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OPTIMAL CSD RESHAPING TOWARDS T2S

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In 2013 all ECB publications feature a motif taken from the €5 banknote.

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Abstract

T2S is the single and harmonised IT platform for securities settlement in central bank money developed by the Eurosystem to promote integration in the European post-trading industry, and will go live in 2015. CSDs joining T2S are thus faced with the decision problem of determining to which degree they should reshape, that is, adapt their own IT infrastructure, human resources and business strategy to T2S. A more complete reshaping entails higher immediate fixed costs, but allows to benefit the most from the cost-reduction allowed by T2S. In this article we use a game theoretic approach to model the strategic choice of the CSDs. We then derive several results from this model. In particular, we give closed-form solutions for the degree of optimal reshaping and the optimal prices set in the unique equilibrium if the time-horizon is finite. In case of an infinite horizon we give a sufficient and necessary condition for the existence of another subgame perfect Nash equilibrium in which CSDs continually delay the decision to reshape. We argue this equilibrium is not robust and provide a condition under which a given CSD will always reshape, whatever the other CSDs' strategy. We note that by adjusting the cost function and the interpretation of the reshaping parameter, the same game theoretic framework can be used to model the decision to join or not to join T2S.

Keywords: TARGET2-Securities (T2S); securities settlement; adaptation costs.

JEL classification: G10, G20, L11

Non technical summary

TARGET2-Securities (T2S) is a project for creating a single European IT-platform for settling securities transactions. It will offer the various national Central Securities Depositories (CSDs) harmonised securities settlement services in central bank money for a single European market. CSDs deciding to join T2S have to adapt their own IT infrastructure to T2S, that is, to "reshape" towards T2S. This adaptation towards T2S requires some investments by CSDs. In practice anything between two extreme cases is possible: in the first extreme scenario, a CSD could apply a "T2S-fees-on-top approach" in which the CSD simply keeps its existing IT infrastructure and puts T2S on top of it. In this case, the CSD will have to replicate all the data from T2S to its own legacy system before it can further operate and provide its services. Such a strategy probably implies no cost savings compared to the status quo and puts the operational costs of T2S on top. This strategy may, however, limit the immediate CSD adaptation costs to T2S. In the other extreme scenario, a CSD could apply a "Greenfield approach" in which it sets up a completely new infrastructure that is entirely shaped to incorporate T2S in the most cost-efficient way. In this case, this CSD could minimise the part of its own costs that come on top of the fees to be paid for the use of T2S, but at the same time it would probably face significant upfront investment costs. Hence, each CSD is faced with an individual strategic decision, and the overall gain in efficiency brought by T2S to the settlement industry depends on each CSD's individual decision as to which degree it will reshape.

Since reshaping involves a significant lump sum investment, CSDs could be tempted to delay the moment when they reshape. Because higher operating costs in the industry justify higher prices and possibly bigger CSD profit margins, there could be an incentive for CSDs to tacitly collude to avoid reshaping. Hence, besides the problem of knowing what is a CSD's individual optimal degree of reshaping is the problem of studying the possibility (and plausibility) of tacit collusion not to reshape.

This study makes use of a simple analytical model, cast into a dynamic game theoretic setting, to answer both issues. A closed-form expression for the optimal degree of reshaping is derived from the model, and simulations are provided as an illustration. These simulations involve ranges of plausible parameters derived from public data: For example, the cost of settling in T2S is taken from the T2S price list published by the ECB and the adaptation costs for a large CSD are assumed within the very broad range of EUR 5 million to EUR 50 million. Clearstream and Euroclear, the two largest European CSD groups, have announced that they intend to charge their users a maximum of EUR 30 million and EUR 25 million, respectively, as adaptation costs to T2S. In particular, these simulations allow to experimentally visualize the sense of variation of the optimal degree of reshaping as a function of the various parameters of the model, such as the market size, the costs per transaction of a given CSD, the price elasticities of the demand of settlement services, etc. A simple condition under which tacit collusion not to reshape is an equilibrium is provided. Interestingly, when this condition fails, and when price competition is assumed in each price-setting stage of the model, there can be no such tacit collusion. The stability of the various equilibria is then discussed. Finally, it is shown analytically that two natural generalisations of the model, i.e. allowing for an arbitrary number of CSDs and introducing a delay in the observability of the reshaping decision of other CSDs, do not affect the results of this paper.

The focus of the paper is on the optimal decision of CSDs for their reshaping towards T2S. The results provide interesting insights about an optimal, non-reversible investment decision that can be applied more broadly in the Industrial Organisation literature.

1 Introduction

1.1 What is T2S?

TARGET2-Securities (T2S) is a project for creating a single European IT-platform for settling securities transactions. Developed and – as of 2015 – operated by the central banks of the euro area (the *Eurosystem*), T2S will offer the various national Central Securities Depositories (CSDs) harmonised securities settlement services in central bank money for a single European market.

The introduction of T2S is a significant change to the European post-trading industry. Today, around 40 CSDs in Europe operate in a domestically-oriented, fragmented and thus largely monopolistic market structure. An analogy often used with T2S is a railway system in which national monopolists own and operate the tracks and the trains. In this analogy, both tracks and trains have a number of national specificities, which makes travelling across borders a costly, lengthy and sometimes risky process because of the frictions associated in particular with non-harmonised tracks.

Today's European settlement industry is in many ways similar to such a fragmented railway system. Already in 2001, a group chaired by Alberto Giovannini published a report that asserted that "inefficiencies in clearing and settlement represent the most primitive and thus the most important barrier to integrated financial markets in Europe"(see Giovannini [13]). Differences for example in technical interfaces, message formats, intraday settlement finality rules, opening days and daily timetables became known as the 15 Giovannini barriers and they have largely remained in place since 2001.

T2S aims at overcoming these fragmentations in European securities settlement. In the image used above, it will provide a single set of tracks on which the railway companies may operate their trains. T2S will thus allow economies of scale, more cost-efficient processes and new business opportunities[1]. The Eurosystem has invited all CSDs in Europe to outsource their settlement services to T2S. By 2012, 22 CSDs have signed a legal agreement ("Framework Agreement") with the Eurosystem, including almost all CSDs of the euro area. While CSDs will keep the legal relationship with their customers and can provide various services to them, the settlement of securities will be technically processed in T2S.

1.2 What is CSD "reshaping" towards T2S?

CSDs need to "reshape" their existing infrastructures to access T2S. This adaptation towards T2S requires some investments by CSDs. In practice anything between two extreme cases is possible: in the first extreme scenario, a CSD could apply a "T2S-fees-on-top approach" in which the CSD simply keeps its existing IT infrastructure and puts T2S on top of it. In this case, the CSD will have to replicate all the data from T2S to its own legacy system before it can further operate and provide its services. Such a strategy probably implies no cost savings compared to the *status quo* and puts the operational costs of T2S on top. This strategy may, however, limit the immediate CSD adaptation costs to T2S.

In the other extreme scenario, a CSD could apply a "Greenfield approach" in which it sets up a completely new infrastructure that is entirely shaped to incorporate T2S in the most cost-efficient way. In this case, this CSD could minimise the part of its own costs that come on top of the fees to be paid for the use of T2S, but at the same time it would probably face significant upfront investment costs.

Hence, European CSDs face crucial strategic decisions: as the participation in T2S is voluntary, should CSDs jump with their trains on the new tracks, i.e. should they join T2S? And, for those who decide to join, to which degree should they adapt – or "reshape" – their current trains, i.e. their own IT platform, their human resources and even their business model to T2S? In particular, CSDs are faced with the problem of deciding what is the optimal pricing scheme in the new post-trade environment. For example, CSDs might be tempted to increase their fees in order to recover their immediate investment costs from adapting to T2S, thus passing these to their customers.

[1] The European Central Bank (ECB) provides information about T2S via the website www.t2s.eu. Non-technical general information on T2S is available in particular in the T2S brochures (`http://www.ecb.europa.eu/paym/t2s/about/brochures/html/index.en.html`) and in some videos in the multimedia room (`http://www.ecb.europa.eu/paym/t2s/about/multimedia/html/index.en.html`).

1.3 Outline of the model and main results

In this article we address these questions using a finite and then infinite dynamic game, where the decision to reshape incurs immediate fixed costs but future potential benefits, stemming both from cost reductions and from increased market shares. The focus of the paper is on the optimal decision of CSD for their reshaping towards T2S. The results provide interesting insights about an optimal, non-reversible investment decision than can be applied more broadly in the Industrial Organisation literature.

In our first (finite-time) model of Section 2, we give all CSDs the choice to reshape only at the beginning of the game, and then determine the unique subgame perfect Nash equilibrium of this game. In particular, we prove that in the absence of capital constraints, CSDs should not increase the price of their transaction services after reshaping (Proposition 1), and should even decrease them correspondingly to the costs-reduction obtained from their reshaping. We derive closed-form formulas giving the optimal, profit-maximizing pricing at each period of the model as well as the degree of optimal reshaping as a function of the main parameters of the model (Theorem 1), and in particular as a function of the costs per transactions of each CSD as well as of the different substitution effect among CSD settlement services.

We carry out simulations with different parameters derived from publicly available data. For example, the cost of settling in T2S is taken from the T2S price list published by the ECB. The costs for fully reshaping a large CSD towards T2S are assumed within the very broad range of EUR 5 million to EUR 50 million. At the end of 2012, Clearstream and Euroclear announced that they intend to charge their users a maximum of EUR 30 million and EUR 25 million, respectively, as adaptation costs to T2S (see Clearstream [4] and Euroclear [10]). Since greater adaptation costs reduce the benefits of reshaping, our parameters are chosen in a conservative way which is detrimental to reshaping. Also the other parameters are based on rough approximations from publicly available data, primarily from the two largest European CSD groups, Clearstream and Euroclear.

In the second model, presented in Section 3.1.2, we modify the rules of the game to allow participating CSDs to reshape at any time. This induces more strategic behaviour on the part of CSDs, because it allows them to design reshaping strategies that are dependent on others' reshaping timing. An important result is an explicit sufficient condition on costs per transactions and price-elasticities under which tacit collusion to perpetually delay the reshaping can occur (Theorem 2). That is, we provide a condition under which there also exists a particular subgame perfect Nash equilibrium in which no existing CSD reshapes towards T2S and all CSDs just puts T2S on top of its current infrastructure, because each CSD expects all other CSDs to behave like this. However, this equilibrium seems rather fragile as it likely to break down as soon as one of the CSDs deviates from it, or as soon as only marginal changes to the basic model are introduced: possibility of new entrants in the settlement industry, observability of reshaping behaviour, or enough homogeneity in cost-efficiency and price-elasticities of the main CSDs. Since all of these effects are likely to be fulfilled in the post-trading market with T2S, it seems unlikely that it will be the optimal strategy for a CSD not to reshape its current infrastructure to T2S. Moreover, this condition we prove to be also necessary (Theorem 3). Hence, we also get a condition under which there can be no collusion to delay the reshaping. The main results are further discussed in Section 3.2.3. They are robust with respect to a delay in the observability of the reshaping of other CSDs as discussed in Section 4.

We also briefly discuss – since it is not the main focus of the article – how to apply this model and hence derive the same type of results with respect to the combined decision to join and reshape. In particular, while we note that tacit collusion for not joining T2S could be one of several possible equilibria, the publicly stated intentions of some CSDs to join T2S imply that this equilibrium will not apply in reality. In particular, Theorem 7 gives a sufficient condition for a CSD to join and to completely reshape no matter what the other CSDs decide.

1.4 Literature review

There has been, until fairly recently, a relative scarcity of research articles on the post-trading industry, at least when compared to the trading industry or to other well-studied network industries such as telecommunications. For example, the 2007 survey by post-trading by Milne [21] includes only papers published after 2001. Milne's conclusion that more research in this field is needed is as valid as ever, not least because of a significant amount of regulatory (such as the Dodd-Frank act in the United States or the European Market Infrastructure Regulation (EMIR) in the European Union) and technical initiatives (such as T2S).

On the empirical side, the papers by Cayseele and Wuyts [2], and by Schmiedel, Malkamaki and Tarkka [24] have received most attention in the literature. They show that the post-trading industry exhibits significant economies of scale and scope.

On the theoretical side, the relevant literature combines elements of the classic theory of industrial organization, as described by, for example, Tirole [27] with more distinctive features of the post-trading industry. It focuses on network effects, two-sided markets and vertical as well as horizontal integration as key features of post-trading infrastructures.

A description of general network effects in an industrial organization setting is given for example by Economides [8]. The participation of additional market participants in a settlement system increases the benefits of participation for other market participants. For example, if market participants can do business with more counterparties, liquidity increases and as a consequence, capital costs tend to decrease (see for example [7], chapter 5). An introductory general discussion of these network effects in the setting of the post-trade industry, with some comparisons with other network industries, can be found in Knieps [16] and an application of the network effect in settlement in Holthausen and Tapking [14].

Post-trading market infrastructures and CSDs in particular are necessary to bring together issuers and investors in the primary market and to allow the exchange of securities between investors in the secondary market. The cost of capital for issuers and the returns of issuers after transaction costs can create a virtuous circle with the liquidity in the market. Hence, post-trading market infrastructures exhibit features of two-sided markets where for example pricing decisions should take into account both sides of the market. For example Kauko [15] is an attempt to capture this feature by designing ad-hoc models to illustrate this dual aspect specific to the post-trading industry.

The majority of the theoretical literature in this field looks at vertical and horizontal integration in trading and post-trading. In some national markets such as Germany or Italy, a single "vertical silo" operates the trading, clearing and settlement infrastructure. Other markets such as France use separate firms for the different elements of the value chain. Some of the firms active in these markets have horizontally integrated their activities across different national markets. A prominent example is Euroclear's ESES-platform (Euroclear Settlement of Euronext-zone Securities). A key difference to the standard industrial organisation literature as in [29], for example, is that market infrastructures with economies of scale and scope and network effects operate at each layer of the value chain (trading, clearing, settlement). Many financial market participants use each infrastructure directly so that the standard perspective of upstream and downstream firms producing goods or services for the final user is not necessarily directly applicable.

Tapking and Yang [26] use a theoretical model for cross-border trading to systematically compare the welfare of different industry structures for securities trading and settlement. In their model, which captures competition and transaction cost effects, welfare is the highest for horizontal integration, while it is higher for vertical integration than for complete separation. Pirrong [22] focuses on the industry structure in a single market and emphasises transaction cost reductions from vertical integration. He argues that the transaction cost benefits can be (partly) offset by increasing differences between the scope economies at the different layers of the value chain. Cherbonnier and Rochet [3] look at a different possibility of vertical integration, namely between a single CSD as provider of settlement and custody and banking services[2] and one of two competing custodian banks. They build on the model developed in Holthausen and Tapking[14] to analyse the welfare effects and optimal regulation of such vertical integration. Cherbonnier and Rochet conclude that the possibility of vertical integration would require the regulation of access pricing which would introduce other inefficiencies. They suggest that competition between several CSDs, which is one of the visions of T2S promoted by the Eurosystem, could limit the need for regulation, but this idea is not included in their formal model.[3]

By looking at the optimal reshaping decision of CSDs, we add another aspect to the literature on post-

[2] The scope of services offered by CSDs differs and can include custody and banking services as some CSDs have a banking licence.

[3] Koeppl and Monnet [17] and Rochet [23] analyse further interesting theoretical models about vertical integration in the post-trading industry.

trading that is closely linked to the industrial organisation literature. In fact, the basic mechanisms of the model remain fairly general and thus contribute to literature on investments, which has also examined the trade-off between paying a lump sum today to reduce costs or improve quality tomorrow. There has also been considerable research in a general setting about investments in productive capacity, as well as in modelling competing firms' strategies (with or without investment). For example, Fine and Freund [11] examine the trade-off faced by firms in investing into a product-specific or a more expensive flexible production capacity. Nevertheless investment in our setting would be dedicated to reducing the average cost-per-transaction, not to aim at a more flexible business. Eliashberg and Steinberg [9] examine the possible strategies of two competing firms with different cost structures. Most of these models were designed for the manufacturing industry. While capacity constraints and thus optimal capacity planning is also relevant for the processing of securities settlement transactions, a key element in the previous literature is the management of the inventory. This does not apply to the production of settlement services. The reader interested in the specific problems of investment in capacity, in particular in high IT-industry, can refer to Wu's survey [31].

One of the closest article to ours in the stream of the literature dealing with investment seems to be Spence [25]. It tackles the problem of the trade-off of paying a higher lump-sum of investment in order to reduce costs per unit of production in the subsequent periods of the game. Although Spence's article allows for very general forms of profit and demand functions and could in theory be applied to many investment decision problems, the investment decision in the article is presented as an investment in research and development, more than in infrastructure. Hence, Spence's main concerns are tied to the potential spillover effects of research on the optimal degree of investment by firms as well as the optimal level of subsidies from the authorities required to maximize aggregate welfare. Another article close to ours is Valletti and Cambini [28] where investment is designed to improve the quality of the services provided by a network, rather than to reduce costs as in our article. This investment can also a positive external effect on the quality of the service offered by other networks. The paper is similar to our article in two different aspects: first, the higher quality attracts more demand, as in our model lower costs allows lower equilibrium prices and ultimately attract more demand. Second, the authors demonstrate that high termination charges, as well as an increased interplay between the quality level of both networks, can lead to collusion to under-invest. A difference with our collusion theorem is that collusion here stems from the direct interplay between the two competitors, either through the termination fees they have to pay to each other's or through the quality spillover of their investment decision, while our tacit collusion theorem comes from the links of competitors as providing partially substitutable services of settlement. For a survey indicating how access prices can be used as an instrument of tacit collusion in a communication network setting, please refer to Vogelsang [30]. For an review of the price-competition in a dynamic setting, please refer to Maskin and Tirole [19].

Our paper obviously also contributes to the literature on T2S. The Eurosystem published an economic feasibility study about T2S in 2007 [5], followed by an economic impact assessment in 2008 [6]. These studies describe and quantify the cost savings and economic benefits associated with T2S for CSDs, their users and final investors and issuers. In particular the later study ([6]) was developed in close cooperation with market participants and the quantitative results are based on data provided by market participants.

There is, however, a significant lack of studies that use analytical models to investigate the competition, network and welfare effects of T2S more specifically. Cales et al [1] is the only paper to our knowledge that is concerned explicitly with T2S. It is based on Matutes and Padilla [20], who use the Salop model of space competition, which is itself a generalization of the Hoteling model used in many articles of the post-trading literature. An advantage of using the Salop model instead of the more classical Hoteling model is to allow for competition between three CSDs providing depository and settlement services to banks. An important assumption is that securities must be deposited in their domestic CSD, whereas banks (the users of CSDs) can choose the CSD for settlement if the CSD is in T2S. Hence, three alternative situations for competition in settlement services can be modelled: the so-called "compatibility case", where all the three CSDs join T2S for settlement, the "incompatibility" case, which corresponds to the current pre-T2S situation and no CSD joining T2S, and the "partial compatibility" case, where precisely two CSDs join T2S while the other stays out. The authors capture the benefits of T2S for cross-border / cross-CSD settlement transactions by a network benefit for banks that increases in the number of CSDs joining T2S. This is an important difference to our model in which we focus on cost-reductions for CSDs that depend on the reshaping of CSDs. They arise from the availability of a single settlement platform allowing for economies of scale.

Cales et al [1] conclude that competition will lead to price decreases if all CSDs join T2S. If only a subset of CSDs joins T2S, the effect on CSD prices depends on the relative importance of the competition and network effects as CSDs in T2S can gain part of the network benefit from banks.[4] Importantly, banks' welfare increases also in this case, as for them the network benefit always dominates the potential price increase of CSDs. Similar to our paper, Cales et al [1] mention a possible coordination problem for CSDs joining T2S and the possibility of tacit collusion among CSDs. However, we look at the possibility and sustainability of tacit collusion in much more detail.

2 A dynamic N-period model for the degree of optimal reshaping

2.1 Motivation

We consider the subset of CSDs having joined T2S and ask the question of the optimal degree of reshaping that they should target. Indeed, reshaping incurs immediate costs, but long term potential benefits, hence a balance should be found between a complete reshaping and a pure juxtaposition of the T2S IT-infrastructure and business model on top of the CSD infrastructure and its own business model, with no costs reduction, neither technological or in the servicing staff, from the participating CSD. Also, the settlement industry is a somewhat competitive environment - and even if it were not, it is expected to become much more competitive with the introduction of T2S. Hence the optimal pricing and reshaping decisions do not only depend on a given CSD's own characteristics but on the similar decisions from others.

2.2 The dynamic game

To try to capture how the degree of optimal reshaping - interpreted in a rational setting as the degree of reshaping in a Nash equilibrium of the game - depends on these various parameters and participants' action, we define the following dynamic game:

1) At the beginning of the game all CSDs having joined T2S simultaneously decide to which extent they will reshape[5]. This is modelled by the choice, for each CSD i, of a pair (a_i, b_i) of real numbers between 0 and 1, each pair being associated with a given adaptation cost $C_{i,adapt}(a_i, b_i)$. The choice of a greater reshaping to reduce fixed costs is modelled by a higher a_i, while the choice of a greater reshaping to reduce transaction costs is modelled by a greater b_i. The adaptation costs function $C_{i,adapt}$ is thus logically increasing in both its parameters, and the maximum possible reshaping corresponds to $(a_i, b_i) = (1, 1)$. Such a reshaping is not necessarily optimal, since although it would produce the

[4] The reason is that in Cales et al [1] demand is perfectly inelastic, the greater utility of being a client of compatible CSDs allows them, in some case, to price their services higher than in the incompatibility case. This result should be taken with care: first, it relies on perfect inelasticity of demand: the banks are not given the opportunity to trade and settle less often in front of higher prices in the model: they have to be some CSD clients no matter the overall prices. Second, Cales et al distinguish in total four ranges of parameters, each with its own implications in terms of profitability. In particular, when the added utility of the network effect (and which relies exclusively on the inter-substitutability of CSDs having joined T2S in terms of their settlement services) is high enough compared to transportation costs (which represents product differentiation and banks 'preferences for their closest CSD), Cales et al find that the CSDs which had opted out would be better off inside T2S. This suggests CSDs which settle products that have very close substitutes in the other CSDs' market would make a very poor bet by staying out of T2S to preserve their natural monopoly on the settlement services (service "bundled" with depository services), in particular in a context of globalization and harmonisation in Europe where many issuers could easily issue in other European countries and thus completely avoid a CSD outside of T2S.

[5] The legal contract governing the relationship between CSDs and the Eurosystem as the provider of T2S services, the Framework Agreement, requires that "The Contracting CSD shall use reasonable efforts to adapt its operational, internal guidelines as well as its processes and related technical systems in order to foster the development of the European post-trading infrastructure, make efficient use of the T2S Services and maintain the Multilateral Character of T2S" (Article 4, Chapter 2). This formulation leaves a very large degree of freedom to CSDs concerning their reshaping.

maximum cost-reduction in the future, it entails a the highest cost at the beginning. We assume CSDs can finance their adaptation costs at the cost of capital r.

2) Each CSD observes the reshaping decision of others, then play repeatedly, for a certain number $N \geq 2$ of times, the price-setting game $G(\widetilde{C}_{i,fixed}, \widetilde{c_i})$ described in the section below, which consists in a simultaneous choice by each CSD i of the price p_i it sets for its settlement service, given the associated costs $(\widetilde{C}_{i,fixed}, \widetilde{c_i})$ assumed in the following table:

period	fixed costs $\widetilde{C}_{i,fixed}$ of CSD i	cost per transaction $\widetilde{c_i}$ of CSD i
1	$(1-a_i)C_{i,fixed} + C_{i,adapt}(a_i, b_i)$	$(1-b_i)c_i + c_{T2S}$
2	$(1-a_i)C_{i,fixed}$	$(1-b_i)c_i + c_{T2S}$
...	...	...
N	$(1-a_i)C_{i,fixed}$	$(1-b_i)c_i + c_{T2S}$

Hence these costs depend on each CSD reshaping decision at the first stage of the game: if CSD i has chosen a (a_i, b_i)-reshaping at the beginning of the game, then it pays immediately $C_{adapt}(a_i, b_i)$, the corresponding adaptation costs, which is assumed increasing in both its variable a_i and its variable b_i. The cost per transaction $\widetilde{c_i}$ consists of the individual CSD's cost c_i after reshaping and a constant fee per transaction c_{T2S} for the use of T2S.

2.3 The price-setting game $G(\widetilde{c_i})$

The model chosen for capturing inter-CSD competition and substitution effect among the settlement services provided by CSDs is the following: at each period of the game the demand for settlement transactions q_i in CSD i is given by

$$q_i = \alpha_i - \gamma_{ii} p_i + \gamma_{ij} p_j$$

where p_i is the average price charged by CSD i for its settlement services, and γ_{ii} and γ_{ij} are nonnegative constants. Note that γ_{ii} is the elasticity of the volume of transaction q_i to the price per settlement transaction p_i, and γ_{ij} is its elasticity to the price per settlement transaction set by CSD j; hence greater competition and substitution effect translates into a greater γ_{ij}.

Remark 1: The demand equation could be normalised in p_i, but keeping the parameter γ_{ii} makes interpretations of the simulation part easier to understand.

Remark 2: We consider in this Remark the many possibilities presented by this very simple stage game model. First, for two CSDs i and j the model consists in assuming:

$$\begin{cases} q_i = \alpha_i - \gamma_{ii} p_i + \gamma_{ij} p_j \\ q_j = \alpha_j - \gamma_{jj} p_j + \gamma_{ji} p_i \end{cases}$$

Hence total demand for settlement services is

$$q = q_i + q_j = \alpha_i + \alpha_j + (\gamma_{ji} - \gamma_{ii}) p_i + (\gamma_{ij} - \gamma_{jj}) p_j$$

The assumption $\gamma_{ji} \leq \gamma_{ii}$ as well as $\gamma_{ij} \leq \gamma_{jj}$ ensures that there is no possible way to increase the price level (p_i, p_j) that would result in a higher *aggregate* demand (because we assume γ_{ii} and γ_{jj} nonnegative the local demand will necessarily be lower). We will thus work under this assumption. Nevertheless, our formal derivations only require the weaker assumptions $\gamma_{ji} \leq \gamma_{ii}\sqrt{2}$ and $\gamma_{ij} \leq \gamma_{jj}\sqrt{2}$.

In terms of competition through price-setting, the model can be interpreted as follows. Consider, for example, a price increase of one unit for the settlement services provided by CSD i, while the price set by the other CSD stays constant. It translates into a loss of demand of γ_{ii} for settlement services offered by CSD i, of which γ_{ji} is re-captured by CSD j: market participants which could do so have reacted by switching to the settlement services provided by j, still the higher prices result in a decrease of $\gamma_{ji} - \gamma_{ii}$ of aggregate settlement demand.

Demand is indeed responsive to price; a lower local price translate in higher settlement demand since transactions that were not profitable before now become profitable; still, we can ensure (if we ever want to) an inelastic *aggregate* demand by assuming the extreme case where $\gamma_{ji} = \gamma_{ii}$ and $\gamma_{ij} = \gamma_{jj}$. Then $q = \alpha_i + \alpha_j$ is constant, whatever the price level, while demand for settlement in a given CSD still responds negatively to prices set by this CSD. The case where settlement services are completely non-substitutable – and where, as a consequence, there can be no competition between CSD i and CSD j – is obtained by setting $\gamma_{ij} = \gamma_{ji} = 0$.

In order not to mix the incentives for the investment / reshaping decision given price competition with the effects of increases in price competition, the main part of the paper does not consider changes in price competition over time. However, Annex 7.5 shows that the set of parameters for which tacit collusion is sustainable in a high competition / substitution environment is larger than in a low competition / substitution environment.

Interpreting CSD j as consisting of all other agents providing settlement services except CSD i, instead of a particular given CSD, is a convenient simplification to generalizing the model to n CSDs and multiplying the number of parameters given the complexity of analytical solutions. The focus is then set on CSD i, while the parameters defining CSD j reflect an average of the parameters for the rest of the settlement industry.[6] In particular, new entrants in the industry will modify the value of these parameters, for example entrance of new low-cost CSDs will certainly affect c_j negatively, since they decrease the aggregate costs in the settlement industry. The price p_j would, following the same idea, be the average price for settlement services in the market, excluding the price set by CSD i.

2.4 Analytical Resolution

For dynamic games, the concept of subgame-perfect Nash equilibrium is particularly relevant, because contrary to the simple Nash Equilibrium (Nash equilibrium) concept, it does not include strategies involving threats that are not credible. This is the reason why it is widely used in economics today and will be used extensively in this article. Note that although it is common to distinguish between strict and weak Nash equilibria, we will often not do so as the limiting conditions that translate into a weak Nash equilibrium are most of the time of very little interest. In fact, we will always talk of Nash equilibrium (resp. subgame perfect Nash equilibrium) when we actually mean strict Nash equilibrium (resp. subgame perfect Nash equilibrium). Solving the game by backward induction as we proceed to do allows to find *all* subgame perfect Nash equilibria. We will see that, because each stage game has only one Nash equilibrium, there exists in fact only one subgame perfect Nash equilibrium: the one where the Nash equilibrium of each subgame is played repeatedly. Only the main results are reported here: the detailed computations are provided in Annex 7.1.

2.4.1 A Cost Structure Assumption (CSA)

We will assume for all our derivations that the costs of settlements are always smaller than the price set by the CSD for the settlement services. This allows to get rid of unwanted corner solutions, where a given CSD would in practice stop all settlement activities (which amount to withdrawing from the market), because its optimal price, that is, the price at which maximizes its profits, still does not yield positive profits. Formally, the CSA states that for any costs $(\widetilde{c_i}, \widetilde{c_j})$ involved in the model, we have $p_i^* - \widetilde{c_i} \geq 0$ and $p_i^* - \widetilde{c_i} \geq 0$, so that no CSD makes a loss by settling an additional transaction. As we will see, at equilibrium of the price-setting stage we have $p_i^* - \widetilde{c_i} = \frac{1}{\gamma_{ii}} q_i^*$, hence the CSA also translates into stating that the quantities produced at equilibrium are positive, i.e. $q_i^* \geq 0$ and $q_j^* \geq 0$. Note the assumption of CSA has to be checked whenever simulations using the equilibrium formulas provided here are performed.

[6] Of course, such simplification does not allow to analyse all potential strategic aspects for such an additional CSD. However, the focus below (see Section 3.2.3) is on the impact of an additional CSD on CSD i by lowering c_j and p_j. Annex 7.3 includes a more formal generalisation of the model to n CSDs under the simplifying assumption of symmetry between the CSDs.

2.4.2 Nash equilibrium of the price-setting game $G(\widetilde{c_i})$

Profits for a given CSD i in a period where the costs per transaction are $\widetilde{c_i}$ are:

$$\pi_i = q_i(p_i - \widetilde{c_i})$$

The best-response $p_i^*(p_j)$ of CSD i to a price p_j from CSD j is thus

$$p_i^*(p_j) = \frac{1}{2\gamma_{ii}}(\alpha_i + \gamma_{ii}\widetilde{c_i} + \gamma_{ij}p_j)$$

and similarly

$$p_j^*(p_i) = \frac{1}{2\gamma_{jj}}(\alpha_j + \gamma_{jj}\widetilde{c_j} + \gamma_{ji}p_i)$$

Corresponding profits are

$$\pi_i(p_i^*(p_j), p_j) = \frac{1}{\gamma_{ii}}\left(\frac{1}{2}(\alpha_i - \gamma_{ii}\widetilde{c_i} + \gamma_{ij}p_j)\right)^2$$

and an analogous formula holds for the maximum profits derived by CSD j for any choice of price p_i by CSD i.

At equilibrium of the stage-game G, the level of prices (p_i, p_j) satisfies:

$$\begin{cases} p_i = p_i^*(p_j) \\ p_j = p_j^*(p_i) \end{cases}$$

Let us denote by p_i^* the price set by CSD i in the equilibrium of the stage-game. Then we get, by solving the above system:

$$\begin{cases} p_i^* = \frac{1}{4\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}}(2\gamma_{jj}\alpha_i + 2\gamma_{ii}\gamma_{jj}\widetilde{c_i} + \gamma_{ij}\alpha_j + \gamma_{ij}\gamma_{jj}\widetilde{c_j}) \\ p_j^* = \frac{1}{4\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}}(2\gamma_{ii}\alpha_j + 2\gamma_{ii}\gamma_{jj}\widetilde{c_j} + \gamma_{ji}\alpha_i + \gamma_{ji}\gamma_{ii}\widetilde{c_i}) \end{cases}$$

Note in passing that the fixed costs do not influence the equilibrium prices of the price-setting game. That is, the model predicts that CSDs should not try to recover the adaptation costs by increasing their price per transaction compared to the situation with no adaptation costs. CSDs will recover their adaptation costs over time from cost reductions and increased demand for settlement services. Indeed, if CSD i deviates from the Nash equilibrium prices by trying to set p_i higher to compensate for its *adaptation* costs, then, assuming CSD j does maximize its profits and play $p_j^*(p_i)$, CSD i does not maximize its profits (This is because it is not playing its best response $p_i = p_i^*(p_j)$ - if it was, then since CSD j plays $p_j = p_j^*(p_i)$ they would indeed both be playing the Nash equilibrium (p_i^*, p_j^*), a contradiction with CSD i deviating from it).

By applying backward induction to the whole repeated game, we see that this stage-result holds for the whole repeated game, that is, the optimal pricing should not take into account the adaptation costs (see the remark at the end of section 2.4.3). In other words, CSDs which try to pass on their own adaptation costs to their clients will be penalized by earning less than if they had just set the equilibrium price they would have set in the absence of adaptation costs[7]:

[7] It is interesting to note that Clearstream and Euroclear, the two largest European CSD groups, published documents in December 2012 (see Clearstream [4] and Euroclear [10]) that informs the clients their respective clients of an additional charge of EUR 0.094 and EUR 0.10, respectively, per instruction to finance the adaptation costs to T2S *before* their respective migration to T2S. (For the Euroclear Group, this charge applies so far only to the ESES platform, covering the Belgian, Dutch and French markets.) Clearstream furthermore announced that they would not add any margin on top of the direct T2S fee (c_{T2S} below). These decisions appear to confirm the prediction of the model that CSDs will not be able to recover adaptation costs by increasing fees after adaptation to the T2S environment. The model does not capture the possibility to raise fees in today's largely monopolistic market structures before the introduction of T2S.

Proposition 1 *In the finite model, no CSD should increase its prices in an attempt to cover its adaptation costs, whatever they are. If some CSD i deviates in at least one period of the repeated game from the price p_i^* it would have chosen in the absence of adaptation costs, it will make strictly less profits than by playing p_i^*. p_i^* is only a function of the costs $(\widetilde{c_i}, \widetilde{c_j})$ of all the CSDs at this period, the different price-elasticities $(\gamma_{ii}, \gamma_{ij}, \gamma_{ji}, \gamma_{jj})$ and the sizes (α_i, α_j) of the demand for each settlement service.*

Our price-setting model has thus clear-cut behavioural consequences concerning the pricing of the settlement services in the periods subsequent to the adaptation period, suggesting that in a competitive setting CSDs will not increase the price of their settlement services if they try to maximize their profits. In particular, it asserts that if players are rational and profit-seeking, then the costs reduction (that is, a decrease in $\widetilde{c_i}$ and $\widetilde{c_j}$) will be correctly reflected in lower prices (since p_i is a decreasing function of $\widetilde{c_i}$ and $\widetilde{c_j}$), and not kept at the pre-reshaping level in an attempt to cover the previous adaptation costs, least of all increased.

Definition 1 *We say that CSDs engage in price competition, or that there is price competition, if they always play the Nash equilibrium (p_i^*, p_j^*) at each price-setting game, given the current costs.*

Hence, price competition rules out tacit collusion to leave prices higher than the unique competitive equilibrium of the stage-game.

2.4.3 Optimal degree of reshaping chosen in the first period of the game

The optimal degree of reshaping in the first period of the game depends on the trade-off between the total profits a CSD can make after reshaping and the adaptation cost for the reshaping.

Total profits: Using the formula expressing $\pi_i = \pi_i(p_i^*(p_j), p_j)$ from the above section and replacing p_j by the equilibrium price p_j^* chosen by CSD j, we find that the profits derived by CSD i in a price-setting stage game where the equilibrium prices are (p_i^*, p_j^*) are selected is:

$$\pi_i^* = (A_i + B_i\widetilde{c_j} - D_i\widetilde{c_i})^2$$

where

$$\begin{aligned}
A_i &= \frac{1}{2\sqrt{\gamma_{ii}}}(\alpha_i + \frac{\gamma_{ij}}{4\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}}(2\gamma_{ii}\alpha_j + \gamma_{ji}\alpha_i)) \\
B_i &= \frac{\sqrt{\gamma_{ii}}\gamma_{ij}\gamma_{jj}}{4\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}} \\
D_i &= \sqrt{\gamma_{ii}}(\frac{2\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}}{4\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}})
\end{aligned}$$

A similar expression holds for π_j^*. Note that our assumption that $\gamma_{ii} \geq \gamma_{ji}$ and $\gamma_{jj} \geq \gamma_{ij}$ (Section 2.3) ensures that $D_i \geq 0$.

Remark: Note in passing that the quantity $A_i + B_i\widetilde{c_j} - D_i\widetilde{c_i}$, in the above expression for π_i^*, is positive for any costs $(\widetilde{c_i}, \widetilde{c_j})$ involved, because by the above $A_i + B_i\widetilde{c_j} - D_i\widetilde{c_i} = p_i^* - \widetilde{c_i} = \frac{1}{\gamma_{ii}}q_i^*$ and that our assumption on costs, that is, the CSA, precisely state that $p_i^* - \widetilde{c_i} \geq 0$ and $p_i^* - \widetilde{c_i} \geq 0$ for all "costs involved" (see Section 2.4.1).

The *total profits* are defined as the profits for the whole game, that is, the discounted sum, by an appropriate discount-factor δ, of the profits obtained at each stage game. For example, total profits for CSD i are

$$\pi_i^{tot} = \sum_{t=1}^{N} \delta^{t-1}\pi_{i,t}$$

provided that $\pi_{i,t}$ are the profits realised by CSD i on the t-th stage game. Note that, as usual, $\delta = \frac{1}{1+r}$, if r is the cost of capital mentioned earlier. Now the total profit π_i^{tot} derived by playing the Nash-equilibrium at each stage game is obtained by replacing in the sum each $\pi_{i,t}$ by the appropriate values of π_i^* (note that π_i^* depends on the fixed and variable costs involved at each period, and that these costs are the same for each period $t \geq 2$):

$$\pi_i^{tot} = \widetilde{\delta}(A_i + ((1-b_j)c_j + c_{T2S})B_i - ((1-b_i)c_i + c_{T2S}))D_i)^2 - C_{i,adapt}(b_i)$$

with

$$\widetilde{\delta} = \begin{cases} N & \text{if } \delta = 1 \\ \frac{1-\delta^N}{1-\delta} & \text{if } 0 \leq \delta < 1 \end{cases}$$

Adaptation costs: In the adaptation cost function $C_{i,adapt}(a_i, b_i)$, the parameter a_i captures the reshaping impact on the CSD's fixed cost; the parameter b_i captures the reshaping impact on the CSD's variable cost per transaction (see Table in Section 2.2).

Assume $C_{i,adapt}(a_i, b_i) = \xi_i b_i^2$ for simplicity and to focus on the tradeoff, in a competitive environment, between paying the lump sum today and benefiting from potential costs-per-transaction reductions tomorrow, and investing very little today while putting the T2S fees c_{T2S} on top of one's cost c_i. Note this yields an increasing, convex cost-function, as is probably the case in real life for the adaptation costs facing a CSD with the costs for reshaping the CSD's fixed cost part normalised to 0.[8]

Lemma and theorem for optimal degree of reshaping: If $\xi_i > \widetilde{\delta} c_i^2 D_i^2$, then the best-response function of CSD i at the first period of the game is

$$b_i^*(b_j) = \min(1, \max(0, \psi_i(A_i + ((1-b_j)c_j + c_{T2S})B_i - (c_i + c_{T2S})D_i))) \qquad (BR)$$

with

$$\psi_i = \frac{\widetilde{\delta} c_i D_i}{\xi_i - \widetilde{\delta} c_i^2 D_i^2}$$

But what if $\xi_i < \widetilde{\delta} c_i^2 D_i^2$? The following Lemma presents the answer:

Lemma 1 *Assume $\xi_i < \widetilde{\delta} c_i^2 D_i^2$. Then if CSDs engage in price competition, the best-response function $b_i^*(b_j)$ is the constant function $b_i^*(b_j) = 1$, which represents a complete reshaping decision from CSD i whatever the degree b_j CSD j choose to reshape.*

The proof of Lemma 1 is contained in Annex 7.1.3.

Remark: Note $\xi_i < \widetilde{\delta} c_i^2 D_i^2$ indicates low adaptation costs compared to the square of the CSD transaction costs times a specified function of cross-elasticities $D_i = D_i(\gamma_{ii}, \gamma_{ij}, \gamma_{ji}, \gamma_{jj})$. Lemma 1 shows these low costs result in a complete reshaping ($b_i = 1$) being always more appropriate, regardless of the other CSD's decision concerning its own degree of reshaping b_j.

Together with the best-response expression in case $\xi_i > \widetilde{\delta} c_i^2 D_i^2$ given by (BR), this allows to prove the following theorem:

[8] Of course, it would be possible to solve the model with other, more general forms for the adaptation-cost function, in particular, choosing costs functions that also depend on a_i or on the size of the price-independent demand α_i. For our adaptation cost function $C_{i,adapt}(a_i, b_i) = \xi_i b_i^2$, since there is no dependency on a_i then trivially at equilibrium $a_i = 1$. Indeed, since decreasing fixed costs do not affect the pricing, as previously noted in Proposition 1, profits are maximized when the fixed costs are minimized, which only occurs in $a_i = 1$.

Theorem 1 *If all CSDs are playing the Nash equilibrium of each following stage-game of the repeated game, then the optimal degrees* (b_i^*, b_j^*) *of reshaping are given by the following formulas:*
(i) If $\xi_i > \widetilde{\delta} c_i^2 D_i^2$ *and* $\xi_j > \widetilde{\delta} c_j^2 D_j^2$*, and assuming the quantities* (b_i^{**}, b_j^{**}) *defined by*

$$
\begin{aligned}
b_i^{**} &= \frac{\psi_i(A_i + (c_j + c_{T2S})B_i - (c_i + c_{T2S})D_i) - B_i\psi_i\psi_j c_j(A_j + (c_i + c_{T2S})B_j - (c_j + c_{T2S})D_j)}{1 - B_iB_j\psi_i\psi_j c_i c_j} \\
b_j^{**} &= \frac{\psi_j(A_j + (c_i + c_{T2S})B_j - (c_j + c_{T2S})D_j) - B_j\psi_i\psi_j c_i(A_i + (c_j + c_{T2S})B_i - (c_i + c_{T2S})D_i)}{1 - B_iB_j\psi_i\psi_j c_i c_j}
\end{aligned}
$$

belong to $[0, 1]$*, then* $(b_i^*, b_j^*) = (b_i^{**}, b_j^{**})$.
(ii) If $\xi_i < \widetilde{\delta} c_i^2 D_i^2$ *and* $\xi_j < \widetilde{\delta} c_j^2 D_j^2)$ *then* $b_i = b_j = 1$.
(iii) If $\xi_i > \widetilde{\delta} c_i^2 D_i^2$ *and* $\xi_j < \widetilde{\delta} c_j^2 D_j^2$ *then* $b_i = b_i^*(1) = \max(0, \min(\psi_i(A_i + c_{T2S}B_i - (c_i + c_{T2S}))D_i)))$ *and* $b_j = 1$.
(iv) If $\xi_i < \widetilde{\delta} c_i^2 D_i^2$ *and* $\xi_j > \widetilde{\delta} c_j^2 D_j^2$ *then* $b_i = 1$ *and* $b_j = b_j^*(1) = \max(0, \min(\psi_j(A_j + c_{T2S}B_j - (c_j + c_{T2S}))D_j)))$.

Proof: The proof consists solely of solving the system:

$$
\begin{cases} b_i = b_i^*(b_j) \\ b_j = b_j^*(b_i) \end{cases}
$$

in the four different cases, of which depends the form of the best-response function, as mentioned above. Detailed computations are provided in Annex 7.1.3.□

Now playing the Nash equilibrium of each stage game, that is, engaging in price competition, certainly yields a subgame-perfect Nash-equilibrium for our finite horizon game. Moreover, since the price-setting games G only have one Nash-equilibrium, and since this is also the case of the reshaping part of the game (step 1), backward-induction shows that this subgame-perfect Nash equilibrium is *de facto* the only one (see the Remark below for a more formal proof). Hence the optimal degree of reshaping in our finite game setting happen to be those given by Theorem 1. This concludes the analytical resolution of the finite game model.

Remark 1: The unique subgame-perfect Nash equilibrium can be derived more formally by following the generalised backward induction procedure described and proven in Mas-Colell et al. [18], p. 277: Let Γ_E denote the extensive form of the game which consists of the reshaping decision in the first period $t = 1$ of the game and subsequent price-setting games G_t in periods $t = 1, ..., N$. All variables are also indexed with t, so that the unique Nash equilibrium in the final subgame is $(p_{i,N}^*, p_{j,N}^*)$. As a consequence, $(p_{i,N-1}^*, p_{j,N-1}^*)$ is also the equilibrium in the reduced game of the preceding period $N - 1$. Continuing this procedure until the first period $t = 1$ gives $(p_{i,1}^*, p_{j,1}^*)$ as the equilibrium. Finally, applying Theorem 1 yields the optimal degrees of reshaping (b_i^*, b_j^*) in $t = 1$. The uniqueness of the Nash equilibrium at each step implies the uniqueness of the subgame-perfect Nash equilibrium of Γ_E derived by backward induction.

Remark 2: The case where $\xi_i > \widetilde{\delta} c_i^2 D_i^2$ and $\xi_j > \widetilde{\delta} c_j^2 D_j^2$ includes potential "corner solutions" which are not covered by the statement of the theorem for simplicity. The complete derivation of all NE, including the so-called "corner-solutions", can be found in the proof of the theorem, in Appendix 7.1.3. Of course, it is these complete results established by distinguishing twelve different cases that should be used for any simulations. Still, the non-corner solution (b_i^{**}, b_j^{**}) quoted by the theorem appears in five out of twelve of these cases.

Remark 3: Notice that, as is often the case in a finite-horizon setting, the model as such does not allow for tacit collusion. Because all the subsequent stage-games which are price-setting games have only one Nash equilibrium and hence only one possible Nash equilibrium payoff vector, backward-induction predicts that repeating the Nash equilibrium of each stage-game is the only subgame perfect

Nash equilibrium of the whole game. Hence the N-period game does not add a lot of insights over the one-period game, that is, the game obtained by setting $N = 1$, except for quantifying the variation of the degree of reshaping to the time-horizon N of the CSDs. We will enlarge the spectrum of the possible strategies by moving to an infinite setting in Section 3, while re-using most of the results proved above.

2.4.4 Simulation results

The model developed previously can be used to produce numerical outputs. With sufficient data concerning prices, costs and settlement volumes, a precise number for the optimal degree of reshaping could be obtained. Because of the insufficient number of data points in our possession, we are not in a position to estimate the building blocks of the model in a methodologically correct way. Hence, the results of this section should better be understood as obtained by *assuming* a numerical range for the various parameters for a large CSD, rather than by *estimating* such ranges from reliable data.

Selection of parameter ranges: Estimations with very limited data on the annual settlement volumes and average prices per transaction of Clearstream and Euroclear[9] were used as a very rough indication to select a plausible order of magnitude for the α- and γ-parameters of the demand functions. Furthermore, we assume for the baseline model of the simulations that the corresponding fixed parameter values are identical for both markets, that is, that $\gamma_{11} = \gamma_{22}$, $\gamma_{12} = \gamma_{21}$, $\alpha_1 = \alpha_2$ and $c_1 = c_2$. The fixed value given to $\gamma_{11}, \gamma_{22}, \gamma_{12}, \gamma_{21}, \alpha_1$ and α_2 as well as the minimum and maximum values reported in the Table below are thus not directly linked to our data estimations.

For the whole the simulation part we choose to work with the costs and prices per settlement *instruction*, in line with the pricing conventions of most CSDs. A settlement transaction generally consists of two separate settlement instructions, so that we could have as well chosen to work with the costs and prices per settlement *transaction*, appropriately adjusting the parameters.

Data on current CSD costs for settlement and on their adaptations costs to T2S are difficult to obtain, too. For the current cost per instruction, i.e. the cost parameters c_1 and c_2, we assume the range between EUR 0.2 and EUR 0.6 as a rough approximation. These figures are based on observed prices for domestic settlement instructions and assume (significant) profit margins, so that the actual average cost of domestic and cross-border instructions is likely to be greater.

In T2S, a settlement instruction will cost EUR 0.15 plus fees for information services which depend on their use, which in turn will depend on the degree and way of a CSDs' adaptation towards T2S. The minimum usage of information services leads to an add-on of EUR 0.012 per instruction, whereas the currently expected average add-on will be EUR 0.042[10]. For the purpose of the simulation, $c_{T2S} = 0.192$ is used.

CSDs' adaptation costs will in reality be highly influenced by their size and their current IT architecture. The more modular their current architecture, the cheaper the adaptation to T2S. This is because T2S can more easily substitute the CSD's current settlement engine with only limited impact on the IT for other services. The more embedded the settlement engine in a CSD's overall architecture, the more costly reshaping becomes. These costs grow with the number of other services that a CSD provides. While these aspects are not directly captured by the model, they justify a very broad range for the adaptation costs $\xi_1 = \xi_2$ of a large CSD, which are assumed between EUR 5 million and EUR 50 million. This range may be set too high even for the largest European CSDs and covers costs for the adaptation of non-settlement related activities. Hence, in the absence of precise data concerning CSDs' costs, the parameters are chosen in a conservative way which reflects a significant variance[11] and is detrimental to reshaping.[12]

[9] The data are taken from the annual reports of Clearstream and Euroclear.

[10] These estimates are based on the report of a T2S pricing workshop with market participants in February 2011, see http://www.ecb.europa.eu/paym/t2s/progress/pdf/ag/mtg13/item-4-3-2011-02-23-report-of-the-pricing-workshop.pdf.

[11] The coefficient of variation, a normalised measure of variability, is greatest for the cost parameter ξ.

[12] In December 2012, Clearstream and Euroclear, the two largest European CSD groups, released publicly (see Clearstream [4] and Euroclear [10]) that they will charge a) EUR 25 million as a maximum to their users for the adaptation costs of Euroclear's ESES platform which covers the Belgian, Dutch and French markets and b) EUR 30 million as maximum "external portion" of Clearstream's T2S investment cost. Their respective total adaptation costs may be somewhat greater.

Thus the main purpose of this section is not to provide a clear-cut answer about the degree of optimal reshaping, but to:

- illustrate how simulations could be used to answer numerically the problem of optimal reshaping, if more data were to become available;
- (partially) solve the problem of the variation of the optimal degree of reshaping b_i with respect to its different arguments. Indeed, it is more illustrative to draw these graphs for some assumed parameter distributions than to try to solve analytically by computing derivatives whose signs are very difficult to determine given the many parameters involved. In particular, the adaptation costs were purposely chosen very high in order to get informative graphs. For example cutting down these high adaptation costs by a factor of 2 would result in a complete reshaping being optimal for almost all other ranges of parameters. This results in graphs being straight lines of equation $b_1 = 1$, a not-so-interesting graph to comment, although it has the strong policy implication of a complete reshaping being the best option.

Below is the Table which summarizes the parameters and parameter ranges assumed for all the simulations in this section.

	min value	max value	fixed value	coefficient of variation
γ_{11}, γ_{22}	150×10^6	230×10^6	190×10^6	0.12
γ_{12}, γ_{21}	90×10^6	150×10^6	120×10^6	0.14
α_1, α_2	100×10^6	180×10^6	140×10^6	0.16
c_1, c_2	0.2	0.6	0.4	0.29
c_{T2S}			0.192	0
N			1	0
$\xi_1 = \xi_2$[13]	5×10^6	50×10^6	27.5×10^6	0.47

Overview of figures: Each of the two-dimensional graph of Figure 1[14] was obtained by selecting a given parameter, displayed on the horizontal axis, and making it vary within the realistic range delimitated by the "min value" and the "max value" indicated in the above Table, while keeping all other parameters constant at the level indicated by the column "fixed values" of the same table. The vertical axis depicts the degree of optimal reshaping, b_1^*. Notably, the magnitude of the impact of each parameter has to be seen in conjunction with its assumed range of variability as evident from the coefficients of variation in the Table above.

Similarly, the three-dimensional graphs of Figure 2 were obtained by selecting two parameters to vary within their range while fixing the others. To interpret these graphs one has to keep in mind they represent the optimal degree of reshaping b_1^* of CSD 1, assuming the other CSD, CSD 2, also acts in an optimal way. Should the other CSD not follow a profit-maximizing rule, the optimal degree of reshaping of CSD 1 would be different, and would need to be computed using the best-response function $b_1^*(b_2)$ given in (BR) with b_2 as an input.

The histogram of Figure 3 shows the distribution of the optimal degree of reshaping b_1^* of CSD 1 assuming each parameter is chosen randomly and independently in the range described by the Table, for 100000 simulations, the uniform distribution being used. We see that, at least for those parameter distributions, complete reshaping is often optimal. A word of caution is warranted there: drawing randomly parameters from their assumed range can certainly give parameter constellations that make no sense. In particular, it could happen that the CSA does not hold. Hence in the histogram of Figure 3 we discarded those parameter constellations for which the CSA does not hold, which amounted to discarding roughly 9% of the total number of parameter constellations drawn.

[13] The same adaptation costs were assumed for the two CSDs at any point in the simulation, which implies that when ξ_1 increases then ξ_2 increases by the same amount.

[14] See Annex 7.6 for all figures based on the simulations.

Finally, the graphs of Figure 4 and 5 depict the results of Monte-Carlo simulations concerning the expected value of the optimal degree of reshaping, subject to knowing each of its parameter in turn. More precisely, for each point to be drawn, about 10 000 of randomly chosen constellations of parameters were chosen, and the average alone was represented. The ranges are still the same as indicated in the Table, and uniform distributions are assumed. The lowest curve of each graph indicates the variance of the simulations carried out to obtain the corresponding expected value of optimal reshaping on the higher curve. Because the shape of the graphs obtained are very similar to those of Figure 1, they provide a positive robustness-check for the sense of variation of the optimal degree of reshaping.

Results of the simulation exercise: We will comment the graphs of Figure 1 by moving from left to right and top to bottom.

The first and second graphs only underline that the greater the domestic market (α_1) or the foreign market (α_2), the higher the optimal degree of reshaping (for a fixed amount of adaptation costs). This is because higher volume of instructions in the market make it easier for a given CSD to recover from its adaptation costs, by benefiting of lower costs per instructions applied to larger volumes. Hence larger markets have higher degree of optimal reshaping than smaller markets, holding other parameters constant. Also, the size of the foreign market is much less an incentive to reshape than the size of the domestic market, as can be seen from the scale of the vertical axis of these graphs.

The third graph indicates that the higher the costs of settlement for CSD 1, c_1, the more incentives it has to reshape, and that there seems to be a linear relation between these two quantities. This confirms intuition, as reshaping cuts part of these costs per settlement instruction.

The fourth graph shows the optimal degree of reshaping b_1 of CSD 1 as a function of the costs per instruction c_2 of its competitor. We see that b_1 is first a (slightly) increasing function of the costs c_2, probably because reshaping allows CSD 1 to put lower prices and draw part of the demand for settlement services originally directed towards CSD 2 to its own market. This positive relation holds up to a certain threshold, with decreasing marginal increases of b_1 as c_2 increase. These decreasing marginal increases are probably due to the strategic decision of CSD 2 itself. Indeed, as we have seen, higher costs per instruction will prompt CSD 2 to reshape to decrease those costs. Past the threshold this effect dominates and the optimal degree of reshaping decreases with higher competitor costs. The optimal degree of reshaping for CSD 1 then becomes flat, corresponding to the situation where CSD 2 has completely reshaped anyway so cannot decrease any further its costs. Hence, when the cost of CSD 2 continues to increase, this has no impact anymore on CSD 1 decision to reshape. Nevertheless, note that all these variations, although interesting to comment analytically, are negligible (vertical scale).

The fifth graph shows that the higher the adaptation cost of a complete reshaping ξ_i, the less useful it is to reshape; this is obvious since the adaptation cost is the price to pay to reshape, which counter-balances future benefits. Hence the higher these costs, the less interesting it is to reshape, holding all other parameters constant. The interesting point is the particular form of this function, which decreases quickly first and then very slowly. This particular shape is probably linked to the convex form assumed for the adaptation cost function. Also noteworthy is the existence of the threshold under which complete reshaping is the optimal solution.

The last four graphs give an idea of the variation of the optimal degree of reshaping with respect to different market elasticities, and hence different equilibrium prices before reshaping.

For example, the sixth graph shows that the optimal degree of reshaping b_1 happens to be a decreasing function of γ_{11}: The effect from cost-saving on each instruction can compensate the relative fewer gains of market share due to lower prices, making reshaping more attractive for lower price sensitivity.[15] Notice that if we were to increase the overall price sensitivity of demand in market 1 by increasing both the parameter γ_{11} and the parameter γ_{12}, the optimal degree of reshaping would increase, as can be seen in the third three-dimensional graph of Figure 2.

The seventh graph indicates that the more CSD 1 is able to capture demand from the market of CSD 2, that is, the more it provides substitutable services, the higher the degree of optimal reshaping of CSD 1. More potential business seems always a good incentive to reshape, as low prices is the main determinant of demand in the building block of our model.

The eighth graph indicates that variations of γ_{21}, which is a parameter characterizing the demand for settlement services of CSD 2, and not of CSD 1, has nevertheless an impact on the optimal degree of reshaping of CSD 1. [16]

The last graph could be explained using the same argument of indirect effects stemming from the reshaping decision of another CSD. Indeed, higher γ_{22} implies less reshaping from CSD 2, as for the case of CSD 1 with γ_{11} explained in the sixth graph, which per se would suggests a higher incentive to reshape for CSD 1. However this effect is dominated by the fact that the price set by CSD 2, even in the absence of reshaping, would be much lower, which, if all other parameters are held constant, translates into less demand for CSD 1 settlement services, and hence less incentive to reshape for CSD 1.

Two other graphs could be drawn: choosing $N > 1$, one could draw the optimal degree of reshaping b_1 as a function of the discount factor δ applied to its future cash flows, and as a function of the time horizon N. These graphs show that the preference of the CSDs for the present has a meaningful impact on the optimal degree of reshaping. Hence, the more patient a given CSD, the more reshaping it will undertake today to derive benefits from it tomorrow, all other things being equal. An interesting point to notice is that making N very large does not necessarily make a complete reshaping the optimal solution: for some parameters values the optimal degree of reshaping converges instead towards a given value inferior to 1.

3 Extending the game to infinity

Although the finite game-theoretic setting described in the previous section is useful in simulations (to add the parameter N or time-horizon of the involved CSDs), and for understanding the model more easily, it is not very rich in terms of subgame perfect Nash equilibrium: as noted earlier, because the game is finite and that each stage game yields only one Nash equilibrium, solving by backward induction to find the subgame-perfect Nash equilibrium yields a single strategy, that is, the one where the unique Nash equilibrium of each stage game is played repeatedly, independently of the past. Hence the myopic profit-maximization strategy, where CSDs set the prices yielding the higher payoffs at each period of the game, is the only subgame-perfect Nash equilibrium.

To allow for other, intertemporal strategies, we need to translate the game to an infinite setting, which basically just means choosing to repeat it indefinitely while assuming $\delta < 1$ to ensure finite payoffs. Of course, playing at each step the Nash equilibrium of the stage game still yields a subgame-perfect Nash

[15] The expectation could be that higher price-sensitivity to its own price would make the price-reduction allowed by cost-reduction through reshaping even more attractive. Nevertheless, this neglects that prices at equilibrium (and thus the volumes at equilibrium) would, in a market with a higher γ_{11}, already be lower than in a market with low γ_{11}. This can be checked analytically by computing the first derivative of p_i^* with respect to γ_{11}, which is negative. Of course, cost-reduction also applies, and reshaping will drop costs, and thus enable lower prices, in the same way as in a low γ_{11} scenario. The graph hence pin point that the positive effect of lowering prices, and hence of reshaping, is less strong in the high γ_{11} scenario than in the low γ_{11} scenario.

[16] Since q_1 is unaffected by $\gamma_{21}$by definition, this impact can only be indirect, due to the strategic decision of CSD 2 to reshape. Indeed, by symmetry, we know from the graph of b_1 as a function of γ_{12} that, when faced with a higher γ_{21}, CSD 2 will reshape more. Hence compared to the low γ_{21} case prices of CSD 2 will be lower and CSD 1, faced with a lower demand for settlement services than in the low γ_{21} case, but still with the same adaptation cost, will find it less profitable to reshape. Hence its lower degree of optimal reshaping. Also interesting is that this effect disappears below a certain threshold (not shown in this graph).

equilibrium, since any player unilaterally deviating from such strategy will be worse off, but there may now be other subgame-perfect Nash equilibrium. Nevertheless, most of other results, such as the formula for the degree of optimal reshaping assuming all CSDs play the Nash equilibrium of each subsequent stage game, still hold for the infinite case. In particular, Lemma 1 as well as Theorem 1 hold as such in the new infinite setting, by taking $\widetilde{\delta} = 1/(1-\delta)$, that is, the limit of the previous $\widetilde{\delta}$ when N tends to infinity. Note the assumption included in Lemma 1 and in Theorem 1 that it gives the optimal reshaping *assuming all players play the Nash equilibrium of each price-setting game afterwards.*

3.1 Delaying or prompting the Reshaping: tacit collusion in the model

3.1.1 Motivation for a new game

In the previous model, CSDs are only allowed to reshape at a single point in time, whereas reshaping is a private decision that is part of the overall business strategy of the CSDs, and thus may be taken at different point in time for each of the CSDs. Importantly, because T2S opens up the domestically-oriented, fragmented and thus largely monopolistic European settlement markets, the decision when to reshape of a given CSD may be dependent on other CSDs' choices. Hence, to further study the possibility of reshaping, we should allow CSDs to reshape at any period of the model. To reach this aim we modify slightly the previous rules of the game as follows:

3.1.2 The new game

Consider the stage game $G_{reshape}$ made up of the three following consecutive steps:

1) CSDs that have not reshaped so far (simultaneously) choose either to reshape now by some degree $b_i > 0$ or not to reshape yet ($b_i = 0$).

2) All CSDs observe the reshaping decisions of others and (simultaneously) set the price of the settlement service they provide.

3) They earn the associated payoffs as implied by the price-setting stage game described in Section 2.3.

The game $G_{reshape}$ is repeated an infinite number of times. The payoff of the whole game is just the discounted payoffs of each stage game, with a discount factor δ (strictly) inferior to 1. Note that the stage-game $G_{reshape}$ is not exactly independent of the past since CSDs which chose to reshape in the past cannot choose to reshape to a different degree in the future. That is, each CSD can only reshape once and the reshaping cannot then be changed in subsequent periods of the game. This impediment is of course unrealistic for a CSD which would like to increase its reshaping but much more close to reality for a CSD who would like to *decrease* its degree of reshaping and recover part of the adaptation costs it had paid in some previous period: this would, as in real-life, not be feasible in our model. Indeed, adaptation costs are costs paid in a lump-sum that cannot be recovered even if a given CSD changes its mind and downsizes the degree of reshaping it wanted. That being said, to allow only reshaping to occur in a precise period and not in progressive (positive) steps is a limitation to our model. Nevertheless, the time frame, that is, the meaning of the "period" of the model, could easily be interpreted as a large enough lapse of time such that reshaping indeed only occurs during one precise period, for each CSD. Moreover, the results we are about to derive assume far-sighted CSDs, which discount the future very little, hence it is unlikely that those results would be affected by the possibility of CSDs to reshape in steps.

An important consequence for a game-theoretic point of view is that although our infinite game consists in playing $G_{reshape}$ over and over an infinite number of times, it does not fall into the usual category of the repeated games because the population of CSDs able to make a decision at step 1) of $G_{reshape}$ is subject to change depending on previous history. For example, all subgames where all CSDs have previously chosen to reshape (to such or such degree) consist solely of playing the price-setting game given this reshaping, that is, step 2 and 3 of $G_{reshape}$, and are thus different from the first $G_{reshape}$ played in the game.

3.2 Two main theorems

3.2.1 Tacit collusion to delay reshaping

We will first show that even for large values of δ, when one would expect that the incentive to reshape is too hard for the CSDs to resist (since they favour future revenues almost as much as the current lump sum they would have to pay for the adaptation costs, and these future revenues tend to infinity when δ tends to 1), there is a subgame-perfect Nash equilibrium in which no CSD reshape when the ratio of the costs $\frac{c_i}{c_j}$ of the involved CSDs satisfies a precise inequality. Indeed, denote by $f_i = f_i(\gamma_{ii}, \gamma_{ij}, \gamma_{ji}, \gamma_{jj})$ the function of the four different price-elasticities of the model defined by:

$$f_i = \frac{B_i}{D_i} = \frac{\gamma_{ij}\gamma_{jj}}{2\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}}.$$

Then we have:

Theorem 2 *Assume $\frac{1}{f_j} < \frac{c_i}{c_j} < f_i$. Then there exists $\overline{\delta} \in]0, 1[$ such that for any discount factor $\delta \in]\overline{\delta}, 1[$, collusion for no reshaping can be sustained in a subgame perfect Nash-equilibrium. The result holds even when price competition is assumed.*

That is, there exists a subgame-perfect Nash equilibrium in which no CSD ever reshape when playing it.

The proof of Theorem 2 is given in Annex 7.2.1 and uses the following trigger strategy (S):

(S) *Do not reshape if no player has reshaped so far. If another player has reshaped and you have not reshaped, then reshape. At each price-setting game (step 2) play the unique Nash equilibrium, given all the costs involved.*

Remark 1: Since $f_i = \frac{B_i}{D_i} = \frac{\gamma_{ij}\gamma_{jj}}{2\gamma_{ii}\gamma_{jj}-\gamma_{ij}\gamma_{ji}}$, the condition $\frac{1}{f_j} < \frac{c_i}{c_j} < f_i$ of the theorem can be re-written directly in terms of the elasticities as $2\frac{\gamma_{jj}}{\gamma_{ji}} - \frac{\gamma_{ij}}{\gamma_{ii}} < \frac{c_i}{c_j} < \frac{\gamma_{ij}\gamma_{jj}}{2\gamma_{ii}\gamma_{jj}-\gamma_{ij}\gamma_{ji}}$. The condition for CSD i to be dissuaded to reshape by CSD j own potential reshaping is:

$$c_j f_i > c_i$$

We see that the larger the costs of CSD j, the more likely this is going to occur. Indeed, CSD j can more convincingly threaten to reshape and engage in tougher price competition if it is currently operating at higher costs than if it is already efficient. Now, for the strategy profile to be a Nash equilibrium, a similar inequality is required for CSD i costs, that is, $c_i f_j > c_j$.

Remark 2: It is possible to determine explicitly a value of $\overline{\delta}$ for which Theorem 2 holds. Indeed, (see Annex 7.2.2) we have that $c_i < c_j f_i$ is a necessary and sufficient condition for the inequality $(*)$ of the Proof of Theorem 2 (given in Annex 7.2.1) to hold. we see that the result of the theorem holds for $\overline{\delta} := \max(1 - \frac{c_i^2 D_i^2}{\xi_i}, 1 - \frac{c_j^2 D_j^2}{\xi_j}, \frac{B^2-A^2-\xi_i}{B^2-C^2-\xi_i}, \frac{B^2-A^2-\xi_j}{B^2-C^2-\xi_j})$ if $\xi < B^2 - C^2$ and for $\overline{\delta} := \max(1 - \frac{c_i^2 D_i^2}{\xi_i}, 1 - \frac{c_j^2 D_j^2}{\xi_j})$ if $\xi > B^2 - C^2$.

Remark 3: Using the expression for $\overline{\delta}$ provided by Remark 2, we see that for adaptation costs high enough, more precisely for $\xi_i > B^2 - C^2$ and $\xi_j > B^2 - C^2$, any further increase in both adaptation costs translates into an increase of our value of $\overline{\delta}$, since $\overline{\delta} := \max(1 - \frac{c_i^2 D_i^2}{\xi_i}, 1 - \frac{c_j^2 D_j^2}{\xi_j})$ and $1 - \frac{c_i^2 D_i^2}{\xi_i}$ (resp.$1 - \frac{c_j^2 D_j^2}{\xi_j}$) are increasing with ξ_i (resp. with ξ_j). This *suggests* that in that case, high adaptation costs do not increase but decrease the chance of collusion among CSDs. More precisely, the more costly the adaptation, the higher the discount factor above which there can be collusion (in the proof), hence the less likely collusion in that setting. This is rather counter-intuitive and stems from the fact that to make tacit collusion possible in our non-cooperative setting, CSDs must be credible when they frighten the others to reshape themselves should any CSD move towards reshaping. This tacit threat limits the benefits derived from reshaping for any deviating CSD. Yet, if reshaping costs are too high, then the threat become less credible, and the possibility of collusion based on that threat diminishes. Hence, a higher $\overline{\delta}$, that is, a greater valuation for the future cash flows compared to the current ones, need to be assumed to make such a threat credible. Now if $\xi_i < B^2 - C^2$ and $\xi_j < B^2 - C^2$, the effect of an increase in adaptation costs is not clear-cut and, even assuming that no better collusion strategies can be designed than our trigger strategy, depends on how the various parameters of the model compare to each other's. Indeed, $\frac{B^2-A^2-\xi_i}{B^2-C^2-\xi_i}$ is, when $\frac{c_i}{c_j} < f_i$, decreasing in ξ_i, hence if $\overline{\delta} := \max(1 - \frac{c_i^2 D_i^2}{\xi}, 1 - \frac{c_j^2 D_j^2}{\xi}, \frac{B^2-A^2-\xi_i}{B^2-C^2-\xi_i}, \frac{B^2-A^2-\xi_j}{B^2-C^2-\xi_j}) = \frac{B^2-A^2-\xi_i}{B^2-C^2-\xi_i} > \max(1 - \frac{c_i^2 D_i^2}{\xi}, 1 - \frac{c_j^2 D_j^2}{\xi}, \frac{B^2-A^2-\xi_j}{B^2-C^2-\xi_j}) =: \delta_0$ then an increase in ξ_i will first translate into a decrease of $\overline{\delta}$ (until $\overline{\delta}$ reaches δ_0) indicating that higher adaptation costs increase the chance of collusion. This is of course because, holding all other factors constant, the higher the adaptation costs a given CSD i faces for itself, the less profitable it is for the CSD to reshape and thus the more likely it is going to collude to delay the reshaping (see inequality $(*)$ in Proof of Theorem 2 given in Annex 7.2.1).

Remark 4: Under the conditions of Theorem 2, that is, when $c_i < f_i c_j$ and $c_j < f_j c_i$, the strategy (S) is, by the proof of the theorem, a Nash equilibrium for high discount factors. Another obvious Nash equilibrium under such conditions is for both CSDs to reshape completely at the first period of the game. Call this strategy (1). Now, it can readily be checked from the form of the one-period payoff function that, for high discount factors, the strategy (S) is (strictly) Pareto-dominated by the strategy (1) if both f_i and f_j are superior to 1. If both f_i and f_j are inferior to 1, then the converse holds: the strategy (S) (strictly) Pareto-dominates the strategy (1). Nevertheless we will prove in the discussion that in such a case (S) cannot be a subgame perfect Nash equilibrium[17]. When f_i is inferior to 1 and f_j superior to 1 or the reverse, then none of this two equilibrium Pareto-dominates the other, since one of the player is always worse off in a situation than in the other.

Remark 5: Theorem 2 corresponds to a situation where higher aggregate costs per transaction translates into higher profit margins for CSDs. Indeed, replacing f_i by its expression as a function of the elasticities it can easily be shown that $c_i < f_i c_j$ is equivalent to the profit margin per transaction $p_i^* - \widetilde{c}_i$ being higher when no CSDs reshape than when both fully reshape.

3.2.2 A sufficient condition for immediate reshaping

Theorem 2 gives a condition, namely that $c_i < f_i c_j$ and $c_j < f_j c_i$, under which there is possibility of collusion not to reshape among CSDs, when they value the future relatively to the present enough. The natural question is then to determine if, when the condition is not fulfilled, reshaping by at least one of the CSD always happens. Interestingly, it turns out to be the case:

Theorem 3 *Assume $c_i > f_i c_j$. Then for any discount-factor close enough to 1, CSD i will always completely reshape in any subgame perfect Nash equilibrium consistent with price competition. In particular, in the presence of price competition, the other CSD j cannot deter CSD i from reshaping in any credible way. More precisely, if $\xi_i < \pi_i(1,0)^2 - \pi_i(1,1)^2$, then for $\delta \geq 1 - \frac{c_i^2 D_i^2}{\xi_i}$ CSD i will always completely reshape, while if $\xi_i > \pi_i(1,0)^2 - \pi_i(1,1)^2$, CSD i will always completely reshape for $\delta \geq \max(1 - \frac{c_i^2 D_i^2}{\xi_i}, \frac{\pi_i(1,0)^2 - \pi_i(0,0)^2 - \xi_i}{\pi_i(1,0)^2 - \pi_i(1,1)^2 - \xi_i})$.*

[17] We need to establish that the conditions $c_i < f_i c_j$ and $c_j < f_j c_i$ are also necessary for (S) to be a subgame perfect Nash equilibrium, and this will be a consequence of the next section 's results. The fact that (S) as a subgame perfect Nash equilibrium cannot Pareto-dominate (1) then easily follows).

Proof. Since $\delta \geq 1 - \frac{c_i^2 D_i^2}{\xi_i}$, we have, by a reasoning similar to the proof of Theorem 2, that whatever the degree $b_j > 0$ CSD j choose to reshape, or if it refrains from reshaping ($b_j = 0$), $b_i = 1$ is the profit maximization choice of CSD i, should it choose to reshape. Note that it does not mean, *a priori*, that CSD i will choose to reshape: it could, in principle, as in Theorem 2, be dissuaded to do so by other CSDs' strategies.

The rest of the proof consists in showing this is never the case. Hence, the results will be that CSD i does choose to reshape, and that it chooses $b_i = 1$, that is, a complete reshaping, when it does so.

Now, the worst *credible* punishment CSD j could inflict on CSD i in order to induce it not to reshape is to reshape as soon as CSD i did by a degree $b_j^*(b_i)$ and engage in price competition afterwards, that is, play repeatedly the Nash equilibrium of the price-setting game. This would yield a payoff for CSD i of $\pi_i(1, b_j^*(b_i))$ instead of the $\pi_i(0,0)$ it received when it was not reshaping or of the $\pi_i(1,0)$ it received when it chose to reshape, in all subsequent price-setting stage games of the repeated game. Since $\pi_i(1, b_j^*(b_i)) \geq \pi_i(1,1)$, if we show that CSD i will still prefer to reshape if it earns, from the moment CSD j punishes it by also reshaping, $\pi_i(1,1)$ instead of $\pi_i(1, b_j^*(b_i))$, we will have proved that CSD i best action is indeed to reshape. But from the proof of Theorem 2, we know that whenever the inverse inequality of $(*)$ holds, CSD i is better off not playing the collusion strategy and reshaping, even though CSD j will punish it by reshaping itself and by engaging in harder price competition afterwards.

Assume by contradiction that $(*)$ holds. Then since by the proof of Remark 2 of Theorem 2 we have that $(*)$ holds for high values of δ if, and only if, $c_i < f_i c_j$, we immediately get a contradiction with $c_i > f_i c_j$. This proves the first part of Theorem 3.

To be more precise and prove the second part of Theorem 3, providing explicit values for the range of δ for which no infinite delaying of the reshaping decision can occur, we need to make a more precise use of Remark 2 of Theorem 2 by separating two cases:

If $\xi_i < B^2 - C^2$ then for $(*)$ to hold for some δ, in particular for some $\delta \geq 1 - \frac{c_i^2 D_i^2}{\xi_i}$, we need $c_i < f_i c_j$, which gives a contradiction with our assumption $c_i > f_i c_j$. Hence $(*)$ does *not* hold for *any* $\delta \geq 1 - \frac{c_i^2 D_i^2}{\xi_i}$, which gives our result than for $\delta \geq 1 - \frac{c_i^2 D_i^2}{\xi_i}$ CSD i will always reshape.

If $\xi_i > B^2 - C^2$ then we need $c_i \leq f_i c_j$ for $(*)$ to hold for $\delta \geq \frac{B^2 - A^2 - \xi_i}{B^2 - C^2 - \xi_i}$. Since by our assumptions $c_i > f_i c_j$, we deduce that for $\delta \geq \max(1 - \frac{c_i^2 D_i^2}{\xi_i}, \frac{B^2 - A^2 - \xi_i}{B^2 - C^2 - \xi_i})$, CSD i will always reshape. This concludes the proof. □

In order to understand Theorem 3 better, we look at some of its consequences in a symmetric setting. We first define explicitly what we mean by "symmetric":

Definition 2 *A market is said symmetric if, and only if, $\alpha_i = \alpha_j$, $\gamma_{ij} = \gamma_{ji}$ and $\gamma_{ii} = \gamma_{jj}$. Two CSDs' transaction costs are symmetric if, and only if, $c_i = c_j$.*

Remark: The condition $\alpha_i = \alpha_j$ is not necessary for any of the propositions below. Indeed, an interesting point to notice is that none of the conditions given in the theorem refers to the parameters α_i and α_j, but only to elasticities. Hence, it is the sensitivity to prices more than the size of the market (of which (α_i, α_j) is a proxy) that matters. We included the condition $\alpha_i = \alpha_j$ in the Definition to reflect a completely symmetric demand function for both CSDs in a symmetric market.

Proposition 2 *Assume that market and CSDs transaction costs are symmetric, that $\gamma_{ij} < \gamma_{ii}$ and that CSDs engage in price competition. Then for any discount-factor $\delta \geq 1 - \frac{c_i^2 D_i^2}{\xi_i}$, both CSDs will completely reshape in any subgame perfect Nash equilibrium.*

Proof. It is enough to show that the condition $c_i > f_i c_j$ of Theorem 3 is satisfied. Because $c_i = c_j$ this condition is equivalent to $1 > f_i$. By definition of $f_i = \frac{\gamma_{ij}\gamma_{jj}}{2\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}}$ this is equivalent to $\gamma_{ij}(\gamma_{jj} + \gamma_{ji}) < 2\gamma_{ii}\gamma_{jj}$, which is true since by symmetry and the usual assumptions we have $\gamma_{ij} < \gamma_{ii}$ and $\gamma_{jj} + \gamma_{ji} < 2\gamma_{jj}$. Hence by Theorem 3 both CSDs will completely reshape in any subgame perfect Nash equilibrium consistent with price competition. □

Note the condition $\gamma_{ij} < \gamma_{ii}$ only translates the property that a price level increase should lead to a lower *aggregate* demand, as explained in Remark 2 of Section 2.3, and is hence a rather natural assumption. From Proposition 2 we can easily get a similar result that predicts no collusion and the reshaping of at least one of the two CSDs in case the CSDs transaction costs are not symmetric:

Proposition 3 *Assume the market is symmetric and that $\gamma_{ij} < \gamma_{ii}$, and that CSDs engage in price competition. Then for all discount-factor $\delta \geq \max(1 - \frac{c_i^2 D_i^2}{\xi_i}, 1 - \frac{c_j^2 D_j^2}{\xi_j})$, the CSD with higher costs will reshape in any subgame perfect Nash equilibrium.*

Proof. Without loss of generality, we can assume CSD i has the highest costs, i.e. $c_i \geq c_j$. Then $\frac{c_i}{c_j} \geq 1$ while by Proposition 2 $1 > f_i$. Hence $\frac{c_i}{c_j} > f_i$ and Theorem 3 implies CSD i will reshape. $\square$.

Admittedly, all these non-collusion results assume price competition in the subsequent periods. Nevertheless, allowing for price collusion does not dissuade CSDs from reshaping, as can be seen in Annex 7.4. Simply put, this is because allowing for price-collusion allows the CSDs to pocket the unit-transaction cost-reductions instead of reflecting them in their prices.

3.2.3 Discussion of the main results

Theorem 2 asserts that for some range of parameters – more precisely, when $c_j f_i > c_i$ and $c_i f_j > c_j$ – there can be a perpetual delaying of the reshaping decision by the CSDs, resulting in none of the CSDs ever reshaping. This is the case because by not reshaping as long as the others do not reshape, a given CSD avoids to pay the adaptation costs, while the extra-rent it would extract through cost-reduction and earning market shares by reshaping seem no longer worth it when it knows that the other CSDs would then also reshape and compete with it on the same grounds. Of course, there is a first mover advantage: the first CSD to reshape would make more profits in the following period. But when the discount factor exceeds a certain threshold, this short-term benefit becomes irrelevant since the high discount factor gives an almost equal weight to the future than to the present.

Also worth noticing, the tacit collusion for not reshaping described by Theorem 2 holds for any value of δ higher than some threshold $\overline{\delta}$, which means no reshaping can potentially occur for far-sighted CSDs which attribute a high value to future payoffs. Hence contrary to our original model of Section 2, where the decision to reshape or not depended on the trade-off between paying now high adaptation costs and benefiting more from transaction costs reduction later, or avoiding this investment now and bearing the high costs per transaction in the subsequent periods of the model, the decision concerning reshaping depends here entirely on the expected strategy of the other CSD[18].

The suboptimal implication of Theorem 2 in terms of efficiency and investors' welfare should be accessed keeping in mind that Theorem 2 does not indicate that the *only* credible (subgame perfect) outcome is the one in which reshaping is indefinitely delayed: even under the assumptions that $c_j f_i > c_i$ and $c_i f_j > c_j$, there may be many more subgame perfect Nash equilibria. For example, one of these other possible subgame perfect Nash equilibrium is the one were CSDs reshape by their respective optimal degree as specified by Theorem 1 at the very first period of the model, and then set prices accordingly. Call this strategy (1), and call (S) any strategy yielding a perpetually delayed reshaping. It can easily be proved that strategy (S) never Pareto-dominates (1) as a Nash-equilibrium. Indeed by the previous results if (S) is a NE it means that $c_i < c_j f_i$ and $c_j < c_i f_j$. But for (S) to Pareto-dominate (1) we must have, by Remark 4 of Section 3.2.1, $f_i < 1$ and $f_j < 1$. This implies $c_i < c_j f_i < c_j$ and $c_j < c_i f_j < c_i$, a

[18] A comparison with the *infinite* version of our first model (Section 2), where the decision to reshape can only be taken in the first period of the model, gives an insight of how much the decision to delay reshaping in Theorem 2 relies on the mere possibility of CSDs to freely choose the time of their reshaping. Typically, in a setting where CSDs can only take the decision to reshape at the first period of the model, not only would there be no tacit collusion to delay reshaping, but, if $\delta \geq 1 - \frac{c_i^2 D_i^2}{\xi}$ – as is assumed in the Proof of Theorem 2 – then by Lemma 1 the reshaping would be *complete* (meaning CSD i would choose $b_i = 1$).

contradiction. On the contrary, we know by Remark 4 of Section 3.2.1 that if $f_i > 1$ and $f_j > 1$, which is the case for example if costs are symmetric ($c_i = c_j$) and if (S) is a NE, that the strategy (1), which implies reshaping in the first period, Pareto-dominates strategy (S). Hence both player will always be better off by reshaping immediately than by delaying ad infinitum the reshaping.

Besides, some other factors are likely to mitigate the results of Theorem 2:

- first, Theorem 2 applies only to the specific constellations of parameters satisfying the rather restrictive conditions that $c_j f_i > c_i$ and $c_i f_j > c_j$. Moreover, we proved in Theorem 3 that these conditions are also necessary, that is, if it ever happens that $c_i \geq f_i c_j$ or that $c_j \geq f_j c_i$, then at least one of CSD i or j will reshape, whatever the other CSD's strategy: there can be no deterrence for no-reshaping in that context, and (S) is not a subgame perfect Nash equilibrium anymore.

- second, Theorem 2 applies as such only for a market with no new entrants. A new entrant could create its infrastructure by optimally adjusting it to T2S following a Greenfield approach, which would be similar to a complete reshaping of an existing CSD. Its investment costs would play a similar role as the adaptation costs in the model, while its transaction cost would be likely to be less than the average cost of the market. Now, if we want to predict the outcome for the whole CSD industry, then as noted previously in Section 2.3, we have to interpret CSD j as representing the whole CSD industry, minus CSD i. The new entrant CSD would drive down the average costs for the whole settlement industry except CSD i, c_j. This dynamic effect of new entrance is thus pushing the inequality $c_j f_i > c_i$, which as we saw is necessary for collusion not to reshape among existing CSDs, towards the opposite inequality, that is, towards $c_j f_i \leq c_i$. As soon as this inequality is satisfied, it is Theorem 3, instead of Theorem 2, which applies, meaning CSD i will itself re-shape (whatever the other CSDs' plans). Hence, new entrants have a positive effect on the decision of reshaping by other, pre-existing, CSDs.

- third, because of the technical irreversibility of reshaping, the strategy supporting the continual delaying of reshaping can only be a "pure" trigger strategy, hence the resulting equilibrium is very fragile: should any player ever deviate from it, all players will reshape and the suboptimal situation is replaced by the optimal one, where costs per transactions are reduced both for CSDs, through a decrease of the costs in the model ($\widetilde{c_i}$ and $\widetilde{c_j}$), and for market participant, through a decrease of the prices (p_i and p_j), they are being charged.

Let us discuss further the condition under which tacit collusion is possible, but in terms of the various price-elasticities involved instead of in terms of the costs as in Remark 1 of Theorem 2. By definition, $f_i = f_i(\gamma_{ii}, \gamma_{ij}, \gamma_{ji}, \gamma_{jj}) = \frac{\gamma_{ij}\gamma_{jj}}{2\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}}$. Computing the first partial derivatives with respect to the relevant parameters, we see that f_i is decreasing in both parameters γ_{ii} and γ_{jj} while it is increasing in both cross-elasticities γ_{ij} and γ_{ji}. Hence, the higher the substitution / competition effect, i.e. the higher γ_{ij} and γ_{ji} compared to γ_{ii} and γ_{jj}, the more likely we end up with $c_i < f_i c_j$ (taking c_i and c_j as given here), and hence with the possibility that CSD j can influence CSD i strategy by refraining from reshaping but threatening to do so should CSD i reshape. If a similar condition holds for CSD i vis-a-vis CSD j, the result is a possible tacit collusion not to reshape. As noted in Remark 1 of Theorem 2, the threat for CSD j to reshape (and punish CSD i by competing at lower prices) is only credible if CSD j current costs are high (more precisely, if $c_j > \frac{c_i}{f_i}$). The deterrence comes precisely from the fact that should CSD j reshape, it will get costs similar to those of CSD i which will thus lose the competitive advantage it had benefited from before. Now, the elasticities γ_{ij} and γ_{ji} are precisely what allow this competitive advantage to be used. Hence, the higher the substitution effects, the more convincing CSD j threat to reshape and draw down its costs, and the more likely the collusion, should a similar condition hold for CSD i vis-a-vis CSD j. Conversely, the lower the substitution effects, the less likely the threat and influence of CSD j decision on CSD i. The extreme case would be completely independent and non-competing CSDs, i.e. $\gamma_{ij} = \gamma_{ji} = 0$, then the decisions to reshape or not by the CSDs are completely independent from one another and solely based on the respective costs.[19]

[19] Similarly to this paragraph, Annex 7.5 shows more formally that the set of parameters for which tacit collusion is sustainable in a high competition / substitution environment is larger than in a low competition / substitution environment.

Hence our model provides the result, counter-intuitive at first sight, that greater competition can actually hinder the reshaping process rather than favouring it. However, this result is based on the assumption that substitution and thus competition effects are constant over time. We expect that explicitly allowing for increasing substitution / competition effects over time, which is expected as a consequence of the introduction of T2S, the CSD regulation and further harmonisation initiatives, would reverse this result. We leave the explicit modelling of increasing competition over time to future research.

4 Introducing delayed observability of the reshaping decision

The previous theorems were derived in a setting where the decision to reshape of other CSDs is immediately observable by all the CSDs in the next step: this comes from our definition of the game in Section 3.1.2. For example, we proved that, in such a game, tacit collusion to delay infinitely the reshaping can be sustained in a subgame perfect Nash equilibrium, even though reshaping would be done straight away if the decision to reshape could only be taken at a single point in time like in the infinite version of the model of Section 2. Important to note is that to design the trigger strategy (S) which is essential to the proof of Theorem 2, CSDs need to be aware of other CSDs decision to reshape. But one could argue that such an information is internal to the reshaping CSD and should not be known to the market (except by interpreting possible price-signals).

Suppose we modify the rules of the game to allow a given CSD to become aware of the decision of other CSD to reshape only N_0 periods after that decision was taken. Then, a similar result holds in this game of imperfect information, which provides some evidence of the robustness of Theorem 2 when the decision to reshape or not become eventually known by the market, possibly after some lag.

Theorem 4 *Assume $\frac{1}{f_j} < \frac{c_i}{c_j} < f_i$ and that the game is modified such that any given CSD only becomes aware of the decision of other CSDs' decision to reshape after playing N_0 times the other stage-game. Then there exists $\overline{\delta} \in]0,1[$ such that for any discount factor $\delta \in]\overline{\delta},1[$, tacit collusion for no reshaping can be sustained in a subgame perfect Nash-equilibrium, even if we assume CSDs engage in price competition.*

The Proof is given in Annex 7.2.3.

Remark: In the proof, inequality $(**)$ becomes[20] $\pi_i(0,0) > (1-\delta^{N_0})\pi_i^{ab} + \delta^{N_0}\pi_i(b_i,1)$ when no adaptation costs are assumed ($\xi_i = \xi_j = 0$). This assumption is only made in this Remark to allow to solve $(**)$[21]:

$$\delta > \sqrt[N_0]{\frac{\pi_i^{ab} - \pi_i(0,0)}{\pi_i^{ab} - \pi_i(b_i,1)}} =: \overline{\delta}$$

Hence the higher N_0, i.e. the less observable the reshaping, the greater the value of $\overline{\delta}$ as obtained in the proof of the theorem. This *suggests*[22] that the less observable the reshaping, the more difficult it becomes to sustain tacit collusion.

We now provide a theorem similar to Theorem 3 in which the decision to reshape always occurred in a delayed observability setting.

Theorem 5 *Assume $c_i > f_i c_j$. Then for any high-enough discount-factor δ, CSD i will always completely reshape in any subgame perfect Nash equilibrium consistent with price competition. In particular, the other CSD j cannot deter CSD i from reshaping in any credible way.*

[20] Note in passing that $\pi_i(0,0) > (1-\delta^{N_0})\pi_i^{ab} + \delta^{N_0}\pi_i(b_i,1)$ is also a sufficient condition for $(**)$ to be true. Hence when $\delta > \overline{\delta}$ then $(**)$ is also true, whatever the adaptation costs, and the proof of the theorem applies.

[21] Indeed, there is no closed-form formula for solving general polynomials of degree more than 4 (Galois).

[22] This only *suggests* it, and does not *prove* it, since nothing indicates that the threshold obtained above is an optimal one (that is, the lowest one). There may exist a threshold independent of N_0 above which tacit collusion occurs in a subgame perfect Nash equilibrium (possibly the strategy profile (S_{N_0}) defined in the proof of Theorem 4, or another strategy profile).

Proof. First we prove that for a high-enough discount-factor δ, CSD i, if it ever chooses to reshape, will choose to reshape completely, that is, will choose $b_i = 1$.

By the proof of Theorem 4 we have that the profit of CSD i, should it choose to reshape by some degree b_i, would be:

$$\pi_i^{dev}(b_i) = \frac{1-\delta^{N_0}}{1-\delta}\pi_i^{ab} - \xi_i b_i^2 + \frac{\delta^{N_0}}{1-\delta}\pi_i(b_i, b_j^*(b_i))$$

Note that, trivially, $\pi_i(b_i, b_j^*(b_i))$ is maximum for $b_i = 1$. We have to prove that $\pi_i^{dev}(b_i)$ is also maximal for $b_i = 1$, knowing that π_i^{ab} is a function of b_i too. From the expression of $\pi_i^{dev}(b_i)$ above we have that for any $b_i \in [0,1]$:

$$(1-\delta)\pi_i^{dev}(b_i) - \delta^{N_0}\pi_i(b_i, b_j^*(b_i)) = (1-\delta^{N_0})\pi_i^{ab} - (1-\delta)\xi_i b_i^2$$

Since $\pi_i(b_i, b_j, p_i, p_j)$ is bounded, so is π_i^{ab}. Furthermore $\xi_i b_i^2$ is also bounded, since $\xi_i b_i^2 \in [0, \xi_i]$. Hence when δ tends to 1, the right-hand side of this equation tends to 0, which proves that whatever b_i, $(1-\delta)\pi_i^{dev}(b_i) - \delta^{N_0}\pi_i(b_i, b_j^*(b_i))$ tends to 0 when δ tends to 1. Clearly, $(1-\delta)\pi_i^{dev}(b_i)$ reaches its maximum as a function of b_i whenever $\pi_i^{dev}(b_i)$ do so. Assume now by contradiction that $\pi_i^{dev}(b_i)$ is maximum for a $\widehat{b_i} < 1$. Then by applying the previous result to both $b_i = 1$ and $b_i = \widehat{b_i}$ we get that both $(1-\delta)\pi_i^{dev}(1) - \delta^{N_0}\pi_i(1, b_j^*(1))$ and $(1-\delta)\pi_i^{dev}(\widehat{b_i}) - \delta^{N_0}\pi_i(\widehat{b_i}, b_j^*(\widehat{b_i}))$ tends to 0 when δ tends to 1, hence the sum of these two quantities, that is,

$$(1-\delta)(\pi_i^{dev}(1) - \pi_i^{dev}(\widehat{b_i})) - \delta^{N_0}(\pi_i(1, b_j^*(1)) - \pi_i(\widehat{b_i}, b_j^*(\widehat{b_i})))$$

also tends to 0 when δ tends to 1. But since $\pi_i(b_i, b_j^*(b_i))$ is maximum *only at* $b_i = 1$, and that $\widehat{b_i} < 1$, we have that $\pi_i(1, b_j^*(1)) - \pi_i(\widehat{b_i}, b_j^*(\widehat{b_i})) =: \eta > 1$, hence $\delta^{N_0}(\pi_i(1, b_j^*(1)) - \pi_i(\widehat{b_i}, b_j^*(\widehat{b_i})))$ tends to η when δ tends to 1. Hence

$$(1-\delta)(\pi_i^{dev}(1) - \pi_i^{dev}(\widehat{b_i}))$$

tends to $\eta > 0$ when δ tends to 1. In particular there exists a δ such that $(1-\delta)(\pi_i^{dev}(1) - \pi_i^{dev}(\widehat{b_i}))$ is close enough to η to be strictly superior to $\eta/2$ and hence strictly positive. Hence $\pi_i^{dev}(1) > \pi_i^{dev}(\widehat{b_i})$, a contradiction with the definition of $\widehat{b_i}$ and the assumption that $\widehat{b_i} < 1$. This proves that $\widehat{b_i} = 1$ and that $\pi_i^{dev}(b_i)$ is maximum for $b_i = 1$.

Note that it does not mean, *a priori*, that CSD i will indeed choose to reshape: it could, in principle, as in Theorem 4, be dissuaded to do so by the other CSD strategy.

The rest of the proof consists in showing this is never the case. Hence, the results will be that CSD i do choose to reshape, and that it chooses $i = 1$, that is, a complete reshaping, when it does so.

Now, the worst *credible* punishment CSD j could inflict on CSD i in order to induce it not to reshape is to reshape as soon as CSD i did by a degree $b_j^*(b_i)$ and engage in price competition afterwards, that is, play repeatedly the Nash equilibrium of the price-setting game. This would yield a payoff for CSD i of $\pi_i(1, b_j^*(b_i))$ instead of the $\pi_i(0,0)$ it received when it was not reshaping or of the $\pi_i(1,0)$ it received when it chose to reshape, in all subsequent price-setting stage game of the repeated game. Since $\pi_i(1, b_j^*(b_i)) \geq \pi_i(1,1)$, if we show that CSD i will still prefer to reshape if it earns, from the moment CSD j punishes it by also reshaping, $\pi_i(1,1)$ instead of $\pi_i(1, b_j^*(b_i))$, we will have proved that CSD i best action is indeed to reshape. But from the proof of Theorem 4, we know that whenever the inverse inequality of $(*)$ holds, CSD i is better off not playing the collusion strategy and reshaping, even though CSD j will punish it by reshaping itself and engaging in harder price competition afterwards. Hence we only need to prove that under our assumptions (that is, that $c_i > f_i c_j$), the inverse inequality of $(*)$ holds.

$c_i > f_i c_j$ implies successively:

$$\begin{array}{rcl}
D_i c_i - B_i c_j & > & 0 \\
\pi_i(0,0) & < & \pi_i(1,1) \\
\pi_i^{ab} - \pi_i(0,0) & > & \pi_i^{ab} - \pi_i(1,1) \\
\delta & < & 1 < \sqrt[N_0]{\dfrac{\pi_i^{ab} - \pi_i(0,0)}{\pi_i^{ab} - \pi_i(1,1)}} \\
\delta^{N_0} & < & \dfrac{\pi_i^{ab} - \pi_i(0,0)}{\pi_i^{ab} - \pi_i(1,1)} \\
\pi_i(0,0) & < & (1-\delta^{N_0})\pi_i^{ab} + \delta^{N_0}\pi_i(1,1)
\end{array}$$

for all discount factor δ. Now since $(1-\delta)\xi_i$ tends to 0 when δ tends to 1, we have that, for δ high enough:

$$\pi_i(0,0) < (1-\delta^{N_0})\pi_i^{ab} + \delta^{N_0}\pi_i(1,1) - (1-\delta)\xi_i$$

and this is precisely the inverse inequality of $(*)$.□

We assumed in this section that the reshaping decision is not observable for N_0 periods, but that it became public knowledge after. Hence, one could argue that working under the assumption of *complete* non-observability of the reshaping decision, where no other CSD than the reshaping CSD itself know about its decision to reshape, would have a different effect on the possibility for CSDs not to reshape. However, there are strong limits to the extent that CSDs can hide information on a crucial strategic decision such as their adaptation to T2S. For example, most CSDs are either user-owned and thus largely transparent to their users or publicly listed companies with certain transparency requirements. Furthermore, the CSD adaptation to T2S may require some adaptation on the side of the CSDs' clients, so CSDs usually involve their clients in major changes to their IT infrastructure. Hence, their decision to reshape or not is likely to be known to the market, possibly after some finite lag N_0.

5 Analysing the question to join or not with the model

In the previous section we have been concerned exclusively with the modelling of the reshaping decision for the CSDs which had chosen to join in the first place. Yet, what about also modelling the decision of joining or not joining T2S?

We can adjust the previous model to address this question by changing the cost per transaction $\widetilde{c_i}$: instead of $\widetilde{c_i} = (1-b_i)c_i + c_{T2S}$ we change the interpretation of parameter $b_i \in [0,1]$ and set $\widetilde{c_i} = (1-b_i)c_i + b_i c_{T2S}$. Hence, $b_i = 0$ corresponds to the decision not to join (and thus not to reshape), yielding costs of $\widetilde{c_i} = c_i$. Any $b_i > 0$ corresponds to the decision to join T2S and to reshape by some degree b_i.The extreme case, $b_i = 1$, represents the decision to join and to *completely* reshape, yielding $\widetilde{c_i} = c_{T2S}$ as before. Hence, the table which gives the relevant cost structures for the price-setting game of each stage changes to:

period	fixed costs $\widetilde{C}_{i,fixed}$ of CSD i	cost per transaction $\widetilde{c_i}$ of CSD i
1	$(1-a_i)C_{i,fixed} + C_{i,adapt}(a_i,b_i)$	$(1-b_i)c_i + b_i c_{T2S}$
2	$(1-a_i)C_{i,fixed}$	$(1-b_i)c_i + b_i c_{T2S}$
...	...	...
N	$(1-a_i)C_{i,fixed}$	$(1-b_i)c_i + b_i c_{T2S}$

That is, the more b_i-reshaping, the more the costs per transaction converge towards the T2S fees. This is because $(1-b_i)c_i + b_i c_{T2S}$ is nothing else that a convex combination of the two types of costs, the CSD i costs c_i and the T2S costs c_{T2S}.

Now, in case $c_i < c_{T2S}$, we obtain the fact that once a CSD has chosen to join T2S, the more it reshapes, the more its costs per transaction increase. But this has no consequences for the equilibrium solutions, since if a CSD has (average) costs per transaction below the T2S fees, then in our model it will choose $b_i = 0$ and thus not join T2S.[23] Abstracting from this corner solution, we will in everything that concerns the modelling of joining or not joining always assume that

$$c_i > c_{T2S}.$$

Note that we can apply our previous model and obtain similar results by just substituting every occurrence of c_i by $c_i - c_{T2S}$ (and, of course, c_j by $c_j - c_{T2S}$) since $(1-b_i)c_i + b_i c_{T2S} = (1-b_i)(c_i - c_{T2S}) + c_{T2S}$. In particular, we get theorems similar to Theorem 2 and 3 of Section 3, but concerning the decision to join or not to join T2S:

Theorem 6 *Assume $\frac{1}{f_j} < \frac{c_i - c_{T2S}}{c_j - c_{T2S}} < f_i$. Then there exists $\overline{\delta} \in]0,1[$ such that for any discount factor $\delta \in]\overline{\delta}, 1[$, collusion for not joining T2S can be sustained in a subgame perfect Nash-equilibrium, even if price competition is assumed.*

That is, there exists a subgame-perfect Nash equilibrium in which no CSD ever join T2S when playing it. The corresponding subgame-perfect Nash equilibrium is of course the following trigger strategy (S'):

(S') *Do not join if no player has joined so far. If another player has joined and yourself has not joined, then join. At each price-setting game (step 2) play the unique Nash equilibrium, given all the costs involved.*

Theorem 7 *Assume $c_i \geq f_i(c_j - c_{T2S}) + c_{T2S}$. Then for any $\delta \geq 1 - \frac{c_i^2 D_i^2}{\xi_i}$, CSD i will always join T2S in any subgame perfect Nash equilibrium consistent with price competition. In particular, the other CSD j cannot deter CSD i from joining T2S in any credible way.*

Hence, translating a parameter of the reshaping model allows to study the decision to join or not. The model focuses on the possible transaction cost savings with T2S. There are other benefits of T2S for the CSDs and their clients that are outside the scope of this model (see, e.g., Eurosystem [6]). They can make participation in T2S beneficial independent of the direct settlement costs.

6 Conclusion

In this article we used a game theoretic approach to model the strategic decision of European CSDs to reshape towards T2S, in both a finite and in an infinite setting, and noted that a similar model and discussion can also apply for the CSD's decision to join or not to join T2S.

In the finite setting, we obtained an explicit formula for the degree of optimal reshaping as well as for the optimal prices set at each period. Trying to recover the fixed adaptation costs by increasing the price per transaction results in decreasing profits for CSDs compared with the optimal solution.

In the infinite setting we provided a condition for the model parameters under which two CSDs, which would normally reshape in the previous setting, delay their reshaping infinitely many times because they are hence able to avoid paying adaptation costs while the incentives to reshape (cost-reduction and earning market shares) are reduced by this tacit collusion. This result is not robust though, in particular with the possibility of new entrants on the market who may push average costs lower and force the pre-existing CSDs to reshape. Indeed, we provide a condition under which a given CSD will always reshape, no matter whether the other CSDs are reshaping or not. Decreasing average costs for the whole market push towards

[23] When the individual CSD's costs are below the T2S fees, the CSD will not join T2S in this second model because joining and reshaping could only result in higher costs. Since there may be non cost-based incentives for a given, cost-efficient CSD to join, the second model does not capture properly the decision about reshaping for all CSDs. Hence, we used the first model, which starts from the subset of CSD having joined T2S (no matter the reasons), as a more appropriate way to express the degree of optimal reshaping.

this condition, and away from tacit collusion. Furthermore, this result is robust with respect to a delayed observability of the reshaping decision.

In particular the results in the infinite horizon model provide interesting insights about an optimal, non-reversible investment decision. These results are applicable in a wider Industrial Organisation context and not limited to the reshaping decision of CSDs.

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7 Annex

7.1 Computations for the dynamic N-period model for the degree of optimal reshaping

7.1.1 Expression of the prices at equilibrium

Profits for CSD i in a period where the fixed costs are $\widetilde{C}_{i,fixed}$ and the costs per transaction are $\widetilde{c_i}$ are:

$$\begin{aligned}\pi_i &= q_i(p_i - \widetilde{c_i}) - \widetilde{C}_{i,fixed} \\ &= (\alpha_i - \gamma_{ii}p_i + \gamma_{ij}p_j)(p_i - \widetilde{c_i}) - \widetilde{C}_{i,fixed}\end{aligned}$$

Now

$$\begin{aligned}\frac{\partial \pi_i}{\partial p_i} &= (\alpha_i - \gamma_{ii}p_i + \gamma_{ij}p_j) + (-\gamma_{ii})(p_i - \widetilde{c_i}) \\ &= -2\gamma_{ii}p_i + \alpha_i + \gamma_{ij}p_j + \gamma_{ii}\widetilde{c_i}\end{aligned}$$

Hence $\frac{\partial \pi_i}{\partial p_i} > 0$ if, and only if, $p_i < \frac{1}{2\gamma_{ii}}(\alpha_i + \gamma_{ij}p_j + \gamma_{ii}\widetilde{c_i})$. This proves that the best-response $p_i^*(p_j)$ of CSD i to a price p_j from CSD j is:

$$p_i^*(p_j) = \frac{1}{2\gamma_{ii}}(\alpha_i + \gamma_{ii}\widetilde{c_i} + \gamma_{ij}p_j)$$

Denoting by (p_i^*, p_i^*) the equilibrium prices of the stage-game G, we have:

$$\begin{cases} p_i^* = p_i^*(p_j) = \frac{1}{2\gamma_{ii}}(\alpha_i + \gamma_{ii}\widetilde{c_i} + \gamma_{ij}p_j) \\ p_j^* = p_j^*(p_i) = \frac{1}{2\gamma_{jj}}(\alpha_j + \gamma_{jj}\widetilde{c_j} + \gamma_{ji}p_i) \end{cases}$$

Hence

$$p_i^* = \frac{1}{2\gamma_{ii}}(\alpha_i + \gamma_{ii}\widetilde{c_i} + \gamma_{ij}(\frac{1}{2\gamma_{jj}}(\alpha_j + \gamma_{jj}\widetilde{c_j} + \gamma_{ji}p_i)))$$

$$4\gamma_{ii}\gamma_{jj}p_i^* = 2\gamma_{jj}\alpha_i + 2\gamma_{jj}\gamma_{ii}\widetilde{c_i} + \gamma_{ij}(\alpha_j + \gamma_{jj}\widetilde{c_j} + \gamma_{ji}p_j)$$

$$(4\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji})p_i^* = 2\gamma_{jj}\alpha_i + 2\gamma_{jj}\gamma_{ii}\widetilde{c_i} + \gamma_{ij}\alpha_j + \gamma_{ij}\gamma_{jj}\widetilde{c_j}$$

$$p_i^* = \frac{1}{4\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}}(2\gamma_{jj}\alpha_i + 2\gamma_{ii}\gamma_{jj}\widetilde{c_i} + \gamma_{ij}\alpha_j + \gamma_{ij}\gamma_{jj}\widetilde{c_j})$$

Similarly,

$$p_j^* = \frac{1}{4\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}}(2\gamma_{ii}\alpha_j + 2\gamma_{ii}\gamma_{jj}\widetilde{c_j} + \gamma_{ji}\alpha_i + \gamma_{ji}\gamma_{ii}\widetilde{c_i})$$

These formulas define the equilibrium prices only when they are superior to the costs involved. Indeed, if $p_i^* < \widetilde{c_i}$ then CSD i optimal response is not to settle any transactions. We will always assume, in what follows, that this is not the case, i.e. that it is always more profitable for a CSD to settle a transaction than to refuse to settle it. This allows to get rid of (unrealistic) corner solutions. Hence, we will always assume $p_i^* \geq \widetilde{c_i}$ and $p_j^* \geq \widetilde{c_j}$ for any costs involved where $p_i^* = p_i^*(\widetilde{c_i}, \widetilde{c_j})$ and $p_j^* = p_j^*(\widetilde{c_i}, \widetilde{c_j})$ are the expressions, dependent on the cost level $(\widetilde{c_i}, \widetilde{c_j})$, given above. This assumption is thus a joint set of assumptions on the parameter of the model and will be used in other part of the article.

7.1.2 Expression of the profits at equilibrium

Replacing in the expression of the profits p_i by $p_i^*(p_j)$ gives:

$$\begin{aligned}
\pi_i(p_i^*(p_j),p_j) &= (\alpha_i - \gamma_{ii}p_i + \gamma_{ij}p_j)(p_i - \widetilde{c}_i) - \widetilde{C}_{i,fixed} \\
&= (\alpha_i - \frac{1}{2}(\alpha_i + \gamma_{ii}\widetilde{c}_i + \gamma_{ij}p_j) + \gamma_{ij}p_j)(\frac{1}{2\gamma_{ii}}(\alpha_i + \gamma_{ii}\widetilde{c}_i + \gamma_{ij}p_j) - \widetilde{c}_i) - \widetilde{C}_{i,fixed} \\
&= \frac{1}{\gamma_{ii}}\left(\frac{1}{2}(\alpha_i - \gamma_{ii}\widetilde{c}_i + \gamma_{ij}p_j)\right)^2 - \widetilde{C}_{i,fixed}
\end{aligned}$$

Then replacing p_j by its expression at equilibrium yields:

$$\begin{aligned}
\pi_i &= \frac{1}{4\gamma_{ii}}\left(\alpha_i - \gamma_{ii}\widetilde{c}_i + \gamma_{ij}p_j\right)^2 - \widetilde{C}_{i,fixed} \\
&= \frac{1}{4\gamma_{ii}}\left(\alpha_i - \gamma_{ii}\widetilde{c}_i + \frac{\gamma_{ij}}{4\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}}(2\gamma_{ii}\alpha_j + 2\gamma_{ii}\gamma_{jj}\widetilde{c}_j + \gamma_{ji}\alpha_i + \gamma_{ji}\gamma_{ii}\widetilde{c}_i)\right)^2 - \widetilde{C}_{i,fixed} \\
&= \frac{1}{4\gamma_{ii}}(\alpha_i + \frac{\gamma_{ij}}{4\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}}(2\gamma_{ii}\alpha_j + \gamma_{ji}\alpha_i) \\
&\quad +(\frac{2\gamma_{ii}\gamma_{ij}\gamma_{jj}}{4\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}} - \gamma_{ii})\widetilde{c}_j + \frac{\gamma_{ij}\gamma_{ji}\gamma_{ii}}{4\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}}\widetilde{c}_i)^2 - \widetilde{C}_{i,fixed} \\
&= (\frac{1}{2\sqrt{\gamma_{ii}}}(\alpha_i + \frac{\gamma_{ij}}{4\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}}(2\gamma_{ii}\alpha_j + \gamma_{ji}\alpha_i)) \\
&\quad + \frac{\sqrt{\gamma_{ii}}\gamma_{ij}\gamma_{jj}}{4\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}}\widetilde{c}_j + \frac{1}{2\sqrt{\gamma_{ii}}}(\frac{\gamma_{ii}\gamma_{ij}\gamma_{ji}}{4\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}} - \gamma_{ii})\widetilde{c}_i)^2 - \widetilde{C}_{i,fixed}
\end{aligned}$$

Now

$$\begin{aligned}
\frac{1}{2\sqrt{\gamma_{ii}}}(\frac{\gamma_{ii}\gamma_{ij}\gamma_{ji}}{4\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}} - \gamma_{ii}) &= \frac{\sqrt{\gamma_{ii}}}{2}(\frac{\gamma_{ij}\gamma_{ji} - 4\gamma_{ii}\gamma_{jj} + \gamma_{ij}\gamma_{ji}}{4\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}}) \\
&= \sqrt{\gamma_{ii}}(\frac{\gamma_{ij}\gamma_{ji} - 2\gamma_{ii}\gamma_{jj}}{4\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}}) \\
&= -\sqrt{\gamma_{ii}}(\frac{2\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}}{4\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}})
\end{aligned}$$

Let

$$\begin{aligned}
A_i &= \frac{1}{2\sqrt{\gamma_{ii}}}(\alpha_i + \frac{\gamma_{ij}}{4\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}}(2\gamma_{ii}\alpha_j + \gamma_{ji}\alpha_i)) \\
B_i &= \frac{\sqrt{\gamma_{ii}}\gamma_{ij}\gamma_{jj}}{4\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}} \\
D_i &= \sqrt{\gamma_{ii}}\frac{2\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}}{4\gamma_{ii}\gamma_{jj} - \gamma_{ij}\gamma_{ji}}
\end{aligned}$$

Now profits can be written in a very compact way as:

$$\pi_i = (A_i + B_i\widetilde{c}_j - D_i\widetilde{c}_i)^2 - \widetilde{C}_{i,fixed}$$

Note in passing that $A_i + B_i\widetilde{c}_j - D_i\widetilde{c}_i$ is positive for any costs $(\widetilde{c}_i, \widetilde{c}_j)$ involved, because by the above $A_i + B_i\widetilde{c}_j - D_i\widetilde{c}_i = p_i - \widetilde{c}_i = \frac{1}{\gamma_{ii}}q_i \geq 0$ by assumption (CSA). This fact will be used later.

Total profits, as defined in Section 2.4.3, are thus:

$$\begin{aligned}
\pi_i^{tot} &= (1 + \delta + ... + \delta^{N-1})\{(A_i + ((1-b_j)c_j + c_{T2S})B_i - ((1-b_i)c_i + c_{T2S})D_i)^2 - (1-a_i)C_{i,fixed}\} \\
&\quad - C_{i,adapt}(a_i, b_i) \\
&= \widetilde{\delta}(A_i + ((1-b_j)c_j + c_{T2S})B_i - ((1-b_i)c_i + c_{T2S})D_i)^2 - \widetilde{\delta}(1-a_i)C_{i,fixed} - C_{i,adapt}(a_i, b_i)
\end{aligned}$$

with

$$\widetilde{\delta} = \begin{cases} N & \text{if } \delta = 1 \\ \frac{1-\delta^N}{1-\delta} & \text{if } 0 \leq \delta < 1 \end{cases}$$

7.1.3 Expression of the degree of reshaping chosen and Proof of Lemma 1 and Theorem 1

Now assuming in the previous expression of π_i^{tot} that $C_{i,adapt}(a_i, b_i) = \xi_i b_i^2$ and $C_{i,fixed} = 0$ for simplicity, we can compute the derivative of π_i^{tot} with respect to the degree of reshaping b_i:

$$\frac{\partial \pi_i^{tot}}{\partial b_i} = 2\widetilde{\delta} c_i D_i (A_i + ((1-b_j)c_j + c_{T2S})B_i - ((1-b_i)c_i + c_{T2S})D_i) - 2\xi_i b_i$$

Hence $\frac{\partial \pi_i^{tot}}{\partial b_i} > 0$ is equivalent to:

$$c_i D_i (A_i + ((1-b_j)c_j + c_{T2S})B_i - (c_i + c_{T2S}))D_i) > \frac{\xi_i - \widetilde{\delta} c_i^2 D_i^2}{\widetilde{\delta}} b_i$$

We can now prove Lemma 1, which is re-state here for convenience for the reader:

Lemma 1 *Assume $\xi_i < \widetilde{\delta} c_i^2 D_i^2$. Then if all CSDs are playing the Nash equilibrium of each following stage-game of the repeated game, the best-response function $b_i^*(b_j)$ is the constant function $b_i^*(b_j) = 1$, which represents a throughout reshaping decision from CSD i whatever the degree b_j CSD j choose to reshape.*

Proof of Lemma 1:

If $\xi_i < \widetilde{\delta} c_i^2 D_i^2$ then the sign of $\frac{\partial \pi_i^{tot}}{\partial b_i}$ indicate that π_i^{tot} is first decreasing, then increasing, hence its maximum is reached either at $b_i = 0$ or at $b_i = 1$ (and possibly at both). But

$$\pi_i^{tot}(0, b_j) < \pi_i^{tot}(1, b_j)$$

is successively equivalent to

$$\begin{aligned} \widetilde{\delta}(A_i + ((1-b_j)c_j + c_{T2S})B_i - (c_i + c_{T2S})D_i)^2 &< \widetilde{\delta}(A_i + ((1-b_j)c_j + c_{T2S})B_i - c_{T2S}D_i)^2 - \xi_i \\ \widetilde{\delta}(A_i + ((1-b_j)c_j + c_{T2S})B_i - (c_i + c_{T2S})D_i)^2 + \xi_i &< \widetilde{\delta}(A_i + ((1-b_j)c_j + c_{T2S})B_i - c_{T2S}D_i)^2 \end{aligned}$$

But the last line is true since we can write:

$$\begin{aligned} & \widetilde{\delta}(A_i + ((1-b_j)c_j + c_{T2S})B_i - (c_i + c_{T2S})D_i)^2 + \xi_i \\ \leq\ & \widetilde{\delta}(A_i + ((1-b_j)c_j + c_{T2S})B_i - (c_i + c_{T2S})D_i)^2 + \widetilde{\delta} c_i^2 D_i^2) \\ \leq\ & \widetilde{\delta}\{(A_i + ((1-b_j)c_j + c_{T2S})B_i - (c_i + c_{T2S})D_i)^2 + (c_i D_i)^2\} \\ \leq\ & \widetilde{\delta}\{(A_i + ((1-b_j)c_j + c_{T2S})B_i - (c_i + c_{T2S})D_i + (c_i D_i))^2 \\ =\ & \widetilde{\delta}\{(A_i + ((1-b_j)c_j + c_{T2S})B_i - c_{T2S}D_i)^2 \end{aligned}$$

The last inequality above was obtained using $a^2 + b^2 \leq (a+b)^2$ for any a, b such that $ab \geq 0$. Here $a = A_i + ((1-b_j)c_j + c_{T2S})B_i - (c_i + c_{T2S})D_i \geq 0$ (since this is equal to a positive factor times the quantity at equilibrium of the stage game) and $b = c_i D_i \geq 0$. Note that this result is independent of the level of reshaping b_j chosen by the other CSD j. This concludes the proof of Lemma 1. □

Assume now that $\xi_i > \widetilde{\delta} c_i^2 D_i^2$. Then the maximum $b_i^{**}(b_j)$ of $\pi_i^{tot}(b_j)$ where b_i is not restricted to $[0, 1]$ is:

$$b_i^{**}(b_j) = \frac{\widetilde{\delta} c_i D_i}{\xi_i - \widetilde{\delta} c_i^2 D_i^2} (A_i + ((1-b_j)c_j + c_{T2S})B_i - (c_i + c_{T2S}))D_i)$$

Note $b_i^{**}(b_j)$ can be greater than 1 or smaller than 0. Letting $\psi_i = \frac{\widetilde{\delta} c_i D_i}{\xi_i - \widetilde{\delta} c_i^2 D_i^2}$, we have

$$\begin{aligned} b_i^{**}(b_j) &= \psi_i(A_i + ((1-b_j)c_j + c_{T2S})B_i - (c_i + c_{T2S}))D_i) \\ &= \psi_i(A_i + (c_j + c_{T2S})B_i - (c_i + c_{T2S})D_i) - \psi_i c_j B_i b_j \end{aligned}$$

Because $\xi_i > \widetilde{\delta} c_i^2 D_i^2$, π_i^{tot} is first increasing, then decreasing, and reaches its (unrestricted) maximum in $b_i^{**}(b_j)$. It is then easy to see that the best-response function $b_i^*(b_j)$ is 1 if $b_i^{**}(b_j) > 1$ and 0 if $b_i^{**}(b_j) < 0$, whereas it is $b_i^{**}(b_j)$ when $b_i^{**}(b_j) \in [0,1]$. This can be summarized by writing:

$$b_i^*(b_j) = \min(1, \max(0, b_i^{**}(b_j)))$$

that is,

$$b_i^*(b_j) = \min(1, \max(0, \psi_i(A_i + (c_j + c_{T2S})B_i - (c_i + c_{T2S})D_i) - \psi_i c_j B_i b_j))$$

We can now proceed to prove Theorem 1.

Proof of Theorem 1:

We will use Lemma 1 extensively to prove part (ii), (iii) and (iv) of the theorem.

(ii) Assume $\xi_i < \widetilde{\delta} c_i^2 D_i^2$ and $\xi_j < \widetilde{\delta} c_j^2 D_j^2$. Then by Lemma 1 applied to both CSD i and j we have $b_i^*(b_j) = 1$ for any b_j and $b_j^*(b_i) = 1$ for any b_i. Hence at equilibrium $b_i^* = 1 = b_j^*$.

(iii) Assume $\xi_i > \widetilde{\delta} c_i^2 D_i^2$ and $\xi_j < \widetilde{\delta} c_j^2 D_j^2$. Lemma 1 applied to CSD j gives $b_j^*(b_i) = 1$ for any b_i, and thus at equilibrium CSD i plays its best-response (as previously computed above) $b_i^*(b_j) = \min(1, \max(0, \psi_i(A_i + (c_j + c_{T2S})B_i - (c_i + c_{T2S}))D_i)))$.

(iv) is similar to (iii) interchanging the roles of i and j. Indeed, if we assume $\xi_i < \widetilde{\delta} c_i^2 D_i^2$ and $\xi_j > \widetilde{\delta} c_j^2 D_j^2$. Lemma 1 applied to CSD i gives $b_i^*(b_j) = 1$ for any b_j, and thus at equilibrium $b_j^*(b_i) = \min(1, \max(0, \psi_i(A_i + (c_j + c_{T2S})B_i - (c_i + c_{T2S}))D_i)))$.

We are thus only left with the task of proving point (i) of Theorem 1.

Assume $\xi_i > \widetilde{\delta} c_i^2 D_i^2$ and $\xi_j > \widetilde{\delta} c_j^2 D_j^2$. Consider the solution (b_i^{**}, b_j^{**}) obtained from solving:

$$\begin{cases} b_i^{**} = b_i^{**}(b_j^{**}) \\ b_j^{**} = b_j^{**}(b_i^{**}) \end{cases}$$

That is, we solve for a Nash equilibrium the unrestricted game. The system is equivalent to

$$\begin{cases} b_i^{**} = \psi_i(A_i + (c_j + c_{T2S})B_i - (c_i + c_{T2S})D_i) - \psi_i c_j B_i b_j^{**} \\ b_j^{**} = \psi_j(A_j + (c_i + c_{T2S})B_j - (c_j + c_{T2S})D_j) - \psi_j c_i B_j b_i^{**} \end{cases}$$

Substituting b_j^* by its expression in function of b_i^* (given by the second equation) in the first equation of the system gives:

$$\begin{aligned} b_i^{**} &= \psi_i(A_i + (c_j + c_{T2S})B_i - (c_i + c_{T2S})D_i) - \psi_i c_j B_i(\psi_j(A_j + (c_i + c_{T2S})B_j - (c_j + c_{T2S})D_j) - \psi_j c_i B_j b_i) \\ b_i^{**} &= \frac{\psi_i(A_i + (c_j + c_{T2S})B_i - (c_i + c_{T2S})D_i) - \psi_i \psi_j c_j B_i(A_j + (c_i + c_{T2S})B_j - (c_j + c_{T2S})D_j)}{1 - \psi_i \psi_j c_i c_j B_i B_j} \end{aligned}$$

This is the optimal, unrestricted, degree b_i^{**} of reshaping chosen by CSD i at equilibrium, assuming $\psi_i \psi_j c_i c_j B_i B_j \neq 1$[24]. A similar formula, interchanging the role of i and j, holds for b_j^{**}.

Now, whenever b_i^{**} and b_j^{**} both belong to $[0,1]$, the solution (b_i^*, b_j^*) of the *restricted* game, that is, where b_i^* and b_j^* are constraint to belong to $[0,1]$, is just the solution of the unrestricted game. That is,

[24] There is, of course, a graphical interpretation to the condition $1 \neq \psi_i \psi_j c_i c_j B_i B_j$. Indeed, if $1 = \psi_i \psi_j c_i c_j B_i B_j$ then the two best-response functions $b_i^{**}(b_j)$ and $b_j^{**}(b_i)$ are parallel when drawn in the (b_i, b_j)-plane, hence they either intercept in all their points or in none. A discussion on what happens in this precise case is carried out in Appendix 7.1.4. For the rest of the article we always assume $1 \neq \psi_i \psi_j c_i c_j B_i B_j$.

$(b_i^*, b_j^*) = (b_i^{**}, b_j^{**})$. This intuitive result can be proved more formally: by definition, (b_i^{**}, b_j^{**}) solves the system

$$\begin{cases} b_i^{**} = b_i^{**}(b_j^{**}) \\ b_j^{**} = b_j^{**}(b_i^{**}) \end{cases}$$

Now, because $b_i^{**}(b_j^{**}) = b_i^{**}$ belongs to $[0,1]$, $b_i^*(b_j^{**}) = \min(1, \max(0, b_i^{**}(b_j^{**}))) = b_i^{**}(b_j^{**}) = b_i^{**}$. Similarly, $b_j^* = b_j^{**}$.

This proves Theorem 1 as such. We know move to provide a complete resolution (including corner-solutions) of the Nash equilibrium of game.

We have proved previously that

$$b_i^{**}(b_j) = \psi_i(A_i + (c_j + c_{T2S})B_i - (c_i + c_{T2S})D_i) - \psi_i c_j B_i b_j$$

For convenience, let

$$\alpha = \psi_i(A_i + (c_j + c_{T2S})B_i - (c_i + c_{T2S})D_i)$$

and

$$\beta = \psi_i c_j B_i$$

Hence

$$b_i^{**}(b_j) = \alpha - \beta b_j$$

Let us look at the signs of the quantities involved. Since $\xi_i > \widetilde{\delta} c_i^2 D_i^2$, we have that $\psi_i = \frac{\widetilde{\delta} c_i D_i}{\xi_i - \widetilde{\delta} c_i^2 D_i^2} > 0$, and by the CSA applied with the costs $(c_i + c_{T2S}, c_j + c_{T2S})$, we have $A_i + (c_j + c_{T2S})B_i - (c_i + c_{T2S})D_i > 0$. Hence, $\alpha > 0$. Similarly, because $\psi_i > 0$ and that the CSA applied with the costs $(c_i + c_{T2S}, c_{T2S})$ gives $A_i + c_{T2S}B_i - (c_i + c_{T2S}) > 0$, we get that $b_i^{**}(1) = \alpha - \beta > 0$.

Similarly, define $\widetilde{\alpha}$ and $\widetilde{\beta}$ such that

$$b_j^{**}(b_i) = \widetilde{\alpha} - \widetilde{\beta} b_i$$

for all b_i. We also have, through the CSA, that $\widetilde{\alpha} > 0$ and $\widetilde{\alpha} - \widetilde{\beta} > 0$.

We can now proceed to a disjunction aimed at capturing all the different types of corner solutions that could occur.

First assume $\alpha < 1$ and $\widetilde{\alpha} < 1$. In that case $b_i^*(b_j) = b_i^{**}(b_j)$ for all $b_j \in [0,1]$ and similarly $b_j^*(b_i) = b_j^{**}(b_i)$. Hence both best-response functions are straight-lines on their whole domain of definition (which is $[0,1]$), and since $b_j^*(b_i)$ is below $b_i^*(b_j)$ at $b_i = \alpha - \beta$ while it is above at $b_i = \alpha$, the two segments intersect in some point in the interior of $[0,1]^2$. Because the unrestricted best-responses $b_i^{**}(b_j)$, $b_j^{**}(b_i)$ coincide with $b_i^*(b_j)$, $b_j^*(b_i)$ this point is precisely (b_i^{**}, b_j^{**}). This proves that the Nash equilibrium, in that case, is unique, and is (b_i^{**}, b_j^{**}).

Assume $\alpha < 1$ and $\widetilde{\alpha} > 1$. Consider the intersection of the two best-response functions with the straight line D of equation $b_i = \alpha - \beta$. Because $b_i^*(1) = \alpha - \beta$, D intersects $b_i^*(b_j)$ at the point of coordinate $(\alpha - \beta, 1)$. Assume $b_j^{**}(\alpha - \beta) \geq 1$. Then $b_i^{**}(b_j)$ and $b_j^{**}(b_i)$ do not intersect in the interior of $[0,1]^2$ and $b_j^*(\alpha - \beta) = 1$, implying that the unique intersection point of the two best-response function is precisely $(\alpha - \beta, 1)$, that is, $(b_i^{**}(1), 1)$. Hence, $(b_i^*, b_j^*) = (b_i^{**}(1), 1)$ is the unique NE. Assume now $b_j^{**}(\alpha - \beta) < 1$. Then $b_j^*(b_i)$ is below $b_i^*(b_j)$ at $b_i = \alpha - \beta$ while it is above at $b_i = \alpha$, and as in the case where $\alpha < 1$ and $\widetilde{\alpha} < 1$ we conclude that (b_i^{**}, b_j^{**}) is the unique NE.

Assume $\alpha > 1$ and $\widetilde{\alpha} < 1$. By the same reasoning as above, interchanging the role of i and j, we get that if $b_i^{**}(\widetilde{\alpha} - \widetilde{\beta}) \geq 1$, then the unique NE is $(b_i^*, b_j^*) = (1, b_j^{**}(1))$ whereas if $b_i^{**}(\widetilde{\alpha} - \widetilde{\beta}) < 1$, it is $(b_i^*, b_j^*) = (b_i^{**}, b_j^{**})$.

The last case is when $\alpha > 1$ and $\widetilde{\alpha} > 1$. To tackle it we need further disjunctions.

First assume $\widetilde{\alpha} - \widetilde{\beta} > 1$ and $\alpha - \beta > 1$. This implies that $b_i^{**}(b_j) > 1$ and $b_j^{**}(b_i) > 1$ for all b_j in $[0,1]$ and all b_i in $[0,1]$. Hence the two best-response functions are $b_i^{**}(b_j) = 1$ and $b_j^{**}(b_i) = 1$ for all b_j in $[0,1]$ and all b_i in $[0,1]$. They intersect in the unique NE, at $(b_i^*, b_j^*) = (1, 1)$.

Second assume $\widetilde{\alpha} - \widetilde{\beta} > 1$ and $\alpha - \beta < 1$. This implies that $b_i^{**}(b_j) > 1$ for all b_j in $[0,1]$ and thus $b_i^{**}(b_j) = 1$ for all b_j in $[0,1]$. This best-response thus intersect $b_i^{**}(b_j)$ at a unique NE, which is $(b_i^*, b_j^*) = (b_i^{**}(1), 1)$.

The third case, in which $\widetilde{\alpha} - \widetilde{\beta} < 1$ and $\alpha - \beta > 1$, is similar to the second case, interchanging i and j. It thus yields a unique NE at $(b_i^*, b_j^*) = (1, b_j^{**}(1))$.

The last case is when $\widetilde{\alpha} - \widetilde{\beta} < 1$ and $\alpha - \beta < 1$. Let A be the point of coordinate $(x_A, 1)$ such that $b_j^*(x_A + \varepsilon) < 1$ for any $\varepsilon > 0$ whereas $b_j^*(x_A) = 1$. Similarly, let B be the point of coordinate $(1, y_B)$ the point such that $b_i^*(y_B + \varepsilon) < 1$ for any $\varepsilon > 0$ whereas $b_i^*(y_B) = 1$. Because x_A solves $1 = b_j^{**}(x_A)$ and y_B solves $1 = b_i^{**}(y_B)$, we easily get $x_A = \frac{\widetilde{\alpha}-1}{\widetilde{\beta}}$ and $y_B = \frac{\alpha-1}{\beta}$. It is then convenient to carry out the discussion depending on the relative value of x_A compared to $\alpha - \beta$ and of y_B compared to $\widetilde{\alpha} - \widetilde{\beta}$.

If $\widetilde{\alpha} - \widetilde{\beta} > y_B$ and $\alpha - \beta < x_A$, then there is a unique NE at $(b_i^*, b_j^*) = (b_i^{**}(1), 1)$.

If $\widetilde{\alpha} - \widetilde{\beta} > y_B$ and $\alpha - \beta > x_A$, then there is a unique NE at $(b_i^*, b_j^*) = (b_i^{**}, b_j^{**})$.

If $\widetilde{\alpha} - \widetilde{\beta} < y_B$ and $\alpha - \beta > x_A$, then there is a unique NE at $(b_i^*, b_j^*) = (1, b_j^{**}(1))$.

If $\widetilde{\alpha} - \widetilde{\beta} < y_B$ and $\alpha - \beta < x_A$, then each of the previous points, that is, $(b_i^{**}(1), 1)$, (b_i^{**}, b_j^{**}) and $(1, b_j^{**}(1))$, are NE, and these are the only NE, since the best-response functions intercept in exactly these three points.

This concludes the disjunction. □

7.1.4 Case where $1 = \psi_i \psi_j c_i c_j B_i B_j$

We assume in this section $1 = \psi_i \psi_j c_i c_j B_i B_j$ and re-use the notations of the previous section. The slope of $b_i^{**}(b_j)$ relatively to the plane (b_i, b_j) is thus precisely $\frac{-1}{\beta}$ and the slope of $b_j^{**}(b_i)$ in this same plane is $\widetilde{-\beta}$. Hence the slopes are equal and the (unrestricted) best-response are parallel if, and only if, $\frac{-1}{\beta} = -\widetilde{\beta}$, i.e. if and only if $1 = \beta\widetilde{\beta}$. Because $\beta = \psi_i c_j B_i$ and $\widetilde{\beta} = \psi_j c_i B_j$, this is equivalent to $1 = \psi_i c_j B_i \psi_j c_i B_j$, that is, precisely our condition. Here we discuss what happens for $b_i^*(b_j)$ and $b_j^*(b_i)$ in that case – the rest of the article always making the implicit assumptions that $1 \neq \psi_i c_j B_i \psi_j c_i B_j$.

We can separate three cases.

Assume first that $\widetilde{\alpha} > \frac{\alpha}{\beta}$. This means the best-response straight-line $b_j^{**}(b_i)$ is "above" $b_j^{**}(b_i)$ in the (b_i, b_j) plane. Now if $b_i^*(1) \geq 0$ the best-response $b_i^*(b_j)$ and $b_j^*(b_i)$ intersect in a single NE at $(b_i^*, b_j^*) = (b_i^{**}(1), 1)$. Otherwise, they do not intersect and there is no NE.

Assume that $\widetilde{\alpha} < \frac{\alpha}{\beta}$. This means the best-response straight-line $b_j^{**}(b_i)$ is "below" $b_j^{**}(b_i)$ in the (b_i, b_j) plane. Now if $b_j^*(1) \geq 0$ then the best-response $b_i^*(b_j)$ and $b_j^*(b_i)$ intersect in a single NE at $(b_i^*, b_j^*) = (1, b_j^{**}(1))$. Otherwise, they do not intersect and there is no NE.

Assume $\widetilde{\alpha} = \frac{\alpha}{\beta}$. Then the two best-responses form a single straight-line and any point of $b_j^{**}(b_i)$ in the interior of $[0,1]^2$ is a NE. There is first an infinity of NE in that case.

To conclude, note that in this case too the CSA still implies $\alpha > 0$ and $\alpha - \beta > 0$ as well as $\widetilde{\alpha} > 0$ and $\widetilde{\alpha} - \widetilde{\beta} > 0$, hence there are no other cases to explore than the one previously stated.

7.2 Proofs of Section 3 results and Theorems

7.2.1 Proof of Theorem 2

Assume $\frac{1}{f_j} < \frac{c_i}{c_j} < f_i$ and consider the following trigger strategy (S):

(S) *Do not reshape if no player has reshaped so far. If another player has reshaped and yourself has not reshaped, then reshape. At each price-setting game (step 2) play the unique Nash equilibrium, given the costs.*

To prove this strategy profile yields a subgame-perfect Nash equilibrium, we show it satisfies the definition of a subgame-perfect Nash equilibrium, that is, it induces a Nash equilibrium on every subgame of the whole game. The whole game consists of two types of subgames: those where at least one player has reshaped in one of the previous periods, and those where no player has reshaped so far.

Consider a subgame of the first type. Then the strategy induced by (S) on this subgame consists of reshaping at stage 1) in case it has not been done yet, and playing the Nash equilibrium of the price-setting game for each stage game. As argued in Section 2.4, this constitutes a Nash equilibrium of the subgame. Indeed, since player's actions do not take the past into account anymore and play repeatedly the Nash equilibrium of the price setting game, any player unilaterally deviating from it would not be better of, by definition of a Nash equilibrium of the stage game and of the payoff for the subgame.

Consider a subgame of the second type. Then the strategy profile induced by (S) on this subgame is precisely (S) itself, and in particular it implies not to reshape at stage 1) of the first period of the subgame, since no one has ever reshaped before. Now does it constitutes a Nash equilibrium of the subgame? Consider first the value V of playing this induced strategy for player i, assuming all other players also play it. V is equal to the sum of the immediate payoff $\pi_i(0,0)$ plus the discounted value of playing this strategy on the next periods of the subgame - which happen conveniently to be equal to our whole subgame. Hence we can write

$$V = \pi_i(0,0) + \delta V$$

which yields

$$V = \frac{\pi_i(0,0)}{1-\delta}$$

Now assume player i would unilaterally deviate from this strategy by reshaping to some extent $b_i > 0$. This entails some immediate reshaping costs $\xi_i b_i^2$. Since in the first period of our subgame other players do not reshape, CSD i immediate payoff ends up being $\pi_i(b_i,0) - \xi_i b_i^2$. But in the following periods all CSDs also reshape and CSD i profit is $\pi_i(b_i, b_j^*(b_i))$. Hence the total profits $\pi_i^{dev}(b_i)$ obtained from unilaterally deviating from the induced strategy by playing $b_i > 0$ at step 1) of the first period of the subgame are

$$\begin{aligned} \pi_i^{dev}(b_i) &= \pi_i(b_i,0) - \xi_i b_i^2 + \delta\pi_i(b_i, b_j^*(b_i)) + \delta^2\pi_i(b_i, b_j^*(b_i)) + ... \\ &= \pi_i(b_i,0) - \xi_i b_i^2 + \frac{\delta}{1-\delta}\pi_i(b_i, b_j^*(b_i)) \end{aligned}$$

which can be written as:

$$\pi_i^{dev}(b_i) = (A_i + (c_j + c_{T2S})B_i - ((1-b_i)c_i + c_{T2S}))D_i)^2 + \frac{\delta}{1-\delta}(A_i + c_{T2S}B_i - ((1-b_i)c_i + c_{T2S}))D_i)^2 - \xi_i b_i^2$$

$$\frac{\partial \pi_i^{dev}}{\partial b_i} = 2c_iD_i(A_i + (c_j + c_{T2S})B_i - ((1-b_i)c_i + c_{T2S}))D_i) + \frac{2\delta}{1-\delta}c_iD_i(A_i + c_{T2S}B_i - ((1-b_i)c_i + c_{T2S}))D_i) - 2\xi_i b_i$$

Rearranging the expression we see that $\frac{\partial \pi_i^{dev}}{\partial b_i} > 0$ if, and only if:

$$(\xi_i - \frac{c_i^2 D_i^2}{1-\delta})b_i < c_iD_i(\frac{1}{1-\delta}A_i + (c_i + \frac{1}{1-\delta}c_{T2S})B_i - \frac{1}{1-\delta}(c_i + c_{T2S})D_i)$$

Now assuming $\delta > 1 - \frac{c_i^2 D_i^2}{\xi_i}$ yields $\xi_i - \frac{c_i^2 D_i^2}{1-\delta} < 0$. Hence the maximum of π_i^{dev} is attained either at $b_i = 0$ or at $b_i = 1$. To decide which value of b_i yields this maximum we just have to compare $\pi_i^{dev}(0)$ with $\pi_i^{dev}(1)$. Assume also $\xi_i < \frac{c_j^2 D_j^2}{1-\delta}$, that is, the similar condition for CSD j; then, because Lemma 1 applies for CSD j (note it does not apply as such for CSD i), a complete reshaping ($b_j = 1$) is always more appropriate for CSD j, whatever the CSD i decision concerning its own degree of reshaping b_i. In particular $b_j^*(0) = b_j^*(1) = 1$, which we will use below.

We will prove that the maximum of π_i^{dev} is attained at $b_i = 1$ using a argument inspired from the proof of Lemma 1 (see Annex 7.1.3). Indeed, $\pi_i^{dev}(0) < \pi_i^{dev}(1)$ is successively equivalent to

$$\begin{aligned} \pi_i(0,0) + \frac{\delta}{1-\delta}\pi_i(0, b_j^*(0)) &< \pi_i(1,0) + \frac{\delta}{1-\delta}\pi_i(1, b_j^*(1)) - \xi_i \\ \pi_i(0,0) + \frac{\delta}{1-\delta}\pi_i(0,1) &< \pi_i(1,0) + \frac{\delta}{1-\delta}\pi_i(1,1) - \xi_i \end{aligned}$$

Setting for convenience:

$$\begin{array}{rcl} A & = & A_i + (c_j + c_{T2S})B_i - (c_i + c_{T2S}))D_i \\ B & = & A_i + c_{T2S}B_i - (c_i + c_{T2S}))D_i \\ A' & = & A_i + (c_j + c_{T2S})B_i - c_{T2S})D_i \\ B' & = & A_i + c_{T2S}B_i - c_{T2S}D_i \end{array}$$

this amounts to

$$A^2 + \frac{\delta}{1-\delta}B^2 < A'^2 + \frac{\delta}{1-\delta}B'^2 - \xi_i$$

Since $\xi_i < \frac{c_j^2 D_j^2}{1-\delta}$, this last line is implied by

$$A^2 + \frac{\delta}{1-\delta}B^2 < A'^2 + \frac{\delta}{1-\delta}B'^2 - \frac{c_j^2 D_j^2}{1-\delta}$$

But this later condition is equivalent, given that $\frac{c_j^2 D_j^2}{1-\delta} = (1 + \frac{\delta}{1-\delta})c_j^2 D_j^2$, to

$$A^2 + c_j^2 D_j^2 + \frac{\delta}{1-\delta}(B^2 + c_j^2 D_j^2) < A'^2 + \frac{\delta}{1-\delta}B'^2$$

which is always true because

$$\begin{cases} A^2 + c_j^2 D_j^2 = A^2 + (c_j D_j)^2 \leq (A + c_j D_j)^2 = A'^2 \\ B^2 + c_j^2 D_j^2 = B^2 + (c_j D_j)^2 \leq (B + c_j D_j)^2 = B'^2 \end{cases}$$

(this is the case because both Ac_jD_j and Bc_jD_j are nonnegative).

This prove that π_i^{dev} is maximum for $b_i = 1$.

Hence not reshaping and conforming to the induced strategy in the subgame yields a higher payoff than deviating from the strategy profile for player i if

$$V > \pi_i^{dev}(1)$$

idem est if

$$\frac{\pi_i(0,0)}{1-\delta} > \pi_i^{dev}(1) = \pi_i(1,0) - \xi_i + \frac{\delta}{1-\delta}\pi_i(1,1)$$

which is equivalent to

$$\pi_i(0,0) > (1-\delta)(\pi_i(1,0) - \xi_i) + \delta\pi_i(1,1) \quad (*)$$

Now if $\pi_i(0,0) > \pi_i(1,1)$, then there exists $\overline{\delta} \in]0,1[$ such that for any discount factor $\delta \in]\overline{\delta}, 1[$, inequality $(*)$ is true. Indeed, when δ tends toward 1, the right-hand side of $(*)$ tends to $\pi_i(1,1)$, so if $\pi_i(0,0) > \pi_i(1,1)$, then, since this inequality is strict, there exist a $\overline{\delta} \in]0,1[$ such that

$$\pi_i(0,0) > (1-\overline{\delta})(\pi_i(1,0) - \xi_i) + \overline{\delta}\pi_i(1,1) > \pi_i(1,1)$$

and then any $\delta \in]\overline{\delta}, 1[$ also satisfies this inequality.

But $\pi_i(0,0) > \pi_i(1,1)$ is equivalent to

$$A_i + (c_j + c_{T2S})B_i - (c_i + c_{T2S})D_i > A_i + c_{T2S}B_i - c_{T2S}D_i$$

idem est

$$c_jB_i - c_iD_i > 0$$

which is precisely the condition $\frac{c_i}{c_j} < \frac{B_i}{D_i}$. Hence if $\frac{c_i}{c_j} < \frac{B_i}{D_i}$, CSD i will not find it profitable to deviate from the strategy induced by (S) on our subgame. We have to assume similarly that $\frac{c_j}{c_i} < \frac{B_j}{D_j}$ such that CSD j will not find it profitable neither. Hence, under the assumptions of the Theorem, the strategy induced by (S) is a Nash equilibrium of the subgame.

This concludes the proof that (S) is a subgame-perfect Nash equilibrium of the whole game.□

7.2.2 Proof of Remark 2 concerning Theorem 2

We want to derive from the proof of Theorem 2 an explicit value for δ. Setting for convenience:

$$\begin{aligned} A &= \sqrt{\pi_i(0,0)} = A_i + (c_j + c_{T2S})B_i - (c_i + c_{T2S})D_i \\ B &= \sqrt{\pi_i(1,0)} = A_i + (c_j + c_{T2S})B_i - c_{T2S}D_i \\ C &= \sqrt{\pi_i(1,1)} = A_i + c_{T2S}B_i - c_{T2S}D_i \end{aligned}$$

inequality $(*)$ of the proof of Theorem 2 is equivalent to:

$$\begin{aligned} A^2 &> (1-\delta)(B^2 - \xi_i) + \delta C^2 \\ A^2 - B^2 &> \delta(C^2 - B^2 + \xi_i) - \xi_i \\ \delta(B^2 - C^2 - \xi_i) &> B^2 - A^2 - \xi_i \end{aligned}$$

If $\xi_i < B^2 - C^2$ then $(*)$ is thus equivalent to:

$$\delta > \frac{B^2 - A^2 - \xi_i}{B^2 - C^2 - \xi_i}$$

This is only possible for at least *one* value of δ if $\frac{B^2-A^2-\xi_i}{B^2-C^2-\xi_i} < 1$ *idem est* if $B^2 - A^2 - \xi_i < B^2 - C^2 - \xi_i$, which amounts to $A > C$, that is, to $c_jB_i - c_iD_i > 0$, which is true since by assumption $c_i < c_jf_i$. $(*)$ then holds for any $\delta > \frac{B^2-A^2-\xi_i}{B^2-C^2-\xi_i}$. Hence looking at the Proof of Theorem 2 again we see that setting $\overline{\delta} := \max(1 - \frac{c_i^2D_i^2}{\xi_i}, 1 - \frac{c_j^2D_j^2}{\xi_j}, \frac{B^2-A^2-\xi_i}{B^2-C^2-\xi_i}, \frac{B^2-A^2-\xi_j}{B^2-C^2-\xi_j})$ gives an explicit value for the $\overline{\delta}$ mentioned in the theorem.

If $\xi_i > B^2 - C^2$ then $(*)$ is equivalent to:

$$\delta < \frac{B^2 - A^2 - \xi_i}{B^2 - C^2 - \xi_i}$$

This only happens for all large enough values of δ if $\frac{B^2-A^2-\xi_i}{B^2-C^2-\xi_i} \geq 1$ *idem est* if $B^2 - A^2 - \xi_i \leq B^2 - C^2 - \xi_i$, which amounts again to the assumption $c_i \leq c_jf_i$. Then it is here enough to set $\overline{\delta} := \max(1 - \frac{c_i^2D_i^2}{\xi_i}, 1 - \frac{c_j^2D_j^2}{\xi_j})$ as an explicit value for $\overline{\delta}$.

So in both situation we see that $(*)$ holds for all values of δ close enough to 1 if, and only if, $c_i < c_jf_i$. More precisely we proved that $c_i < c_jf_i$ is equivalent to: "for any ξ_i there exists $\overline{\delta} \in]0,1[$ such that for any $\delta \in]\overline{\delta}, 1[$, $(*)$ holds."

This Remark will be useful for the proof of Theorem 3.

Note in passing that $\frac{B^2-A^2-\xi_i}{B^2-C^2-\xi_i}$ can be simplified by rewriting it as:

$$\frac{(B-A)(B+A) - \xi_i}{(B-C)(B+C) - \xi_i} = \frac{c_iD_i(c_jB_i - c_iD_i) - \xi_i}{c_jB_i(2A_i + (c_j + 2c_{T2S})B_i - 2c_{T2S}D_i) - \xi_i}$$

7.2.3 Proof of Theorem 4

Let N_0 be the time necessary for a given CSD to become aware of another CSD's reshaping. We adapt the proof of Theorem 2 to this new game of imperfect information by using the following strategy (S_{N_0}):

(S_{N_0}) *If it is not common belief that at least one player has reshaped so far, then do not reshape and play what you believe is the Nash equilibrium of the price-setting game (that is, play the Nash equilibrium of the price-setting game where costs corresponds to costs without reshaping for all of the players). If* *it is common belief that some player has reshaped, then reshape completely and always play the Nash equilibrium of each price-setting game.*

To prove this strategy profile yields a subgame-perfect Nash equilibrium, we show, as in the proof of Theorem 2, that it satisfies the definition of a subgame-perfect Nash equilibrium, that is, it induces a Nash equilibrium on every subgame of the whole game. The whole game consists of two types of subgames: those where it is known by all CSDs that at least one CSD has reshaped in one of the previous periods, and those where no CSD is commonly known to have reshaped so far.

Consider first a subgame T of the first type. Then the strategy induced by (S_{N_0}) on the subgame T consists of reshaping at stage 1) of the first stage-game $G_{reshape}$ of T in case it has not been done yet, and then playing repeatedly the Nash equilibrium of the price-setting game at each subsequent stage-game. Consider a given CSD i and assume all other CSDs than i are playing according to the induced strategy. We proceed to determine CSD i's best-response – the goal being to prove that i's best-response is also to conform to the induced strategy.

First, since in the subgame T other CSDs are repeatedly playing the Nash equilibrium of the price-setting game, it is easy to see that CSD i's best strategy is to play each time the Nash equilibrium of the price-setting game given its costs, regardless of the degree b_i of reshaping it had possibly chosen and whenever it had possibly chosen to reshape. Indeed, if CSD i deviates in at least one period from playing the Nash equilibrium of the price-setting game, then, by definition of the Nash equilibrium of the price-setting game, CSD i profits on this period would be lower, and so will be its profits for the whole game, since the profits for the whole game are just the discounted payoffs of the profits at each stage game. Hence, in any of its best-responses, CSD i will always choose to play the Nash equilibrium of each price-setting game.

Second, by Lemma 1, if $\delta > \frac{c_i^2 D_i^2}{\xi_i}$, CSD i cannot avoid to reshape at some point. Indeed, Lemma 1 for $\delta > \frac{c_i^2 D_i^2}{\xi_i}$ predicts a higher profit for CSD i if it chooses to reshape in the first period of the model than if it choose to never reshape. Note that this does *not* imply, as such, that the optimal outcome is to reshape immediately (that is, in the first period of the subgame T), since Lemma 1 only compares the outcome of reshaping *in the first period* with no reshaping at all. But this proves that reshaping must indeed occurs in any best-response to other CSDs' strategy (possibly not in the first period of the subgame but in a later period). All that is left to prove is that the reshaping is complete (meaning $b_i = 1$ is chosen) and that it occurs in the first period of the subgame T, which we will call period 0. Let t_0 be the period in which reshaping occurs.

By Lemma 1 again, but applied to the subgame of T starting at time t_0 in which CSD i choose to reshape, we see that, because $\delta > \frac{c_i^2 D_i^2}{\xi_i}$, $b_i = 1$ is optimal. Hence reshaping, when it occurs, is always complete.

Assume that reshaping occurs at a period $t_0 > 0$. Then the additional profits from reshaping one period earlier would have been:

$$-\xi_i + \delta\xi_i + P$$

where P is the additional profits in one period obtained from a low cost situation compared to a high cost situation, *idem est*

$$\begin{aligned} P &= (A_i + c_{T2S}B_i - c_{T2S}D_i)^2 - (A_i + c_{T2S}B_i - (c_i + c_{T2S})D_i)^2 \\ &= 2c_iD_i(A_i + c_{T2S}B_i - c_{T2S}D_i) - (c_iD_i)^2 \\ &= c_iD_i\{(A_i + c_{T2S}B_i - c_{T2S}D_i) + (A_i + c_{T2S}B_i - (c_{T2S} + c_i)D_i)\} \end{aligned}$$

It can be seen, either from the first expression or from the last one (sum of two positive quantities), than $P > 0$. Hence, for any δ satisfying $1 > \delta > 1 - \frac{P}{\xi_i}$, we have $-\xi_i + \delta\xi_i + P > 0$, a contradiction since we assumed i was playing its best-response. We conclude that the reshaping occurs at time 0 in T. Hence CSD i's unique best-response is to conform too to the induced strategy. This shows that the strategy (S_{N_0}) induces a Nash equilibrium on any subgame of the first type.

Consider now a subgame U of the second type. Then the strategy profile induced by (S_{N_0}) on this subgame is precisely (S_{N_0}) itself, and in particular it implies not to reshape at stage 1) of the first period of the subgame, since no one has ever reshaped before. Now does it constitute a Nash equilibrium of the subgame? Consider first the value V of playing this induced strategy for CSD i, assuming all other CSDs also play it. V is equal to the sum of the immediate payoff $\pi_i(0,0)$ plus the discounted value of playing this strategy on the next periods of the subgame - which happen conveniently to be equal to our whole subgame. Hence we can write

$$V = \pi_i(0,0) + \delta V$$

which yields

$$V = \frac{\pi_i(0,0)}{1-\delta}$$

Now assume CSD i would unilaterally deviate from this strategy by reshaping to some extent $b_i > 0$. Since in the N_0 following periods of the subgame U other CSDs do not reshape and play the Nash equilibrium of the price-setting game in which they are unaware of CSD i new costs, the payoff at each of these periods is denoted by π_i^{ab} where the index ab stands for "abnormal", the abnormality being that other CSDs do not know the true costs of CSD i and are thus in reality not playing the Nash equilibrium of the *relevant* price-setting game. But after N_0 periods all CSDs also reshape and CSD i profit is then only $\pi_i(b_i, b_j^*(b_i))$. Hence the total profits $\pi_i^{dev}(b_i)$ obtained from unilaterally deviating from the induced strategy by playing $b_i > 0$ at step 1) of the first period of the subgame is:

$$\begin{aligned}\pi_i^{dev}(b_i) &= \pi_i^{ab} - \xi_i b_i^2 + \delta\pi_i^{ab} + ... + \delta^{N_0-1}\pi_i^{ab} + \delta^{N_0}\pi_i(b_i, b_j^*(b_i)) + \delta^{N_0+1}\pi_i(b_i, b_j^*(b_i)) + ... \\ &= \frac{1-\delta^{N_0}}{1-\delta}\pi_i^{ab} - \xi_i b_i^2 + \frac{\delta^{N_0}}{1-\delta}\pi_i(b_i, b_j^*(b_i))\end{aligned}$$

The maximum value of π_i^{ab} assuming a given b_i and that other CSDs follow the induced strategy could be computed explicitly, then $\pi_i^{dev}(b_i)$ could be derived with respect to b_i to find its maximum with respect to b_i and the best-possible deviation thus obtained: this was the method of the proof of Theorem 2. But to avoid additional computations in this proof we will use the fact that π_i^{ab} is bounded, since the *general* payoff-function $\pi_i(b_i, b_j, p_i, p_j)$ is itself bounded.

No reshaping and conforming to the induced strategy in the subgame yields a higher payoff than deviating from the strategy profile for CSD i if, and only if,

$$V > \pi_i^{dev}(b_i)$$

i. e.:

$$\frac{\pi_i(0,0)}{1-\delta} > \frac{1-\delta^{N_0}}{1-\delta}\pi_i^{ab} - \xi_i b_i^2 + \frac{\delta^{N_0}}{1-\delta}\pi_i(b_i, b_j^*(b_i))$$

which is equivalent to

$$\pi_i(0,0) > (1-\delta^{N_0})\pi_i^{ab} - (1-\delta)\xi_i b_i^2 + \delta^{N_0}\pi_i(b_i, b_j^*(b_i)) \quad (*)$$

Now notice that for $\delta \geq \frac{c_j^2 D_j^2}{\xi_j}$, applying Lemma 1 to the infinite subgame of our game starting when CSD j becomes aware of CSD i reshaping decision yields $b_j^*(b_i) = 1$ for all b_i. Note that the same Lemma cannot be applied as such to CSD i, even if $\delta \geq \frac{c_i^2 D_i^2}{\xi_j}$ (because of the first N_0 periods of the subgame in which CSD i benefits from a first-mover advantage). We will actually avoid to determine explicitly the reshaping degree of CSD i here[25].

Since $b_j^*(b_i) = 1$ inequality $(*)$ is equivalent to:

$$\pi_i(0,0) > (1-\delta^{N_0})\pi_i^{ab} - (1-\delta)\xi_i b_i^2 + \delta^{N_0}\pi_i(b_i, 1) \quad (**)$$

Now if $\pi_i(0,0) > \pi_i(b_i, 1)$, then there exists $\overline{\delta} \in]0,1[$ such that for any discount factor $\delta \in]\overline{\delta}, 1[$, inequality $(**)$ is true. Indeed, when δ tends toward 1, the right-hand side of $(**)$ tends to $\pi_i(b_i, 1)$

[25] Indeed, for this particular proof we do not need to find b_i explicitly, although it will be shown in the proof of Theorem 4 that $b_i = 1$ for values of δ high enough.

because π_i^{ab} is bounded and $1-\delta^{N_0}$ tends to 0. So if $\pi_i(0,0) > \pi_i(b_i,1)$, then, since this inequality is strict, there exist a $\overline{\delta} \in]0,1[$ such that

$$\pi_i(0,0) > (1-\overline{\delta}^{N_0})\pi_i(1,0) - (1-\delta)\xi_i b_i^2 + \overline{\delta}^{N_0}\pi_i(b_i,1) > \pi_i(b_i,1)$$

and then any $\delta \in]\overline{\delta},1[$ will also satisfies this inequality.

But $\pi_i(0,0) > \pi_i(b_i,1)$ is equivalent to

$$A_i + (c_j + c_{T2S})B_i - (c_i + c_{T2S})D_i > A_i + c_{T2S}B_i - ((1-b_i)c_i + c_{T2S})D_i$$

which is equivalent to

$$c_jB_i - b_ic_iD_i > 0$$

idem est

$$c_jB_i > b_ic_iD_i$$

Since $b_i \leq 1$ and $c_iD_i \geq 0$, we see that this last inequality is implied by the condition $c_jB_i > c_iD_i$, which is precisely the condition $\frac{c_i}{c_j} < \frac{B_i}{D_i} = f_i$ assumed in the theorem. Hence if $\frac{c_i}{c_j} < \frac{B_i}{D_i}$, CSD i will not find it profitable to deviate from the strategy induced by (S_{N_0}) on our subgame U. We have to assume similarly that $\frac{c_j}{c_i} < \frac{B_j}{D_j}$ such that CSD j will not find it profitable neither. Hence, under the assumptions of the theorem, the strategy induced by (S_{N_0}) is a Nash equilibrium of the subgame U.

This concludes the proof that (S) is a subgame-perfect Nash equilibrium of the whole game.□

7.3 Generalising to n CSDs

7.3.1 When CSDs are given the choice to reshape in the first period of the model only

Although throughout the article we favoured the view, when dealing with many CSDs, to focus on a given CSD i and interpret the other CSD j as representing the overall market (minus CSD i), the model could easily be extended to more than two firms, provided some symmetric assumptions are made in order to keep the complexity of the computations down. Indeed, the general model for n CSD has its demand for CSD i settlement services given by:

$$q_i = \alpha_i - \gamma_{ii}p_i + \sum_{j \neq i} \gamma_{ij}p_j$$

We will in the whole section assume that the cross-elasticities are all equal:

$$\gamma_{ij} = \gamma_{kl} := \gamma'$$

for any $i \neq j$ and $k \neq l$, as well as

$$\gamma_{ii} = \gamma_{jj} = \gamma$$

for any i, j.

With this equation demands become:

$$q_i = \alpha_i - \gamma p_i + \gamma' \sum_{j \neq i} p_j$$

and profits can be written as before as

$$\pi_i = (p_i - \widetilde{c}_i)q_i - \widetilde{C}_i$$

For a given vector a of length n always denote by a_{-i} the vector of length $n-1$ obtained from a by deleting the ith element of a. For example if p is the vector of prices $(p_1, ..., p_n)$ then p_{-i} the vector $(p_1, ..., p_{i-1}, p_{i+1}, ...p_n)$. The best-response function for the price-setting stage is:

$$p_i^*(p_{-i}) = \frac{1}{2\gamma}(\alpha_i + \gamma\widetilde{c}_i + \gamma' \sum_{j \neq i} p_j)$$

Solving the system $\{p_i^* = p_i^*(p_{-i}^*),\ i \in \{1,...,n\}\}$(for example, by summing all equations to find an expression for $\sum_j p_j^*$) gives:

$$p_i^* = \frac{(2\gamma - (n-2)\gamma')(\alpha_i + \gamma\widetilde{c}_i) + \gamma' \sum_{j\neq i}(\alpha_j + \gamma\widetilde{c}_j)}{(2\gamma + \gamma')(2\gamma - (n-1)\gamma')}$$

We notice that at equilibrium $p_i^* - \widetilde{c}_i = \frac{1}{\gamma} q_i^*$. Hence $\pi_i^* = \gamma(p_i^* - \widetilde{c}_i)^2 - \widetilde{C}_i$, yielding

$$\pi_i^* = (A_i + B\sum_{j\neq i}\widetilde{c}_j - D\widetilde{c}_i)^2 - C_i$$

with

$$\begin{aligned} A_i &= \frac{\sqrt{\gamma}\{(2\gamma - (n-2)\gamma')\alpha_i + \gamma' \sum_{j\neq i}\widetilde{\alpha}_j\}}{(2\gamma + \gamma')(2\gamma - (n-1)\gamma')} \\ B &= \frac{\gamma\gamma'\sqrt{\gamma}}{(2\gamma + \gamma')(2\gamma - (n-1)\gamma')} \\ D &= \frac{\sqrt{\gamma}\{2\gamma^2 - (n-2)\gamma\gamma' - (n-1)\gamma'^2\}}{(2\gamma + \gamma')(2\gamma - (n-1)\gamma')} \end{aligned}$$

We thus deduce π_i^{tot}:

$$\pi_i^{tot} = \widetilde{\delta}(A_i + (\sum_{j\neq i}((1-b_j)c_j + c_{T2S}))B - ((1-b_i)c_i + c_{T2S}))D)^2 - \widetilde{\delta}(1-a_i)C_{i,fixed} - C_{i,adapt}(a_i, b_i)$$

with

$$\widetilde{\delta} = \begin{cases} N & \text{if } \delta = 1 \\ \frac{1-\delta^N}{1-\delta} & \text{if } 0 \leq \delta < 1 \end{cases}$$

Let $b = (b_1, ...b_n)$. Assuming $C_{i,adapt}(a_i, b_i) = \xi_i b_i^2$, this expression can be derived with respect to b_i to find the best-response function in terms of reshaping in a N period game or in an infinite game with $\delta < 1$. Notice in passing we do not assume symmetric adaptation costs, since ξ_i is allowed to depend on CSD i.

In particular, we have an equivalent to Lemma 1:

Lemma 2 *Assume $\xi_i < \widetilde{\delta} c_i^2 D_i^2$. Then if CSDs engage in price competition, the best-response function $b_i^*(b_j)$ is the constant function $b_i^*(b_{-i}) = 1$, which represents a complete reshaping decision from CSD i whatever the degrees b_j, $j \neq i$ chosen by the other CSDs.*

Proof: Because $\xi_i < \widetilde{\delta} c_i^2 D_i^2$ the function π_i^{tot} is first decreasing in b_i, then increasing. Hence its maximum is either reached at $b_i = 0$ or at $b_i = 1$. We prove that $\pi_i^{tot}(0, b_{-i}) < \pi_i^{tot}(1, b_{-i})$ for any b_{-i}.

$\pi_i^{tot}(0, b_{-i}) < \pi_i^{tot}(1, b_{-i})$ is equivalent to

$$\widetilde{\delta}(A_i + (\sum_{j\neq i} c_j + c_{T2S})B - (c_i + c_{T2S})D)^2 + \xi_i < \widetilde{\delta}(A_i + (\sum_{j\neq i} c_j + c_{T2S})B - c_{T2S}D)^2$$

But since $a^2 + b^2 < (a+b)^2$ for any $ab > 0$, we have

$$\widetilde{\delta}(A_i + (\sum_{j\neq i} c_j + c_{T2S})B - (c_i + c_{T2S})D)^2 + \widetilde{\delta} c_i^2 D^2 < \widetilde{\delta}(A_i + (\sum_{j\neq i} c_j + c_{T2S})B - (c_i + c_{T2S})D + c_i D)^2$$

and this inequality implies the above, since $\xi_i < \widetilde{\delta} c_i^2 D^2$.$\square$

Assume now $\xi_i < \widetilde{\delta} c_i^2 D^2$ for all i. Then π_i^{tot} reaches its maximum in some point $b_i^{**}(b_{-i}) = \psi_i(A_i + (\sum_{j\neq i} c_j + c_{T2S})B - (c_i + c_{T2S})D)) - \psi_i B \sum_{j\neq i} b_j$. Solving for the (unrestricted) solution to the system $\{b_i^{**} = b_i^{**}(b_{-i}^{**}),\ i \in \{1,...,n\}\}$ yields:

$$b_i^{**} = \frac{((1 - \frac{1}{\psi_i}\sum_{j\neq i}\psi_j)\beta_i + \sum_{j\neq i}\beta_j}{1 + \psi_i B \sum_{j\neq i}\frac{\psi_j(1-\psi_i B)}{\psi_i(1-\psi_j B)}}$$

with $\beta_i := \psi_i(A_i + (\sum_{j\neq i} c_j + c_{T2S})B - (c_i + c_{T2S})D)$.

This proves the following theorem (note we will not consider all the corner solutions as in the two-CSDs case, but just assume the previously computed quantities belongs indeed to the action set of each CSD:

Theorem 8 *Assume CSDs are engaging in price competition, and that $\xi_i > \widetilde{\delta} c_i^2 D_i^2$ and $b_i^{**} \in [0,1]$ for each CSD i. Then the optimal degree of reshaping is given by $b_i^* = b_i^{**}$.*

Simpler formula, assuming symmetric variable costs for CSDs. In order to provide a less complex formula, we could have assumed perfect symmetry of the CSDs and markets. In particular, CSDs are assumed to have the same costs c_i. We can derive very easily a simpler expression for b_i^* under the assumptions of Theorem 8. Indeed by symmetry $b_i^{**} = b_j^{**}$ for any CSD i, j hence,

$$\begin{aligned} b_i^{**} &= \beta_i - \psi_i B \sum_{j\neq i} b_j^{**} \\ &= \beta_i - \psi_i B(n-1) b_i^{**} \end{aligned}$$

Hence

$$b_i^{**} = \frac{\psi_i}{1 + (n-1)\psi_i B}(A_i + ((n-1)B - D)(c_i + c_{T2S})$$

One can check this is indeed, for $n = 2$, an equivalent formula to the one given by Theorem 1, assuming $\gamma_{jj} =: \gamma$, $\gamma_{ij} := \gamma'$ for any $i \neq j$.

Remark: using this formula, by computing the derivative of b_i^{**} with respect to the number n of CSDs, one can show that provided the market are large enough (i.e α_i high enough), or that the costs $c_i + c_{T2S}$ are high enough, the degree of optimal reshaping is an increasing function of the number of CSDs in the market. More precisely, in the symmetric case b_i^{**} is a (strictly) increasing function of n if, and only if,

$$(c_i + c_{T2S})(1 - \psi_i D) > A_i \psi_i$$

7.3.2 When CSDs are given the choice to reshape at any period: collusion theorem and immediate reshaping

Using very similar arguments to the ones employed in the proofs of Theorems 2 and 3, one could prove:

Theorem 9 *If $c_i < f_i(\sum_{j\neq i} c_j)$ for all i and the discount factor is high enough, a collusion-type theorem holds: there exists a subgame perfect Nash equilibrium where CSDs indefinitely delay reshaping. The strategy sustaining it is the same as in Theorem 2.*

Theorem 10 *If CSD i's costs satisfy $c_i > f_i(\sum_{j\neq i} c_j)$, and the discount factor is high enough, and assuming CSDs engage in price competition, then CSD i will always reshape in any subgame perfect Nash equilibrium.*

Note that, mathematically, it *looks* as if increased number of CSDs would tend to re-enforce the possibility of tacit collusion not to reshape, as $\sum_{j \neq i} c_j$ grows with n. Nevertheless the fact that the strategy sustaining collusion in the proof of the theorem needs this assumption do not mean there is no other strategies, with less stringent assumptions, that sustain collusion. Hence strictly speaking nothing can be deduced about the likelihood of collusion from the assumption $c_i < f_i(\sum_{j \neq i} c_j)$. Moreover, the strategy employed in the proof, that is, that each CSD assumes other CSDs' strategy is to reshape only in case some competitor has reshaped, become a less and less realistic assumption as the number of competitors in the market grows. Divergence of views on the market as well as on other market participants strategy is likely to drive the real process outside of the equilibrium path of strategy (S), resulting in CSDs reshaping.

7.4 Nash equilibrium with collusion in prices

7.4.1 Introduction

In this paper we focused our attention on tacit collusion in the decision to reshape towards T2S. Most of the literature since Friedman [12] has concentrated on tacit collusion in prices. This paper has instead focused on the question of the consequences of price competition on the incentives for reshaping towards T2S, including potential collusion therein. This focus can be justified because the European securities settlement industry will benefit from increased competition by three complementary factors after the introduction of T2S: first, the introduction of T2S will overcome a number of technical barriers to cross-border settlement and provide a single platform on which CSDs will be able to compete; second, the CSD regulation will ensure common legal standards for securities settlement in the European Union, including free choice of CSDs for users and a 'passport'-concept for authorised CSDs to provide their services in other EU member states; and third, a number of other initiatives contribute to further harmonisation of technical, legal and regulatory aspects and the associated business processes[26].

Expectations of strong price competition after the introduction of T2S appear to be confirmed by a recent announcement by Clearstream, with roughly 40% of the expected settlement volumes the biggest single CSD group in T2S, that Clearstream would not add any margin on top of the direct T2S fee (c_{T2S} in our model) after their migration to T2S (see Clearstream [4]). In order not to mix the incentives for the investment / reshaping decision given price competition with the effects of increases in price competition, we have assumed no changes in price competition over time. The degree of price competition is indirectly captured in the cross-sensitivity parameter γ_{12} and γ_{21}. We expect that explicitly modelling an increase in competition over time would further increase the benefits of reshaping towards T2S, but leave the explicit modelling for future research.

Another interesting theoretical question, closely linked to the literature following Friedman [12], is to find other Nash-equilibria of the whole game for which collusion also appears at the price level. Characterising all Nash equilibria seems an ambitious task, as many strategies become possible when the assumption of price competition is removed. Hence, we will only consider a few such equilibria, and under some simplifying assumptions. This nevertheless helps cast some light on the more complex real world situation where CSDs can potentially collude both in the timing of the reshaping, in its degree, and in the price levels fixed after reshaping. Price-collusion does not necessarily imply a greater potential for collusion in delaying the decision to reshape. This is because by colluding on prices CSDs can pocket the unit-cost-reduction stemming from reshaping rather than passing them to the market.

7.4.2 Simplifying assumptions

1) For simplicity, we will assume all CSDs' parameters are symmetric Hence we will adopt the same notations as in Annex 7.3 with

$$\gamma_{ij} = \gamma_{kl} =: \gamma'$$

[26] See, e.g., the third progress report of the T2S Harmonisation Steering Group, available at http://www.ecb.europa.eu/paym/t2s/pdf/Third_T2S_Harmonisation_Progress_Report.pdf.

for any $i \neq j$ and $k \neq l$, and

$$\gamma_{ii} = \gamma_{jj} =: \gamma$$

for any i, j, but also:

$$\begin{aligned} \alpha_i &= \alpha_j =: \alpha \\ c_i &= c_j =: c \end{aligned}$$

for any i, j.

2) We will also restrict ourselves to *symmetric* strategies. This makes the resolution easier and can to some extent be justified on the grounds of symmetric conditions, although of course symmetric parameters do not dispel the possibility of the existence of non-symmetric NE.

7.4.3 Analytical resolution

By definition, in an *explicit* collusion situation, prices are set jointly so as to maximize the aggregate profit. Hence in a symmetric setting we can expect each CSD's individual profits explicitly colluding to be usually higher than in a competitive equilibrium. Nevertheless, because of demand elasticity it could happen that such profits are just equal to, and no greater than, the ones made in the competitive equilibrium. Lemma 3 below gives the condition for which profits are *strictly* higher, as this will be needed for the proof of Theorem 12.

Lemma 3 *Assume the same current unit costs for both CSDs, that is, $\widetilde{c}_i = \widetilde{c}_j =: \widetilde{c}$. Then the one-stage profit in a (explicit) price collusion of the price-setting game is greater than the (unique) NE of the price setting game if, and only if,*

$$\alpha\left(\frac{1}{2(\gamma - \gamma')^{\frac{3}{2}}} - \frac{\sqrt{\gamma}}{2\gamma + \gamma'}\right) > \left(\frac{1}{2}\frac{1}{\sqrt{\gamma - \gamma'}} + \frac{\sqrt{\gamma}(\gamma\gamma' - 2\gamma^2 - \gamma'^2)}{4\gamma^2 - \gamma'^2}\right)\widetilde{c} \qquad (C)$$

Proof: We assume the same current unit costs for both CSDs. This would be the consequence, for example, of same individual unit-cost as implied by our symmetric assumption *and* of the same degree of prior reshaping. Then $\widetilde{c}_i = c_{T2S} + (1 - b_i)c = c_{T2S} + (1 - b_j)c = \widetilde{c}_j =: \widetilde{c}$, and profits in the price setting stage obtained **under competition** can be expressed as:

$$\pi_i^* = (A + B\widetilde{c}_j - D\widetilde{c}_i)^2 = (A + (B - D)\widetilde{c}_i)^2 =: \pi^*.$$

with

$$\begin{aligned} A &= \frac{\alpha\sqrt{\gamma}}{2\gamma + \gamma'} \\ B &= \frac{\gamma^{\frac{3}{2}}\gamma'}{(4\gamma^2 - \gamma')} \\ D &= \frac{\sqrt{\gamma}(2\gamma^2 - \gamma'^2)}{4\gamma^2 - \gamma'^2} \end{aligned}$$

Let's assume **price-collusion** in one of the stage price setting game (note this cannot result in a NE of the one-period game). Because we are only looking at symmetric equilibria, this is tantamount to looking at identical prices $p_i = p_j = p$ that a monopoly consisting of both CSDs would set. Profits to each of the CSD for setting a common price p is

$$\pi = (\alpha + (\gamma - \gamma')p)(p - \widetilde{c})$$

The maximum is obtained for

$$p = \frac{\widetilde{c}}{2} + \frac{\alpha}{2(\gamma - \gamma')}$$

This results in an individual profit of:

$$\pi = \frac{1}{4}\frac{1}{\gamma - \gamma'}(\frac{\alpha}{\gamma - \gamma'} - \widetilde{c})^2$$

Now, profits can be higher in the price-collusion case if, and only if,

$$\pi > \pi^*$$

This can be re-written as:

$$\frac{1}{2}\frac{1}{\sqrt{\gamma - \gamma'}}(\frac{\alpha}{\gamma - \gamma'} - \widetilde{c}) > A + (B - D)\widetilde{c}$$

Replacing A, B and C by their value and re-arranging yields precisely the condition (C).

Remark 11 *If we denote by p^* the price corresponding to price competition, we can express the difference of prices of p^* and p, as*

$$p - p^* = \frac{\gamma'}{2(2\gamma + \gamma')}\widetilde{c} + \frac{\alpha\gamma'}{2(\gamma - \gamma')(2\gamma - \gamma')}$$

Hence, the higher the unit-cost, the higher the potential price charged by the de facto monopoly compared to the competitive equilibrium price. p is here the price which maximizes profits in a monopoly. Nevertheless as in what follows the (tacit) collusion is obtained not by assuming a monopoly setting prices but by indefinitely repeating the price-setting game and punishing the other CSD in case it deviates by playing the competitive price, and any price between p^* and p can be sustained in the equilibria we will define.

Theorem 12 *Assume that $\widetilde{c} := c_{T2S}$ satisfies (C). Let p be any price strictly higher than $p^* = \frac{\alpha}{2\gamma - \gamma'} + \frac{\gamma c_{T2S}}{2\gamma + \gamma'}$ but no greater than $\frac{c_{T2S}}{2} + \frac{\alpha}{2(\gamma - \gamma')}$. Then for discount factors high enough, there exists a subgame perfect Nash equilibrium where CSDs collude in prices after reshaping.*

Proof: Let $p \in]\frac{\alpha}{2\gamma - \gamma'} + \frac{\gamma c_{T2S}}{2\gamma + \gamma'}; \frac{c_{T2S}}{2} + \frac{\alpha}{2(\gamma - \gamma')}]$. Consider the following strategy, that we denote by (S_{col}): *"If you have not reshaped yet, then fully reshape immediately. Then, if someone has not reshaped yet then play p^* in each of the subsequent price-setting stage. Otherwise play p as long as everyone plays p, and play p^* indefinitely otherwise."*

We will prove (S_{col}) is a subgame-perfect Nash equilibrium. We need first the following Lemma:

Lemma 4 *For high enough discount factors, a given CSD will fully reshape at the beginning of the game whether it plans to follow the strategy (S_{col}) or to defect. That is, it will always choose to reshape with a degree of reshaping equal to 1 in the first period of the game.*

Proof of Lemma: Let b be the degree of reshaping which maximizes total profits of CSD i in a game where price-collusion is assumed to happen.

$$\begin{aligned} \pi_i^{tot} &= \widetilde{\delta}(A + ((1-b)c_j + c_{T2S}))B - ((1-b)c_i + c_{T2S}))D)^2 \\ &= \widetilde{\delta}(A + ((1-b)c + c_{T2S}))(B - D))^2 \end{aligned}$$

by symmetry. But

$$\begin{aligned} D - B &= \frac{\sqrt{\gamma}}{4\gamma^2 - \gamma'^2}(2\gamma^2 - \gamma'^2 - \gamma\gamma') \\ &= \frac{\sqrt{\gamma}}{4\gamma^2 - \gamma'^2}(\gamma^2 - \gamma'^2 + \gamma(\gamma - \gamma')) \end{aligned}$$

with $\gamma^2 - \gamma'^2 \geq 0$ and $\gamma - \gamma' \geq 0$ since $\gamma \geq \gamma'$ by assumption. Hence $D - B \geq 0$ and π_i^{tot} is maximum for $b = 1$.

Now assume a given CSD wants to deviate from the strategy. Because the CSD non-deviating from it will then switch to the non-cooperative mode (playing the competitive price equilibrium of each subsequent price-setting game), the optimal degree of reshaping is precisely the one already derived in the corpus of this paper. When δ tends to 1, $\widetilde{\delta} = 1/(1-\delta)$ tends to $+\infty$ and so $\xi_i < \widetilde{\delta} c_i^2 D_i^2$. Hence, by Lemma 1:

$$b^* = 1$$

This completes the proof of Lemma 4.

We now resume the proof of Theorem 12. Let H be a subgame of the game. Let always be i the CSD playing on the considered subgame H according to the strategy (S'_{col}) induced by (S_{col}) on H.

If H is **on** the equilibrium path, it can only be of one of the following type:

- type A: H contains the first stage *idem est* H is the whole game. Then, by Lemma 4, playing $b_j = 1$ is optimal for CSD j. After the reshaping phase, because CSD i is following (S_{col}), we are left with a simple infinitely repeated game where one of the player (CSD i) is playing a simple grim trigger strategy to promote playing $p > p^*$. Because condition (C) ensures playing p indeed results in *strictly* higher payoffs than playing the competitive equilibrium price p^* for each of the stage game, the trigger strategy is a NE on this subgame for high-discount factors. Hence the best-response answer, for high enough discount factors, is for CSD j to play p too, and punish i if it deviates. Overall, CSD j is thus also conforming with (S_{col}) on H.

- type B: H does not contain the first stage. Because H lies on the equilibrium path by assumption the reshaping for both CSDs has already occurred in the first stage of the game. Hence H again only consists of the price-setting game being repeated an infinite number of times. Condition (C) coupled with high enough discount factors imply the simple grim trigger strategy induced by the strategy (S_{col}) on the subgame H is a Nash equilibrium.

If H is **off** the equilibrium path, it can be of the following type:

- type C: the history leading to H is one where the CSD i has not reshaped yet then since we assume i will follow the strategy (S'_{col}) induced by (S_{col}) on H, CSD i will immediately reshape with $b_i = 1$. Discarding the history leading to H gives a game identical to the whole game, that is, to the type A subgame. The same reasoning applies and CSD j's best-response is again to conform with (S_{col}).

- type D: the history leading to H is one where the CSD i has reshaped. Then either j has reshaped at the *same* point in time and played p from that point in time till now, and then i's strategy is a grim-trigger strategy for which the strategy induced by (S_{col}) on H is again j's, or not. If j has played another move than the one advocated by (S_{col}) during the history leading to H then i is playing the competitive price equilibrium p^* of the price-setting stage indefinitely. Clearly, j's best-response is p^*, since p^* is the unique Nash equilibrium of the stage game. This achieves the proof of subgame perfection.

Remark 13 *The following strategy also sustains the price-collusion Nash equilibrium, and leads to the same equilibrium path: "Reshape in the first period of the game completely. If everyone has reshaped in the first period and played until now p in the price-setting game then play p. Otherwise play p^*." The proof is somehow simpler than for the strategy (S_{col}) we chose since it is clear Type C and D collapse together. Nevertheless the strategy (S_{col}) is more adapted to allow a delay in the decision to reshape. Also, as soon as p is chosen close to the optimal $p^* = \frac{\widetilde{c}}{2} + \frac{\alpha}{2(\gamma-\gamma')}$ where p^* is the price-setting stage game Nash equilibrium for current costs $\widetilde{c}$, there is little incentive for CSDs to delay the reshaping: for high discount rates a Pareto-dominating subgame-perfect Nash equilibrium is indeed to reshape immediately and completely, while price-colluding on each subsequent price-setting stage by playing exactly $p(\widetilde{c}) = \frac{\widetilde{c}}{2} + \frac{\alpha}{2(\gamma-\gamma')}$ for $\widetilde{c} = c_{T2S} + c$. Thus, the unit-cost reductions obtained from reshaping are partly absorbed by the CSDs themselves and not passed to the market. Because $\widetilde{c}$ decrease with reshaping, the market still has a decreased colluding price compared to a (hypothetical, not modelled here) situation of prior price-collusion with higher costs. If the prior situation was closer to $p^*(c)$, the prices may increase as a result of the new price-collusion.*

7.5 Discussion on the effect of greater competition on the relevance tacit collusion theorem

The model captures competition effects through the cross-elasticities γ_{ij}, which reflect a (fixed) substitution effect assumed between the two different markets. Indeed, as noticed earlier, for $\gamma_{ij} < \gamma_{jj}$, an increase of one unit of the price p_j leads to a fall of γ_{jj} of volume of CSD j, but to an increase of volume of γ_{ij} for CSD i. In the extreme case where $\gamma_{ij} = \gamma_{jj}$, an increase of one unit of the price p_j leads to no fall in the aggregate volume of settlements, which can be interpreted as "the whole demand for settlement services having left market j due to the price increase moves to market i". Hence, the closer γ_{ij} is to γ_{jj}, the greater the substitution effect. Now, it is interesting to wonder if a greater competition (substitution) effect invalidates the tacit collusion theorem. That is, does greater competition decrease the chances of tacit collusion to avoid reshaping? Interestingly, the answer is that it does quite the opposite: the distribution of individual CSDs' costs satisfying the condition of the tacit collusion theorem 2 becomes larger with greater competition. To show this, let us for convenience assume

$$\gamma_{ij} = \gamma_{kl} =: \gamma'$$

for any $i \neq j$ and $k \neq l$, and

$$\gamma_{ii} = \gamma_{jj} =: \gamma$$

for any i, j. The condition under which there is a subgame perfect Nash equilibrium in which CSDs indefinitely delay the moment of their reshaping, i.e. tacitly collude not to reshape, is:

$$\frac{1}{f_j} < \frac{c_i}{c_j} < f_i$$

with

$$f_i = f_j = \frac{\gamma\gamma'}{2\gamma^2 - \gamma'^2}$$

Hence the condition becomes, if we let $z = \frac{\gamma'}{\gamma}$

$$\frac{2}{z} - z < \frac{c_i}{c_j} < (\frac{2}{z} - z)^{-1}$$

Now as explained previously, greater competition is translated by the model parameter γ' coming closer to γ. That is, by an increase by some factor $a > 1$ of the ratio $z = \frac{\gamma'}{\gamma}$, which becomes $az > z$ (with, of course, $az < 1$). The range of admissible ratios $\frac{c_i}{c_j}$ actually becomes larger when z increases to az, because $\frac{2}{az} - az < \frac{2}{z} - z$, as well as $(\frac{2}{z} - z)^{-1} < (\frac{2}{az} - az)^{-1}$. This is because $\frac{2}{az} - az < \frac{2}{z} - z$ is equivalent to $2(\frac{1-a}{a}) < z^2(a-1)$ which is trivially true for $a > 1$ since $2(\frac{1-a}{a}) < 0 < z^2(a-1)$. This concludes the proof.

Therefore, the set of parameters for which tacit collusion is sustainable in a high competition / substitution environment is larger than in a low competition / substitution environment.

7.6 Graphs from simulations of Section 2.4.4

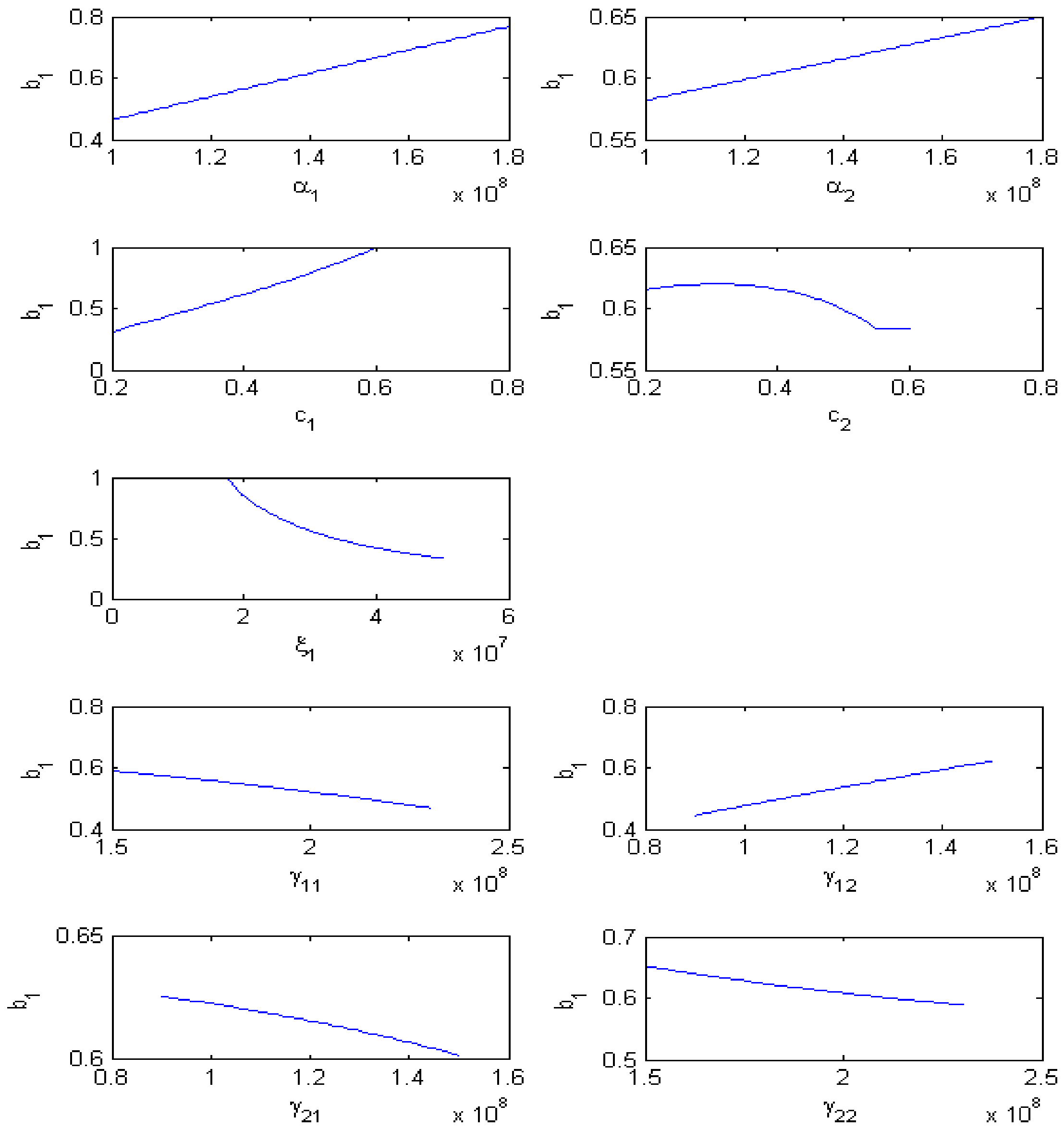

Figure 1: Two-dimensional graphs of the optimal reshaping function of CSD 1

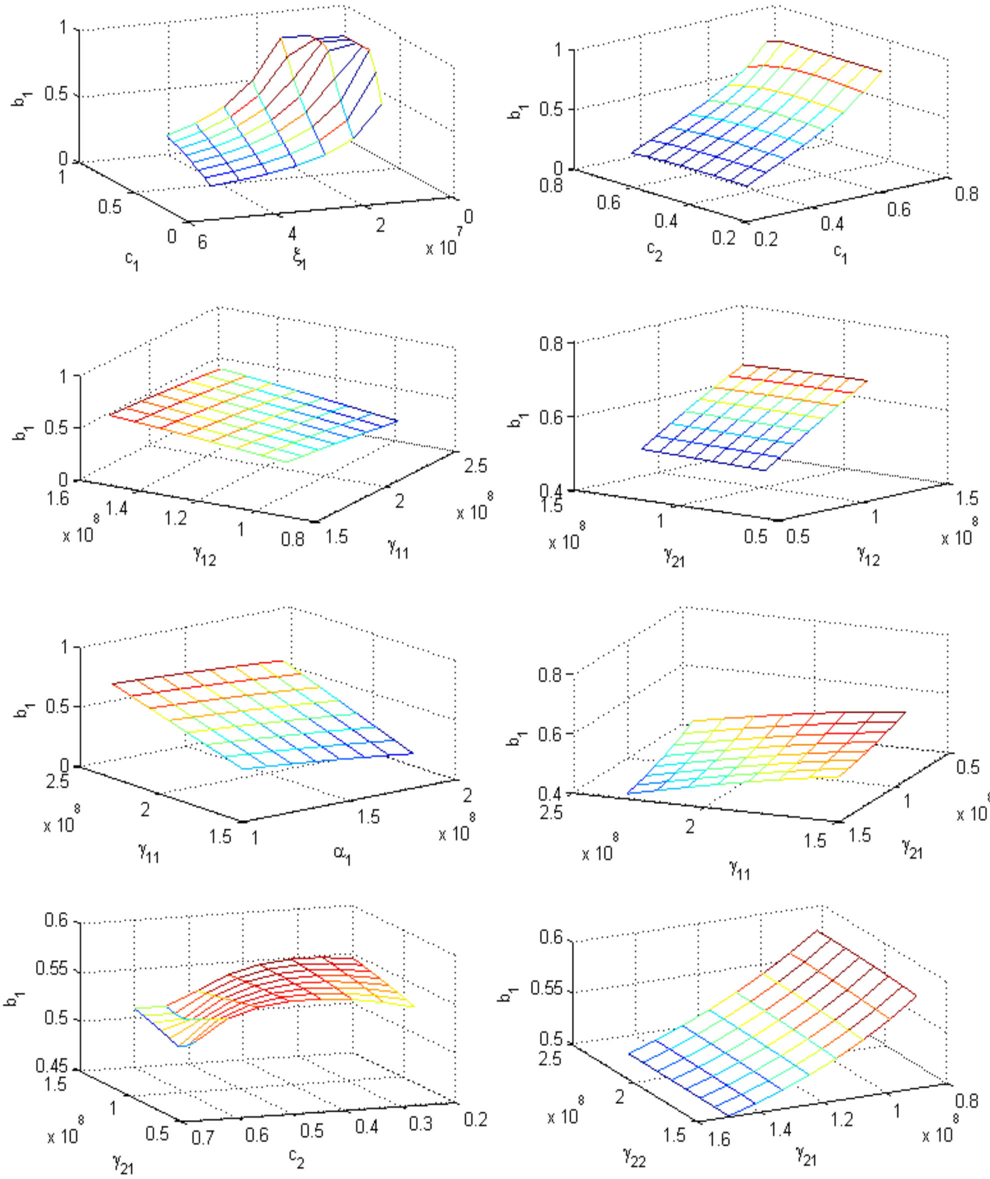

Figure 2: Three-dimensional graphs of the optimal reshaping function of CSD 1

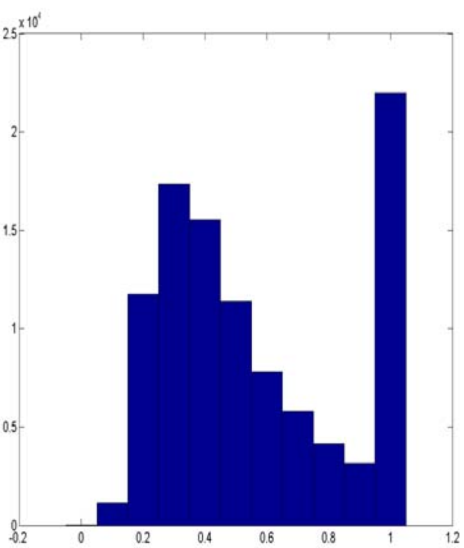

Figure 3: Histogram of the optimal reshaping function of CSD 1

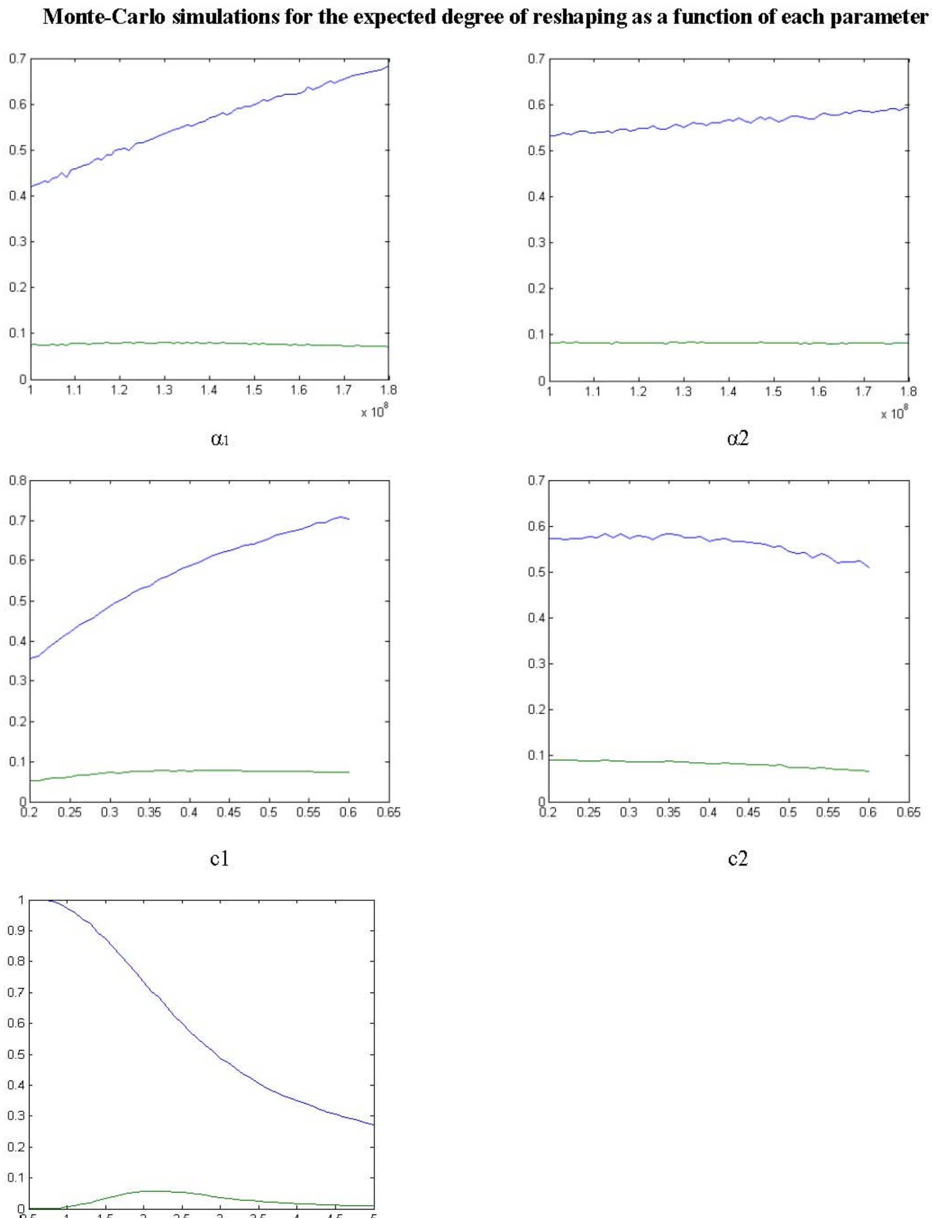

Figure 4: Average value (top line of each graph) and variance (bottom line of each graph) of the degree of reshaping (10000 simulations for each point)

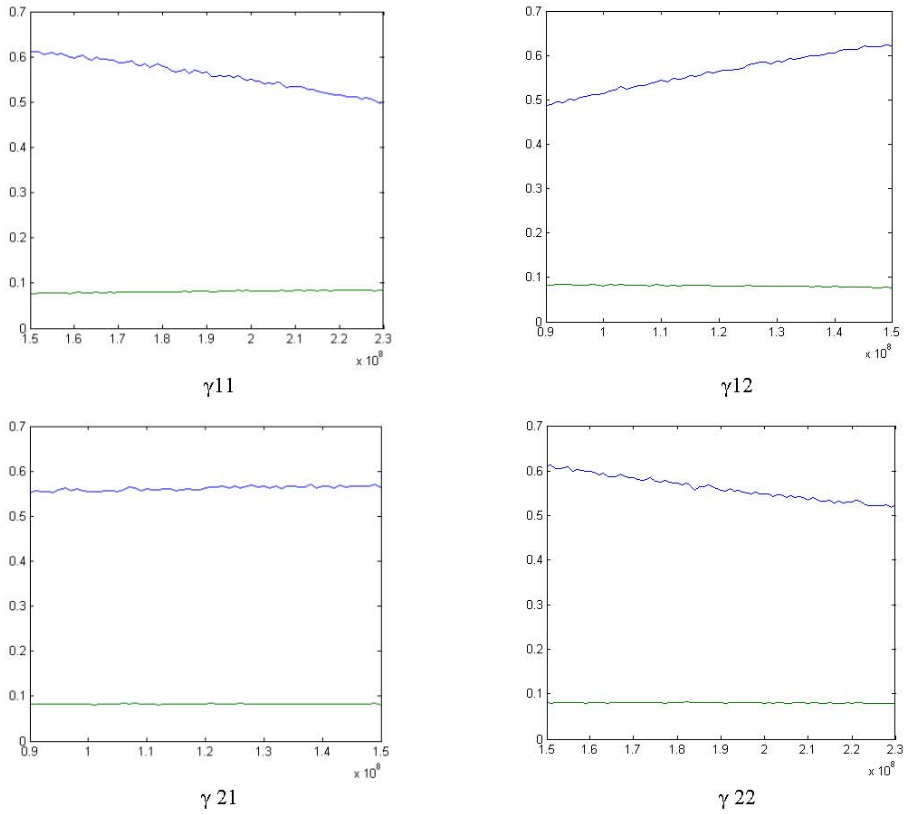

Figure 5: Average value (top line of each graph) and variance (bottom line of each graph) of the degree of reshaping (10000 simulations for each point)