\left[\int_0^T e^{-\rho t} u(C^i(t), H)\mathrm{d}t + e^{-\rho T}\frac{W(T)^{1-\gamma}}{1-\gamma}\right], \quad i = l, h. \tag{10}$$

The associated system of Hamilton-Jacobi-Bellman equations is the following:[10]

$$\rho V^r(\cdot,l) = \sup_{C^l}\left\{u(C^l,H) + \mathcal{D}V^r(\cdot,l) + \theta^{l\to h}(V^r(\cdot,h) - V^r(\cdot,l))\right\}, \tag{11}$$

$$\rho V^r(\cdot,h) = \sup_{C^h}\left\{u(C^h,H) + \mathcal{D}V^r(\cdot,h) + \theta^{h\to l}(V^r(\cdot,l) - V^r(\cdot,h))\right\}, \tag{12}$$

where

$$\mathcal{D}V^r(\cdot,i) = (rB+(1-g)L^i - C^i - \alpha H\times P)V^r_A(\cdot,i) + \mu_P P V^r_P(\cdot,i) + V^r_t(\cdot,i) + \frac{\sigma_P^2}{2}P^2 V^r_{PP}(\cdot,i), \quad i=l,h, \tag{13}$$

and $\theta^{i\to j}(V^r(\cdot,j) - V^r(\cdot,i))$ reflects the impact of labor income on the value functions. This term is the product of the instantaneous probability of a regime shift, from i to j, and the change in the value function occurring after a regime shift.

Note that the rental cost has two components: $\alpha = (r-\mu_P)+\lambda$. The first component, $r-\mu_P$, is the user cost of the house.[11] The second component, λ, is the rent risk premium. Following Sinai and Souleles (2005), I set the rent risk premium, λ, such that the agent is indifferent *ex ante* to owning with a mortgage or renting.[12],[13]

[10]Thereafter, the notation $V(\cdot,i)$ refers to $V(A(t),P(t),t,i)$.

[11]I derive the user cost, $r-\mu_P$, assuming that the landlord (i) is risk neutral, (ii) is endowed with enough wealth to afford the house purchase, (iii) is only exposed to the house price fluctuations. Hence, equilibrium in the housing market occurs when the expected cost of owning a house equals that of renting. Equating the *ex ante* utilities of renting and owning:

$$\underbrace{W(0) - \mathrm{E}\left[\int_0^T e^{-r(T-t)}\tilde{\alpha}H\times P(t)\mathrm{d}t\right]}_{\text{Renting}} = \underbrace{W(0) - H\times P(0) + \mathrm{E}\left[e^{-rT}H\times P(T)\right]}_{\text{Owning}}, \tag{14}$$

I obtain that $\tilde{\alpha} = (r-\mu_P)$, where r is the cost of foregone interest that the landlord could have earned on the risk free asset, B. It is adjusted to include the offsetting benefit given by the expected capital gain, μ_P, on the house value, $H\times P$. Alternatively, the user cost can be derived assuming that the current house value, $H\times P$, is equal to the expected present value of the future rents. The user cost is determined imposing that the expected value of capital gain on house value and rent is equal to the risk free rate over the interval $\mathrm{d}t$:

$$\mathrm{E}\left[\frac{\mathrm{d}(H\times P)}{H\times P} + \frac{\alpha H\times P\mathrm{d}t}{H\times P}\right] = r\mathrm{d}t.$$

[12]In order to derive it, I could explicitly model the landlord problem, assuming that the landlord is a risk averse agent who receives the rent as cash flow, who is borrowing from the lender in order to be the owner of the house, who receives a stochastic labor income flow, who is consuming the housing services of another house and so on. This approach would add unnecessary complications to the model, requiring the solution of the landlord's optimal control problem, while not adding to the generality of the problem.

[13]Henderson and Ioannides (1983) derive a full equilibrium model of the choice between owning and renting where the tenant may not properly care for the property. Because of limited information, landlords cannot distinguish *ex ante* good tenants from bad tenants. Under such circumstances, landlords charge rents which reflect average

Solving for the first order condition, I obtain the optimal numeraire consumption:

$$C^i = \left(\frac{H^{-(1-\beta)(1-\gamma)} V_A^r(\cdot, i)}{\beta} \right)^{1/(\beta(1-\gamma)-1)}, \quad i = l, h. \tag{15}$$

The optimal numeraire consumption, C^i, is subject to the constraint that the cash on hand position must not be negative at any time t: $A(t) \geq 0$. If the agent meets the constraint with equality at some date t, she is forced to leave the house at that date, and is left with zero housing and numeraire consumption from then on.

3.2 Owning

3.2.1 Owner's problem

The equation describing the evolution of the cash on hand is:

$$\mathrm{d}A = \big(rB + (1-g)L^i - C^i - N\big)\,\mathrm{d}t, \quad i = l, h. \tag{16}$$

As in the renter's problem, the cash on hand dynamic takes two different forms, depending on whether or not the level of labor income is L^l or L^h. The other terms represent respectively the return on the risk free asset rB, the numeraire consumption C^i and the mortgage coupon N.

The owner's problem is:

$$\begin{aligned} V^{om}(A(0), P(0), 0, i) = \sup_{\omega, \tau, C^i(t)} \mathrm{E} \Bigg[& \int_0^T e^{-\rho t} u(C^i(t), H) \mathrm{d}t \\ & + \underbrace{\mathbf{1}_{\{\tau \leq T\}} e^{-\rho\tau} V^r(A(\tau), P(\tau), \tau, i)}_{\text{Default}} + \mathbf{1}_{\{\tau > T\}} e^{-\rho T} \frac{W(T)^{1-\gamma}}{1-\gamma} \Bigg], \quad i = l, h. \end{aligned} \tag{17}$$

maintenance costs across potential tenants. It follows that tenants who have a predisposition to maintain their home pay rents which exceed the marginal costs they impose on landlords. These tenants have the incentive to own the house avoiding externalities in the rental market.

The associated system of Hamilton-Jacobi-Bellman equations is:

$$\rho V^{om}(\cdot, l) = \sup_{C^l} \left\{ u(C^l, H) + \mathcal{D}V^{om}(\cdot, l) + \theta^{l \to h}(V^{om}(\cdot, h) - V^{om}(\cdot, l)) \right\}, \tag{18}$$

$$\rho V^{om}(\cdot, h) = \sup_{C^h} \left\{ u(C^h, H) + \mathcal{D}V^{om}(\cdot, h) + \theta^{h \to l}(V^{om}(\cdot, l) - V^{om}(\cdot, h)) \right\}, \tag{19}$$

where

$$\begin{aligned} \mathcal{D}V^{om}(\cdot, i) &= (rB + (1-g)L^i - C^i - N)V_A^{om}(\cdot, i) + \mu_P P V_P^{om}(\cdot, i) \\ &\quad + V_t^{om}(\cdot, i) + \frac{\sigma_P^2}{2} P^2 V_{PP}^{om}(\cdot, i), \quad i = l, h, \end{aligned} \tag{20}$$

over a region where it is not optimal to default.

In the first period, the agent chooses the down payment, ω. Thus, the optimal down payment ω^* determines the optimal amount of leverage the agent takes on, maximizing the indirect utility of owning given the initial cash on hand, $A(0)$, the initial house price, $P(0)$, and the initial labor income in the high regime, $L^h(0)$.

If it is optimal to default, the agent loses the house and incurs the additional cost, $\epsilon H \times P(\tau)$, due to transaction and legal costs. However, she disregards the mortgage balance $F(\tau)$. Because the agent is forced into the rental market, the indirect utility at default is denoted by $V^r(\cdot, i)$.

Then, solving for the first order condition, I obtain the optimal numeraire consumption C^i:

$$C^i = \left(\frac{H^{-(1-\beta)(1-\gamma)} V_A^{om}(\cdot, i)}{\beta} \right)^{1/(\beta(1-\gamma)-1)}, \quad i = l, h. \tag{21}$$

As for the renter's case, the optimal numeraire consumption, C^i, is subject to the constraint that the cash on hand must not be negative at any time t: $A(t) \geq 0$.

3.2.2 Mortgage valuation

In this section, I derive the mortgage fair value $M(A^*, P, t, i)$, $i = l, h$. The contract is a fixed rate fully amortizing contract. Paying the mortgage coupon N, the agent gradually increases her equity share in the house value due to amortization. After signing the contract, I assume that the agent or the lender cannot re-negotiate the mortgage. Specifically, the default premium κ is determined at the contract's origination and it is constant.

I assume that the lenders take the optimal default strategy of the owner as given. The market is also competitive, implying that the mortgage coupon N is set equal to the full mortgage coupon so that the mortgage is issued at par. Moreover, the lenders adopt a contingent claims approach to fairly price the mortgage contract. The solution approach is that of option pricing analysis. Traditionally, in option pricing literature, prices of contingent claims are based on arbitrage arguments. However, such an approach requires assumptions about the liquidity of underlying assets. In the case of mortgage contracts, such arbitrage arguments are questionable. One underlying asset is a house which is subject to substantial transaction costs and an inability to be sold short. The second is labor income for which contingent claims contracts are not observed in reality for reasons of moral hazard and adverse selection. Alternatively, although an appropriate model must be chosen, an equilibrium approach relaxes the tradability assumptions needed for arbitrage pricing. For simplicity, I assume risk neutrality, so that all assets are priced to yield an expected rate of return equal to the risk free rate, r.[14] This restrictive assumption can be relaxed by adjusting the drift rates to account for a risk premium in the manner of Cox and Ross (1976).

Consider the instantaneous return on $M(A^*, P, t, i)$, $i = l, h$, over a region in which the mortgage has not yet defaulted. The value of the mortgage depends on A^*, which is the owner's cash on hand accounting for the optimal numeraire consumption C^i, the owner's labor income, L^i, and the house price, P. The lender receives a cash inflow due to the coupon payment of N over the interval $\mathrm{d}t$. The total expected return on $M(A^*, P, t, i)$ per unit of time μ_M is defined as

$$\mathrm{E}\left[\frac{\mathrm{d}M(A^*, P, t, i)}{M(A^*, P, t, i)} + \frac{N\mathrm{d}t}{M(A^*, P, t, i)}\right] = \mu_M \mathrm{d}t, \quad i = l, h. \tag{22}$$

Setting the expected return μ_M to the risk free rate r and simplifying yields the following equilibrium partial differential equation:

$$rM(\cdot, l) = N + \mathcal{D}M(\cdot, l) + \theta^{l \to h}(M(\cdot, h) - M(\cdot, l)), \tag{23}$$

$$rM(\cdot, h) = N + \mathcal{D}M(\cdot, h) + \theta^{h \to l}(M(\cdot, l) - M(\cdot, h)), \tag{24}$$

[14] The assumption is the same made by Grenadier (1996) who proposes a unified framework for determining the equilibrium credit spread on commercial leases subject to default risk.

where

$$\mathcal{D}M(\cdot,i) = (rA + (1-g)L^i - C^i - N)M_{A^*}(\cdot,i) + \mu_P P M_P(\cdot,i) + M_t(\cdot,i) \\ + \frac{\sigma_P^2}{2}P^2 M_{PP}(\cdot,i), \quad i = l,h. \tag{25}$$

The boundary conditions are:

$$M(A^*(T), P(T), T, i) = 0, \tag{26}$$

$$\lim_{P\to\infty} M(A^*(t), P(t), t, i) = \widetilde{M}(t), \tag{27}$$

$i = l,h$. The boundary condition (26) is the terminal condition, reflecting the residual mortgage balance at the terminal date T. The boundary condition (27) says that when the house price goes up, default no longer occurs: $\widetilde{M}(t)$ is the value of a bond with the same promised cash flows as the mortgage, but with no house price dependence.

At default, the boundary condition is:

$$M(A^*(\tau), P(\tau), \tau, i) = (1-\psi)H \times P(A(\tau), \tau, i), \quad i = l,h. \tag{28}$$

The lender seizes the house and recovers $H \times P(A(\tau), \tau, i)$ net of the loss $\psi H \times P(A(\tau), \tau, i)$.

4 Model calibration

I use the Survey of Income and Program Participation (SIPP) of the U.S. Census Bureau from 1997 to 2005 to estimate Equation (4), which describes labor income as a function of age and other characteristics. SIPP tracks approximately $30,000$ households. During the period considered, information was collected from three distinct groups of households; the first was interviewed during the years $1996-2000$ (four times), the second during the years $2001-2003$ (three times), and the latter during the years $2004-2006$ (two times). During its active period, each panel is interviewed every year. SIPP over-samples from areas with high concentration of low income households, which should be taken into account when interpreting the results. However, its longitudinal feature is desirable as it enables the analysis of dynamic characteristics, such as changes in income and household and family composition, or housing dynamics.

To estimate the labor income process, I follow Cocco, Gomes, and Maenhout (2005). I only consider male headed households to eliminate the potential effect of differences in age profile and a reduced number of available observations for female headed households. I also restrict the sample to household heads who are still in the labor force and aged between 25 and 65. I define labor income as the total reported income for the head of the household, deflated by the Consumer Price Index with 1997 as base year. I use the number of completed years of education from the SIPP individual data to split the sample in two mutually exclusive categories: (i) high school graduates, including individuals that spent some years in college but didn't obtain a degree; and (ii) college graduates.

The labor income process, as described in Equation (4), is a function $f(t, e_j, D_j(t))$. The term j identifies the household, e_j is the education level of the household head and $D_j(t)$ is a vector of demographic characteristics, such as marital status, household composition and whether the head of the household is retired or not, but excluding age and individual fixed effects. The function f is assumed to be additively separable in t and $D_j(t)$. The function f is estimated following a two-step procedure. First, I regress the logarithm of income on age dummies, a dummy for each possible age between 25 and 65, individual fixed effects and the set of demographic variables. In this stage, I only consider household heads who own a house and are employed, because I model unemployment separately as a different regime.

In the second stage, following Cocco, Gomes, and Maenhout (2005), age enters the labor income process as a third-order polynomial of age

$$f(\text{age}) = \mu_0 + \mu_1 \times \text{age} + \mu_2 \times \text{age}^2 + \mu_3 \times \text{age}^3. \tag{29}$$

Hence, I regress the age dummies coefficients from the estimated labor income process estimated in the first stage to get the coefficients μ of the polynomial itself. Figure 1 depicts the fit of a third order-polynomial for the two groups, college and high school graduate respectively. I use these age profiles for my calibration exercise.

Insert Figure 1 here

I estimate the labor income parameters that govern the low labor income regime separately. I use SIPP variables relating to unemployment status to estimate these parameters. I consider male household heads who were unemployed for a period of time during the year and the probability of such an event occurring within my sample. I define the sample probability of experiencing an unemployment spell within a year as the fraction of individuals experiencing unemployment in a given year, regardless of the length of the unemployment, over the number of individuals employed during the previous year.[15] Table 1 reports, by education group, the probability of experiencing a spell of unemployment within a year, $\theta^{h\to l}(e_j)$, the average yearly labor income for individuals with and without an unemployment spell, and the average income replacement rate during unemployment calculated (using the latter two variables), $\phi(e_j)$.

Insert Table 1 here

As expected, the probability of unemployment, $\theta^{h\to l}(e_j)$, is significantly higher for the second education group, confirming that its short-term income risk is higher: unemployment probability for high school graduates is 13.9 percent versus 7.42 percent for college graduates. Interestingly, the higher the level of education the higher the fraction of labor income household heads lose when unemployed, with a replacement rate of 55.97 percent versus 74.58 percent for high school graduates. I define the probability of finding a new job, within the year following in which an unemployment spell is recorded, as the proportion of individuals who find a job in any given year experiencing an unemployment spell in the previous year on individuals who are unemployed in the previous year. The probability of finding a job within a year $\theta^{l\to h}(e_j)$ is very similar across the two groups. The parameters of the calibrated labor income are also reported in Panel A of Table 2. Finally, I set the income tax rate, g, equal to 20 percent.

Insert Table 2 here

For each education group, I set the house value in the first period, $H \times P(0)$, using the house value's estimates reported by homeowners in SIPP. When considering male headed households employed and aged 25, I find that the average house value is approximately $\$180,000$ for the college graduates and approximately $\$128,000$ for the high school graduates. As reported in Panel

[15]The SIPP survey register unemployment under the condition that this lasts at least six months.

B of Table 2, I set the house price appreciation parameter, μ_P, at 1.6 percent as a baseline value and 2.6 percent as an alternative value.

As widely documented in literature, it is difficult to pin down house price volatility. The following two main alternatives might be considered: (i) house price indexes; or (ii) self-reported house price surveys such as SIPP. Using house price indexes usually leads to low estimates due to inertia in house price indexes.[16] As for house prices coming from surveys, the same household needs to be present in consecutive annual interviews and to have reported not having moved since the previous year, hence leading to a large measurement error. For my model, I prefer considering alternative parameterizations for the house price process, therefore setting the house price volatility at 10 percent as a baseline value and 15 percent as an alternative value.[17]

For the baseline parameterization, I assume a relative risk aversion, γ, of 2. The parameter β, measuring how much the agent values housing consumption relative to other goods, is set at 0.3 consistent with the average proportion of household housing expenditure in the U.S.[18] In addition, I assume the rate of time preference, ρ, is 3 percent, and the risk free rate, r. I set the initial age at 25 and the terminal age at 55, thus with a mortgage maturity of 30 years. To complete the model baseline parameterization, I assume a lender's default costs, ψ, of 28 percent. This value is based on Campbell, Giglio, and Pathak (2011), who found that, in Massachusetts state over the period $1987-2008$, houses in foreclosure were sold on average with a 28 percent discount. Finally, I assume that the transaction cost, incurred by the agent when defaulting, is 5 percent of the house value at default, $\epsilon H \times P(\tau)$. The figure includes legal fees, time cost of searching and the direct cost of moving. These parameters are listed in Panel C of Table 2.

[16] Wallace (2011) has recently emphasized that indices generate downwardly biased estimates of the idiosyncratic volatility of prices around the index and thus systematically undervalue embedded default options in mortgage products.

[17] Campbell and Cocco (2003) estimate a standard deviation parameter of the house price of 16.2 percent, while Campbell and Cocco (2003) estimate a standard deviation parameter of the house price of 11.5 percent; Cocco (2005) estimates a standard deviation of 6.2 percent. Yao and Zhang (2005) set the standard deviation at 10 percent.

[18] While Cocco (2005) sets β at 0.1, Yao and Zhang (2005) assume that β equals 0.2.

5 Results

5.1 Benchmark parameters

The goal is to solve the problem of owning with a mortgage versus renting to identify the optimal down payment ω^*, the default premium κ, and the rent risk premium λ, so that (i) the agent is indifferent *ex ante* as to owning with a mortgage or renting a house, $H \times P(0)$, when she is employed (the high labor income regime); and (ii) the mortgage fair value equals the mortgage face value. The optimal down payment and the two premia depend on the level of the labor income $L^h(0)$, the cash on hand $A(0)$ and the house price $P(0)$, and the main model's parameters. For convenience, because the agent with college (high school) education finances the housing purchase of $\$180,000$ ($\$128,000$), I set $H = 180$ ($H = 128$), square feet, and $P(0) = \$1,000$ per square foot. In addition, I assume that before signing the fixed rate mortgage contract, the agent is endowed with an initial level of cash on hand equal to the labor income received in the first year, $A(0^-) = L^h(0^-)$. Hence, the down payment is assumed to come from the agent's current income. The agent with college degree has a higher level of initial cash on hand than the agent with high school degree, but she has to finance a more expensive purchase. Later, I will relax this assumption, by allowing the agent to buy the house without the need to take leverage.

Figures 2 and 3 graphically represent the numerical solution of the model. The space $(A(0), P(0))$ spans all the possible combinations of values of A and P in the first period. The space is divided into three areas: (I) owning with a mortgage, (II) renting and (III) defaulting and renting. The existence and location of these areas depends on my parameter assumptions. The owning with a mortgage area (I) borders the renting area (II) in which owning with a mortgage dominates renting, $V^{om}(\cdot, h) > V^r(\cdot, h)$.

Insert Figure 2 here

In Figure 2, X corresponds to the solution of the problem for the college educated agent. When assuming a house price drift, μ_P, of 1.67 percent and a house price standard deviation, σ_P, of 10 percent, the solution of the problem for the college educated agent is an optimal down payment ω^* of 7.5 percent, for a fixed rate mortgage with coupon payments embedding a default premium κ of 0.51 percent, while the rental contract embeds a rent risk premium λ of 2.33 (see Panel A Table

3). The agent borrows 92.5 percent of the house value $H \times P(0) = \$180,000$, hence the mortgage balance, $F(0)$, is $\$166,500$ in the first period.

The down payment provides only a partial representation of the amount of leverage the agent takes on. I find that the loan to labor income ratio, LTI, is approximately 5.65 (10.10) in the high (low) labor income regime and the mortgage payment to net labor income ratio, MTI, is approximately 30.51 (54.51) percent in the high (low) labor income regime. I use income tax rate of 20 percent when I calculate the LTI and MTI ratios. The results are also reported in Panel A of Table 3 in Columns (5) and (7) for the college education group.

Insert Table 3 here

When making the same assumptions in terms of house price drift and standard deviation, the solution for an individual with a high school education is an optimal down payment ω^* of 5 percent, a default premium κ of 0.64 percent, and a rent risk premium λ of 2.18 percent (see Panel B of Table 3). In order to get a sense of the size of mortgages in relation to labor income, I calculate the ratio of mortgage payment to labor income, both in the first year of the mortgage, MTI, and averaged over the life of the mortgage, $\overline{MTI}$. For a college educated individual, the payment on a $\$166,500$ mortgage amounts to 30.51 percent of net income in the first year and 19.33 percent of net income on average over the life of the mortgage in the high labor income regime, while for high school educated individual the payment on a $\$121,600$ mortgage is 26.18 percent of the net labor income initially and 17.51 percent of net labor income on average in the high labor income regime. The higher human capital of the college educated agent makes him more willing to take a higher mortgage balance at the beginning because mortgage payment becomes more affordable in the subsequent years.

Figure 3 graphically represents the default boundaries for the agent with a college education. The agent optimally determines at which point to default based on whether default or continuing to pay mortgage coupons provides her greater expected continuation utility. Specifically, in the Figure 3, two default boundaries are represented: $H \times P(A(\tau), \tau, h)$ is associated with the high labor income regime, while $H \times P(A(\tau), \tau, l)$ with the low labor income regime. Because of the possibility of a regime shift, the optimal default threshold in each regime reflects the possibility of default in the other regime. Therefore, the agent has to determine a default policy in each regime, while taking into account the optimal default policy in the other regime.

Insert Figure 3 here

Negative equity is necessary but not a sufficient condition for declaring default. The default boundary is decreasing in $A(\tau)$. Intuitively, the benefit of defaulting in terms of cash on hand is higher for lower house prices: the agent loses the house value at default $H \times P(A(\tau), \tau, i)$ but she is no longer responsible for the mortgage balance $F(\tau)$. The probability of default increases as the housr value lowers, until reaching certainty at sufficiently low house value. As a result, the agent does not default as soon as equity is negative because the default option includes not only the right to terminate the loan contract now but also the right to terminate in the future. The agent exercises the default option when it is "in the money" and only when the level of cash on hand is substantially low.

In addition, an implication of the model is that labor income and borrowing constraints affect the agent's default behavior. In the presence of borrowing constraints, when the labor income switches from the high to the low regime, the agent exercises her default option in correspondence with higher house prices but with same amount of cash on hand, although labor income is still sufficient to make the required mortgage payment. As consequence, the area delimited by $H \times P(A(\tau), \tau, l)$ (low labor income regime) is wider that the one delimited by $H \times P(A(\tau), \tau, h)$ (high labor income regime). The prediction is consistent to the empirical analysis that suggests that trigger events, such as divorce, loss of a job, or accidents affect default behavior (see Vandell (1995), Deng, Quigley, and Van Order (2000) and Foote, Gerardi, and Willen (2008)). Borrowing constraints exacerbate a negative labor income shock inducing the agent with large mortgage payments relative to labor income to face the choice between severe consumption cutback and default and then to move into the rental market with a higher buffer of savings. This model's prediction is consistent with the findings of Elul et al. (2010) who provided empirical evidence of the importance of liquidity considerations for mortgage default decisions.

In Column (9) of Table 3, I also report the probabilities of default within the contract maturity, 30 years, for both labor income scenarios and for both education groups. This information is of much greater interest than the non observable default premium lenders ask in exchange for the default option.[19] In order to obtain such a probability of default measure, the solution of owning, renting and mortgage problem is a prerequisite, since it is by comparing the owning to renting choice that a homeowner decides whether to continue or default the mortgage. The approach is similar to the one developed in Kau, Keenan, and Kim (1994). Appendix B provides a detailed discussion on how I calculate the probability of default.

The magnitude of the default probabilities is by no means outside the range of values suggested by literature (see Kau, Keenan, and Kim (1994) and more recently Campbell and Cocco (2011)[20]). The probability of default for the college educated agent is 8.50 (9.25) percent in the high (low) labor income regime, while I obtain a probability of 10.46 (10.97) percent for the high school educated agent. As expected, the probability of default increases when the agent takes more leverage and experiences unemployment. The probability of default for a college educated individual is lower because she is taking a lower leveraged position. The probability of becoming unemployed is lower than that of a high school educated one, 7.73 percent and 13.9 percent respectively, although she can experience a more severe drop of 55.97 percent in labor income. In this scenario, the mortgage payment is 54.51 percent of the net labor income in the first period but it is 33.54 percent of the net labor income on average. In contrast, for the high school educated agent, the temporary decline in labor income is lower, 74.58 percent, but the probability of such event is higher, 13.9 percent. It is interesting to notice that the default probability in the high and low labor income regime for the high school educated individual are very close due to the limited drop in the labor income she faces in the case of unemployment. I will investigate later how the labor income parameters affect the optimal solution.

Then, I present the probabilities of default at monthly intervals cumulatively over time, from the origination to the expiration of the mortgage in Figure 4. Similar to previous studies, the probability of default rises slowly at the very beginning, due to the fact it is unlikely that house value will decrease sharply in a such short amount of time. This effect soon disappears, and it is within the

[19]In practice, the mortgage yield of a fixed rate mortgage incorporates both a default and prepayment option.

[20]Campbell and Cocco (2011) report a probability of default of 2.6 and 3.9 percent for a fixed rate mortgage with a LTV of 90 and 95 percent respectively and a LTI of 4.5. In a previous version of their paper, they report a default probability of 10.7 and 13.6 percent respectively for the same base line parameters. The difference is due to the assumption they make on the rental payment. In the current version of their paper, they add a property tax rate of 1.5 percent and a property maintenance cost of 2.5 percent to the user cost. Instead, in the previous version of the paper, they just assume a rental premium of 2 percent. Because the higher cost of renting makes the default option less valuable, the default probabilities are lower.

first ten years that the likelihood of default is the greatest. Thereafter, the probability of default trails off, as the mortgage has been partly paid off and equity has accumulated. In my model, labor income risk and borrowing constraints provide an additional channel. The homeowner chooses to default sooner when faced with negative labor income shocks, since the mortgage payment risk is higher early in life when labor income and cash on hand is low.

Insert Figure 4 here

5.2 House prices

Varying the house price standard deviation from 10 to 15 percent and the house price expected appreciation from 1.67 to 2.67 percent impacts the agent's leveraged position. The down payment, ω^*, and the house price standard deviation, σ_P, are positively related. As the house price standard deviation increases, the agent faces the choice between paying a higher default premium in the mortgage payment due to higher chances of default and a higher rent risk premium in the rental contract due to the more volatile rental payments. The model's prediction is a natural consequence of the fact that the agent is taking leverage to finance the house purchase. On one hand, a higher down payment decreases the volatility of the equity share in the house and hence decreases the default premium making the cost of leverage less onerous. On the other hand, the higher the down payment the higher the rent risk premium has to be to make the agent *ex ante* indifferent between owning with a mortgage and renting the same house. Overall, an increase in the house price standard deviation contributes to a decrease in leverage, although the default premium and the probability of default increase for both education groups.

Specifically, the LTV decreases from 92.5 to 86 percent for the college educated agent and from 95 to 87.5 percent for the high school educated one. At the same time, the default premium increases from 0.51 to 0.89 percent for the college education group and form 0.64 to 1.06 percent for the high school one. In Figure 5, I present the probabilities of default from the origination to the the expiration of the mortgage at monthly intervals cumulatively over time. Interestingly, a higher house price volatility does not lead to a uniformly higher probability of default over time. The probability of default rises more slowly at the beginning when house prices are more volatile due to two effects. The first is that the higher the house price volatility the higher is the down payment.

As a result, a higher equity stake at mortgage origination decreases the probability of default. The second is provided by options analysis; the higher the house price volatility, the greater the value of the default option. But higher house price volatilities are associated with lower default triggers and optimal default does not occur until the homeowner is substantially "underwater" in respect of the mortgage. Hence, the higher the house price volatility, the higher the chance that house prices are to rise and then the homeowner would have strictly positive home equity. Overall, the default probability at origination of the loan within the next 30 years increases from 8.5 to 9.47 percent in the high labor income regime for the college educated agent and from 10.46 to 10.69 percent in the high labor income regime the high school educated one.

Insert Figure 5 here

In Table 3, I also present results that show the impact of house price appreciation. Higher house price appreciation and relatively low house price volatility result in highly leveraged positions for both education groups. *Ceteris paribus*, higher house price appreciation decreases the probability of default. Housing becomes a more attractive investment, so that individuals are less willing to leave their home. This channel contributes to an increase in the leveraged position for both education groups, keeping default probabilities close to those obtained with lower house price appreciation.

Due to the subprime crisis, the relationship between housing appreciation and required down payment has recently attracted the media and academias attention. Most of recent literature claims that the boom-bust cycle of house prices in some U.S. states, such as California or Florida, was mainly due to some some exogenous shock to credit supply, like the expansion of subprime lending (see Mian and Sufi (2009)) or lax underwriting (see Keys et al. (2010)) that was encouraged by mortgages' securitization. However, the causality could have run in the other direction.[21] My model suggests that if lenders had had optimistic views on house price dynamics in terms of low house price volatility and/or high expected house price growth rate, then they may have loosened underwriting standards offering low down payments contracts. Therefore, we should expect to find that U.S. households took more leveraged positions during the recent house price boom.

Campbell and Cocco (2011) highlighted that there was an increase in the average LTV in the years before the crisis at U.S. level, but to a point that does not seem high by historical standards.

[21] I thank an anonymous referee for suggesting this model implication.

Instead, there was a large increase in the LTI ratio from an average of 3.3 during the 1980s and 1990s to a value as high as 4.5 in the mid 2000s. Figure 6 plots the LTV and LTI ratios for U.S. and for four U.S. states (California, Florida, Indiana and Texas) over the 1996-2006 period.[22],[23] Interestingly, we observe a stable LTV ratio and an increasing LTI ratio over the time period considered for these four U.S. states, but what is particularly striking is the dramatic increase in the LTI ratio of California and Florida over the $2003 - 2006$ period.

Insert Figure 6 here

These empirical facts are not fully consistent to my model predictions. First, higher house price appreciation should lead to an increase in both LTV and LTI ratios. One explanation is that I assume for tractability that housing is fixed. If the agent is allowed to adjust the housing services, the amount she borrows to buy a bigger or smaller house may vary, together with the possibility that the LTV might stay relatively constant and the LTI might increase or decrease, should the agent buy a larger or smaller house. In my model, the default premium directly depends on the housing size and the mortgage amount. Therefore, I would need to identify the default premium and the rent risk premium relating to different combinations of housing and mortgage amounts and verify whether the agent could get a higher indirect utility with a different combination of housing and mortgage amount.[24] There may be interesting interactions between housing choices and leverage choice. The agent may be willing to buy a bigger house taking on mortgage because she has income and expects to be able to pay for the house over time. At the same time, the housing purchase price would be relatively high resulting in a higher LTI and in a relevant mortgage payment that has to be paid over the course of several years. The more leverage the agent takes on by increasing the mortgage to fund the house purchase, the greater the risk that an adverse surprise will lead to default. From lenders' prospective, the mortgage contract necessarily has a higher risk of default which must be priced in the coupon. As a result, the cost of mortgage has a major impact on the amount of leverage the agent will take on and then on the housing size she will buy. This extension

[22] The LTV data are from the Monthly Interest Rate Survey (MIRS), while the LTI data are calculated as the ratio of the average loan amount obtained form the same survey to the average household income obtained from SIPP data.

[23] I pick these four U.S. states because they are the most representative in the SIPP data

[24] The goal is to get the optimal house size H^* in the first period. Identifying H^* requires running the numerical algorithm for different levels of housing stock H and verifying at which level of the housing stock, H, the agent gets the highest indirect utility.

is left for future research.

Finally, an explanation for the dramatic increase in the LTI ratio in some U.S. states is that I consider a simple diffusion process for house prices.[25] Corradin, Fillat, and Vergaraa-Alert (2011) recently considered a time-varying drift term, governed by a Markov process, in the house price dynamics, generalizing the classic Grossman and Laroque (1990) model of the optimal portfolio choice with housing and transaction costs. Their theoretical results, corroborated by empirical findings, indicate that the optimal trading policies of homeowners are time varying and decrease or alternatively increase, when house prices are expected to rise or fall respectively. They, then, found there are important differences in expected growth rates, spread between the highest and lowest growth rate, and timing of high and low growth rates phases across different U.S. census divisions and states. In a set up where the expected housing growth rate is allowed to switch across regimes, the amount of leverage would depend on the housing growth regime, the housing return in each regime and the probability of switching across regimes. This channel might potentially explain the heterogeneity we observe in the LTV and LTI patterns across U.S. states. In particular the dramatic increase in the LTI ratio of California and Florida might be due to the steep rise in house prices observed over the period $2003-2006$.

Finally, house price volatility affecting the mortgage characteristics in terms of down payment and coupon payment also has consequences on numeraire consumption C. Figure 7 shows consumption rules for the college educated agent as an owner with a mortgage for the two levels of house price standard deviations, 10 and 15 percent respectively. Consumption is positive, increasing and concave in cash on hand. For low levels of cash on hand, the agent exhibits buffer-stock behavior: the consumption function starts along the 45-degree line, where the agent consumes everything. At some critical level of cash on hand, the precise value depends on the problem's parameters, it is optimal for the agent to save keeping something for the future.[26]

Insert Figure 7 here

The impact of owning a house with a mortgage is substantial on numeraire consumption when house prices are highly volatile. The agent, as an owner, consumes more higher levels of cash on

[25]This assumption is also made in the mortgage pricing and household finance literature.

[26]The consumption behavior derived is consistent with the one derived in the buffer-stock models literature (see Zeldes (1989), Deaton (1991), Carroll (1997) and Gourinchas and Parker (2002)).

hand when house price is more volatile. A higher down payment decreases initial cash on hand but when the homeowner's liquidity improves due to labor income increases and lower mortgage payments, she is willing to consume more.

5.3 Labor income

In my model, the source of income risk is unemployment, or the probability of a large decline in labor income. In order to evaluate the extent to which my results vary, I solve the model (i) increasing the probability of switching from a high to a low labor income regime, $\theta^{h \to l}$; (ii) decreasing the probability of switching from a low to a high labor income regime, $\theta^{l \to h}$; and (iii) decreasing the labor income replacement rate, ϕ. The new parameters are derived in the following way: for each education group, I double $\theta^{h \to l}$, I decrease by half $\theta^{l \to h}$ and I pick the 25 percentile of the empirical distribution of income drops observed in SIPP. All three scenarios consider a more risky labor income and as a consequence the agent takes less leverage due to the higher default premium lenders accordingly charge. These results have to be compared to those obtained with a house price drift, μ_P, of 1.67 percent and standard deviation, σ_P, of 10 percent (see Table 3).

First, I consider the impact of a higher probability of becoming unemployed. Because the persistence in regimes reflects the opportunity of defaulting in one regime versus the other, the two default boundaries are affected by its changes. A higher probability of switching from high to low labor income regime increases the opportunity of defaulting in the high regime as well. The LTV decreases for both educational groups, specifically from 92.5 percent to 91.8 percent for a college educated agent and from 95 percent to 94 percent for a high school educated one (see Table 4). The default premium slightly increases from 0.51 (0.64) to 0.55 (0.74) percent for the college (high school) educated agent. Then, the probability of default varies in both high and low labor income regimes for both education groups, from 8.5 (9.25) percent in the high (low) labor income regime to 8.69 (9.37) percent for a college educated agent and from 10.46 (10.97) percent in the high (low) labor income regime to 10.54 (10.79) percent for a high school educated one.

Insert Table 4 here

Second, I consider the impact of unemployment persistence on the default strategy. Specifically, a higher persistence of low labor regime increases the opportunity of defaulting in low regime because the expected duration of being unemployed increases. *Ceteris paribus*, the increasing opportunity of defaulting increases the default premium. Consequently, the agent provides a higher down payment. Nevertheless, the default premium slightly increases from 0.51 (0.64) to 0.54 (0.70) percent for a college (high school) educated agent. The probability of default in the low labor income regime increases from 9.25 to 10.84 percent for a college educated agent and from 10.97 to 12.83 percent for a high school educated one. Furthermore, a higher risk of unemployment persistence also uniformly increases the probability of default over time. This effect is clearly visible from Figure 8, where I present the probability of default cumulatively over time for the low labor income regime for the base case and this scenario for both education groups.

Insert Figure 8 here

Finally, I consider a higher drop in labor income. Specifically, the labor income replacement rate $\phi(e_j)$ is 39.77 percent, instead of 55.97 percent, for a college educated agent, and 46.33 percent, instead of 74.58 percent, for a high school educated one. In this scenario, when labor income switches from the high to the low labor income regime, the MTI and the average $\overline{MTI}$ ratios substantially increase in the low regime due to the dramatic drop of labor income relatively to the mortgage payment. As a result, borrowing constraints are more binding, making the default option more attractive. The probability of default dramatically increases in the low labor income regime, from 9.25 to 17.05 percent for a college educated agent and from 10.97 to 15.58 percent for a high school educated one.

In Figure 9, I present the probability of default cumulatively over time for the low labor income regime as the base case and the scenario where a more severe drop in labor income is expected. The probability of default rises quickly at the very beginning, owing to the fact that borrowing constraints are more binding in the first years of a mortgage life. This effect is more apparent for the college educated agent where the MTI ratio increases from 54.51 to 73.44 percent. The likelihood of default is the greatest within the first four years. In this scenario, borrowing constraints exacerbate the negative labor income shock making the default option very valuable and inducing an agent with large mortgage payments relative to labor income to face the choice between severe consumption cutback and default followed by a move into the rental market with a higher buffer of savings.

Insert Figure 9 here

5.4 Owning vs renting decision

The model sheds light on the choice between owning and renting. In literature, Sinai and Souleles (2005) addressed the question of under which circumstances an agent is better off owning. They identified a hedging benefit to owning rather than renting should an agent have a long horizon and be exposed to volatile rents. Their claim that owning is a natural hedge against rent fluctuations is intuitive, but is built on the assumption that an agent does not need to take a mortgage, because she is endowed with enough wealth to pay the full purchase price. They abstract from how the agent finances a house purchase.[27]

The risk premium for renting, λ, measures the risk associated with renting. Because owning provides the benefit of avoiding the rent shocks, in equilibrium the rent risk premium is bid into rental payments with a positive sign. Hence, *ceteris paribus*, the rent risk premium tends to increase the demand for home ownership. In my model, the default premium, κ, measures the risk associated with the cost of owning a house and this reduces the amount of leverage the agent is willing to take and hence the demand for owner-occupied housing. For instance, if the down payment ω decreases, increasing the default premium κ, the rent risk premium λ must decrease sufficiently to keep the owner indifferent between owning and renting. Because leverage makes owning a house less attractive, the rent risk becomes decreasingly small relative to the default risk.

To illustrate this point, I assume that the agent is endowed with an initial level of cash on hand that is enough to buy the house without using leverage, as in Sinai and Souleles (2005). Specifically, I assume that the initial cash on hand is equal to the labor income the agent receives in the first year plus an endowment corresponding to the house value in the first period, $A(0^-) = L^h(0^-) + H \times P(0)$. Then, to obtain the rent risk premium that makes the agent indifferent between owning and renting, I assume the agent does not take leverage but she withdraws the entire amount $H \times P(0)$ from her

[27] Their argument is built on two other main assumptions as well. The first assumption is that the agent does not earn any risky labor income which might be positively correlated with rent fluctuations. In a partial equilibrium framework, Ortalo-Magné and Rady (2002) analyze an agent's tenure choice under uncertainty of income and house prices. They show that renting becomes relatively more attractive than homeownership as the covariance between labor income and house price increases. Davidoff (2006) shows that an agent with mean-variance preferences optimally purchases less housing as the covariance between labor income and housing prices rises. The second assumption is that they rule out fixed-price, long-term, lease contracts (whereby the tenant could lock in fixed rental payments for the duration of her tenancy).

saving account. Then, I solve the model for both education groups varying the house price process parameters as in Table 3. Table 5 Column (1) reports the rent risk premium when the agent does not finance the house purchase using leverage, while Column (2) reports the rent risk premium reported in Table 3. As expected, I obtain a higher rent risk premium in any house price scenario considered for both education groups. Hence, my results support the argument that ruling out the effect of taking costly leverage on owning a house significantly biases the Sinai and Souleles (2005) results in favor of owning over renting.

Insert Table 5 here

6 Conclusions

I propose a life-cycle model where a finitely lived risk averse agent finances her housing investment taking leverage. The main innovation of this paper is to derive jointly the down payment, the default premium and the rent risk premium in a dynamic setting where the agent is exposed to both stochastic labor income and stochastic house prices. An agent finances the home purchase by means of a fixed rate mortgage and she has the option to default on the mortgage itself, moving into the rental market. Risk neutral lenders efficiently price mortgages charging a default premium to compensate themselves for expected losses due to default on a mortgage. As a result, mortgage value and amount of leverage are closely linked. An alternative is for the agent to rent the same house, paying a rent fully adjustable to house prices. The rent risk premium is set such that the agent is indifferent *ex ante* between owning with a mortgage and renting.

An important implication of the model is that the down payment and the house price volatility are positively related. As house price volatility increases, a higher down payment decreases the volatility of the equity share in a house and hence decreases the default premium making the cost of leverage less onerous. A further novel implication is that labor income risk and borrowing constraints directly affect household leverage and default strategy. In the presence of borrowing constraints, a higher risk of unemployment persistence and/or a substantial drop in the labor income decreases the leverage the agent takes. Finally, ruling out the effect of taking costly leverage on owning a house significantly biases the results in favor of owning over renting.

Future research could test these predictions empirically. The recent steep rise of U.S. house prices has received a lot of attention in the media and in literature, but recent literature documents substantial differences in expected growth rates, spread between highest and lowest growth rates, and timing of high and low growth rates phases across U.S. states during the same period. It could be empirically investigated whether these variations substantially affected the leverage U.S. households took.

Finally, it would be interesting to extend the model by endogenizing the housing choice. I treat this choice as exogenous, for simplicity, but it would be interesting to allow the agent to choose the housing services in the first period. The model's prediction is that the cost of mortgage has a major impact on the amount of leverage. In reality, household's decision is more complex because she has to jointly decide on how much housing stock to buy and how much leverage to take. Such an extension could establish new links between housing choices and leverage choices.

A Numerical approach

The problems of both the owner and the renter as well as the mortgage valuation cannot be solved analytically. I adopt a finite difference scheme following Kushner and Dupuis (2002).[28]

The goal is to solve a system of six partial differential equations: equations (18)-(19) (owning), equations (11)-(12) (renting), and equations(23)-(24) (mortgage valuation). For the equations (18), (19), (11) and (12), I obtain the optimal numeraire consumption of the owner and the renter respectively. To obtain the mortgage fair value M, I need to consider the default strategy of the owner and the value of the collateral at default. At the terminal date T the mortgage value is $M(T) = F(T) = 0$. Prior to maturity, the mortgage value satisfies the system of partial differential equations (23)-(24). The default strategy is derived solving simultaneously the system of partial differential equations (18)-(19) (owning) and (11)-(12) (renting) respectively, at any time step Δt, and verifying whether the agent does or does not exercise the default option. Conditional on the optimal exercise policy, the lender receives the mortgage coupon N or seizes the house, at net of the dead weight losses: $(1-\psi)H \times P(A(\tau), \tau, i)$, $i = l, h$. Hence, the partial differential equations (18)-(19) (owning) and (23)-(24) (mortgage valuation) have to satisfy the associated default boundary conditions. I identify two default boundaries $H \times P(A(\tau), \tau, i)$ associated with the high regime and low regime. Finally, I solve the model pinning down the down payment ω^*, the default premium, κ, and the rent risk premium, λ, given the levels of state variables $A(0)$ and $P(0)$ and the labor income regime.

The problem can be formulated as a standard finite-horizon dynamic program and solved by backward induction. At the terminal date T, the solution is trivial: consume everything. I solve for the indirect utility V at age $T - \Delta t$ conditional on cash on hand, income, house price and the (degenerate) policy rule at T. Next, I need to solve for numeraire consumption C^i at any time t and $i = l, h$. Usually, the consumption policy is derived (i) using the first order condition $\left(H^{-(1-\beta)(1-\gamma)}V_A(\cdot, i)/\beta\right)^{1/(\beta(1-\gamma)-1)}$ and approximating $V_A(\cdot, i)$ with its finite difference representation; (ii) discretizing the space of available policies and using simple grid search methods to find the optimal policy at each iteration. However, the first one lacks sufficient reliability in the presence of borrowing constraints as in my case, while the second one has the disadvantage that is very slow and not feasible if accurate solutions are necessary. Instead, I exploit the fact that the first order condition $C^i = \left(H^{-(1-\beta)(1-\gamma)}V_A(\cdot, i)/\beta\right)^{1/(\beta(1-\gamma)-1)}$ provides a simple mapping between C^i and $V_A(\cdot, i)$. Therefore, I take advantage of this relation by deriving the partial differential equation associated with the optimal numeraire consumption C^i. I provide the derivation for the rental case in high regime. Differentiating the Hamilton-Jacobi-Bellman equation:

$$\rho V^r(\cdot, h) = \sup_{C^h} \left\{u(C^h, H) + \mathcal{D}V^r(\cdot, h) + \theta_h(V^r(\cdot, l) - V^r(\cdot, h))\right\},$$

where

$$\mathcal{D}V^r(\cdot, h) = (rA + L^h - C^h - \alpha H \times P)V_A^r(\cdot, h) + \mu_P P V_P^r(\cdot, h) + V_t^r(\cdot, h) + \frac{\sigma_P^2}{2} P^2 V_{PP}^r(\cdot, h),$$

with respect to A, I obtain

$$\begin{aligned} 0 = & (r-\rho)V_A^r(\cdot, h) + \theta_h(V_A^r(\cdot, l) - V_A^r(\cdot, h)) + V_{tA}^r(\cdot, h) \\ & + (rA + L^h - C^h - \alpha H \times P)V_{AA}^r(\cdot, h) + \mu_P P V_{PA}^r(\cdot, h) + \frac{\sigma_P^2}{2} P_t^2 V_{PPA}^r(\cdot, h). \end{aligned} \tag{A-1}$$

Using the first order condition $V_A^r(\cdot, h) = \beta H^{(1-\beta)(1-\gamma)}(C^h)^{(-1+\beta-\beta\gamma)}$, tedious algebra produces the

[28] The approach was adopted in portfolio choice problems by Fitzpatrick and Fleming (1991), Hindy, Huang, and Zhu (1997), Munk (2000), Øksendal and Sulem (2007) and Budhiraja and Ross (2007); in capital structure problems by Titman, Tompaidis, and Tsyplakov (2004).

following dynamics

$$
\begin{aligned}
0 &= (-\tilde{C}_P^2((2+\beta(3+\beta(\gamma-1))(\gamma-1))\sigma_P^2 P^2)/(2\tilde{C}) - (r-\rho)\tilde{C} \\
&+ \theta^{h\to l}(\tilde{C} - \tilde{C}^{2-\beta+\beta\gamma}\hat{C}^{-1+\beta-\beta\gamma}) + ((1-\beta+\beta\gamma)(-\tilde{C} + (1-g)L_t^h + rA - \alpha H \times P))\tilde{C}_A \\
&+ \mu_P P(1-\beta+\beta\gamma)\tilde{C}_P + (1-\beta+\beta\gamma)\tilde{C}_t + \frac{\sigma_P^2}{2}(1-\beta+\beta\gamma)P^2\tilde{C}_{PP}, \qquad \text{(A-2)}
\end{aligned}
$$

where $\tilde{C} = C(A(t), P(t), t, h)$ and $\hat{C} = C(A(t), P(t), t, l)$. Following Candler (1999) I treat the non-linear terms 'explicitly', thus resolving the problem according to a finite difference approach (see Kushner and Dupuis (2002)). I impose that $C(A(t), P(t), t, h) \leq A(t)$. The terminal boundary condition is $C(A(T), P(T), T, h) = C(A(T), P(T), T, l) = A(T) + H \times P(T)$.

The system of the Hamilton-Jacobi-Bellman equations are evaluated on a discrete state space grid. For W, I set the lower bound of the domain at $A_{\min} = 1,000\$$ and the upper bound at $A_{\max} = 250,000\$$. For P, I use $P_{\min} = 1\$$ and $P_{\max} = 3,000\$$. The boundary conditions are treated as follows. Economic intuition does not offer exact boundary conditions at $P_{\min}$, $P_{\max}$ and $A_{\max}$. Then, I impose that $V_A(A_{\max}, P, t, i) = V_P(A, P_{\max}, t, i) = V_P(A, P_{\min}, t, i) = 0$ and $C_A(A_{\max}, P, t, i) = C_P(A, P_{\max}, t, i) = C_P(A, P_{\min}, t, i) = 0$, $i = l, h$.

B Determination of the Probability of Default

The solution of the owning, renting and mortgage problem is a prerequisite to obtain the probability of default, since it is by comparing the indirect utility of owning to the indirect utility of renting that the agent decides whether to continue the mortgage payments or default. I evaluate the probability the homeowner is into her default region expressed as

$$
u(A^*(t), P(t), t, i) = \text{Prob}(A^*(\tau), P(\tau), \tau, i) \quad \in \quad D \quad \text{for} \quad \text{some} \quad \tau > t, \qquad \text{(A-3)}
$$

where $A^*(t)$ is the owner's cash on hand accounting for the optimal numeraire consumption $C(A^*(t), P(t), t, i)$, the owner's labor income $L^i(t)$ and the house price $P(t)$ at current time t. Such probability satisfies the Kolmogorov backward equation that is

$$
\begin{aligned}
0 &= \mathcal{D}u(\cdot, l) + \theta^{h\to l}(u(\cdot, h) - u(\cdot, l)), \qquad \text{(A-4)} \\
0 &= \mathcal{D}u(\cdot, h) + \theta^{l\to h}(u(\cdot, l) - u(\cdot, h)), \qquad \text{(A-5)}
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{D}u(\cdot, i) &= (rA_t + (1-g)L_t^i - C_t^i - N)u_{A^*}(\cdot, i) + \mu_P P_t u_P(\cdot, i) + u_t(\cdot, i) \\
&+ \frac{\sigma_P^2}{2} P_t^2 u_{PP}(\cdot, i), \quad i = l, h. \qquad \text{(A-6)}
\end{aligned}
$$

The boundary conditions are:

$$
\begin{aligned}
u(A^*(T), P(T), T, i) &= 0, \qquad \text{(A-7)} \\
u(A^*(\tau), P(\tau), \tau, i) &= 1, \qquad \text{(A-8)} \\
u(A^*(t), P(t), t, i) &= 0, \qquad \text{(A-9)}
\end{aligned}
$$

for $i = l, h$. These conditions merely state that default has probability one if the agent arrives at a default region and probability zero if the agent continues to the mortgage maturity.

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	High School Graduates	College Graduates
Expected annual labor income without unemployment spells in a Year	$37,167	$54,723
Likelihood of experiencing unemployment in a Year	0.1390	0.0742
Expected annual labor income with Unemployment	$27,719	$30,628
Replacement rate of income with versus without Unemployment	0.7458	0.5597
Likelihood of finding a job in a Year	0.4172	0.4467

Table 1: Expected income, likelihood of experiencing unemployment in a year, unemployment income, replacement rate of average with versus without unemployment and likelihood of finding a job in a year, by levels of education. Source: SIPP 1997 – 2005.

Variable	Symbol	Value
Panel A - Labor income parameters		
College graduates		
Level of labor income ($ thousands)	L_0^h	36.8035
Likelihood of experiencing unemployment	$\theta^{h\to l}$	0.0742
Likelihood of finding a job	$\theta^{l\to h}$	0.4467
Labor income replacement rate	ϕ	0.5597
Labor income drift (I) - L_t^h	μ_0	-4.0295
Labor income drift (II) - L_t^h	μ_1	0.2787
Labor income drift (III) - L_t^h	μ_2	-0.0530
Labor income drift (IV) - L_t^h	μ_3	0.0030
High school graduates		
Level of labor income ($ thousands)	L_0^h	31.8058
Likelihood of experiencing unemployment	$\theta^{h\to l}$	0.1390
Likelihood of finding a job	$\theta^{l\to h}$	0.4172
Labor income replacement rate	ϕ	0.7458
Labor income drift (I) - L_t^h	μ_0	-0.6689
Labor income drift (II) - L_t^h	μ_1	0.0358
Labor income drift (III) - L_t^h	μ_2	0.0006
Labor income drift (IV) - L_t^h	μ_3	-0.0006
Tax rate	g	0.2
Panel B - Housing parameters		
House stock ($ thousands) - High School Graduates	H	180
House stock ($ thousands) - College Graduates	H	128
House price drift	μ_P	0.016
House price standard deviation	σ_P	0.10
Panel C - Other parameters		
Relative risk aversion	γ	2
House flow services	β	0.30
Time preference	ρ	0.03
Risk free rate	r	0.03
Time horizon (years)	T	30
Lender's deadweight loss	ψ	0.28
Borrower's deadweight loss	ϵ	0.05

Table 2: This table reports the calibrated and estimated parameters.

	(1) Down Payment ω^*	(2) Default Premium κ	(3) Rent Risk Premium λ	(4) LTV $F/(H \times P)$	(5) LTI F/L	(6) $\overline{LTI}$ $F/\bar{L}$	(7) MTI N/L	(8) $\overline{MTI}$ $N/\bar{L}$	(9) Prob Default u
Panel A - College education									
$\mu_P = 1.67\% - \sigma_P = 10\%$	7.5%	0.51%	2.33%	92.5%	5.65	3.58	30.51%	19.33%	8.50%
					(10.10)	(6.40)	(54.51%)	(33.54%)	(9.25%)
$\mu_P = 1.67\% - \sigma_P = 15\%$	14%	0.89%	2.92%	86%	5.25	3.33	29.69%	18.81%	9.47%
					(9.39)	(5.95)	(53.05%)	(33.62%)	(9.73%)
$\mu_P = 2.67\% - \sigma_P = 10\%$	5.5%	0.47%	3.02%	94.5%	5.77	3.66	31.01%	19.65%	7.55%
					(10.32)	(6.54)	(55.41%)	(35.11%)	(10.47%)
$\mu_P = 2.67\% - \sigma_P = 15\%$	12.5%	0.83%	3.57%	87.5%	5.34	3.39	30%	19.01%	7.95%
					(9.55)	(6.05)	(53.61%)	(33.97%)	(8.94%)
Panel B - High school education									
$\mu_P = 1.67\% - \sigma_P = 10\%$	5%	0.64%	2.18%	95%	4.77	3.19	26.18%	17.51%	10.46%
					(6.40)	(4.28)	(35.11%)	(23.47%)	(10.97%)
$\mu_P = 1.67\% - \sigma_P = 15\%$	12.5%	1.06%	2.61%	87.5%	4.40	2.94	25.38%	16.97%	10.69%
					(5.90)	(3.94)	(34.03%)	(22.75%)	(10.96%)
$\mu_P = 2.67\% - \sigma_P = 10\%$	3%	0.63%	2.84%	97%	4.87	3.26	26.70%	17.85%	8.87%
					(6.54)	(4.37)	(35.80%)	(23.94%)	(10.70%)
$\mu_P = 2.67\% - \sigma_P = 15\%$	7.5%	1.10%	3.18%	92.5%	4.65	3.11	26.98%	18.04%	10.95%
					(6.23)	(4.17)	(36.18%)	(24.19%)	(11.96%)

Table 3: This table reports the model results for different parameter configurations of the house price process. Panel A reports the model results for the college education group, while panel B for the high school one. Numbers reported in parenthesis refer to the low labor income scenario. Columns (1), (2) and (3) display the optimal solution to the problem: the optimal down payment ω^*, the default premium κ and the rent risk premium λ. Column (4) reports the loan to value ratio, LTV, Column (5) the loan to labor income ratio in the first year of the mortgage, LTI, Column (6) the loan to labor income ratio averaged over the life of the mortgage, $\overline{LTI}$, Column (7) the mortgage payment to labor income ratio in the first year of the mortgage, MTI, Column (8) the mortgage payment to labor income ratio averaged over the life of the mortgage, $\overline{MTI}$, Column (9) the default probability at the origination of the loan within the next 30 years.

	(1) Down Payment ω^*	(2) Default Premium κ	(3) Rent Risk Premium λ	(4) LTV $F/(H \times P)$	(5) LTI F/L	(6) $\overline{LTI}$ $F/\bar{L}$	(7) MTI N/L	(8) $\overline{MTI}$ $N/\bar{L}$	(9) Prob Default u
Panel A - College education									
$\theta^{h \to l} = 0.1484$ (0.0742)	8.2%	0.55%	2.39%	91.8%	5.61	3.55	30.43%	19.28%	8.69%
					(10.02)	(6.35)	(54.38%)	(34.46%)	(9.37%)
$\theta^{l \to h} = 0.2233$ (0.4467)	9.5%	0.54%	2.32%	90.5%	5.53	3.50	29.95%	18.98%	8.58%
					(9.88)	(6.26)	(53.51%)	(33.91%)	(10.84%)
$\phi = 0.3977$ (0.5597)	11%	0.47%	2.21%	89%	5.44	3.44	29.20%	18.51%	8.44%
					(13.68)	(8.67)	(73.44%)	(46.54%)	(17.05%)
Panel B - High school education									
$\theta^{h \to l} = 0.2780$ (0.1390)	6%	0.74%	2.18%	94%	4.72	3.16	26.24%	17.54%	10.54%
					(6.34)	(4.23)	(35.18%)	(23.52%)	(10.79%)
$\theta^{l \to h} = 0.2086$ (0.4172)	7.5%	0.70%	2.15%	92.5%	4.65	3.11	25.69%	17.17%	11.05%
					(6.23)	(4.17)	(34.44%)	(23.03%)	(12.83%)
$\phi = 0.4633$ (0.7458)	9.5%	0.66%	2.4%	90.5%	4.55	3.04	25.02%	16.73%	10.72%
					(9.82)	(6.57)	(54.02%)	(36.12%)	(15.58%)

Table 4: This table reports the model results for different parameter configurations of the labor income process. Panel A reports the model results for the college education group, while panel B for the high school one. Numbers reported in parenthesis refer to the low labor income scenario. Columns (1), (2) and (3) display the optimal solution to the problem: the optimal down payment ω^*, the default premium κ and the rent risk premium λ. Column (4) reports the loan to value ratio, LTV, Column (5) the loan to labor income ratio in the first year of the mortgage, LTI, Column (6) the loan to labor income ratio averaged over the life of the mortgage, $\overline{LTI}$, Column (7) the mortgage payment to labor income ratio in the first year of the mortgage, MTI, Column (8) the mortgage payment to labor income ratio averaged over the life of the mortgage, $\overline{MTI}$, Column (9) the default probability at the origination of the loan within the next 30 years.

	(1) Rent Risk Premium (No Leverage)	(2) Rent Risk Premium (Leverage)
Panel A - College education		
$\mu_P = 1.67\% - \sigma_P = 10\%$	2.65%	2.33%
$\mu_P = 1.67\% - \sigma_P = 15\%$	3.33%	2.92%
$\mu_P = 2.67\% - \sigma_P = 10\%$	3.48%	3.02%
$\mu_P = 2.67\% - \sigma_P = 15\%$	4.02%	3.57%
Panel B - High school education		
$\mu_P = 1.67\% - \sigma_P = 10\%$	2.92%	2.18%
$\mu_P = 1.67\% - \sigma_P = 15\%$	3.41%	2.61%
$\mu_P = 2.67\% - \sigma_P = 10\%$	3.55%	2.84%
$\mu_P = 2.67\% - \sigma_P = 15\%$	3.96%	3.18%

Table 5: This table reports the rent risk premium for different parameter configurations of the house price process. Panel A reports the results for the college education group, while panel B for the high school one. Column (1) reports the rent risk premium when the agent does not finance the house purchase using leverage, assuming that the agent's cash on hand in the first period is equal to the labor income in the first year plus an endowment corresponding the the house value in the first period. Column (2) reports the rent risk premium as provided in Table 3 for comparison purposes.

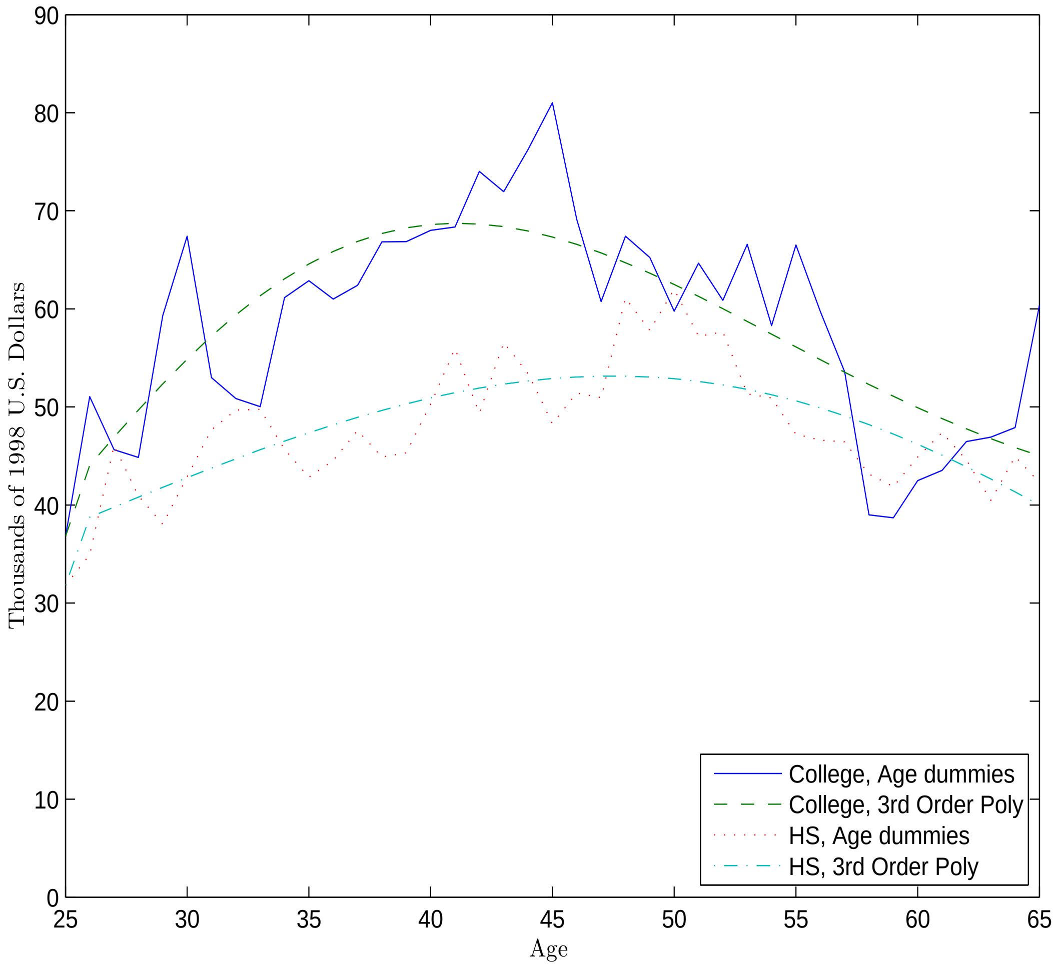


Figure 1: The figure depicts labor income processes estimated from SIPP for two different education groups: household heads (i) with high school education (including individuals that spent some year in college but without obtaining a degree); and (ii) with college graduate education. For each group, the figure plots the estimated age dummies and a fitted third-order polynomial.

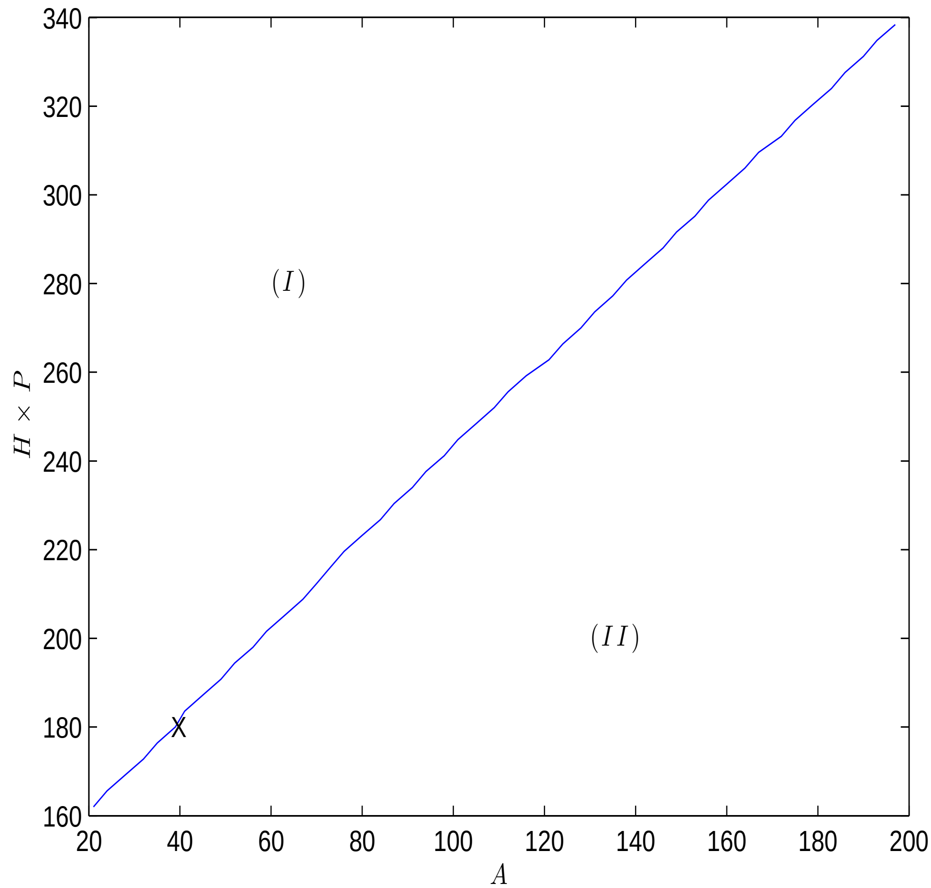


Figure 2: The figure represents the numerical solution of the model. In the owning with a mortgage area (I) owning with a mortgage dominates renting, $V^{om}(A(0), P(0), 0, h) > V^{r}(A(0), P(0), 0, h)$. X corresponds to the solution of the problem for the college educated agent: $\omega^* = 7.5$ percent, $\kappa = 0.51$ percent and $\lambda = 2.33$ percent, assuming a house price drift, μ_P, of 1.67 percent and standard deviation, σ_P, of 10 percent. All the other parameters are at their base values (see Table 2).

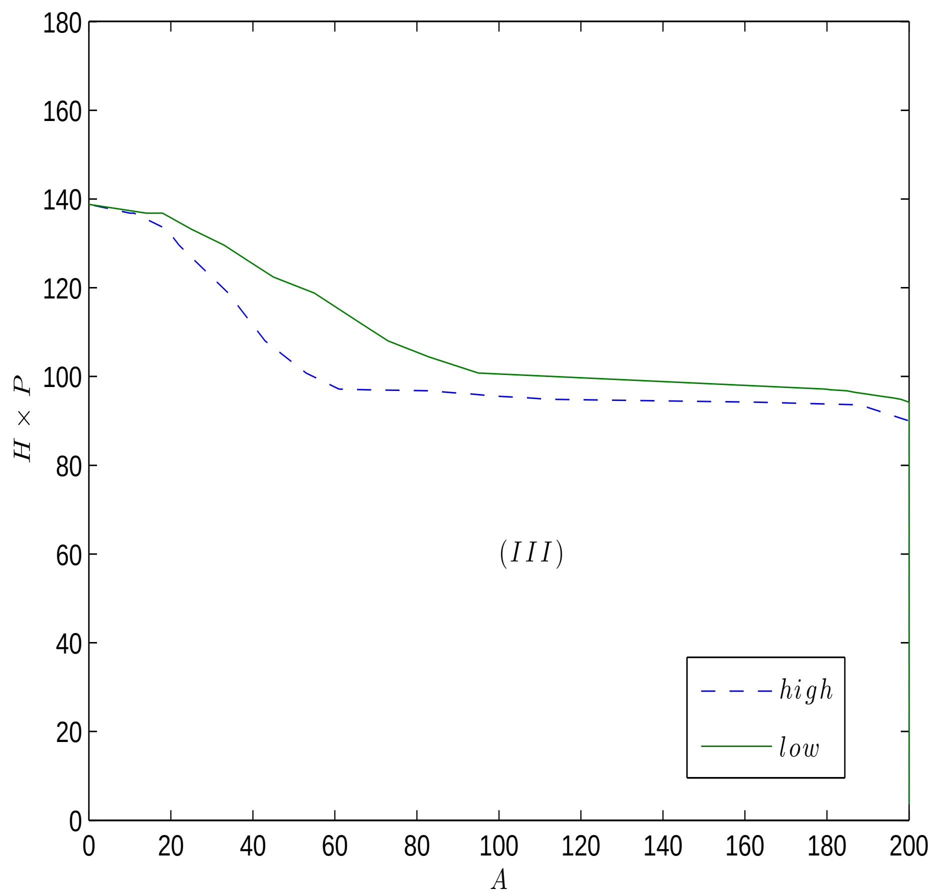


Figure 3: The figure represents the two default boundaries corresponding to the solution of the problem for the college educated agent in high (dotted line) and low (continuous line) labor income regime. (III) identifies the defaulting and renting area, assuming a house price drift, μ_P, of 1.67 percent and a standard deviation, σ_P, of 10 percent. All the other parameters are at their base values (see Table 2).

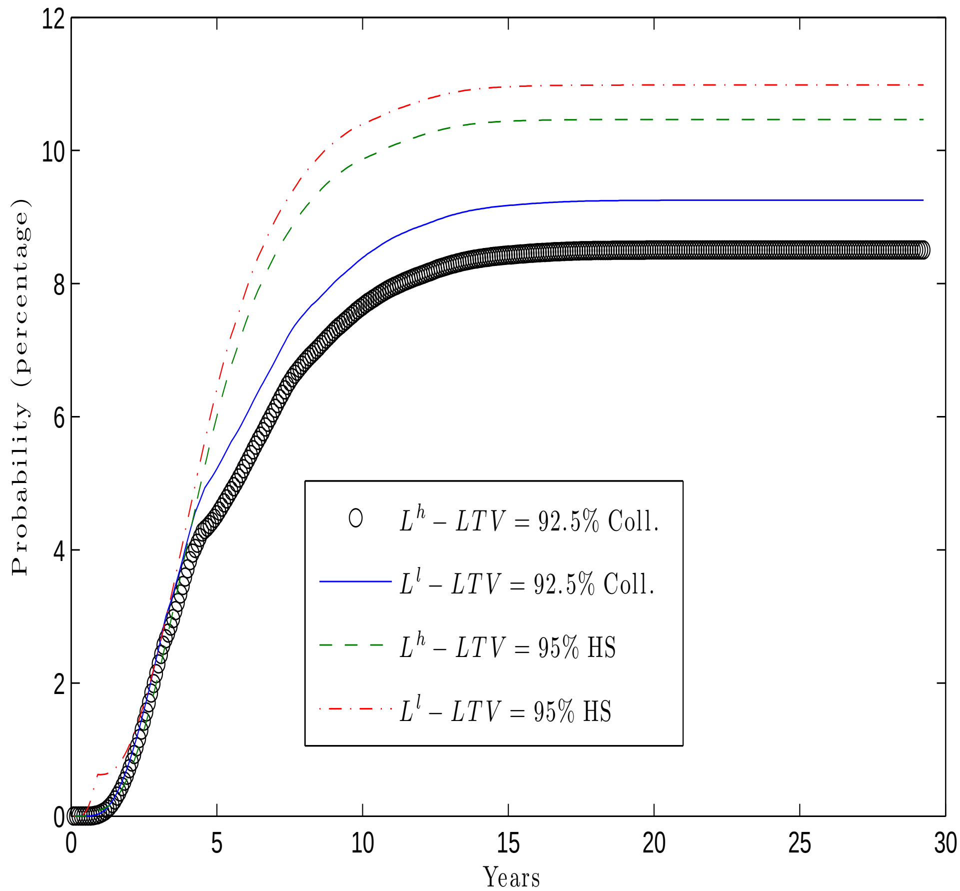


Figure 4: Cumulative probability of default in the high and low labor income regime for each education group, assuming a house price drift, μ_P, of 1.67 percent and a standard deviation, σ_P, of 10 percent. All the other parameters are at their base values (see Table 2). The cumulative probability of default at five years, for example, represents the probability at the origination of the loan that the mortgage defaults within the first five years.

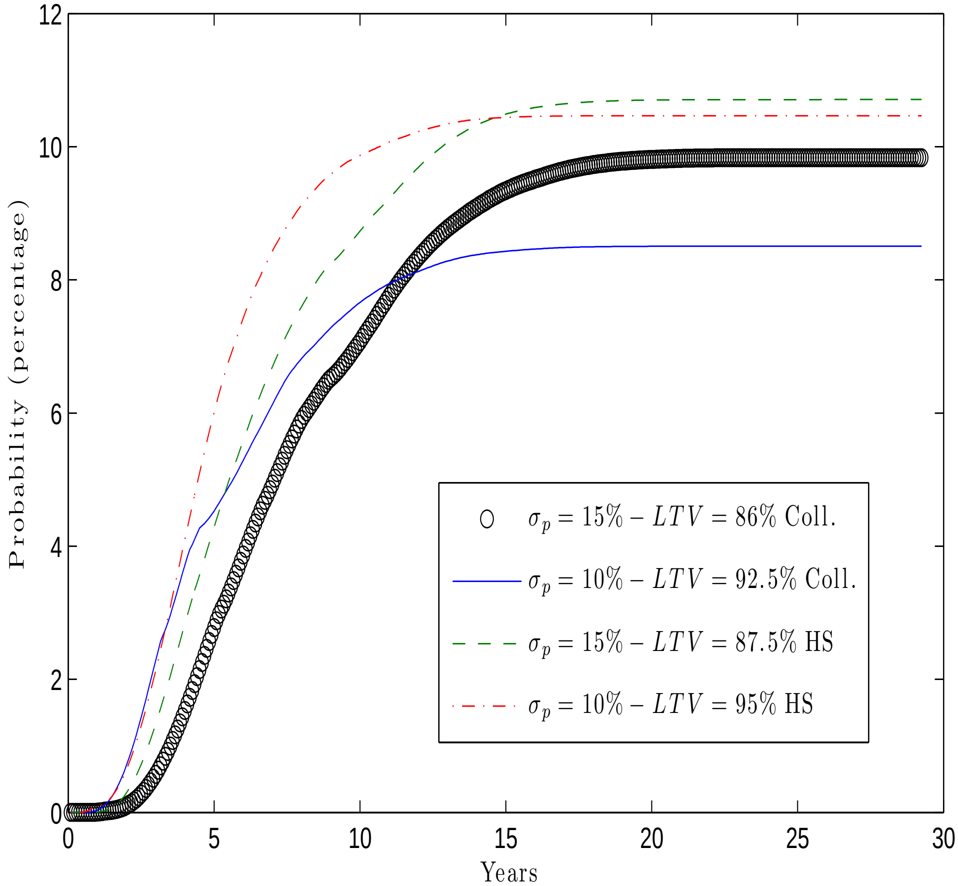
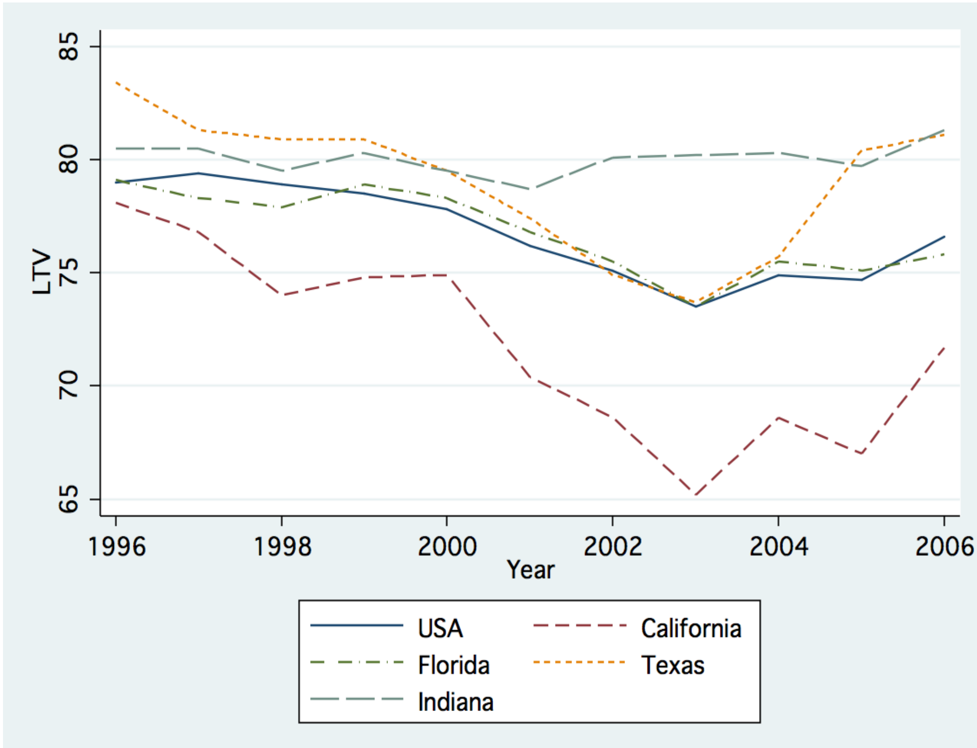


Figure 5: Cumulative probability of default in the high labor income regime for each education group, assuming a house price drift, μ_P, of 1.67 percent and a standard deviation, σ_P, of 10 and 15 percent. All the other parameters are at their base values (see Table 2). The cumulative probability of default at five years, for example, represents the probability at the origination of the loan that the mortgage defaults within the first five years.



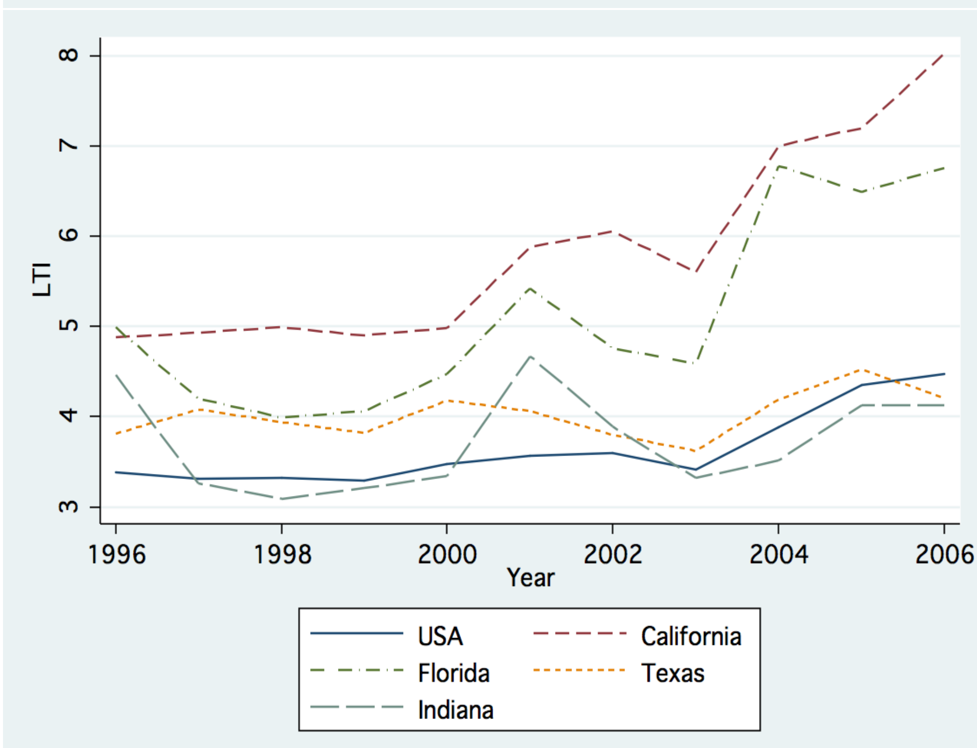


Figure 6: The figure plots the LTV ratio, in the upper panel, and the LTI ratio, in the lower panel, for U.S. and four U.S. states (California, Florida, Indiana and Texas) over the $1996-2006$ period. LTV data are from the Monthly Interest Rate Survey (MIRS), while LTI data are calculated as the ratio of the average loan amount obtained form the same survey to the average household income obtained from SIPP data.

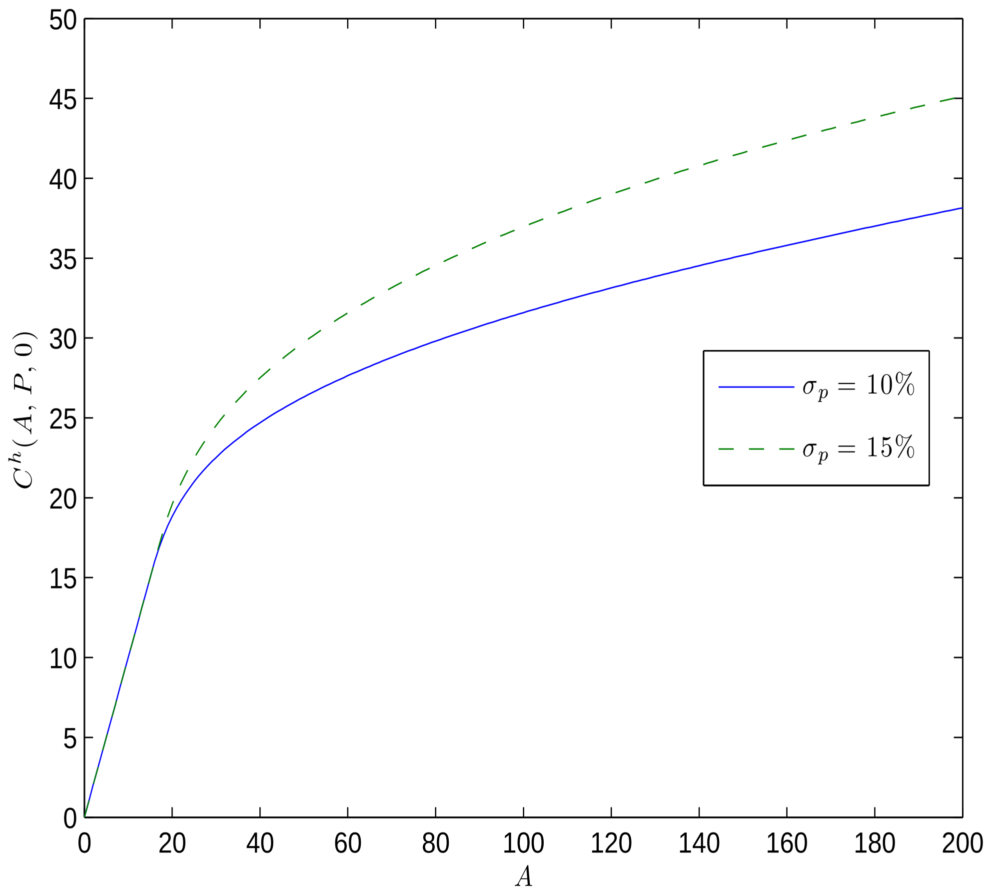


Figure 7: The figure represents the optimal consumption rule for a college educated agent, as a homeowner, as function of the cash on hand, A, in the high labor income regime, given a house value, $H \times P(0)$, of \$180,000. I assume a house price drift, μ_P, of 1.67 percent and a standard deviation, σ_P, of 10 (continuous line) and 15 (dotted line) percent. All the other parameters are at their base values (see Table 2).

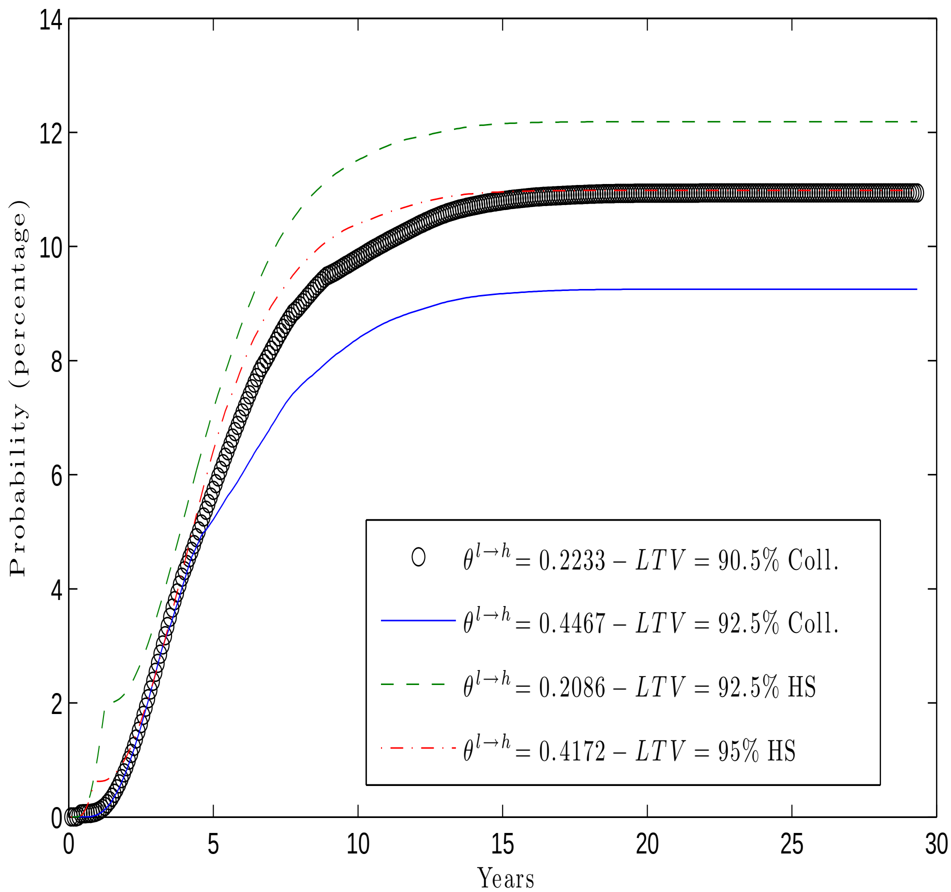


Figure 8: Cumulative probability of default in the low labor income regime for two different probabilities of switching from the low to the high labor income regime, $\theta^{l\to h}$, for each education group, assuming a house price drift, μ_P, of 1.67 percent and a standard deviation, σ_P, of 10 percent. All the other parameters are at their base values (see Table 2). The cumulative probability of default at five years, for example, represents the probability at the origination of the loan that the mortgage defaults within the first five years.

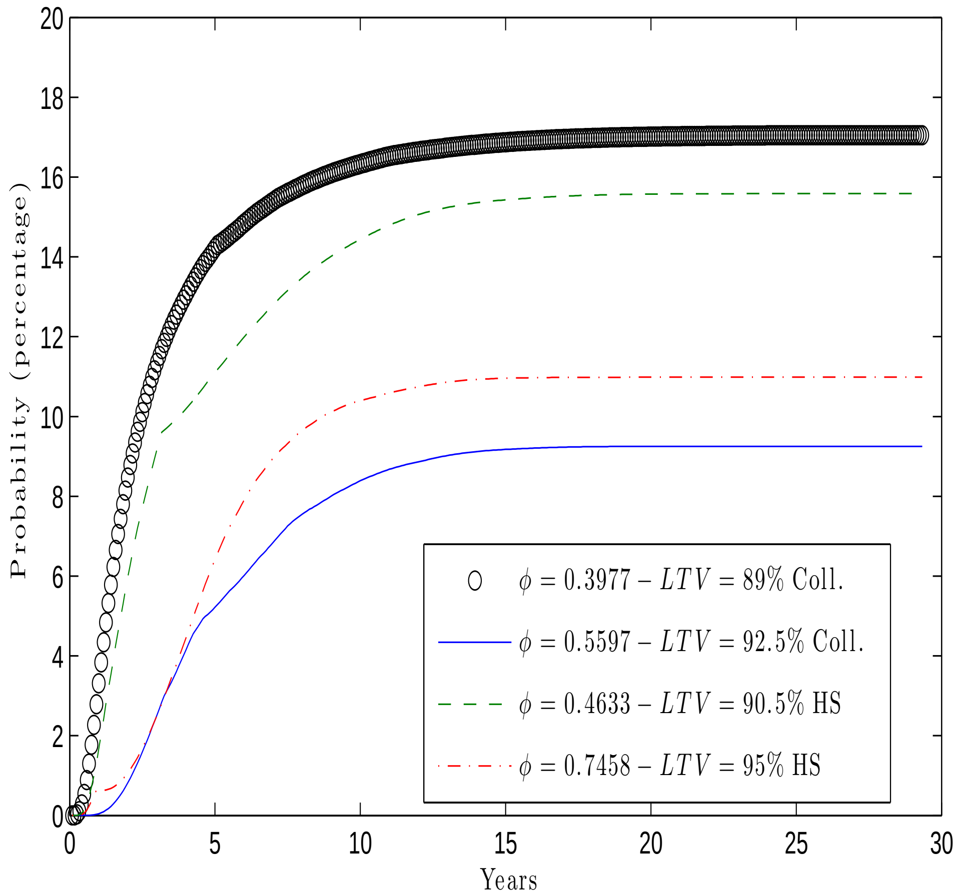


Figure 9: Cumulative probability of default in the low labor income regime for two different labor income replacement rates, ϕ, for each education group, assuming a house price drift, μ_P, of 1.67 percent and a standard deviation, σ_P, of 10 percent. All the other parameters are at their base values (see Table 2). The cumulative probability of default at five years, for example, represents the probability at the origination of the loan that the mortgage defaults within the first five years.