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ESTIMATING GVAR WEIGHT MATRICES

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In 2013 all ECB publications feature a motif taken from the €5 banknote.

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Abstract

This paper aims to illustrate how weight matrices that are needed to construct foreign variable vectors in Global Vector Autoregressive (GVAR) models can be estimated jointly with the GVAR's parameters. An application to real GDP and consumption expenditure price inflation as well as a controlled Monte Carlo simulation serve to highlight that 1) In the application at hand, the estimated weights differ for some countries significantly from trade-based ones that are traditionally employed in that context; 2) misspecified weights might bias the GVAR estimate and therefore distort its dynamics; 3) using estimated GVAR weights instead of trade-based ones (to the extent that they differ and the latter bias the global model estimates) shall enhance the out-of-sample forecast performance of the GVAR. Devising a method for estimating GVAR weights is particularly useful for contexts in which it is not obvious how weights could otherwise be constructed from data.

Keywords: Global macroeconometric modeling, models with panel data, forecasting and simulation

JEL classification: C33, C53, C61, E17

Non-technical summary

As of yet, for estimating the local models of a Global Vector Autoregressive (GVAR) model, respectively for solving the global model, the tradition has been to construct weights, which are needed to that end, based on external data sources. Applications to broad macroeconomic aggregates such as GDP or inflation, for instance, used to employ trade (exports and imports) based weight matrices. For financially oriented applications, alternatives involving various types of financial asset exposures have been suggested.

The present paper aims to illustrate how GVAR weights can be estimated directly along with all other parameters of a GVAR. In an application to real GDP and personal consumption expenditure prices for a panel of 18 countries, estimated weights are compared to trade-based ones. A controlled Monte Carlo simulation with calibrations inherited from the empirical setting serves to assess the extent to which ill-suited weight matrices can bias the global model estimates. The experiment confirms that distorted weights can bias the global model and would therefore blur its simulated dynamics.

The same application is also used to simulate out-of-sample forecasts over a test period of 18 quarters for horizons of 1-4 quarters to show that estimated weights can improve the out-of-sample forecast precision significantly for the majority of countries, with the gain in precision for both GDP and inflation ranging around 22% for specific countries and approaching 10% on average across countries.

While the estimation of GVAR weights is likely to entail substantial uncertainty in sample sizes that are typical for macroeconometric applications, it might be useful for two reasons: first, to cross-check whether fix weight parameters from external data sources are adequate (in principle, it remains advantageous to employ fix weights because one would not introduce additional parameter uncertainty, with the objective of reducing parameter uncertainty being the very rationale of the traditional GVAR methodology). Second, a strategy for estimating weights can be useful for applications in which it is not obvious how fix weights could otherwise be constructed from data.

1 Introduction

As an econometric approach to modeling the increasing economic interdependencies across countries, the Global Vector Autoregressive (GVAR) model methodology has gained widespread interest in recent years [see e.g. [21], [22], and [7] for initial methodological and empirical contributions]. Interlinkages between countries can be modeled by combining a set of country-specific VARs that contain weighted foreign variable vectors. The approach allows modeling simultaneously a large number of countries, while accommodating as well a broad set of economic variables in one model which would if modeled in an otherwise unrestricted conventional VAR be unfeasible to estimate due to a too high number of parameters. Empirical applications of GVARs are meanwhile quite numerous.[1]

As to the question of what to base GVAR weights upon, which are needed to construct the foreign variable vectors in the GVAR's local sub-models, the tradition has been to employ trade data when the application involved broad macroeconomic aggregates such as GDPs, price inflation, monetary policy variables, and the like (see e.g. [7]). An alternative has been suggested for financially oriented applications, e.g. by [17] and [3] who construct weights by referring to financial asset exposures, including elements such as portfolio equity, direct investment, portfolio debt, and others. A discussion paper by [9] explores, for the context of its application, a range of different strategies for weight matrix construction, including trade and different types of financial exposure to then assess the model performance under different schemes.

Factor models (see e.g. [24]), in that context, can be seen as a related approach to estimating weights. Principal component estimation is usually employed to estimate factors; however, weights that would be proportional to them can become negative or exceed one, thus are not necessarily bound on the zero-one interval and should therefore not be referred to as *weights* (they are called *factor loadings* instead). The same applies to Partial Least-Squares (PLS) methods applied in that context.[2] The main difference between factor methods and PLS is that only the latter explicitly takes the relation between dependent variables and independent factors into account. In

[1] Recent applications are e.g. [10], [3], [4], [2], [1], and [11].

[2] PLS was first introduced in [26]. For applications see e.g. [13] and [8] who assess various factor and shrinkage methods (in that paper not in conjunction with GVARs though) with respect to their forecast performance of New Zealand's GDP.

that sense, the GVAR weight estimation scheme as proposed in the present paper is more closely related to PLS since the estimation of weights will be targeted to the explained variance of dependent variables, specifically to that of the GVAR's global set of endogenous model variables.

Though weight estimation as such has therefore been deemed relevant for developing large-scale econometric models, direct estimation of a GVAR along with its weights has not been addressed yet. This paper aims to demonstrate that empirically estimated weight matrices may well differ from weights based for instance on trade flows, a finding that confirms also the conclusions from [8]. The purpose is then to assess the extent to which ill-suited weight matrices can bias the GVAR's dynamics as well as to highlight that the forecast performance of a GVAR may suffer from distorted weights.[3] A strategy for estimating GVAR weights might also be useful for applications in which it is not obvious how weights could otherwise be constructed from data, as for instance in cases when a GVAR is set up for banks, or mixed country and bank cross-sections as illustrated in [14].

2 Model setting

2.1 Local models

The global model is assumed to comprise N cross-section items that are indexed by $i = 1, 2, ..., N$.

A set of item-specific endogenous variables are collected in a $k_i \times 1$ vector $\mathbf{y}_{it}$ which is related to a number of autoregressive lags up to P and a $k_i^* \times 1$ vector of foreign variables $\mathbf{y}_{it}^*$ that enters the model time-contemporaneously and with a number of lags up to Q, that is,

$$\mathbf{y}_{it} = \mathbf{a}_{i0} + \mathbf{a}_{i1} t + \sum_{p=1}^{P} \mathbf{\Phi}_{ip} \mathbf{y}_{i,t-p} + \sum_{q=0}^{Q} \mathbf{\Lambda}_{iq} \mathbf{y}_{i,t-q}^* + \mathbf{\Psi} \mathbf{d}_t + \epsilon_{it} \qquad (1)$$

[3] A toolbox for estimating and solving the GVAR model including the weights, as well as the scripts and functions set up for conducting the simulation and forecast exercises presented in later chapters is available from the author on request.

where $\mathbf{a}_{i0}$, $\mathbf{a}_{i1}$, $\mathbf{\Phi}_{ip}$, $\mathbf{\Lambda}_{iq}$, and $\mathbf{\Psi}$ are coefficient matrices of size $k_i \times 1$, $k_i \times 1$, $k_i \times k_i$, $k_i \times k_i^*$, and $d_i \times 1$ respectively. The vector $\mathbf{d}_t$ contains global weakly exogenous variables. It is assumed that ϵ_{it} is i.i.d. with zero mean and covariance matrix $\mathbf{\Sigma}_{ii}$.

2.2 Estimating weight matrices

Weights w_{ijk} are needed to construct the foreign variable vectors $\mathbf{y}_{it}^*$ in all local models. For a reference item i, the weights assigned to all other items $j = 1, ..., N$ that are used for model variable k shall sum to unity.

$$\sum_{j=1}^{N} w_{ijk} = 1 \forall i, k \tag{2}$$

Moreover, the w_{iik} for all $i = 1, ..., N$ equal zero and the w_{ijk}'s should be non-negative. The w_{ijk} can be collected in K matrices of size $N \times N$ whose columns each sum to one.

$$\mathbf{w}_k = \begin{bmatrix} w_{00k} & w_{10k} & ... & w_{N0k} \\ w_{01k} & & & ... \\ ... & & & \\ w_{0Nk} & ... & & w_{NNk} \end{bmatrix} \tag{3}$$

A hypothetical thought about two sets of weights, corresponding respectively to the 'true' and some distorted weights deviating from the truth, suggests that distorted weights may have the potential to induce omitted variable bias, conditional on the assumptions for it to occur holding true (the omitted variable would need to be a determinant of a domestic, dependent variable and at the same time be correlated with independent variables included in the model). A way of inducing such bias would be to assign a too small (or a zero) weight to a foreign variable that should actually be receiving a significant positive weight. The second of the conditions for omitted variable bias to occur is then rather likely to hold in particular for GVAR applications, in which an omitted variable from a first equation is explicitly allowed to correlate with right-hand side variables from that same equation at another point in the GVAR equation system.

An alternative strategy for obtaining weights would be to estimate them along with the other parameters of the GVAR which would be accomplished by minimizing the sum of squared residuals from a local model subject to the constraints that its set of weights are non-negative and sum to unity. That is,

$$\underset{\Gamma_i, w_{ijk}}{min} \sum_{t=1}^{T} \epsilon_{it}^2 \tag{4}$$

subject to

$$w_{ijk} \geq 0, j = 0, ..., N, k = 1, ..., K$$

and

$$\textstyle\sum_{j=0}^{N} w_{ijk} = 1, k = 1, ..., K$$

where Γ_i comprises all local model coefficients contained in $\mathbf{a}_{i0}$, $\mathbf{a}_{i1}$, $\mathbf{\Phi}_{ip}$, $\mathbf{\Lambda}_{iq}$, $\mathbf{\Psi}$, and $\mathbf{d}_t$. The minimization problem for item i would exclude w_{ii} and set that to zero.

For the weight parameters to be estimable, an identifying assumption is that a significant relation between a local model's endogenous variables and some foreign variables must exist. In general, for K foreign variable vectors in K equations of a local model, at least one significant relation shall exist for each foreign variable vector (with respect to contemporaneous or some lagged vectors) for the weights to be estimable. In the hypothetical case that, say, a country's domestic variables do not relate at all to foreign variables, the weights would become nuisance parameters because changing them would leave the likelihood of that local model unaffected. The likelihood surface of the objective function with respect to the weights would in that case be flat.

For minimizing the constrained objective, an iterative, numerical optimization has been implemented, using a sequential quadratic programming method to solve the constrained multivariate function. Useful entry points to the literature on sequential quadratic programming are [15], [23] and [16]. The estimation is conducted item by item in the cross-section (from the weight matrix perspective therefore column by column) and at item level jointly for the system of local model equations if $K > 1$.

For a general treatment of regression problems that are subject to constraints (non-linear ones in general and both of equality and inequality-type) I refer to [25] who discusses the underlying assumptions and investigates the asymptotic behavior of least-squares estimators for such general constrained cases. Deriving a nonlinear, constrained estimator's asymptotic properties is complicated for two reasons: first, an optimal solution to a nonlinear objective function does not in general have a closed-form solution and can be derived only via some optimization algorithm[4]; Second, for a model involving inequality constraints, as for the weights in the present application, error bounds cannot be computed via the usual t-statistics, i.e. as a ratio of a mean estimate and its standard error (which one in principle could compute from the inverse Hessian matrix that is involved at the quadratic programming stage) because one would not account for the boundary constraints that were imposed. If some weight estimate was close to or at zero (or one), an estimated standard error might suggest that the weight could fall below zero (exceed one) and thereby violate the constraint.

Three approaches are conceivable to deal with such cases: First, confidence limits can be computed by means of inversion techniques that involve either a Wald or a likelihood ratio statistic. For the former see e.g. [19], for the latter e.g. [6]. Second, Bayesian inference methods can be employed which would involve a weighted bootstrap of the posterior distribution of the parameters (see e.g. [18]). Third, an unweighted bootstrap (parametric or nonparametric) can be used to generate a large number of pseudo-data samples from the model to then re-estimate the parameters to obtain their distribution and selected moments thereof, respectively. That third approach is used to obtain error bounds for the weights in this paper. The implemented bootstrap to draw from the residuals is nonparametric, thus no distributional assumptions are imposed on either the marginal distributions or the copula that together constitute the joint distribution of the global model's residuals.

2.3 Global solution

Solving for the global model follows now the standard procedure whereby local models need to be properly reformatted and stacked, involving the weight matrices that are either taken or estimated as outlined above. The description of how to solve the global model will in the following be brief. For details that are omitted I refer to [21].

[4] A nonlinear model, objective function respectively, shall be defined as one whose partial derivatives with respect to model parameters remain a nonlinear function of the model parameters. See e.g. [12].

A country-specific $(k_i + k_i^*) \times 1$ vector $\mathbf{z}_{it}$ is defined as follows.

$$\mathbf{z}_{it} = \begin{bmatrix} \mathbf{y}_{it} \\ \mathbf{y}_{it}^* \end{bmatrix} \tag{5}$$

The local models in equation (1) can then be reformulated.

$$\mathbf{A}_{0i}\mathbf{z}_{it} = \mathbf{a}_{i0} + \mathbf{a}_{i1}t + \mathbf{A}_{1i}\mathbf{z}_{i,t-1} + ... + \mathbf{A}_{Pi}\mathbf{z}_{i,t-P} + \epsilon_{it} \tag{6}$$

where it is assumed for ease of notation in the following that $P = Q$ and the global exogenous variable vector $\mathbf{d}_t$ be empty. The $\mathbf{A}_{ip}$ coefficient matrices are all of size $k_i \times (k_i + k_i^*)$ and have the following form.

$$\begin{aligned} \mathbf{A}_{i0} &= (\mathbf{I}_{k_i}, -\mathbf{\Lambda}_{i0}) \\ \mathbf{A}_{i1} &= (\mathbf{\Phi}_{i1}, \mathbf{\Lambda}_{i1}) \\ &... \\ \mathbf{A}_{iP} &= (\mathbf{\Phi}_{iP}, \mathbf{\Lambda}_{iP}) \end{aligned} \tag{7}$$

The endogenous variables across items in the cross-section are stacked in one global vector $\mathbf{y}_t$ which is of size $k \times 1$ where $k = \sum_{i=1}^{N} k_i$. Here, we need to map the local variable vectors $\mathbf{z}_{it}$ to the global endogenous variable vector $\mathbf{y}_t$ which is accomplished via $(k_i \times k_i^*) \times k$ link matrices $\mathbf{W}_i$. With $\mathbf{z}_{it} = \mathbf{W}_i\mathbf{y}_t$ we can rewrite the model once more.

$$\mathbf{A}_{i0}\mathbf{W}_i\mathbf{y}_t = \mathbf{a}_{i0} + \mathbf{a}_{i1}t + \mathbf{A}_{i1}\mathbf{W}_i\mathbf{y}_{t-1} + ... + \mathbf{A}_{iP}\mathbf{W}_i\mathbf{y}_{t-P} + \epsilon_{it} \tag{8}$$

Now, we move from item-specific models to the global model by stacking the former in one global system, that is,

$$\mathbf{G}_0\mathbf{y}_t = \mathbf{a}_0 + \mathbf{a}_1 t + \mathbf{G}_1\mathbf{y}_{t-1} + ... + \mathbf{G}_P\mathbf{y}_{t-P} + \epsilon_t \tag{9}$$

where the $\mathbf{G}_{0,...,P}$ matrices are of size $k \times k$ and have the following form.

$$(\mathbf{G}_0, ..., \mathbf{G}_P) = \begin{pmatrix} \mathbf{A}_{01}\mathbf{W}_1 \\ \mathbf{A}_{02}\mathbf{W}_2 \\ ... \\ \mathbf{A}_{0N}\mathbf{W}_N \end{pmatrix}, ..., \begin{pmatrix} \mathbf{A}_{P1}\mathbf{W}_1 \\ \mathbf{A}_{P2}\mathbf{W}_2 \\ ... \\ \mathbf{A}_{PN}\mathbf{W}_N \end{pmatrix} \tag{10}$$

A reduced form of the global model is finally obtained by pre-multiplying the system with the inverse of $\mathbf{G}_0$. This representation is observationally equivalent to the model in (1) and can now be used for forecasting and impulse response analysis.

$$\mathbf{y}_t = \mathbf{G}_0^{-1}\mathbf{a}_0 + \mathbf{G}_0^{-1}\mathbf{a}_1 t + \mathbf{G}_0^{-1}\mathbf{G}_1\mathbf{y}_{t-1} + ... + \mathbf{G}_0^{-1}\mathbf{G}_P\mathbf{y}_{t-P} + \mathbf{G}_0^{-1}\epsilon_t \tag{11}$$

3 An application

3.1 Model and weight matrix estimates

The global model that is set up to exemplify the proposed weight matrix estimation scheme comprises 18 countries and two variables: real GDP and personal consumption expenditure prices (retrieved from OECD databases), thus has 36 equations. All variables are modeled in quarter-on-quarter (QoQ) logarithmic differences so as to render them stationary. Table 1 summarizes the countries contained in the model as well as basic summary statistics of the model variables.

Three otherwise identically structured GVARs have been set up, having i) an estimated common weight matrix, ii) two estimated weight matrices for GDP and inflation separately, and iii) trade weights computed from the sum of bilateral nominal exports and imports as of 2005 taken from the IMF direction-of-trade statistics.[5]

The lag structure of the local models has been set by means of a specification search which considered all conceivable combinations of numbers for autoregressive lags (zero to two) and foreign variable vectors (one to two and the contemporaneous vectors

[5]Robustness checks with trade weights from other years or averages of trade weights over the sample period confirm that the results are not very sensitive to the choice of the benchmark trade matrix. Trade weights generally do not change much over time.

being forced) for each country model. The specification search was conducted for the three model schemes separately, where for five of 18 countries (AT, BE, DE, FR, US) the suggestion based on the benchmark GVAR with trade weights would have been different compared to the two GVARs with estimated weights (for the latter two the suggested lag structures were the same for 14 of 18 countries). The rule to decide for a common lag structure was to adopt the maximum of the suggestion respectively for the autoregressive and foreign variable vectors across the three model schemes per local model.[6]

The three GVARs were estimated and solved based on the sample from 1996Q1-2012Q2 (66 observations). Figure 1 summarizes the trade-based and the estimated weights in bar graph format.[7] They are further collected in tables: Table 3 reports the trade weights; Tables 4-6 show mean, lower bounds (5%), and upper bounds (95%) of the estimated common weight matrix; Tables 7-9 and 10-12 summarize mean and bound estimates for the two variable-specific estimation schemes, for GDP and inflation separately. Further, Table 13 reports the mean absolute deviations between any two pairs of weight matrices, with corresponding ranks to help identify the countries for which deviations are the largest / the smallest. Finally, Table 14 summarizes how many of the trade-based weights fall into (respectively exceed upper or fall below lower bounds of) the estimated weights under the two estimation schemes.

When comparing e.g. the GVAR common estimate with trade weights (Tables 3, 4, and 13), NL, FR, and DE (ranks 16-18) stand out as the countries for which the common estimates come closest to their trade-implied weights. For NO, IE and DK (ranks 1-3), deviations are most pronounced. E.g. DK's weight on NL equals 7.6% and 15.7% based on trade and the GVAR common estimate, respectively. Another sharp drop can be observed for instance for CA, where its trade-based weight for the US would equal 87.4% and the common weight estimate approximately 58.9%. Overall, 56% of the trade-based weights fall into the estimated error bounds, thus 44% remain outside (14% below lower and 30% above upper bounds, see Table 14).

[6]For the Monte Carlo simulation in Section 3.2, it was of avail to operate with identical lag structures for the three model schemes.

[7]The identifying assumption discussed in the previous subsection has been found to hold for all country models; in each local model and for either of the two estimation schemes, the coefficients on either contemporaneous or lagged foreign variable vectors were found to be significant at least at a 10% level.

With regard to the variable-specific weight estimates (Tables 7 and 10), the results suggest that estimates for GDP and price inflation separately differ in many cases quite markedly. See for instance DK's weights for SE. The weights for GDP and inflation against SE would equal respectively 2.4% and 16.1%. A second example is IE's weight on ES, with weights for GDP and inflation equalling 39.1% and 0%, respectively. Overall, the comparison of trade-based to variable-specific weights (Table 14), suggests that 52% and 54% of the benchmark weights fall into the estimated bounds for GDP and inflation, respectively.

3.2 Monte Carlo simulation

A first (of three) aspects that this sub-section aims to address is that employing **weight matrices which deviate from a true weight matrix can induce bias and therefore distort the global model dynamics**. Two stochastic simulation exercises were conducted to highlight this point:

1. Simulate 5,000 artificial pseudo-data samples each of size equal to the original sample (66 observations) from the GVAR involving a weight matrix estimate that is *common* for all model variables, which is considered the true Data Generating Process (DGP). Based on all pseudo-data sets, re-estimate and solve the GVAR once by estimating the GVAR parameters along with a common weight matrix and once using the benchmark weight matrix based on trade flows.

2. Simulate 5,000 artificial pseudo-data samples each of size equal to the original sample (66 observations) from the GVAR involving a weight matrix estimate that is *specific* to the model variables, which is considered the true DGP. Based on all pseudo-data sets, re-estimate and solve the GVAR once by estimating the GVAR parameters along with variable-specific weight matrices, once by estimating along with a common weight matrix, and once using the benchmark weight matrix based on trade flows.

Despite being artificial, the simulation is therefore designed to assess the extent to which misspecified weights may have biased the global model estimate in a concrete application, by aligning the DGP with empirical sample size and estimates. Moreover, the simulation is designed so as to take proper account of the uncertainty that surrounds the estimated weights (by re-estimating the GVAR weights, either common or variable-specific) on all pseudo-data samples.[8]

Upon re-estimation of the GVARs on all pseudo-data sets, the resulting sets of distributions for each global model coefficient, from a pair of models one of which was the correct (corresponding to the true DGP) and one the model employing an ill-suited weight matrix, were opposed. Two tests were conducted to assess the deviation of any two parameter distributions: a two-tailed F-test for the hypothesis that any two distributions had equal variances as well as a two-tailed t-test of the Null that any two distributions had equal means.

Results from the assessment of model parameter distributions are summarized in Figures 2 and 3. Table 15 shows the average number of coefficients from the global models that were judged to be deviating with respect to mean and variance conditional on a 1%, 5%, and 10% significance level. A distinction is made between the overall parameter space and a reduced one excluding all intercepts, since the latter play no role for the dynamic properties of the global model.[9]

The fraction of coefficients for which their mean was different at the 1% level equals an approximate 70.4% under the first simulation scheme. Excluding intercepts hardly changes that ratio (to 70.0%) because the number of intercepts (36) is small compared to the overall number of global model coefficients (2,664).

When considering the variable-specific weighting scheme the true DGP, trade-based weights would induce significant bias for about 78.3% of the model coefficients. Despite a common estimate reducing that ratio, it would still distort a considerable portion of means of around 59.3% of the model coefficients. Overall, the simulation results suggest that a significant portion of the GVAR's parameter space is distorted when employing

[8] A high performance computing network has been employed to parallelize the simulation rounds on 64 cores (using Matlab's parallel computing toolbox). The runtime for 5,000 rounds including steps 1. and 2. amounted to about 1 day and 2.5 hours, thus would have been lasting ca 66 days on a local PC's Matlab referring to one core. The runtime for obtaining one estimate of the global model (36 equations) under the common and variable-specific weights modus equal about four and seven minutes, respectively.

[9] Intercepts (together with all other coefficients in the model) do influence the model fit and residuals and thereby affect for instance a generalized impulse shock and response profile at the outset of a shock simulation horizon.

misspecified weight matrices.

A second aspect, related to the first, concerns the **implications for dynamic impulse responses**. Coefficient estimates, in conjunction with weights, have a bearing on two features of dynamic impulse responses that one would simulate from the global model: i) since they determine the residuals of the model and thereby their covariance structure, coefficients influence the characteristics of a shock profile on impact, i.e. the magnitude of an assumed shock (if aligned with, say, one standard deviation of the shock variable's residuals) as well as the magnitude and signs of shock responses[10]; ii) the shape of the dynamic responses and thereby the magnitude and sign of cumulative responses in the long-run.

If the global model's coefficients, weights respectively, were biased, then the model's dynamic shock responses might get distorted in these two respects. To assess how material such distortions might be for the empirical application at hand, a systematic generalized impulse response simulation was conducted for each of the three model schemes (the common weight matrix, variable-specific weight matrices, and the benchmark trade weights), to let each of the $N \times K$ variables once be the shock origin and record all $N \times K$ variables' responses.[11]

Upon simulation of all dynamic responses, three pairs of models were compared: i) the GVAR based on common weights versus trade weights, ii) the model based on variable-specific weights versus trade weights, and iii) the GVAR based on specific weights versus common weights. Four indicators were then computed for each pair:

1) The portion of identically signed generalized impulses on impact (excluding the number $N \times K$ of assumed shocks, because they got the same sign by assumption);

2) The portion of identically signed cumulative impulse responses in the long-run;

3) The root mean square deviation based on the generalized impulses on impact (excluding the number $N \times K$ of assumed shocks, because they got the same size by assumption);

[10] Unless one was to operate with non-factorized impulse responses, in which case a shock to one variable would be assumed not to induce responses for other variables on impact.

[11] See [20] for details about the generalized impulse response concept.

4) The root mean square deviation based on cumulative impulse responses in the long-run.

Table 16 reports the results for 1) and 2). For shock responses on impact and for cumulative ones in the long-run, the portion of identical signs ranges between 82.9% and 87.8%, suggesting for about 12.2% to 17.1% of the responses for a pair of models to have opposite signs.[12]

While from the evaluation presented in Table 16 one might conclude that the portion of incorrectly signed responses for pairs of models is limited, the root mean square deviations in Table 17 appear to be sizable. Particularly pronounced are the deviations for the long-run responses from the specific weights based model and the trade-based one: For GDP, the error estimate amounts to 65.7%, which is 76 times the standard deviation of QoQ GDP growth rates across countries. For inflation, too, a sizable deviation of 40.8% is reported, amounting to 84 times the historical cross-country average. For the responses at $T = 0$, deviations are smaller, though still sizable, with multiples relative to the historical standard deviations ranging between 16 and 24.

A third aspect to be addressed concerns the **additional coefficient uncertainty that the weight estimation scheme entails**. An additional stochastic simulation serves to highlight the point:

1. Simulate 5,000 artificial pseudo-data samples each of size equal to the original sample (66 observations) from the GVAR based on the *benchmark trade weights*, which is considered the true DGP.

2. Based on all pseudo-data sets, re-estimate and solve the GVAR once by estimating the GVAR parameters along with a common weight matrix, once along with variable-specific weights, and once using the benchmark weight matrix based on trade flows.

When estimating the GVAR including its weights in the second step, the resulting weight estimates do not differ significantly from the trade weights (nor do the estimates for the implied parameters of the global model), which proofs that the weight

[12] A full catalogue of all simulated generalized impulse responses underlying these summary measures is available on request.

estimation routine has been properly implemented. The point is instead to focus on the variances (the uncertainty) of the estimates for the implied global parameters when either considering the weights free or not free of uncertainty. Table 18 reports for how many of the coefficients' variances we measure a significant deviation when using the common weight or specific weight estimation scheme on the trade-based pseudo data. At a 10% confidence level, 90.1% of the variances are significantly different when employing the common weight estimation method. For the subset of the coefficient space at that critical significance level, the variances were on average about twice as large under the common weight estimation scheme. When instead using the variable-specific weight estimation method, 97.3% of the coefficients variances would be significantly different, with the corresponding variance factor estimated at 3.17. The results confirm that estimating the GVAR weight matrices comes at the cost of increased variance of the overall parameter space.

3.3 Out-of-sample forecasting

For assessing the potential of estimated versus external weights to influence the forecast performance of a GVAR, a pseudo-out-of-sample forecast exercise was conducted.

The three GVARs were estimated on a reduced sample from 1996Q1-2007Q1 and estimates then held fix and used to produce a set of 1- to 4-quarter ahead forecasts for the period from 2008Q1-2012Q2.[13] The first three intermediate forecasts for within 2007 (Q2-Q4) were neglected to let the evaluation be based on a common test-sample, with the same underlying number of 1- to 4-quarter ahead predictions (18 observations per horizon).

The evaluation of the simulated forecasts is based on Root Mean Square Errors (RMSE), which for the two GVARs involving estimated weights will be expressed as ratios to the benchmark GVAR's RMSE with trade-based weights. The ratios are accompanied by a Clark-West test statistic ([5]) that indicates whether a gain in performance was significant from a statistical viewpoint. RMSE ratios are also provided for the GVAR using estimated variable-specific weights relative to the one with the

[13]Weight estimates based on the reduced sample are not reported but have been compared to the full-sample based ones presented. Deviations are rather small. Detailed results are available upon request.

common weight matrix estimate.

For the evaluation of simulated forecasts beyond the 1-quarter horizon, the QoQ forecasts as well as respective reference data were cumulated. It is not meaningful otherwise to evaluate some predicted QoQ change farther out along the forecast horizon.

Figures 4 and 5 show the absolute RMSEs across variables and horizons. Figures 6-11 present the corresponding RMSE ratios in bar graph format, where the colors of the bars reflect the outcome of the test for equal predictive accuracy. Table 19 reports the averages of RMSE ratios across countries for respective models and variables.

Looking at the performance for GDP with common estimates versus trade weights (Figure 6), RMSE ratios for all horizons are less than one, except for NO up to the 3-quarter horizon, indicating that the involvement of estimated weights improved GDP forecast performance for a considerable portion of countries. At the intermediate horizons of 1 and 2 quarters, ratios approach 0.78 (for DK), thus imply approx. 22% improvement compared to the benchmark. On average across countries and the four horizons, the gain in performance amounts to about 10%. Between 13 to 15 of the improvements were measured to be significant at least at the 10% level.

For price inflation (Figure 7), gains are comparable to those of GDP. For 1-step ahead forecasts, the ratios for FR and BE equal 0.79 and 0.85, with between seven to eleven of the improvements being statistically significant and the average ratio across horizons equalling 0.9.

While the performance of the variable-specific weight GVAR relative to the benchmark GVAR (Figures 8 and 9) appear to be performing rather similarly to the common weights scheme, the results in Figures 10 and 11, opposing the variable-specific to the common weight matrix GVAR, reveal that gains from estimating variable-specific weights are rather limited. RMSE ratios across horizons equal 1.11 and 1.10 for GDP and inflation, respectively, indicating a deterioration in forecast performance of about 10% on average.

4 Conclusions

The aim of the paper was to illustrate how a GVAR can be estimated jointly with its weight matrices and to highlight that misspecified weights can bias the global model and impinge on its forecast performance.

Results from a controlled Monte Carlo simulation, with true and alternative model specifications inherited from empirical estimates based on a panel of 18 countries for GDP and personal expenditure price inflation, suggest that estimated weights differ from trade-based ones in most countries to an extent that would let the trade weights bias the global model significantly. While the results of the stochastic simulations of the type presented in Section 3.2 shall be seen as specific to the empirical setting chosen in this paper, they can be conducted more generally in other empirical settings to assess how material the deviations between a set of benchmark as opposed to estimated weights are.

The evaluation of simulated out-of-sample forecasts further highlights the role that GVAR weights play in influencing the GVAR's performance: common and variable-specific weight estimates improve predictive accuracy compared to a trade-based model for the majority of countries significantly (up to 22% for specific countries and about 10% on average across countries and horizons). For the application to GDP and inflation, the GVAR with variable-specific weight estimates did not improve forecast precision relative to the one involving the common weight matrix estimate.

While it shall remain advantageous, in principle, to operate a GVAR with fix weights from the viewpoint of compressing parameter uncertainty, a strategy for estimating GVAR weights can therefore be seen as a supplementary step for assessing the adequacy of external data based weights. Besides mitigating biases, a strategy for estimating GVAR weights can be useful for applications in which it is not obvious how weights could otherwise be constructed from data, as for instance in cases when a GVAR is set up for banks, or mixed country and bank cross-sections as illustrated in [14].

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Table 1: Overview model variables, countries and basic summary statistics

		Real GDP QoQ [YER]				Expenditure prices QoQ [CED]			
		Mean	STD	Min	Max	Mean	STD	Min	Max
AT	Austria	0.50	0.64	-1.88	1.51	0.45	0.28	-0.21	1.11
AU	Australia	0.80	0.58	-0.73	2.70	0.55	0.36	-0.20	2.40
BE	Belgium	0.44	0.62	-2.12	1.55	0.49	0.49	-0.90	1.35
CA	Canada	0.64	0.64	-2.02	1.65	0.39	0.31	-0.68	1.13
CH	Switzerland	0.45	0.61	-2.25	1.90	0.17	0.36	-0.87	1.17
DE	Germany	0.34	0.88	-4.16	2.19	0.30	0.32	-0.82	1.11
DK	Denmark	0.30	1.23	-2.45	3.82	0.48	0.40	-0.42	1.54
ES	Spain	0.59	0.65	-1.57	1.55	0.61	0.51	-1.09	1.61
FI	Finland	0.64	1.36	-6.81	3.29	0.49	0.78	-1.15	2.69
FR	France	0.39	0.53	-1.72	1.22	0.37	0.34	-0.55	1.18
IE	Ireland	1.02	2.03	-3.72	7.08	0.47	1.15	-5.92	1.95
IT	Italy	0.16	0.76	-3.65	1.39	0.58	0.30	-0.63	1.05
NL	Netherlands	0.50	0.71	-2.25	2.00	0.51	0.43	-0.83	1.92
NO	Norway	0.56	1.11	-1.98	3.32	0.48	0.82	-1.32	2.68
PT	Portugal	0.33	0.91	-2.33	2.21	0.62	0.51	-1.29	1.74
SE	Sweden	0.64	1.00	-3.82	2.42	0.35	0.48	-0.73	1.48
UK	United Kingdom	0.52	0.71	-2.11	1.72	0.54	0.50	-0.63	2.24
US	United States	0.59	0.69	-2.33	1.93	0.51	0.39	-1.43	1.12

Note: All statistics are based on logarithmic quarter-on-quarter differences times 100 based on the sample period 1996Q1-2012Q2.

Table 2: Model structure and performance

			R-square						Durbin-Watson statistic					
	Lag structure		YER			CED			YER			CED		
	P	Q	Benchmark	Common	Specific	Benchmark	Common	Specific	Benchmark	Common	Specific	Benchmark	Common	Specific
AT	2	2	0.834	0.847	0.851	0.793	0.791	0.789	1.722	1.729	1.787	1.716	1.691	1.713
AU	1	0	0.306	0.312	0.319	0.210	0.238	0.230	1.762	1.751	1.767	1.888	1.961	1.951
BE	2	0	0.727	0.779	0.786	0.509	0.628	0.645	1.943	1.980	2.013	1.915	2.049	2.004
CA	1	0	0.646	0.665	0.665	0.433	0.435	0.429	2.263	2.181	2.164	1.887	2.005	1.960
CH	1	0	0.550	0.581	0.678	0.406	0.426	0.524	1.861	1.835	1.872	1.794	1.718	1.858
DE	1	1	0.624	0.628	0.755	0.499	0.481	0.392	1.939	1.956	1.952	2.133	2.104	2.248
DK	1	0	0.459	0.561	0.581	0.175	0.134	0.226	2.436	2.462	2.435	2.116	2.001	2.080
ES	2	2	0.870	0.867	0.871	0.646	0.653	0.640	2.410	2.417	2.431	2.115	2.057	2.093
FI	1	0	0.625	0.684	0.739	0.105	0.083	0.126	2.517	2.498	2.391	2.080	2.070	2.047
FR	1	0	0.775	0.780	0.777	0.780	0.783	0.792	2.206	2.264	2.288	1.718	1.740	1.760
IE	1	0	0.358	0.467	0.503	0.389	0.388	0.448	2.287	2.297	2.344	2.335	2.303	2.389
IT	1	0	0.718	0.777	0.775	0.638	0.602	0.649	1.839	1.915	1.871	1.932	1.889	2.129
NL	1	0	0.698	0.708	0.699	0.225	0.247	0.242	1.967	2.005	1.942	2.062	2.105	2.068
NO	1	1	0.245	0.356	0.470	0.195	0.228	0.220	1.973	1.841	1.823	2.097	2.138	2.059
PT	1	0	0.363	0.462	0.498	0.565	0.537	0.624	2.074	2.000	2.063	2.368	2.443	2.313
SE	1	0	0.531	0.592	0.607	0.231	0.253	0.253	1.912	1.812	1.755	2.156	2.173	2.192
UK	1	0	0.659	0.690	0.713	0.256	0.301	0.351	1.833	1.803	1.947	2.056	2.078	2.050
US	2	0	0.590	0.659	0.673	0.594	0.662	0.697	2.008	1.900	1.947	1.706	1.899	1.702

Note: P and Q refer to the number of autoregressive and weighted foreign variable vectors, respectively.

Table 3: Weight matrix - Trade-based

	AT	AU	BE	CA	CH	DE	DK	ES	FI	FR	IE	IT	NL	NO	PT	SE	UK	US
AT	0.0	1.2	1.0	0.3	5.0	7.3	1.3	1.2	1.4	1.3	0.5	4.0	1.4	0.5	0.8	1.3	1.0	0.9
AU	0.4	0.0	0.5	0.5	0.6	0.7	0.7	0.5	1.2	0.6	0.7	1.0	0.5	0.2	0.2	1.0	1.5	2.2
BE	2.6	2.6	0.0	0.6	3.1	8.1	3.1	4.8	4.3	12.6	12.3	5.7	16.1	3.0	4.0	5.4	7.1	3.1
CA	0.8	4.2	0.7	0.0	1.2	0.9	0.8	0.6	1.2	0.8	0.5	1.0	0.6	4.0	0.4	1.0	2.4	48.1
CH	6.6	2.1	1.2	0.5	0.0	5.8	1.4	2.2	1.4	3.8	3.1	5.5	1.6	0.6	0.9	1.3	2.9	2.3
DE	53.0	11.9	23.3	2.1	32.7	0.0	24.4	19.7	21.3	23.7	9.6	24.4	29.9	15.4	16.8	18.4	17.5	11.6
DK	0.8	1.3	0.8	0.3	0.9	2.4	0.0	1.0	5.5	1.1	1.2	1.3	1.9	5.8	0.9	10.6	1.7	0.7
ES	2.6	2.6	3.6	0.4	4.2	6.3	3.2	0.0	3.1	11.6	3.1	9.3	4.3	2.8	36.2	3.0	5.5	1.5
FI	0.8	1.7	0.7	0.3	0.7	1.7	3.2	0.7	0.0	0.7	0.5	0.9	1.5	2.3	0.8	7.9	1.1	0.6
FR	5.4	7.1	18.3	1.2	12.0	14.5	6.3	23.8	5.7	0.0	6.4	17.7	10.2	8.9	12.9	6.5	11.6	5.5
IE	0.5	2.5	4.8	0.4	2.6	2.2	1.7	1.6	1.1	1.6	0.0	1.4	1.6	2.1	0.9	1.2	7.7	3.6
IT	10.5	6.8	5.4	1.0	12.6	9.7	4.8	12.2	5.0	11.9	4.0	0.0	5.8	3.6	6.5	4.4	5.9	4.2
NL	3.8	3.8	18.6	0.6	5.3	10.9	7.6	6.1	8.7	7.4	5.5	6.5	0.0	9.4	5.3	7.1	9.0	4.0
NO	0.3	0.5	1.2	1.2	0.4	2.2	7.5	1.1	3.7	1.3	1.5	0.8	2.0	0.0	0.9	10.9	4.6	0.9
PT	0.4	0.2	0.8	0.1	0.6	1.2	1.2	8.3	0.8	1.5	0.4	1.2	0.9	0.6	0.0	0.6	1.0	0.3
SE	1.5	2.9	2.1	0.4	1.2	3.1	17.3	1.6	19.8	1.7	1.3	1.8	2.6	11.1	1.4	0.0	2.8	1.7
UK	4.2	12.7	9.5	2.9	6.3	10.9	9.5	9.9	8.9	9.9	29.7	8.4	10.8	22.5	7.1	9.7	0.0	8.7
US	5.5	35.8	7.5	87.4	10.7	11.9	6.0	4.6	7.1	8.5	19.9	9.1	8.3	7.3	4.0	9.7	16.9	0.0

Note: Weights are computed based on the sum of nominal exports and imports, taken from the IMF Direction-of-Trade statistics as of 2005. Each column sums to 100 percent.

Table 4: Weight matrix estimate - Common for all model variables

	AT	AU	BE	CA	CH	DE	DK	ES	FI	FR	IE	IT	NL	NO	PT	SE	UK	US
AT	0.0	3.3	11.6	3.3	7.2	5.6	3.4	3.7	0.0	3.1	0.0	4.2	4.0	0.0	5.5	0.0	2.4	0.6
AU	0.7	0.0	0.1	3.1	3.7	3.3	0.0	3.3	1.2	3.7	0.1	0.0	0.3	21.8	1.7	0.0	6.7	4.3
BE	6.4	5.3	0.0	1.3	8.2	9.1	0.0	4.4	3.4	10.4	13.2	9.3	12.3	0.0	1.6	6.2	4.3	8.0
CA	0.8	3.8	0.2	0.0	2.7	2.8	7.7	4.5	1.2	4.0	0.1	4.6	2.4	0.5	4.1	13.3	3.3	37.7
CH	9.1	3.8	16.3	1.5	0.0	7.4	0.0	4.1	0.6	6.0	18.4	8.7	2.1	0.0	1.1	14.6	6.2	11.7
DE	36.3	8.4	14.6	1.8	23.5	0.0	11.0	13.2	21.6	17.9	0.0	18.6	24.6	0.0	0.0	17.4	19.4	1.7
DK	1.6	2.6	0.0	3.0	0.2	4.5	0.0	3.2	0.0	1.2	8.1	0.0	2.9	4.4	13.5	0.0	1.9	0.0
ES	3.4	4.9	4.8	4.0	3.0	5.3	11.5	0.0	10.7	10.0	37.5	12.8	6.9	1.8	27.1	0.0	9.0	4.1
FI	0.7	2.3	0.0	1.1	3.6	2.7	0.0	0.6	0.0	1.7	0.0	6.7	5.0	11.6	8.9	0.4	2.3	0.0
FR	5.4	7.6	19.5	1.8	11.7	12.4	1.5	20.1	4.6	0.0	0.3	16.4	9.6	6.3	7.0	0.0	13.5	9.1
IE	2.7	4.3	1.2	2.8	0.0	1.8	8.7	1.6	0.0	1.2	0.0	0.1	4.0	0.0	2.4	1.6	1.1	0.0
IT	7.9	6.0	11.6	3.3	11.7	10.2	0.0	10.5	12.0	11.0	0.0	0.0	4.6	0.0	6.2	4.6	9.9	0.0
NL	6.7	4.1	6.3	3.8	2.8	10.7	15.7	5.6	10.3	5.5	0.0	0.6	0.0	1.5	13.7	0.0	0.0	0.0
NO	0.3	3.3	0.0	1.0	0.3	1.5	9.4	1.2	7.5	1.3	0.0	0.7	1.4	0.0	4.0	13.2	0.8	0.0
PT	3.5	2.3	0.0	3.3	0.7	1.4	15.6	6.0	13.3	3.5	6.8	4.8	4.1	23.9	0.0	0.0	0.1	0.0
SE	4.7	2.0	0.0	3.5	4.5	3.1	7.8	2.9	9.8	1.5	1.1	3.1	1.7	25.0	0.0	0.0	3.2	11.4
UK	3.1	9.2	0.0	2.4	5.1	8.4	0.0	6.8	3.7	9.0	14.2	9.4	7.5	3.1	0.0	6.3	0.0	11.3
US	6.7	26.8	13.7	58.9	11.2	9.7	7.5	8.2	0.0	8.8	0.0	0.0	6.6	0.0	3.3	22.5	15.8	0.0

Note: Weights are estimated jointly with the GVAR's parameters and are the same for all endogenous model variables. Each column sums to 100 percent.

Table 5: Weight matrix estimate - Lower bounds (5%) - Common for all model variables

	AT	AU	BE	CA	CH	DE	DK	ES	FI	FR	IE	IT	NL	NO	PT	SE	UK	US
AT		0.0	11.1	0.2	0.0	0.0	0.0	3.2	0.0	3.0	0.0	0.0	0.5	0.0	0.0	0.0	0.0	0.0
AU	0.7		0.0	2.3	0.0	0.0	0.0	1.6	0.0	3.4	0.0	0.0	0.0	0.0	0.0	0.0	4.6	2.3
BE	1.6	0.5		0.2	3.7	0.1	0.0	1.6	0.0	10.3	0.0	6.6	7.0	0.0	0.0	0.0	0.0	4.8
CA	0.8	0.0	0.0		0.0	0.0	0.0	3.9	0.0	2.5	0.0	1.6	0.0	0.0	0.0	0.0	0.3	32.1
CH	2.3	1.9	15.8	0.0		0.0	0.0	0.0	0.0	5.9	0.0	7.4	0.0	0.0	0.0	0.6	1.6	10.5
DE	9.1	0.0	11.1	1.5	0.0		0.0	10.9	7.3	17.8	0.0	11.6	20.3	0.0	0.0	6.5	14.5	0.0
DK	1.6	0.0	0.0	0.0	0.0	0.0		1.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
ES	3.4	1.9	1.1	2.0	0.0	0.0	0.0		0.0	9.9	0.0	11.0	4.4	0.0	6.6	0.0	3.4	0.1
FI	0.6	0.0	0.0	0.2	0.0	0.0	0.0	0.0		1.5	0.0	0.8	0.0	0.7	0.0	0.0	0.0	0.0
FR	1.4	5.9	19.2	0.9	5.0	3.1	0.0	19.5	0.0		0.0	15.4	0.8	0.0	0.0	0.0	12.2	7.9
IE	2.7	0.0	0.0	0.0	0.0	0.0	0.9	0.0	0.0	0.0		0.0	0.0	0.0	0.0	0.0	0.0	0.0
IT	2.0	0.0	11.4	2.9	3.7	5.4	0.0	9.9	0.0	10.9	0.0		0.0	0.0	0.0	0.0	5.4	0.0
NL	1.7	0.0	2.6	3.3	0.0	6.2	0.0	4.5	0.0	4.9	0.0	0.0		0.0	2.9	0.0	0.0	0.0
NO	0.3	0.0	0.0	0.0	0.0	0.0	0.7	0.0	0.0	1.0	0.0	0.0	0.0		0.0	4.1	0.0	0.0
PT	3.5	0.0	0.0	2.9	0.0	0.0	0.0	4.5	0.0	1.1	0.0	0.0	1.6	0.0		0.0	0.0	0.0
SE	4.7	0.0	0.0	3.2	0.1	0.0	0.0	0.0	0.0	0.7	0.0	0.0	0.0	2.7	0.0		0.0	3.0
UK	3.1	0.0	0.0	1.0	0.0	4.0	0.0	4.0	0.0	9.0	0.0	6.0	0.0	0.0	0.0	0.0		0.0
US	1.7	16.8	12.8	58.0	7.3	0.0	0.0	7.0	0.0	8.5	0.0	0.0	0.7	0.0	0.0	2.0	9.9	

Note: Weights are estimated jointly with the GVAR's parameters and are the same for all endogenous model variables. For details as to the method for computing the bounds see Section 2.2.

Table 6: Weight matrix estimate - Upper bounds (95%) - Common for all model variables

	AT	AU	BE	CA	CH	DE	DK	ES	FI	FR	IE	IT	NL	NO	PT	SE	UK	US
AT	0.0	6.3	12.2	3.7	11.5	12.9	14.9	5.1	0.0	3.8	0.0	6.8	8.1	0.0	17.1	0.0	5.2	2.4
AU	0.7	0.0	0.3	4.1	9.6	11.2	0.0	4.9	3.9	3.9	0.7	0.0	1.9	37.6	7.1	0.0	10.3	9.2
BE	17.1	9.4	0.0	1.9	18.2	17.7	0.0	5.6	14.8	11.3	35.3	12.5	16.3	0.0	7.1	14.0	6.8	11.3
CA	0.8	5.4	0.4	0.0	5.3	7.8	27.6	6.8	6.1	4.2	1.0	7.4	5.8	2.6	12.2	26.2	5.9	40.2
CH	24.2	7.7	19.1	1.8	0.0	19.6	0.0	4.9	3.2	6.4	49.7	11.5	6.5	0.0	6.8	24.7	9.0	16.9
DE	96.5	11.7	15.3	3.6	36.9	0.0	29.0	13.8	34.2	18.0	0.0	21.7	29.5	0.0	0.0	27.8	24.0	2.5
DK	1.6	7.9	0.0	3.5	0.5	11.8	0.0	4.8	0.0	1.3	18.9	0.0	6.7	10.6	25.0	0.0	4.4	0.0
ES	3.5	9.1	5.4	4.5	6.6	32.2	26.3	0.0	33.1	10.9	71.9	15.9	11.2	7.4	39.3	0.0	12.4	7.1
FI	0.7	6.3	0.0	3.4	9.8	5.7	0.0	1.7	0.0	2.0	0.0	10.2	9.9	23.0	18.7	1.4	6.2	0.0
FR	14.4	10.6	20.2	2.1	17.9	19.3	7.9	22.3	17.7	0.0	1.8	18.3	12.8	20.4	17.8	0.0	16.3	11.5
IE	2.8	9.7	1.5	3.6	0.0	8.4	16.9	4.1	0.0	1.4	0.0	0.2	12.6	0.0	6.1	7.1	6.0	0.0
IT	20.9	7.7	12.6	5.2	18.9	16.5	0.0	11.8	30.5	11.6	0.0	0.0	13.8	0.0	16.3	11.2	12.9	0.0
NL	17.7	5.9	6.8	4.4	6.6	16.9	33.2	7.3	26.9	5.7	0.0	1.9	0.0	6.7	28.1	0.0	0.0	0.0
NO	0.3	8.5	0.0	2.5	1.7	10.2	18.9	3.0	15.7	3.4	0.0	3.4	4.9	0.0	13.4	22.3	4.1	0.1
PT	3.5	8.8	0.2	5.6	1.9	7.8	32.4	7.5	29.3	3.8	19.5	10.1	8.4	48.4	0.0	0.0	0.5	0.0
SE	4.7	3.9	0.0	5.5	12.4	7.3	22.5	3.8	20.3	1.7	4.7	6.5	6.6	42.1	0.0	0.0	6.9	14.0
UK	3.1	15.9	0.0	3.0	9.7	13.7	0.0	8.5	18.5	9.6	36.7	11.9	17.6	15.6	0.0	15.4	0.0	14.1
US	17.7	29.3	14.7	59.8	20.0	19.2	20.3	10.2	0.0	8.9	0.0	0.0	12.5	0.0	11.4	34.4	18.4	0.0

Note: Weights are estimated jointly with the GVAR's parameters and are the same for all endogenous model variables. For details as to the method for computing the bounds see Section 2.2.

Table 7: Weight matrix estimate - Specific for real GDP

	AT	AU	BE	CA	CH	DE	DK	ES	FI	FR	IE	IT	NL	NO	PT	SE	UK	US
AT	0.0	1.6	15.1	2.3	22.2	6.7	0.9	2.7	0.0	1.5	0.0	6.4	3.8	0.0	7.6	2.4	0.0	0.0
AU	0.4	0.0	0.0	0.5	6.1	3.5	0.0	3.6	5.4	3.5	6.2	0.0	0.3	18.4	0.0	0.0	14.3	7.7
BE	4.1	4.3	0.0	2.5	22.1	9.0	0.0	6.9	0.1	11.4	10.5	7.7	12.8	0.0	0.0	0.0	4.1	6.7
CA	2.6	5.7	2.1	0.0	0.0	9.8	13.6	1.0	17.4	3.1	3.0	3.6	2.8	0.0	6.0	14.7	0.0	36.7
CH	8.9	2.9	14.0	2.5	0.0	0.0	0.0	3.3	0.0	5.5	11.2	8.3	3.2	0.0	0.0	13.4	0.0	10.0
DE	35.8	8.7	11.8	3.3	8.9	0.0	6.9	16.6	8.5	17.1	0.0	19.3	20.5	0.0	0.0	9.5	24.6	0.0
DK	1.4	1.1	0.0	3.1	0.0	0.3	0.0	4.0	0.0	3.2	7.5	0.0	2.2	0.0	10.3	0.0	0.0	0.0
ES	2.6	5.5	2.0	0.9	0.0	0.0	12.7	0.0	10.0	11.8	39.1	12.6	4.7	0.0	26.9	0.0	11.5	0.8
FI	0.5	3.0	3.5	3.3	0.0	5.6	0.0	0.8	0.0	1.7	0.0	8.1	3.0	5.3	17.3	8.9	0.0	0.0
FR	5.0	5.6	12.9	3.9	5.4	11.6	0.0	17.2	0.0	0.0	18.4	16.1	9.8	29.4	0.0	2.8	18.0	8.1
IE	5.2	4.2	3.5	2.0	0.1	0.0	7.9	2.4	0.0	1.7	0.0	0.0	3.7	0.0	2.1	2.1	0.0	0.0
IT	9.5	5.0	7.9	3.0	13.9	13.2	0.0	10.5	26.8	9.4	0.0	0.0	4.4	0.0	0.0	8.4	8.0	0.0
NL	5.2	4.1	12.5	3.2	0.2	25.3	32.6	4.2	6.7	6.7	0.0	0.8	0.0	4.1	25.1	0.0	0.0	0.0
NO	0.2	3.0	0.0	3.0	0.0	0.0	4.9	2.7	6.3	2.0	0.0	1.7	3.9	0.0	4.7	11.0	0.0	0.0
PT	0.7	3.1	0.0	3.2	0.0	0.0	7.7	7.5	9.5	2.2	2.9	3.0	3.0	21.0	0.0	0.0	0.0	0.0
SE	4.2	3.2	0.0	0.9	7.3	0.0	2.4	3.4	9.2	2.2	1.2	3.0	5.0	18.8	0.0	0.0	0.0	12.1
UK	7.6	12.4	3.8	2.0	0.2	10.5	0.0	10.2	0.0	10.0	0.0	9.3	9.5	3.0	0.0	4.2	0.0	17.9
US	6.1	26.8	10.9	60.4	13.5	4.5	10.6	3.1	0.0	7.0	0.0	0.0	7.6	0.0	0.0	22.5	19.5	0.0

Note: Weights are estimated jointly with the GVAR's parameters and are specific to real GDP in the model. Each column sums to 100 percent.

Table 8: Weight matrix estimate - Lower bounds (5%) - Specific for real GDP

	AT	AU	BE	CA	CH	DE	DK	ES	FI	FR	IE	IT	NL	NO	PT	SE	UK	US
AT		0.0	10.7	0.0	13.8	0.0	0.0	1.3	0.0	0.8	0.0	0.0	0.5	0.0	0.0	0.0	0.0	0.0
AU	0.2		0.0	0.0	1.7	0.0	0.0	0.0	0.0	1.2	0.0	0.0	0.0	4.9	0.0	0.0	9.5	1.0
BE	4.1	0.0		0.8	19.6	0.0	0.0	5.3	0.0	10.8	0.0	0.6	4.1	0.0	0.0	0.0	0.0	0.0
CA	2.5	0.0	0.0		0.0	0.0	0.0	0.0	0.0	2.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	28.4
CH	1.6	0.0	10.1	0.0		0.0	0.0	0.0	0.0	4.4	0.0	5.5	0.0	0.0	0.0	0.0	0.0	4.4
DE	6.4	0.0	1.5	2.0	0.0		0.0	11.4	0.0	16.7	0.0	8.4	15.3	0.0	0.0	0.0	15.5	0.0
DK	1.3	0.0	0.0	0.0	0.0	0.0		0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0
ES	2.5	0.0	0.0	0.0	0.0	0.0	0.0		0.0	10.6	0.0	8.2	1.6	0.0	0.0	0.0	6.5	0.0
FI	0.5	0.0	0.0	0.3	0.0	0.0	0.0	0.0		0.0	0.0	0.3	0.0	0.0	0.4	0.0	0.0	0.0
FR	5.0	0.0	7.6	1.5	0.0	1.6	0.0	16.0	0.0		0.0	11.5	0.0	0.0	0.0	0.0	14.8	0.7
IE	5.1	0.0	0.0	0.0	0.0	0.0	0.5	0.0	0.0	0.0		0.0	0.0	0.0	0.0	0.0	0.0	0.0
IT	1.7	0.0	4.5	2.1	6.2	4.7	0.0	9.1	0.0	8.9	0.0		0.0	0.0	0.0	0.0	0.4	0.0
NL	5.1	0.0	6.9	0.1	0.0	16.7	0.0	0.7	0.0	5.0	0.0	0.0		0.0	0.7	0.0	0.0	0.0
NO	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.6	0.0	0.0	0.0		0.0	0.1	0.0	0.0
PT	0.6	0.0	0.0	2.1	0.0	0.0	0.0	3.2	0.0	0.0	0.0	0.0	0.8	5.1		0.0	0.0	0.0
SE	0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.1	0.0	0.2	0.0		0.0	0.0
UK	7.6	7.4	0.0	0.4	0.0	0.6	0.0	9.0	0.0	9.6	0.0	1.8	0.0	0.0	0.0	0.0		7.3
US	6.1	16.4	1.2	58.6	9.4	0.0	0.0	0.6	0.0	6.2	0.0	0.0	0.0	0.0	0.0	0.0	12.7	

Note: Weights are estimated jointly with the GVAR's parameters and are the same for all endogenous model variables. For details as to the method for computing the bounds see Section 2.2.

Table 9: Weight matrix estimate - Upper bounds (95%) - Specific for real GDP

	AT	AU	BE	CA	CH	DE	DK	ES	FI	FR	IE	IT	NL	NO	PT	SE	UK	US
AT		6.0	20.8	3.0	25.9	15.9	3.7	4.9	0.0	2.5	0.0	14.0	8.0	0.0	26.8	11.4	0.0	0.0
AU	0.4		0.0	1.3	12.1	12.7	0.0	6.3	15.3	4.1	23.2	0.0	1.5	32.4	0.0	0.0	19.6	14.7
BE	13.6	9.8		3.6	30.0	16.1	0.0	8.9	0.3	12.3	26.8	13.3	18.6	0.0	0.0	0.0	7.1	12.9
CA	2.6	12.6	5.2		0.0	19.3	49.6	2.4	48.8	3.5	15.7	8.6	8.2	0.0	14.8	33.6	0.0	46.6
CH	29.2	8.0	20.0	3.4		0.0	0.0	5.2	0.0	6.2	31.6	14.4	9.9	0.0	0.0	29.4	0.0	17.7
DE	100.0	16.7	17.0	5.1	14.2		26.7	18.6	18.4	18.4	0.0	24.1	29.9	0.0	0.0	29.3	28.4	0.0
DK	1.4	4.7	0.0	4.4	0.0	1.9		7.4	0.0	3.7	15.4	0.0	5.2	0.0	24.9	0.0	0.0	0.0
ES	2.6	11.3	5.1	1.5	0.0	0.0	37.0		26.5	12.3	72.0	19.6	9.6	0.0	47.8	0.0	16.8	4.0
FI	0.8	10.3	8.6	5.6	0.0	13.1	0.0	2.1		2.3	0.0	15.3	9.0	12.3	32.1	25.0	0.0	0.0
FR	16.5	9.8	15.3	4.9	7.3	19.8	0.0	18.6	0.0		80.6	19.8	15.5	68.0	0.0	10.3	22.3	13.4
IE	5.2	13.0	6.9	3.2	0.5	0.0	16.6	6.7	0.0	4.1		0.0	9.4	0.0	6.0	8.1	0.0	0.0
IT	31.4	9.4	12.4	5.0	20.0	21.9	0.0	13.1	54.7	11.5	0.0		15.0	0.0	0.0	18.6	12.0	0.0
NL	17.0	8.1	16.6	4.0	1.1	34.9	66.6	6.5	21.9	7.0	0.0	3.2		13.8	41.5	0.0	0.0	0.0
NO	0.2	10.3	0.0	4.8	0.0	0.0	12.9	5.4	13.7	4.2	0.0	6.8	11.0		12.4	23.2	0.0	0.0
PT	0.7	8.8	0.0	6.8	0.0	0.0	22.2	10.4	22.8	2.5	11.0	8.1	6.1	37.0		0.0	0.0	0.0
SE	4.2	8.5	0.0	2.8	10.8	0.0	10.0	6.7	26.5	3.0	7.0	7.0	15.2	36.2	0.0		0.0	20.1
UK	25.0	24.3	9.0	3.3	0.8	18.8	0.0	12.7	0.0	11.9	0.0	15.7	17.3	20.0	0.0	13.1		23.9
US	20.2	37.8	16.1	62.2	23.0	14.7	28.5	4.5	0.0	7.8	0.0	0.0	14.5	0.0	0.0	49.8	24.0	

Note: Weights are estimated jointly with the GVAR's parameters and are the same for all endogenous model variables. For details as to the method for computing the bounds see Section 2.2.

Table 10: Weight matrix estimate - Specific for expenditure price inflation

	AT	AU	BE	CA	CH	DE	DK	ES	FI	FR	IE	IT	NL	NO	PT	SE	UK	US
AT	0.0	5.3	6.0	3.8	0.3	8.7	7.8	2.0	12.5	5.3	5.9	2.9	4.5	0.0	9.9	0.2	5.2	4.1
AU	1.6	0.0	0.9	0.4	0.4	0.0	0.6	3.2	0.0	1.6	0.0	5.3	2.5	0.0	2.3	0.0	0.0	0.0
BE	5.5	2.7	0.0	3.3	33.6	9.4	4.2	4.7	0.0	11.0	0.0	4.8	12.3	0.0	6.6	13.7	1.9	6.7
CA	2.4	3.3	1.8	0.0	0.0	0.8	0.2	4.8	0.0	2.0	0.0	4.1	2.3	0.0	0.1	4.7	7.2	33.4
CH	7.6	5.5	15.2	3.7	0.0	6.3	0.0	5.4	30.5	3.3	5.0	4.8	3.1	0.0	0.1	7.9	9.1	8.6
DE	37.1	9.5	16.5	4.2	12.0	0.0	1.3	14.8	3.1	19.8	50.3	16.7	22.6	0.0	2.2	23.7	15.2	8.8
DK	2.8	1.0	0.0	1.1	0.3	0.0	0.0	2.2	0.0	1.3	0.0	2.4	1.3	24.0	0.1	11.3	12.7	3.3
ES	3.0	5.1	5.5	2.9	2.9	0.0	9.7	0.0	0.0	12.2	0.0	10.6	7.1	2.0	18.2	0.4	6.0	3.6
FI	4.1	2.9	0.0	0.4	7.3	5.1	0.0	0.8	0.0	2.4	0.0	1.2	3.6	4.5	0.0	0.0	7.0	0.0
FR	4.4	6.4	26.2	3.8	14.0	9.7	9.0	19.9	0.0	0.0	0.0	14.4	9.9	31.6	7.6	2.2	14.2	8.1
IE	4.3	2.8	0.0	3.4	0.0	0.0	3.9	1.5	13.1	1.5	0.0	5.6	1.4	14.5	15.5	0.0	0.0	0.0
IT	8.9	8.8	11.3	2.2	0.0	3.6	4.3	12.5	0.0	11.8	15.8	0.0	4.8	0.0	5.9	0.0	5.8	6.5
NL	3.1	3.0	1.7	2.0	0.0	12.2	0.0	5.2	19.0	5.9	0.0	4.5	0.0	0.0	7.1	0.0	0.0	5.6
NO	0.9	0.7	0.0	1.1	0.0	0.0	10.0	1.4	0.0	2.8	0.0	1.5	2.0	0.0	0.0	11.3	0.5	0.2
PT	3.3	4.4	2.3	3.0	0.0	0.0	12.9	6.5	0.0	1.1	2.4	4.0	3.1	0.0	0.0	0.0	6.0	4.7
SE	3.2	3.0	0.0	1.1	10.0	5.7	16.1	3.3	0.0	3.4	0.0	2.6	3.3	20.6	0.0	0.0	1.1	3.6
UK	3.9	9.0	0.0	3.4	0.0	24.8	16.3	6.9	21.8	6.8	20.6	7.2	7.4	2.7	15.7	10.1	0.0	2.7
US	3.9	26.5	12.4	60.2	19.2	13.5	3.7	5.0	0.0	7.8	0.0	7.6	8.9	0.0	8.7	14.3	7.9	0.0

Note: Weights are estimated jointly with the GVAR's parameters and are specific to personal expenditure price inflation in the model. Each column sums to 100 percent.

Table 11: Weight matrix estimate - Lower bounds (5%) - Specific for expenditure price inflation

	AT	AU	BE	CA	CH	DE	DK	ES	FI	FR	IE	IT	NL	NO	PT	SE	UK	US
AT		0.0	1.8	0.0	0.0	0.0	0.0	0.0	0.0	4.5	0.0	1.9	0.5	0.0	0.0	0.0	0.0	0.0
AU	1.5		0.0	0.1	0.0	0.0	0.0	0.4	0.0	1.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
BE	1.1	0.8		0.0	29.8	0.0	0.0	0.0	0.0	10.6	0.0	3.3	5.6	0.0	0.0	0.0	0.0	3.8
CA	0.5	0.0	0.0		0.0	0.0	0.0	0.3	0.0	0.9	0.0	2.9	0.1	0.0	0.0	0.0	0.0	30.0
CH	1.5	0.5	12.0	0.0		0.0	0.0	0.0	0.0	3.1	0.0	3.5	0.0	0.0	0.0	0.0	0.0	6.0
DE	7.3	3.2	6.3	2.6	0.0		0.0	8.7	0.0	18.5	0.0	14.9	20.1	0.0	0.0	10.7	6.3	0.0
DK	2.8	0.0	0.0	0.7	0.0	0.0		0.0	0.0	0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
ES	3.0	0.0	0.0	1.0	0.0	0.0	0.0		0.0	11.7	0.0	7.4	2.9	0.0	0.9	0.0	0.0	0.7
FI	3.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0		1.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
FR	4.4	2.3	18.9	0.0	9.3	4.3	0.0	17.6	0.0		0.0	13.0	5.1	0.0	0.0	0.0	6.9	5.7
IE	4.2	0.0	0.0	2.0	0.0	0.0	0.0	0.0	0.3	0.0		0.0	0.0	0.0	2.4	0.0	0.0	0.0
IT	1.7	0.0	8.0	1.8	0.0	1.0	0.0	10.3	0.0	11.5	0.0		0.1	0.0	0.0	0.0	0.2	0.0
NL	0.6	0.0	0.0	0.1	0.0	3.4	0.0	0.0	0.0	4.7	0.0	0.0		0.0	0.0	0.0	0.0	0.0
NO	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.3	0.0	0.0	0.0		0.0	0.0	0.0	0.0
PT	3.2	1.8	0.0	0.6	0.0	0.0	0.0	2.9	0.0	0.1	0.0	2.6	0.8	0.0		0.0	0.0	0.0
SE	3.2	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.3	0.0		0.0	0.6
UK	3.8	0.0	0.0	0.0	0.0	13.1	0.0	0.0	0.0	5.5	0.0	4.5	0.0	0.0	0.2	0.0		0.0
US	0.8	0.0	6.2	58.8	12.9	0.0	0.0	2.9	0.0	6.9	0.0	0.0	5.3	0.0	0.0	4.8	0.0	

Note: Weights are estimated jointly with the GVAR's parameters and are the same for all endogenous model variables. For details as to the method for computing the bounds see Section 2.2.

Table 12: Weight matrix estimate - Upper bounds (95%) - Specific for expenditure price inflation

	AT	AU	BE	CA	CH	DE	DK	ES	FI	FR	IE	IT	NL	NO	PT	SE	UK	US
AT		13.6	7.8	4.9	0.3	20.3	30.8	4.2	31.9	8.6	31.9	3.5	7.9	0.0	23.4	0.8	27.0	13.3
AU	2.0		4.9	1.6	0.6	0.0	3.1	6.6	0.0	2.3	0.0	19.6	8.8	0.0	8.4	0.0	0.1	0.0
BE	5.5	7.1		4.6	44.5	19.2	19.2	7.0	0.0	13.1	0.0	6.2	16.3	0.0	20.6	41.2	4.2	11.9
CA	2.5	5.3	5.2		0.0	1.8	0.6	13.3	0.0	2.2	0.0	6.3	4.6	0.1	0.2	8.7	32.8	36.5
CH	23.0	16.3	21.4	4.6		21.0	0.0	10.2	77.7	4.1	26.9	5.8	8.6	0.0	0.6	15.9	46.0	11.2
DE	100.0	14.0	23.2	5.3	16.3		5.4	16.8	8.9	20.2	100.0	17.8	26.2	0.0	13.1	46.7	18.1	31.7
DK	3.0	4.1	0.0	2.1	1.3	0.0		4.2	0.0	1.5	0.0	9.4	3.1	66.5	0.1	36.6	45.4	18.0
ES	3.1	11.0	15.9	6.6	16.5	0.0	20.7		0.0	14.1	0.0	12.5	15.2	8.5	28.6	2.3	10.2	8.6
FI	4.2	10.4	0.0	0.6	24.8	14.8	0.0	2.2		4.9	0.0	2.9	10.6	12.4	0.0	0.0	19.6	0.0
FR	13.3	11.9	39.1	4.5	24.9	14.3	42.7	26.7	0.0		0.0	15.3	12.6	66.7	38.0	12.2	16.8	10.6
IE	4.3	8.1	0.0	9.1	0.0	0.0	13.8	4.4	29.0	2.2		14.6	4.6	36.3	29.1	0.0	0.0	0.0
IT	26.9	15.0	18.7	3.1	0.0	5.3	22.6	16.2	0.0	13.1	62.7		8.1	0.0	16.0	0.0	8.8	40.8
NL	3.1	8.3	4.0	3.3	0.0	18.5	0.0	11.8	47.3	6.3	0.0	14.2		0.0	13.3	0.0	0.0	15.3
NO	1.0	2.1	0.0	1.6	0.0	0.0	26.6	3.6	0.0	5.1	0.0	4.5	4.7		0.0	27.7	1.9	0.6
PT	3.4	13.0	9.9	3.9	0.0	0.0	35.5	10.1	0.0	1.3	10.0	6.6	6.9	0.0		0.0	33.2	17.5
SE	3.2	5.9	0.3	3.2	17.6	13.3	43.6	6.3	0.0	4.1	0.0	5.0	6.9	38.8	0.0		4.1	5.5
UK	11.7	15.8	0.0	5.7	0.0	39.0	54.2	9.9	54.5	7.1	54.7	8.9	15.8	10.1	35.8	34.5		6.9
US	11.8	33.3	18.4	61.0	24.4	30.6	11.7	10.3	0.0	8.2	0.0	31.9	13.9	0.0	31.3	21.6	11.3	

Note: Weights are estimated jointly with the GVAR's parameters and are the same for all endogenous model variables. For details as to the method for computing the bounds see Section 2.2.

Table 13: Deviations between weight matrices

Mean abs. deviation	AT	AU	BE	CA	CH	DE	DK	ES	FI	FR	IE	IT	NL	NO	PT	SE	UK	US
Common vs benchmark	2.43	2.12	4.78	3.46	2.20	1.29	5.93	2.13	3.87	1.61	7.54	3.03	2.08	8.02	5.16	4.95	2.85	4.10
Specific YER vs benchmark	2.23	1.91	4.35	3.28	6.12	4.05	7.58	1.84	7.11	1.48	8.58	2.95	1.99	8.22	7.42	5.38	4.36	4.59
Specific CED vs benchmark	2.50	2.13	5.02	3.22	6.65	3.07	4.71	1.72	8.87	1.40	7.27	2.17	1.97	7.66	5.12	3.55	4.75	3.61
Specific YER vs common	1.22	1.03	2.51	1.36	4.57	3.64	3.17	1.74	4.14	1.04	3.38	0.65	1.29	3.03	2.89	2.30	2.76	1.25
Specific CED vs common	1.40	1.35	1.64	1.42	5.31	3.56	5.43	0.80	9.68	1.45	9.23	2.99	1.06	7.00	5.54	3.74	3.73	3.64

Corresponding ranks	AT	AU	BE	CA	CH	DE	DK	ES	FI	FR	IE	IT	NL	NO	PT	SE	UK	US
Common vs benchmark	12	15	6	9	13	18	3	14	8	17	2	10	16	1	4	5	11	7
Specific YER vs benchmark	14	16	10	12	6	11	3	17	5	18	1	13	15	2	4	7	9	8
Specific CED vs benchmark	13	15	6	11	4	12	8	17	1	18	3	14	16	2	5	10	7	9
Specific YER vs common	15	17	9	12	1	3	5	11	2	16	4	18	13	6	7	10	8	14
Specific CED vs common	15	16	12	14	6	10	5	18	1	13	2	11	17	3	4	7	8	9

Note: Mean absolute deviations are expressed in percentage points. Ranks are ascending. The benchmark refers to the GVAR with trade-based weights.

Table 14: Weight matrix comparison

	Common		Specific for YER		Specific for CED	
	Abs.	Rel.	Abs.	Rel.	Abs.	Rel.
Benchmark below lower bound	42	14%	32	10%	34	11%
Benchmark above upper bound	92	30%	116	38%	106	35%
Benchmark within bounds	172	56%	158	52%	166	54%

Note: The underlying weight estimates can be found in Tables 3-12.

Table 15: Portion of distorted coefficient means/variances when misspecifying GVAR weight matrices

	Mean			Variance		
Significance level	1%	5%	10%	1%	5%	10%
			Including intercepts			
Common WM vs benchmark	0.704	0.758	0.794	0.781	0.829	0.855
Specific WMs vs benchmark	0.783	0.834	0.857	0.815	0.865	0.893
Specific WMs vs common WM	0.593	0.690	0.745	0.380	0.453	0.502
			Excluding intercepts			
Common WM vs benchmark	0.700	0.755	0.791	0.778	0.827	0.853
Specific WMs vs benchmark	0.780	0.832	0.855	0.812	0.863	0.891
Specific WMs vs common WM	0.587	0.686	0.741	0.372	0.445	0.495

Note: The table reports the shares of model coefficients that are biased with respect to mean or variance at respective significance levels. Overall, each global model contains 2,664 coefficients. The number of intercepts that is excluded in the lower part of the table equals 36.

Table 16: Sign equivalence of impulse responses from GVAR models under different weight matrix schemes

Model 1	Model 2	On impact		Long-run	
		Abs.	Rel.	Abs.	Rel.
Common WM	Benchmark WM	1098	87.1%	1121	86.5%
Specific WM	Benchmark WM	1056	83.8%	1074	82.9%
Specific WM	Common WM	1106	87.8%	1137	87.7%

Note: The table reports the absolute and relative number of sign equivalences of generalized shock/response profiles on impact (for 1,290 cells of any pair of two residuals covariance matrices, excluding variances) as well as for long-run cumulative responses (for 1,296 shock/response constellations of $N \times K$ model variables).

Table 17: Root mean square errors of impulse responses from GVAR models under different weight matrix schemes

Common WM	Model 2	GDP			INF		
		STD	T=0	T=Inf	STD	T=0	T=Inf
Common WM	Benchmark WM	0.009	0.138	0.487	0.005	0.075	0.238
Specific WM	Benchmark WM	0.009	0.212	0.657	0.005	0.100	0.408
Specific WM	Common WM	0.009	0.158	0.447	0.005	0.095	0.289

Note: The table reports root mean square errors based on generalized shock/response profiles on impact (T=0), in the long-run (T=Inf), as well as historical standard deviations of the two model variables, all on average across the 18 countries.

Table 18: Coefficient uncertainty under different weight estimation schemes

	Variance			Variance ratio		
Significance level	1%	5%	10%	1%	5%	10%
			Including intercepts			
Common WM vs benchmark	0.840	0.883	0.901	2.099	2.036	2.012
Specific WM vs benchmark	0.951	0.965	0.973	3.226	3.191	3.169
			Excluding intercepts			
Common WM vs benchmark	0.841	0.883	0.900	2.115	2.052	2.028
Specific WM vs benchmark	0.951	0.964	0.973	3.254	3.219	3.197

Note: The table reports the shares of model coefficients whose variance at respective confidence levels differ significantly. The variance ratios reported in the right part of the table are computed on the subset of global model coefficients that was found to differ significantly at respective confidence levels. Overall, the global model contains 2,664 coefficients. The number of intercepts that is excluded in the lower part of the table equals 36. See Section 3.2 for details.

Table 19: RMSE ratios on average across countries

Common to benchmark

Horizon	1	2	3	4	Average
YER	0.91	0.88	0.89	0.90	0.90
CED	0.92	0.89	0.89	0.91	0.90

Specific to benchmark

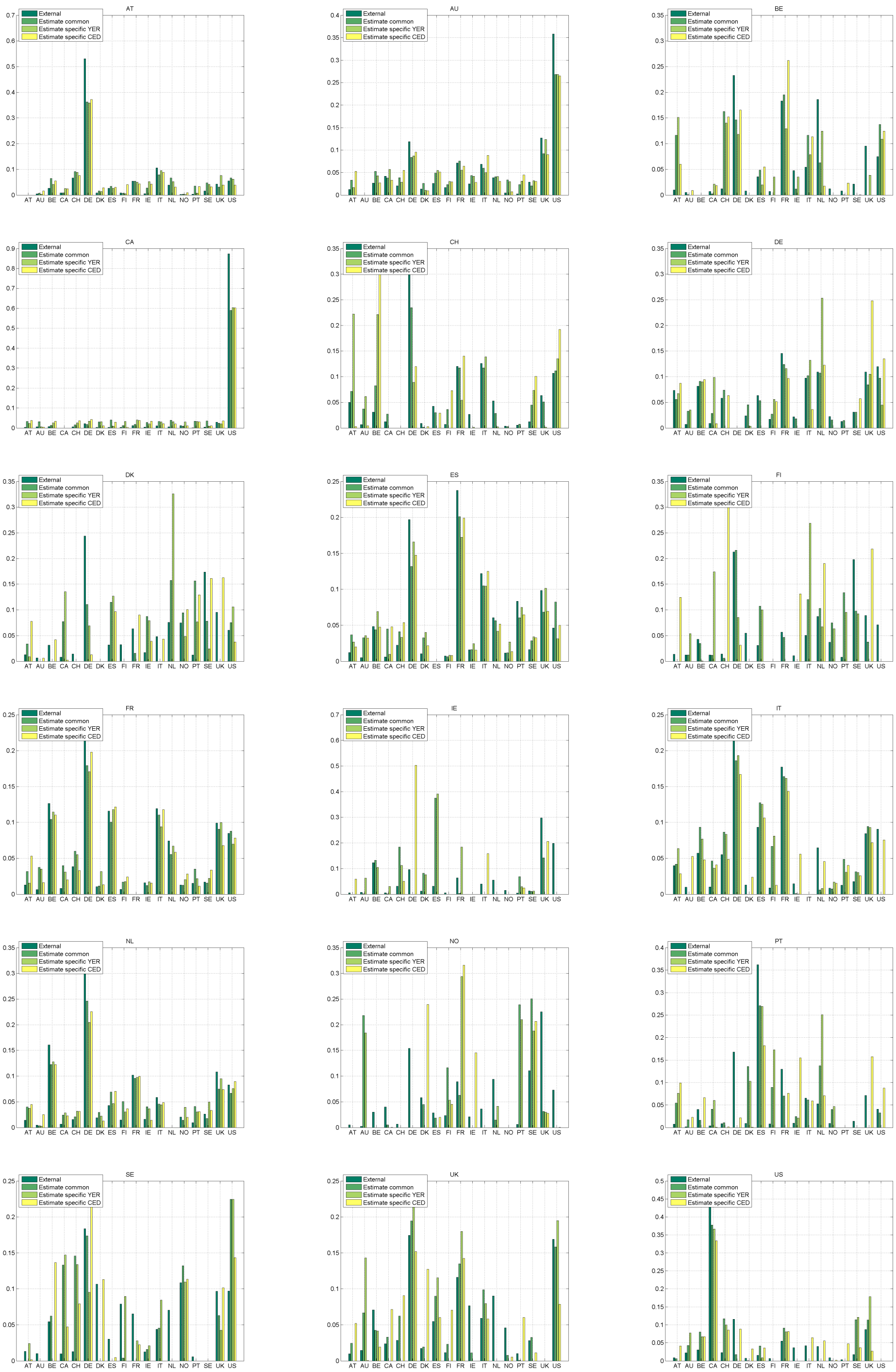
	1	2	3	4	Average
YER	1.01	0.95	1.01	1.02	1.00
CED	0.98	0.96	1.00	1.03	0.99

Specific to common

	1	2	3	4	Average
YER	1.11	1.08	1.14	1.13	1.11
CED	1.07	1.07	1.12	1.14	1.10

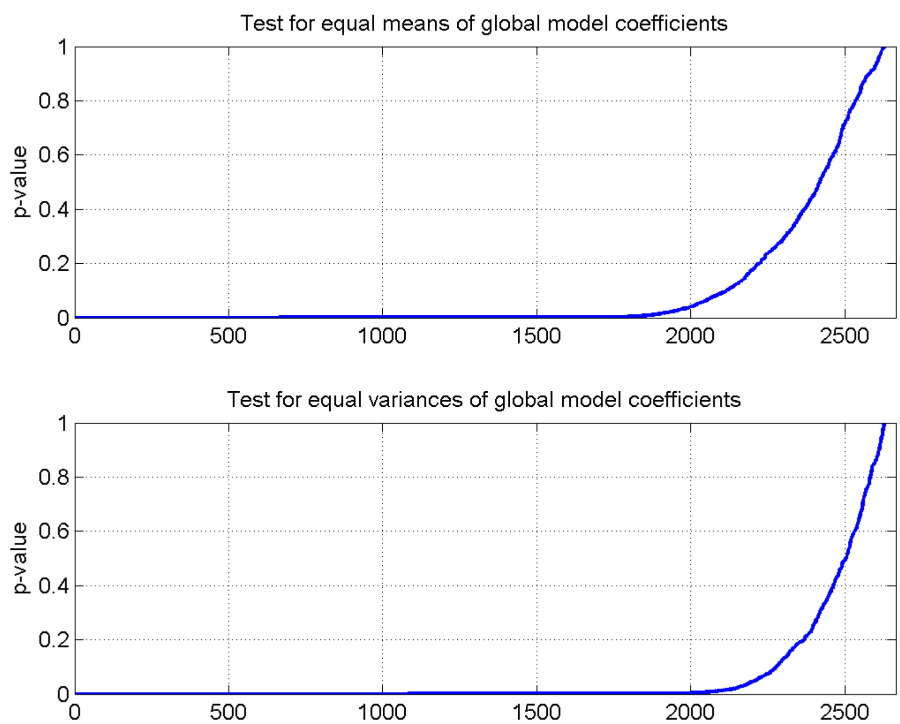
Note: The table reports average RMSE ratios for across countries for the out-of-sample test period from 2008Q1-2012Q2 (18 quarters). The underlying RMSE ratios for individual countries are presented in Figures 6-11.

Figure 1: Trade-based versus estimated GVAR weights



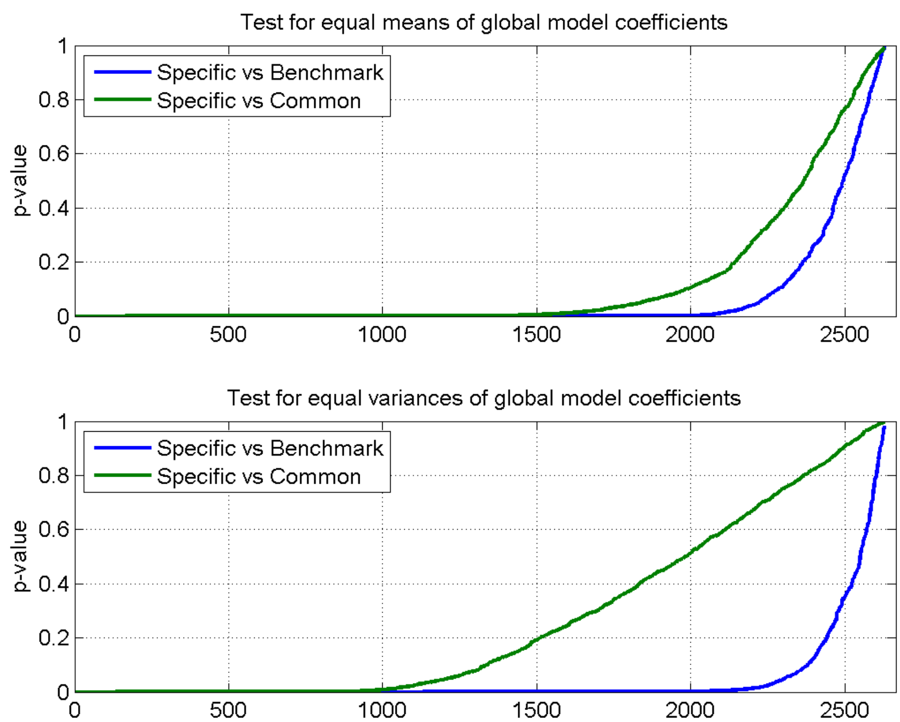
Note: See Tables 3, 4, 7, and 10 for the underlying data in tabular format.

Figure 2: Assessing bias in parameter distributions - GVAR w/ correct vs GVAR benchmark w/ ill-suited weight matrix



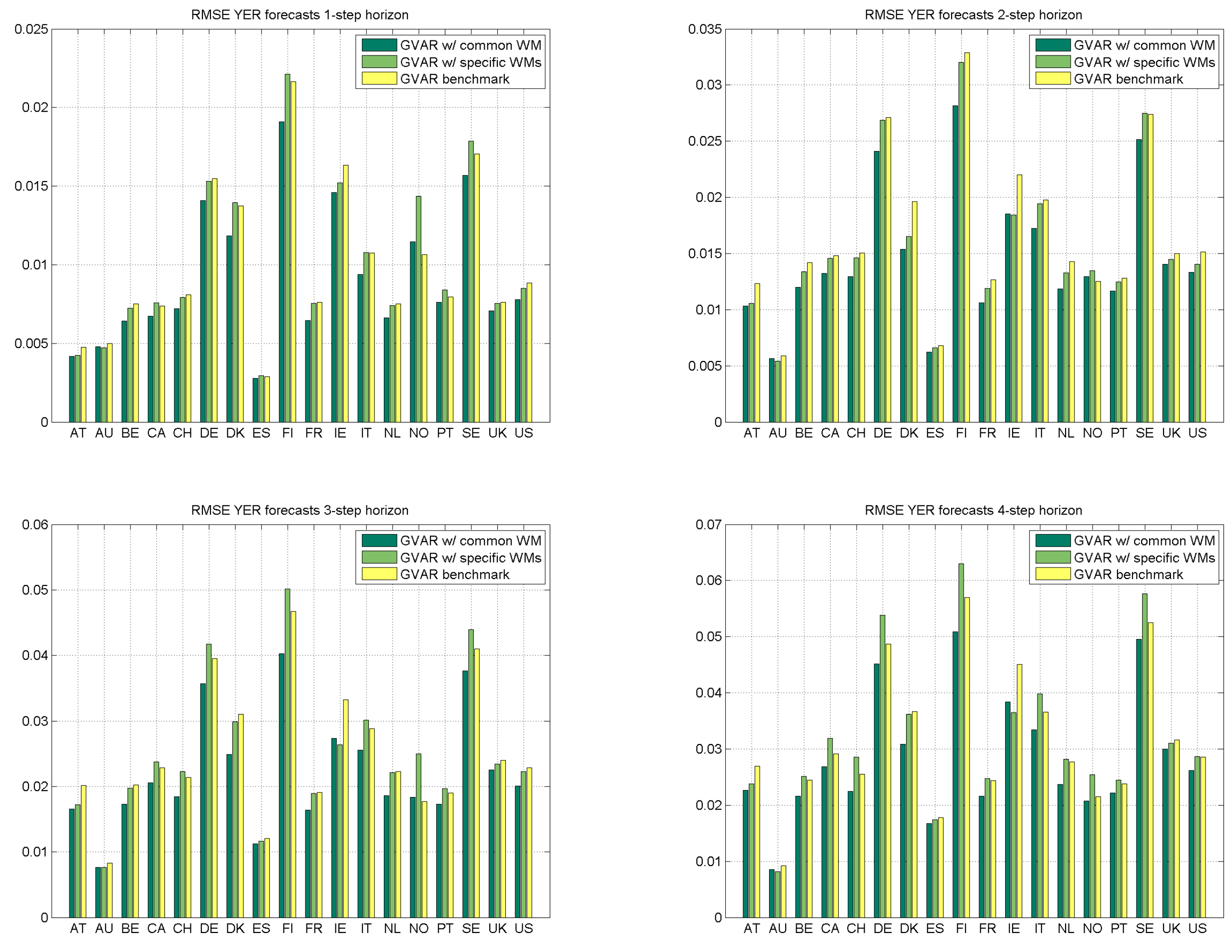
Note: A GVAR with a common weight matrix was the true DGP under which pseudo-data samples were generated. A benchmark GVAR with a knowingly ill-suited weight matrix was estimated on the pseudo data samples. The resulting global parameter distributions were tested for equal mean and variance; the resulting p-values are plotted here (sorted ascending). P-values close to zero indicate strong evidence against the Null of either equal means or equal variances, i.e. bias in that sense. Overall, the global model contains 2,664 coefficients (excluding residual covariance matrix). See text for details.

Figure 3: Assessing bias in parameter distributions - GVAR w/ correct specific weight matrices versus GVAR benchmark and GVAR w/ common ill-suited weight matrix



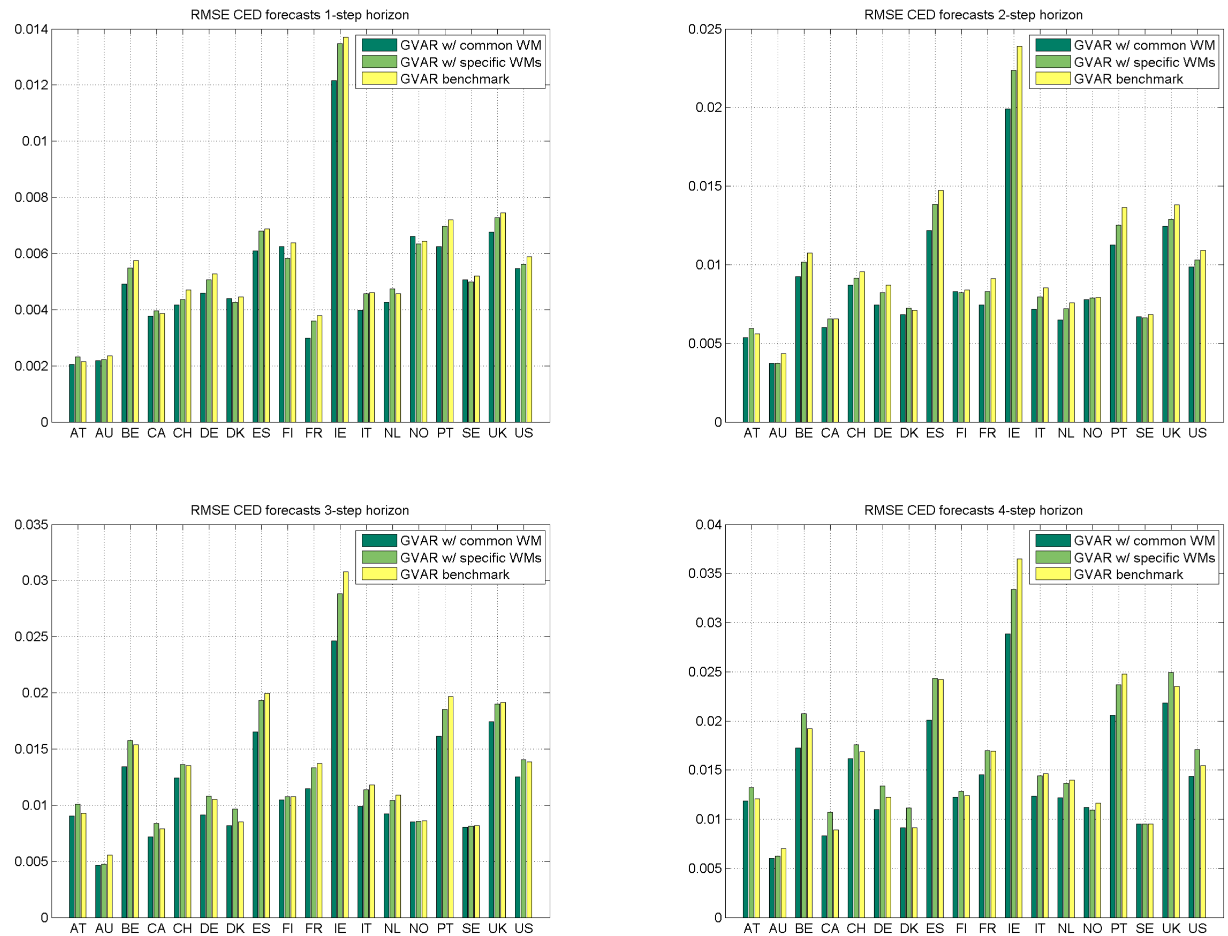
Note: A GVAR with variable-specific weight matrices for the two model variables was the true DGP under which pseudo-data samples were generated. A benchmark GVAR and a GVAR with a common weight matrix estimate were estimated on the pseudo data samples. The resulting global parameter distributions were tested for equal mean and variance; the resulting p-values are plotted here (sorted ascending). P-values close to zero indicate strong evidence against the Null of either equal means or equal variances, i.e. bias in that sense. Overall, the global model contains 2,664 coefficients (excluding residual covariance matrix). See text for details.

Figure 4: RMSE, Real GDP, Out-of-sample forecast performance



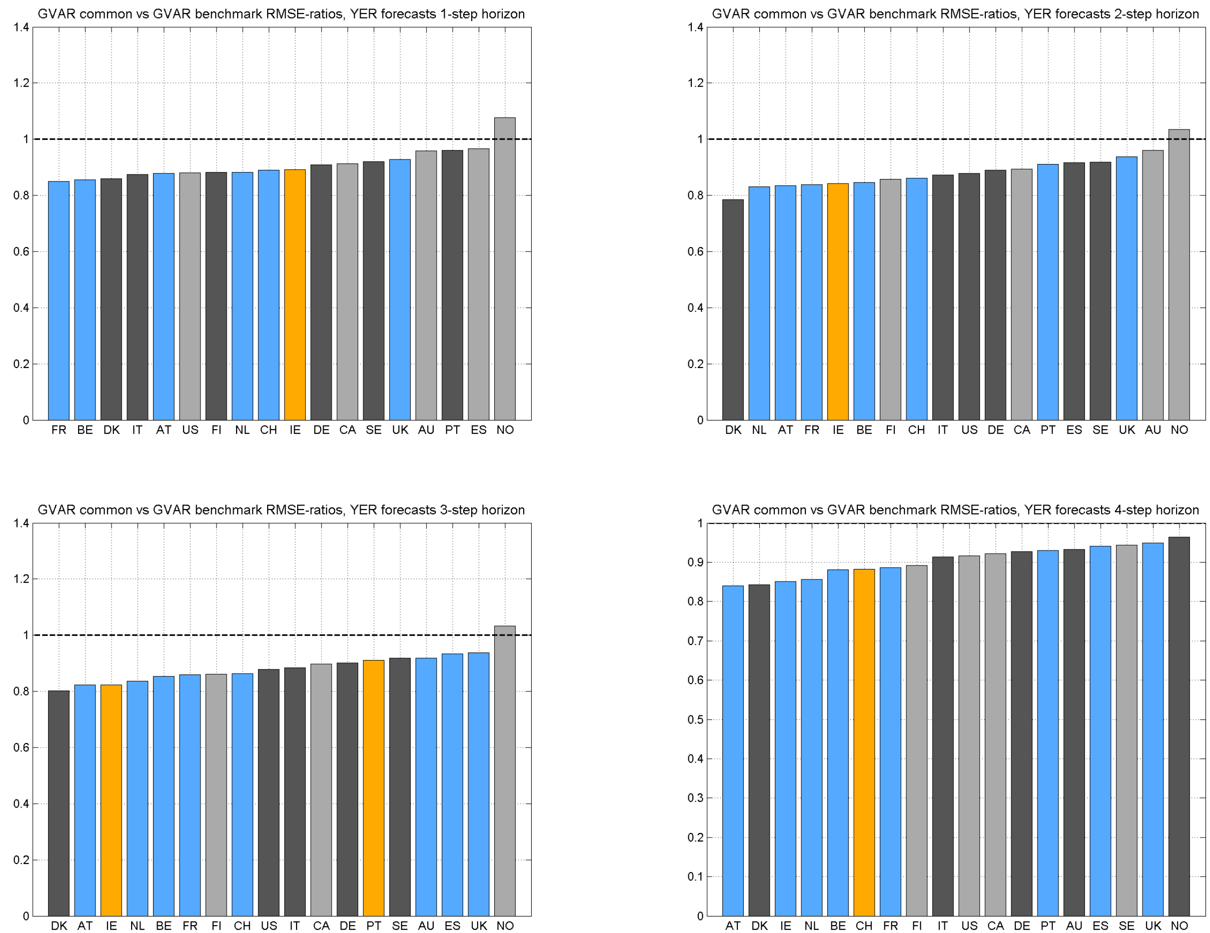
Note: The RMSE are expressed in log differences. For horizons exceeding 1-step, the forecasts and respective reference data were cumulated up to the relevant horizon.

Figure 5: RMSE, Expenditure price inflation, Out-of-sample forecast performance



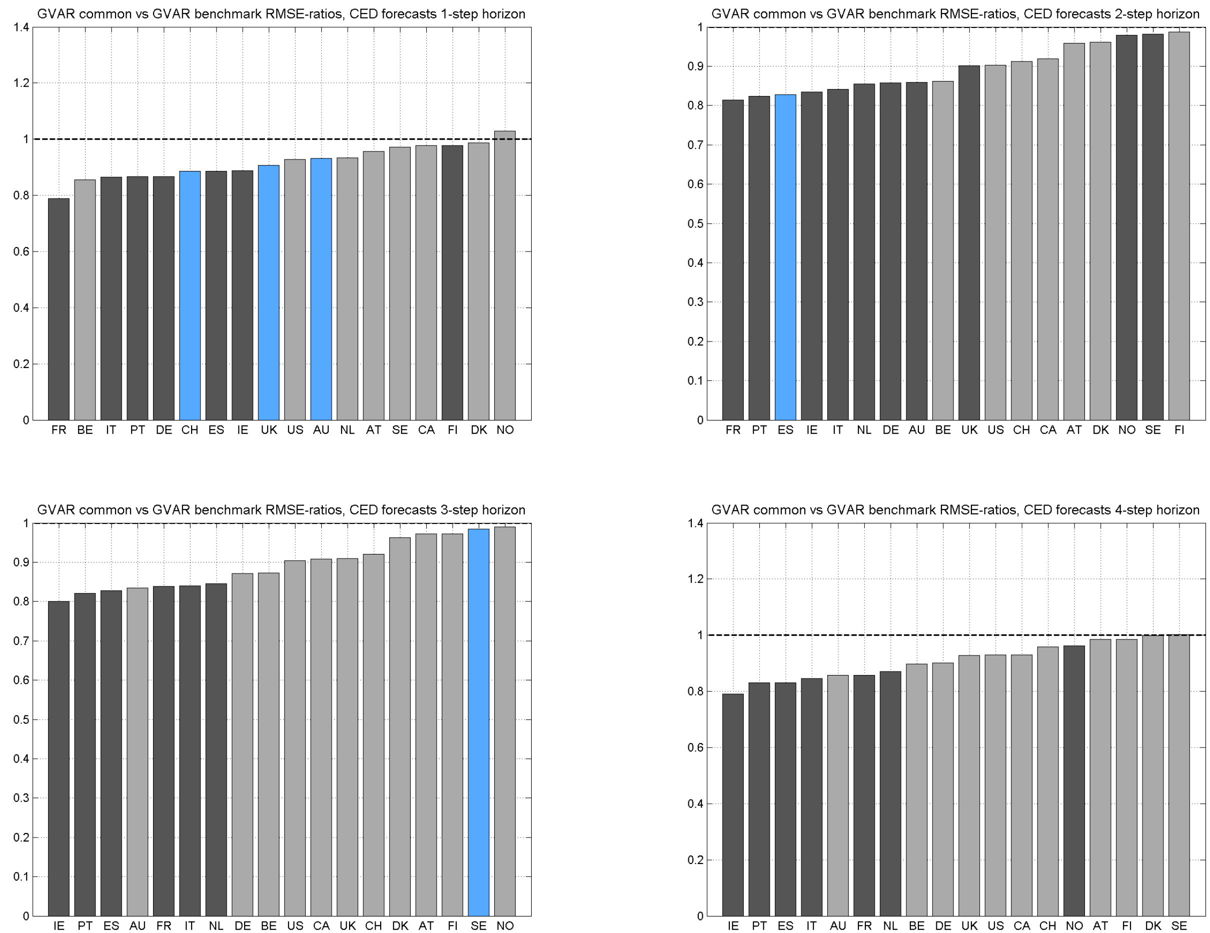
Note: The RMSE are expressed in log differences. For horizons exceeding 1-step, the forecasts and respective reference data were cumulated up to the relevant horizon.

Figure 6: GVAR common WM vs GVAR benchmark, Real GDP, Out-of-sample forecast performance



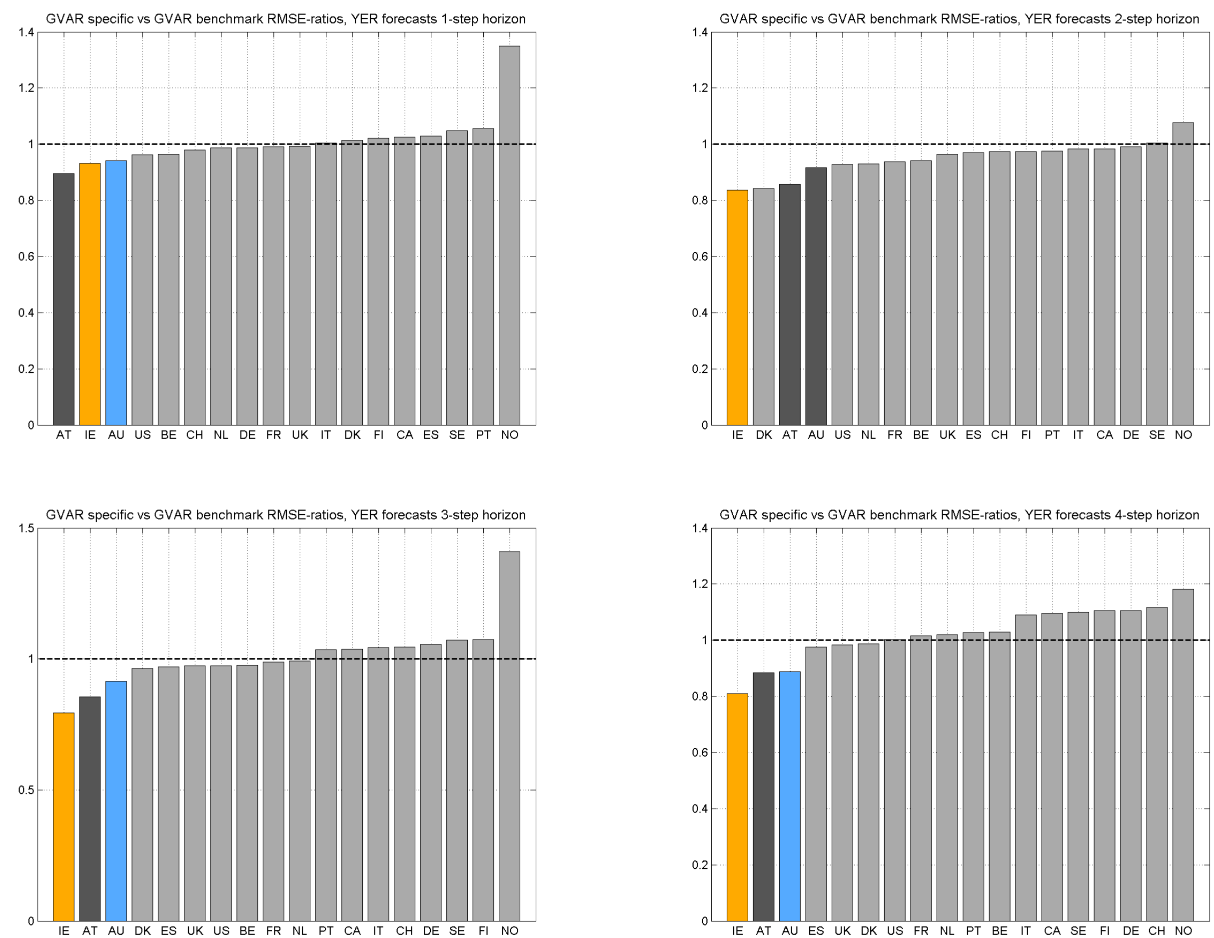
Note: Ratios smaller one indicate that the model (first mentioned) outperformed the benchmark (second mentioned) by one minus the ratio times 100 in percent. The colors of the bars indicate whether the relative gain in performance was significant according to the Clark-West (2007) test-statistic. Orange, blue and dark grey indicate significance at the 1%, 5%, and 10% probability levels.

Figure 7: GVAR common WM vs GVAR benchmark, Expenditure price inflation, Out-of-sample forecast performance



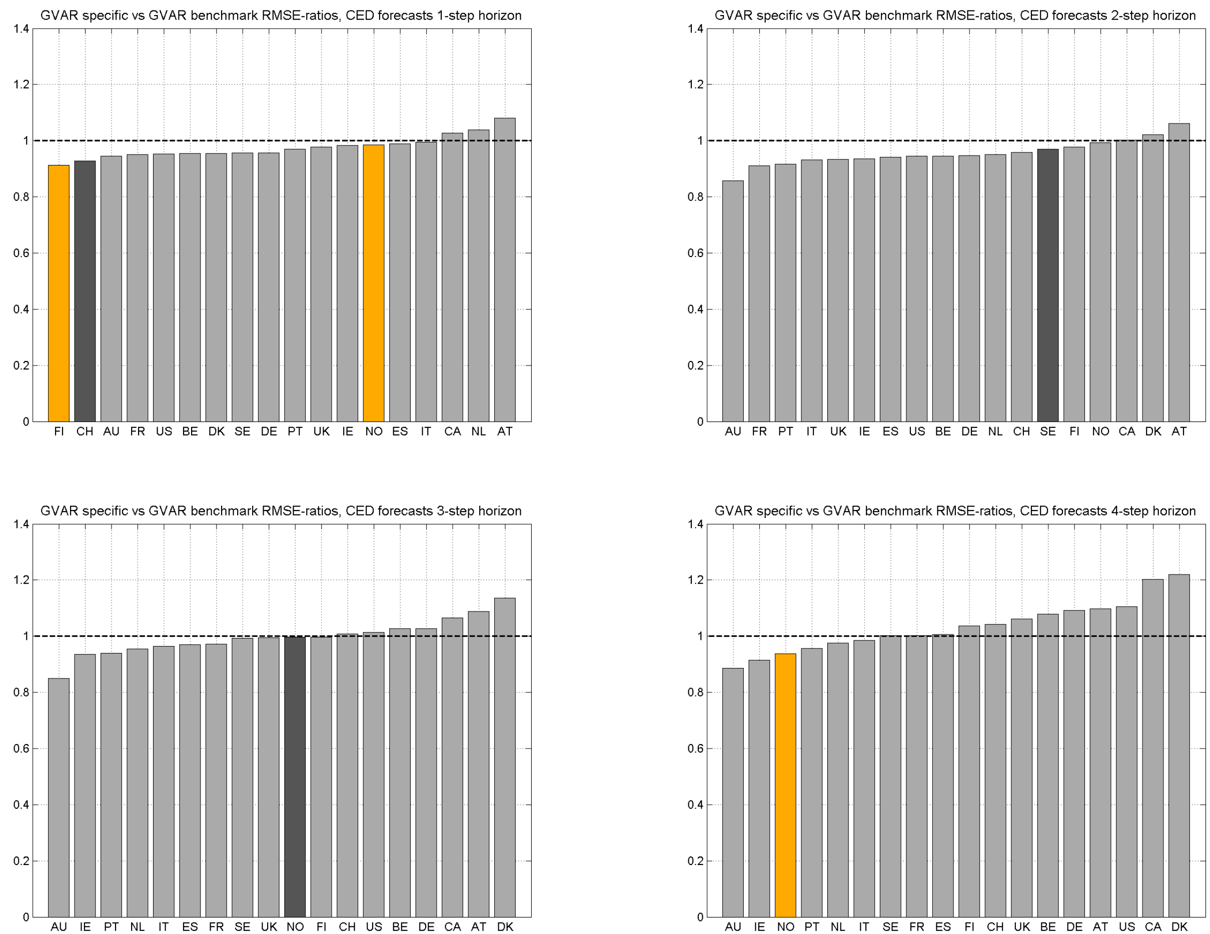
Note: Ratios smaller one indicate that the model (first mentioned) outperformed the benchmark (second mentioned) by one minus the ratio times 100 in percent. The colors of the bars indicate whether the relative gain in performance was significant according to the Clark-West (2007) test-statistic. Orange, blue and dark grey indicate significance at the 1%, 5%, and 10% probability levels.

Figure 8: GVAR variable-specific WMs vs GVAR benchmark, Real GDP, Out-of-sample forecast performance



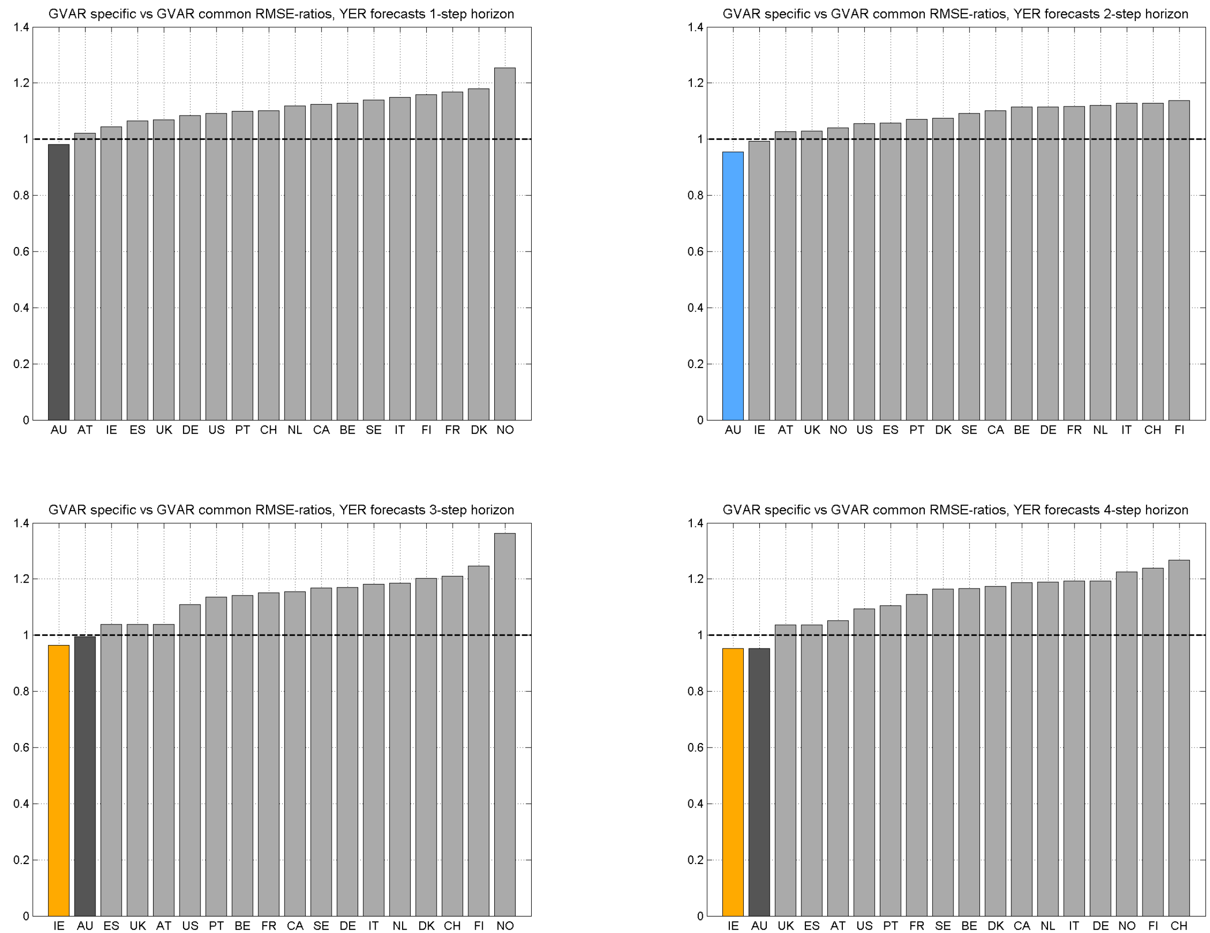
Note: Ratios smaller one indicate that the model (first mentioned) outperformed the benchmark (second mentioned) by one minus the ratio times 100 in percent. The colors of the bars indicate whether the relative gain in performance was significant according to the Clark-West (2007) test-statistic. Orange, blue and dark grey indicate significance at the 1%, 5%, and 10% probability levels.

Figure 9: GVAR variable-specific WMs vs GVAR benchmark, Expenditure price inflation, Out-of-sample forecast performance



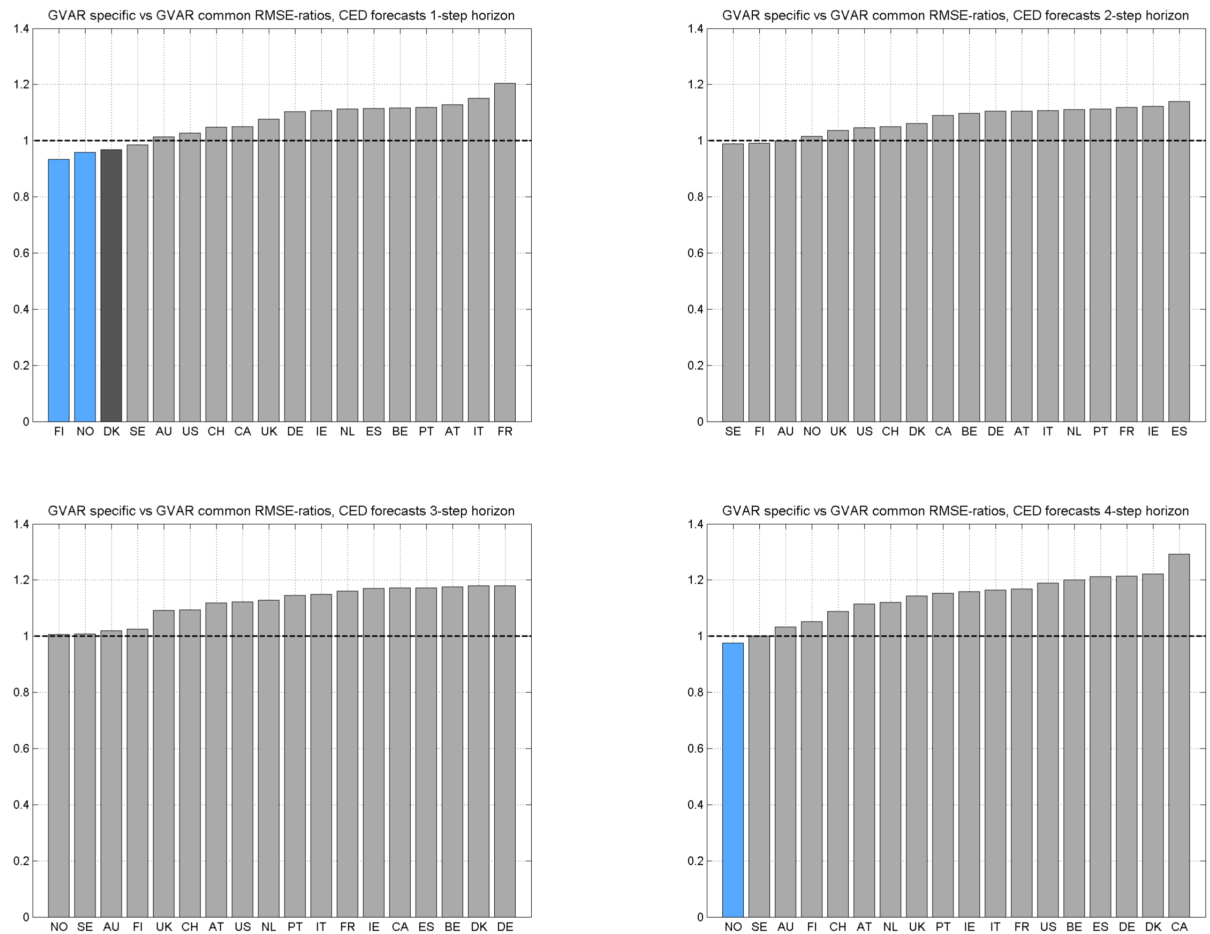
Note: Ratios smaller one indicate that the model (first mentioned) outperformed the benchmark (second mentioned) by one minus the ratio times 100 in percent. The colors of the bars indicate whether the relative gain in performance was significant according to the Clark-West (2007) test-statistic. Orange, blue and dark grey indicate significance at the 1%, 5%, and 10% probability levels.

Figure 10: GVAR variable-specific WMs vs GVAR common WM, Real GDP, Out-of-sample forecast performance



Note: Ratios smaller one indicate that the model (first mentioned) outperformed the benchmark (second mentioned) by one minus the ratio times 100 in percent. The colors of the bars indicate whether the relative gain in performance was significant according to the Clark-West (2007) test-statistic. Orange, blue and dark grey indicate significance at the 1%, 5%, and 10% probability levels.

Figure 11: GVAR variable-specific WMs vs GVAR common WM, Expenditure price inflation, Out-of-sample forecast performance



Note: Ratios smaller one indicate that the model (first mentioned) outperformed the benchmark (second mentioned) by one minus the ratio times 100 in percent. The colors of the bars indicate whether the relative gain in performance was significant according to the Clark-West (2007) test-statistic. Orange, blue and dark grey indicate significance at the 1%, 5%, and 10% probability levels.