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STRATEGIC SELECTION OF RISK MODELS AND BANK CAPITAL REGULATION

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In 2014 all ECB publications feature a motif taken from the €20 banknote.

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Abstract

The regulatory use of banks' internal models aims at making capital requirements more accurate and reducing regulatory arbitrage, but may also give banks incentives to choose their risk models strategically. Current policy answers to this problem include the use of risk-weight floors and leverage ratios. I show that banks for which those are binding reduce their credit supply, which drives interest rates up, invites other banks to adopt optimistic models and possibly increases aggregate risk in the banking sector. Instead, the strategic use of risk models can be avoided by imposing penalties on banks with low risk-weights when they suffer abnormal losses or bailing out defaulting banks that truthfully reported high risk measures. If such selective bail-outs are not desirable, second-best capital requirements still rely on internal models, but less than in the first-best.

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Keywords: Basel risk-weights, internal risk models, leverage ratio, tail risk.

Non-technical summary

The prudential regulation of banks makes an important use of risk measures produced by the banks' internal risk models, in particular in the regulatory framework developed by the Basel Committee on Banking Supervision, where the risk weights associated to different assets can be computed using an internal model approved by the regulator. A topic of particular interest is the hypothesis that some banks may strategically choose risk models so as to get more favorable risk weights and economize on capital, especially when they are expected to cope with higher capital ratios.

The goal of this paper is to build a flexible theoretical framework to derive empirical implications about strategic model selection and discuss various policy options, taking into account several realistic constraints. It is shown in particular that some proposed tools such as floors or leverage ratios can have counter-intuitive equilibrium effects, which may limit their usefulness to avoid the strategic use of risk models.

I use a simple equilibrium model where banks are intermediaries between depositors and final borrowers. Due to deposit insurance and limited liability, banks tend to take on too much risk, which a benevolent regulator tries to avoid by imposing capital requirements. The optimal capital requirement should depend on the riskiness of the loans made by the bank, which is evaluated by an internal risk model, i.e. a probability distribution over the different levels of default that can arise in a portfolio of loans. Several models could be true in this economy. Banks have more information than the regulator about which models are more realistic, which is the reason why the regulator wants to use the banks' expertise in the first place, giving rise to an asymmetric information problem.

I first study a stylized model of the current regulatory situation, where banks choose which risk model to apply, send risk measures to the regulator based on this model, and are then imposed a capital requirement depending on these measures. Banks have an incentive to use more optimistic risk models to save on capital, but this will depend on the equilibrium interest rate they can charge on loans.

The joint determination of prices and model choice in equilibrium leads to a counter-intuitive effect of the regulation. If the regulator is concerned that some banks are using optimistic models to bypass capital requirements and uses a model-independent floor on capital ratios (e.g. a leverage ratio), loan supply and risk will decrease for banks constrained by the new floor. But this implies that interest rates on loans increase, inviting more conservative banks to step in and report optimistic risk measures in order to lend up to the maximum. If demand elasticity on the credit market is not too high, risk in the banking sector can counter-intuitively increase.

While model-independent floors on capital requirements can be useful, they are not a very natural solution to the asymmetric information problem between banks and regulators. I study a mechanism relying on ex post penalties imposed on banks that suffer high losses after having reported low risk-weighted assets. This is a powerful tool to make banks report their risk measures truthfully.

Several constraints have to be taken into account however. One of them is the interaction between the banks' limited liability and the fact that different risk models may differ mostly in their predictions for extreme levels of losses. A bank may already be in default when the regulator realizes that the risk weights it reported were too optimistic. It is then necessary to ensure a positive payoff to shareholders of a defaulting bank that reported conservative risk measures. The cost of this transfer can be recouped via a tax on the bank when it survives.

Such positive transfers have appealing properties in an asymmetric information context, but may have other undesirable properties or not be credible. If such bail-outs are not feasible, then ensuring the use of adequate risk models comes at the cost of an informational rent left to banks. In order to reduce this rent, it is optimal to implement capital requirements that are higher or less risk-sensitive than if the regulator always knew the true risk model.

Some currently debated policies aim at reducing the reliance of the Basel III framework on internal models. Unless the information contained in these models is already available to the regulator, this necessarily comes at the cost of a less risk-sensitive regulation. A more ambitious avenue would be to keep using internal models to make the regulation more risk-sensitive, while giving bank supervisors more tools to ensure the figures reported are unbiased. Such tools can be costly, so that the optimal solution depends on a fine trade-off between risk-sensitivity and costs, among others. The present paper develops a simple framework in which several policy options can be compared under various realistic constraints.

1 Introduction

Many examples during the recent crisis revealed that the risk models used by financial institutions often made them blind to extreme risks. Dowd *et al.* (2008) illustrate in the case of market risk the extent to which some older models used in practice were flawed: "25-sigmas events" happened several times in a row in August 2007, although they were supposed to occur once in every 10^{135} years.

The regulation of banks relies heavily on internal models to compute risk-sensitive capital requirements, which may bias the development of risk models. This concern, expressed for instance by Danielsson (2008), is shared by investors, as shows a recent study by Barclays Equity Research (Samuels, Harrison, and Rajkotia (2012))[1]: more than half of the investors surveyed do not trust risk weightings, 80% think the way the banks' risk models work is a significant driver of major differences between European banks' risk weightings, and even more think model discretion should be removed.

While the problem of strategic selection of risk models is now recognized, there is no consensus on how to solve it. The present paper offers a tractable analytical framework to study how models are strategically chosen. I show in particular that current policy reforms such as using non risk-weighted leverage ratios[2] can sometimes have unintended consequences. I then derive second-best solutions under realistic constraints and compare various policy options. The framework relies on an equilibrium model of the credit market, where banks face a capital constraint that depends on the risk measures they report to the regulator, who may also impose ex post penalties on misreporting banks. Importantly, both regulatory choices and market prices drive the banks' decisions to adopt particular risk models.

I first analyze the equilibrium choice of risk models in a stylized representation of the current regulatory environment, and focus on the impact of adding non risk-based regulatory constraints to capital requirements based on internal models. Proposition 2 shows that such a policy can actually *increase* the risk that a bank defaults. Banks using optimistic

[1] See also "Investors lose faith in risk measures" by B. Masters, *Financial Times*, 24.05.12.

[2] See the joint press release of the Federal Reserve Board, the FDIC and the OCC on July 9, 2013: http://www.federalreserve.gov/newsevents/press/bcreg/20130709a.htm. Bundesbank's Vice-President S. Lautenschläger expressed a more skeptical view of such tools on October 21, 2013: http://www.bundesbank.de/Redaktion/EN/Reden/2013/2013_10_21_lautenschlaeger.html.

models to get lower capital requirements can lend less if they face an additional constraint that they cannot bypass by using such a model. Since regulation is studied in a market equilibrium context, the interest rate on loans increases, so that banks that were previously more cautious have higher incentives to also use optimistic models and supply the loans that their constrained competitors no longer provide. Due to the wider bypassing of the capital requirements constraint, the average default risk of banks can increase. This is a simple yet important caveat for current reforms such as the Collins amendment in the United States, floors based on Basel II's standardized approach or leverage ratios as complements to capital requirements. The analysis also delivers new testable predictions about the choice of risk models, and shows that counter-cyclical capital ratios can turn out to be pro-cyclical.

To restore investors' confidence in risk weights, regulators need to solve a hidden information problem. In the second part of the paper, I study a mechanism where penalties punish banks using optimistic models when unlikely levels of losses occur and show how to reach the first-best outcome (Proposition 4). In contrast to the previous literature, I then explicitly introduce the possibility that internal models differ mainly in their predictions about tail risk. In such a case, a bank can report a very optimistic model and be allowed a high leverage. The regulator cannot learn that a model was too optimistic unless losses in the tail materialize, in which case the highly leveraged bank is in default and cannot be punished. The first-best outcome can still be reached by ensuring a positive payoff to the shareholders of a defaulting bank that truthfully reported high risk weights ex ante (Proposition 5). A commitment to bail out troubled banks that reported conservative risk measures early enough, but not the ones that reported low risk weighted assets and are thus likely to have artificially deflated them, is a powerful tool to avoid misreporting. The costs of the bail-out can be covered (on average) via taxing some of the profits banks make in good states of the world.

Such bail-outs may have other undesirable properties however[3]. If they are not feasible, the second best solution is either to have capital requirements so high that all types of banks can be punished, or to leave an informational rent to banks reporting conservative risk mea-

[3]Moreover, it is important for the mechanism to work that the regulator can commit to bailing out only banks that seem to have honestly reported their risk measures. A credible alternative must thus be available to deal with the others, which is the general problem of dealing with the resolution of large and systemic financial institutions, for instance through bail-ins.

sures and decrease the risk-sensitivity of capital requirements to reduce this rent. Which solution is optimal depends on the regulator's information about risk models and the cost of public funds.

The problem can be illustrated with the example of credit risk. To compute the capital requirement to be held for a corporate, sovereign or bank exposure, a bank under the Basel II or the Basel III framework can opt for the "advanced-internal ratings based approach", in which case its internal models will be used to produce inputs that will enter a regulatory formula defining minimum capital requirements. Tarashev (2008) compares for different classes of bonds the regulatory capital obtained with different academic models. On a sample of BBB-rated bonds for instance, a bank choosing the most optimistic model would have to keep 18% less capital than with the most pessimistic one.

Such differences can give rise to a strategic selection of models. The paper derives empirical implications about this selection (Implications 1 and 2). An exogenous increase in the demand for banks' loans may cause more banks to use optimistic models, as proxied e.g. by the proportion of assets for which banks choose to adopt the "internal ratings based approach" (IRB) of Basel instead of the "standardized" approach (SA). Tightening regulatory constraints, both in the regulated sector or in a competing "shadow" banking sector, has the same impact. Finally, good candidate models to bypass the regulation are those which are proven to be over-optimistic only when the situation is so bad that the regulator cannot be too harsh on banks, leading to a form of herding as in Acharya and Yorulmazer (2008).

The analysis can be applied to all types of risks that are regulated using internal models, but credit risk may be particularly relevant. Credit risk models are extremely difficult to backtest due to their time horizon (typically one year) and the scarcity of available data. Advanced tests can be used (Lopez and Saidenberg (2000)), but the problem of insufficient data cannot be completely bypassed. An additional problem, evidenced by Rajan, Seru, and Vig (2013), is that banks have incentives to extend loans to borrowers whose risk characteristics are poorly reflected in risk weights, so that existing models endogenously become too optimistic[4].

[4]The same problem also affects approaches that do not rely on internal models.

With some amendments, the hidden information framework developed here may be used to study other situations where an agent may strategically use internal models: for instance stress-testing exercises, the assessment of a pool of loans by a credit rating agency, the compensation of a trader or a division based on risk estimates.[5]

Related literature. Debates surrounding the introduction of internal measures for the regulation of market risk in the 1996 Basel amendment triggered an early literature on possible mechanisms to ensure that banks would report their risk measures truthfully: Kupiec and O'Brien (1995), Lucas (2001), and later Cuoco and Liu (2006) and Marshall and Prescott (2006) simulated penalty-based mechanisms to show they could reduce regulatory arbitrage. My framework is somewhat simpler so as to be more tractable, which allows to derive analytically what are the *optimal* mechanisms under a variety of realistic constraints.

Blum (2008) derives an optimal regulation in a simple framework of strategic model selection, and concludes to the necessity of a leverage ratio as a complement to the Basel approach. The present paper arrives at an opposite conclusion however. In Blum (2008) there are two types of banks and two levels of loan repayment. What is optimal in that model is actually to have higher capital requirements than first-best for banks reporting low-risk measures. With two types only there is no difference between this policy and a constraint on the leverage ratio, but the conclusion does not hold with more than two types. The present paper offers a richer description of model uncertainty, which leads to quite different results about the optimal regulation. Moreover, as the asymmetric information problem is embedded in a market equilibrium model, substitution effects between banks, leading to unintended consequences of non-risk based ratios, can be taken into account.[6]

An additional contribution of the present paper is to explicitly introduce asymmetric information about tail risk in a model of bank regulation, and interact it with limited liability. A few recent papers have also looked at the regulation of tail risks specifically, but under moral hazard, in particular Biais *et al.* (2010), and Perotti, Ratnovski, and Vlahu (2011).

A recent paper by Mariathasan and Merrouche (2012) empirically supports the hypothesis

[5] The "London Whale" is a good illustration, see for instance "JP Morgan manipulated VAR and CRM models at London whale unit - Senate report" by D. Wood, *Risk Magazine*, 15.03.13.

[6] Other models of competition between leveraged banks include Herring and Vankudre (1987), Matutes and Vives (2000) and Bolt and Tieman (2004).

that risk models are used strategically: banks switching to the IRB approach are more likely to improve their capital ratios when they have low capital and are less tightly supervised, that is when their incentives for misreporting risk measures seem higher. The present paper suggests to also look at regulatory tightenings and market developments as possible determinants of the use of internal models. Other empirical studies such as Carey and Hrycay (2001) and Jacobson, Linde, and Roszbach (2006) show the potential for regulatory arbitrage offered by internal models, but no direct evidence of such arbitrage. Berkowitz and O'Brien (2002) and Perignon and Smith (2010) show that VaRs reported for market risk were actually *too conservative*, implying that the penalty mechanism for market risk has an impact and that penalties may actually be too high. Finally, two recent studies by the Basel Committee on Banking Supervision (BCBS (2013a), BCBS (2013b)) show the dispersion of risk weights across banks and conclude that part of it is indeed due to different modeling assumptions[7].

Closely related are also papers studying the choice between Basel's SA and IRBA, such as Repullo and Suarez (2004), Ruthenberg and Landskroner (2008), Antao and Lacerda (2011), Hakenes and Schnabel (2011) and Feess and Hege (2011). The latter two are the most related as they show interesting interactions between the two regulatory options in a market equilibrium model, but their focus is not on model choice inside IRB.

This paper finally contributes to the wider question of understanding how economic agents choose (biased) models. Hong, Stein, and Yu (2007) for instance study agents relying on partial models and shifting from one to the other depending on their observations. Few papers look at situations where the demand for models is not directly derived from their predictive power, an exception being Millo and MacKenzie (2009) who study the usefulness of simple risk management models for internal communication.

The remainder of the paper is organized as follows: section 2 develops the general framework; section 3 derives empirical implications from a stylized representation of the current regulation; section 4 studies a regulation based on penalties and discusses policy implications; section 5 discusses robustness questions and extensions that are made available in a separate Internet Appendix.[8] All figures are in the Appendix A.8.

[7]See also EBA (2013).

[8]Available at http://sites.google.com/site/jecolliardengl/RB-Appendix.pdf.

2 Framework

Three elements are needed to study the strategic choice of models: regulated financial intermediation, model uncertainty and asymmetric information between the intermediaries and the regulator. The assumptions and possible extensions are discussed in section 5.1.

Agents and assets. -*Borrowers* need to finance risky projects which can either succeed or fail. The gross return on the nth project is $\rho(n)$ if it is successful, where $\rho(.)$ is a strictly decreasing function. A random proportion t of projects will fail, where t follows a distribution defined below. Failed projects yield 0, leading to the borrower's default. For a given gross interest rate r, the demand for loans is denoted $D(r)$. By definition we have $D = \rho^{-1}$, and I assume $D(1) = +\infty$ and $\lim_{+\infty} D(r) = 0$.

-*Depositors* have a large initial wealth W that they can invest in a safe asset yielding the exogenous risk-free rate r_0 with certainty, or lend to financial intermediaries, but not directly to borrowers. Their deposits are assumed to be fully insured, so that they provide an elastic supply of deposits at the gross rate r_0.

-*Intermediaries* (banks) initially own K (equity) and can borrow M from depositors at r_0. They choose a quantity L to lend to borrowers at rate r, and can also invest in diversified activities yielding the safe rate r_0. They are protected by limited liability and face capital requirements. Intermediaries investing only at r_0 will be seen as sticking to non credit banking activities, or "safe" activities.

There is a continuum $[0, 1]$ of each type of agents, all risk-neutral and price-takers on a competitive market. The capital requirements are set by a benevolent *regulator* whose behavior is described below. Throughout the paper a female pronoun refers to the regulator, and a male pronoun to an intermediary.

Model uncertainty. There exists a family of cdfs $\{F(., \tilde{\sigma}), \tilde{\sigma} \in [\underline{\sigma}, \overline{\sigma}]\}$, with support over $[0, 1]$, which represents the set of plausible risk models to describe the distribution of t, the proportion of defaulting borrowers. This family of cdfs indexed by $\tilde{\sigma}$ can be interpreted as one model with different parameterizations, or models from different families. Denote $\{f(., \tilde{\sigma})\}$ the corresponding pdfs.

The correct risk model of the economy, σ, is randomly selected by nature in $[\underline{\sigma}, \overline{\sigma}]$ according to some cdf $\Psi(.)$, with associated density $\psi(.)$. The actual proportion t of defaulting borrowers then follows the distribution $F(t, \sigma)$.

It will be convenient to assume that F is twice-continuously differentiable in both arguments. Moreover, for some results it will be useful to assume that the family $\{F(., \tilde{\sigma})\}$ satisfies the monotone likelihood ratio property, which implies in particular that models with a low $\tilde{\sigma}$ give risk estimates unambiguously more optimistic than models with a high $\tilde{\sigma}$:

$$\forall t_0, t_1, \sigma_0, \sigma_1 \text{ with } t_1 \geq t_0,\ \sigma_1 \geq \sigma_0,\ \frac{f(t_1, \sigma_1)}{f(t_1, \sigma_0)} \geq \frac{f(t_0, \sigma_1)}{f(t_0, \sigma_0)} \tag{MLRP}$$

Regulation. For a given σ, total welfare in the economy can be written as:

$$\mathcal{V}(L, \sigma) = \underbrace{r_0 W}_{\text{Investors}} \underbrace{+\mathbb{E}_\sigma(1-t)\left(\int_0^L \rho(u)du - rL\right)}_{\text{Borrowers}} + \underbrace{\int_0^1 [rL(1-t) - r_0(L-K)]f(t,\sigma)dt}_{\text{Intermediaries and deposit insurer}} \tag{1}$$

This is maximized when the interest rate is equal to the break-even rate $r^e(\sigma)$:

$$\rho(L) = r^e(\sigma) = \frac{r_0}{\mathbb{E}_\sigma(1-t)} \tag{2}$$

where the operator $\mathbb{E}_\sigma$ denotes an expectation according to the distribution $F(., \sigma)$. Due to limited liability and the absence of market discipline exercised by depositors, banks will borrow and lend too much, as they do not take into account the losses to the deposit insurer and thus maximize $\int_0^1 \max(0, rL(1-t) - r_0(L-K))f(t,\sigma)dt$. This model fits the classical "representation hypothesis" (Dewatripont and Tirole (1994)) according to which the regulator represents the interests of the deposit insurance fund, which itself substitutes for uninformed and scattered depositors who could not efficiently exercise market discipline.

The core of the Basel approach is to control excessive risk-taking with capital requirements. Given that the optimal amount of loans depends on the true level of risk, capital requirements should be risk-sensitive and, in this framework, model-sensitive. Under complete information, the regulator would use a menu of capital requirements $\alpha(.)$ such that when the true model is σ an intermediary has to satisfy the constraint $K/L \geq \alpha(\sigma)$. α will be interpreted as a minimum capital requirement, but equivalently $\alpha(\sigma)L$ times some constant can be seen as risk-weighted assets, with risk-weights computed according to the bank's internal model. I will assume throughout the paper that $D(r^e(\overline{\sigma})) > K$, so that for all values of σ it would be suboptimal to require all intermediaries to be unleveraged.

The purpose of using the banks' internal models for determining capital ratios is to rely on their supposedly better knowledge of risk models, which depends here on the width of the interval $[\underline{\sigma}, \overline{\sigma}]$ and on Ψ.[9] This is by definition a situation of asymmetric information: the regulator would like to set capital requirements based on better risk measures, but this gives incentives to banks to strategically misreport optimistic models. To avoid such a possibility, the current regulatory framework foresees a number of safeguards that can all be included in the following representation of the game between the supervisor and the intermediaries:

-T=0 The regulator specifies a formula linking any model σ to a capital ratio $\alpha(\sigma)$, as well as penalties $T(\sigma', t)$ to be paid by the intermediary if it uses model σ' and t defaults realize.
-T=1 The true model σ is drawn from $\Psi(.)$ and observed by intermediaries. They can remain unleveraged or report a model $\sigma' \in [\underline{\sigma}, \overline{\sigma}]$ at an arbitrarily small cost $\zeta > 0$. [10]
-T=2 The intermediary chooses a supply of loans L and a demand for deposits M maximizing profit, such that $K/L \geq \alpha(\sigma')$ and $L \leq M + K$, taking prices as given. r, M and L are simultaneously determined by competitive equilibrium conditions.
-T=3 A proportion t of borrowers default, drawn from $F(., \sigma)$. Payoffs are realized, intermediaries pay the penalties $T(\sigma', t)$.

This simple game incorporates most tools actually used by the regulator. The capital ratio $\alpha(.)$ links a bank's model to capital requirements, as for instance in the regulatory formula used for credit risk. $\alpha(.)$ can incorporate additional measures of the regulator, such as floors, regulatory multipliers or add-ons. Moreover, a prohibitively high $\alpha(\sigma')$ for a particular model means that model σ' fails to get supervisory approval, based on a comparison with "industry standards", required assumptions of the model, performance of the model on historical data and so on. The approval decision is thus embedded in the definition of $\alpha(.)$. Finally, the

[9] The regulator's prior is for instance very uninformative if Ψ is uniform.
[10] ζ is the cost of developing an internal model, and implies that such a model will be used only if it is a source of additional profit.

penalties T are a generalization of the penalty mechanism used for market risk models. The timeline of the model is summed up in the following figure:

[Insert Fig. 1 here.]

3 Model choice and market equilibrium

I first solve the model under the assumption that the regulator does not use any penalty mechanism. Such penalties are not used in practice for credit risk models. Rather, current responses to model optimism rely on floors on risk weights, which can be seen as constraints on the menu of capital requirements $\alpha(.)$. This section thus develops a positive analysis of the strategic adoption of overoptimistic models, and of the possible consequences of current reforms. Section 4 adopts a normative perspective and discuss other regulatory options.

3.1 The intermediary's program

Consider the program of an intermediary in $T = 2$, taking r_0, r as given with $r \geq r_0$, and facing a constraint $K/L \geq \alpha$. Since depositors ask for a return of r_0, it never pays off to borrow $M > 0$ from depositors and invest at r_0. Thus we have either $L = M + K$ with M possibly zero, or the intermediary invests his equity in safe activities and $L = M = 0$. Due to limited liability, his realized profit if he lends L and a proportion t of borrowers do not repay can be written as $\max\big(0, r(1-t)L - r_0 M\big)$. The intermediary cannot repay his debt if there have been too many defaults in his portfolio, that is if:

$$t > \theta(r, K/L), \text{ with } \theta(r, \alpha) = 1 - \frac{r_0}{r}(1-\alpha) \tag{3}$$

$\theta(r, K/L)$ is the maximum proportion of losses that an intermediary can sustain without defaulting. Denoting $\pi(L, \sigma)$ the intermediary's expected profit if he chooses $L > 0$, we have

$$\pi(L, \sigma) = \int_0^{\theta(r,K/L)} [r(1-t)L - r_0(L-K)] f(t, \sigma) dt \tag{4}$$

The Appendix A.1 shows the following lemma, due to the convexity of the profit function as a result of limited liability:

Lemma 1. *Denote* (M^*, L^*) *the solution, and* $\bar{r}(\alpha,\sigma) < r^e(\sigma)$ *uniquely defined by:*

$$r_0 = \frac{1}{\alpha}\int_0^{\theta(\alpha,\bar{r}(\alpha,\sigma))} [\bar{r}(\alpha,\sigma)(1-t) - r_0(1-\alpha)] f(t,\sigma)dt \tag{5}$$

If $r > \bar{r}(\alpha,\sigma)$ *then* $L^* = K/\alpha, M^* = K(1-\alpha)/\alpha$*; if* $r < \bar{r}(\alpha,\sigma)$ *then* $M^* = L^* = 0$*, the intermediary is indifferent between both solutions in case of equality.*

The intermediary uses the maximum leverage allowed by the regulation if r is high enough to compensate for the high risk of defaulting, and otherwise does not borrow but invests in safe activities only. The cut-off interest rate $\bar{r}$ is lower than $r^e(\sigma)$: the intermediary is ready to extend loans with negative net present values, as some losses are borne by the deposit insurance fund.

Example. Consider the following example, kept for illustration throughout the paper. $\{F(t,\tilde{\sigma})\}$ is a family of Beta distributions with parameters $a = 3.5$, $\tilde{\sigma} = 1/\tilde{b}$, $\tilde{b} \hookrightarrow \mathcal{U}([13, 50])$. Assume the true b is 31.5, $K = 1$, $r_0 = 1$, and $D(r) = \frac{1}{r-1}$.

Fig. 2 (left) plots an intermediary's profit as a function of L when defaults follow the true Beta distribution and $r = 1.107$, slightly lower than $r^e(\sigma) = 1.111$. The intermediary's choice depends on the regulatory constraint: if allowed $L \le K/\alpha(\sigma')$ (a leverage of 14 in the figure) he chooses maximum leverage, whereas with the tighter constraint $L \le K/\alpha(\sigma)$ (a leverage of 9) he does not invest in loans.

[Insert Fig. 2 here.]

3.2 Model choice

With no penalties an intermediary is free to report any model $\sigma' \in [\underline{\sigma}, \overline{\sigma}]$, anticipating he will have to face the constraint $K/L \ge \alpha(\sigma')$. The regulator still chooses optimally the menu $\alpha(.)$, anticipating the strategic choice of intermediaries. The situation is equivalent to a "delegation game" (Holmstrom (1977) and Alonso and Matouschek (2008)) in which banks are offered a set of attainable leverage ratios from which they can choose.[11]

Intermediaries' choice at $T = 1$. Let us consider the subgame starting in $T = 1$ after the true σ is revealed to intermediaries. It must be the case in equilibrium that they have no incentive to change their borrowing or lending behavior, nor to change the model they report. For any interest rate r in $T = 2$, we know from Lemma 1 that an intermediary who reports σ' chooses either $L = 0$ or $L = K/\alpha(\sigma')$. In the former case it is better not to use any internal model in $T = 1$, thus saving the cost ζ. In the latter case the constraint $L \leq K/\alpha(\sigma')$ is binding, so that the intermediary deviates unless $\sigma' = argmin\ \alpha(.)$. In equilibrium intermediaries thus either stick to safe activities or choose a risk model giving minimum capital requirements.[12] Denoting $\bar{\alpha} = \min \alpha(.)$, we have the following results:

Proposition 1. *Starting at $T = 1$ and for a given σ and $\bar{\alpha}$, in the unique equilibrium a proportion μ of intermediaries choose σ' s.t. $\alpha(\sigma') = \bar{\alpha}$, the others remain unleveraged.*
μ decreases in σ, and increases if the demand function shifts from D to $\tilde{D} \geq D$.

Proof: the supply of loans is $\mu K/\bar{\alpha}$ and must match the demand $D(r)$. $\bar{r}(\bar{\alpha}, \sigma)$ is the interest rate making intermediaries indifferent between their two strategies. Loan supply is zero for $r < \bar{r}(\bar{\alpha}, \sigma)$, $K/\bar{\alpha}$ for $r > \bar{r}(\bar{\alpha}, \sigma)$, and any intermediate value for $r = \bar{r}(\bar{\alpha}, \sigma)$. The crossing of this increasing supply curve with demand defines the unique equilibrium, as on Fig. 2 (right). If demand crosses supply on its horizontal part, then $r = \bar{r}(\bar{\alpha}, \sigma)$ and μ is defined by $\mu = \bar{\alpha} D(\bar{r}(\bar{\alpha}, \sigma))/K$. As $D(.)$ does not affect the definition of $\bar{r}(\bar{\alpha}, \sigma)$ an increase in demand thus leads to a higher μ, while σ only has an impact on $\bar{r}(\bar{\alpha}, \sigma)$. As already mentioned, the right-hand side of (5) is increasing in r, it is thus enough to show that it is decreasing in σ, which is done in the Appendix A.2. ■

Since the equilibrium is unique, I denote $r^*(\bar{\alpha}, \sigma)$ the equilibrium interest rate on loans, $\mu(\bar{\alpha}, \sigma)$ the proportion of intermediaries with an optimistic risk model and $p_d(\bar{\alpha}, \sigma)$ the ex-

[11] An interesting difference with these papers is that the problem is embedded in a market equilibrium, where incentives are affected by market prices that react to the regulation.

[12] Section 5 discusses an extension in which all models are used in equilibrium, with a bias towards more optimistic ones.

pected proportion of defaulting intermediaries in equilibrium, where:

$$p_d(\bar{\alpha}, \sigma) = \mu(\bar{\alpha}, \sigma) \left[1 - F\left(\theta(\bar{\alpha}, r^*(\bar{\alpha}, \sigma)), \sigma\right)\right] \tag{6}$$

This is simply the product of the proportion of leveraged intermediaries and their individual default probabilities. When $\mu(\bar{\alpha}, \sigma) < 1$, increasing demand leaves $r^*(\bar{\alpha}, \sigma)$ and thus the second term unchanged, while the first term increases. This implies the following:

Corollary 1. *When $\mu(\bar{\alpha}, \sigma) < 1$, p_d increases if demand shifts from D to $\tilde{D} \geq D$.*

The proposition and the corollary illustrate the role of demand in giving incentives to choose a model: if (i) all intermediaries use an optimistic model, they are able to use a high leverage and the supply of loans is high, implying a low interest rate on loans. This is an equilibrium if and only if the interest rate is still high enough for leverage to be profitable, that is if demand is high. Conversely, if (ii) few intermediaries choose to maximize their leverage, the supply of loans is low and the interest rate high. If it were too high then intermediaries would strictly prefer using an optimistic model and a high leverage, to avoid this demand has to be low. An increase in demand then leads to a wider adoption of optimistic models and a higher risk in the banking sector.

A key question for the regulator is how the equilibrium is affected by $\bar{\alpha}$, which can be interpreted as a floor on risk-weights or capital requirements:

Proposition 2. *If the elasticity of the demand for loans is lower than 1, tightening capital requirements increases the proportion $\mu(\bar{\alpha}, \sigma)$ of intermediaries with an optimistic model. If $\mu(\bar{\alpha}, \sigma)$ was initially low, the average probability $p_d(\bar{\alpha}, \sigma)$ that a bank defaults increases.*

Intuitively, there are three effects when the regulator increases $\bar{\alpha}$. First, choosing the most optimistic model is less profitable because it allows less leverage, so that the interest rate making intermediaries indifferent between using an optimistic model and choosing a safe strategy is higher. Second, due to the higher interest rate the demand for loans is lower. Third, a leveraged intermediary can lend less so that for a given proportion of leveraged intermediaries, supply is lower. If demand is not too elastic, supply has to increase to restore equilibrium, which is done via an increase in μ. If μ is small to start with, this effect dominates the fact that each leveraged intermediary is made safer by the higher $\bar{\alpha}$, so that the average default probability actually increases due to the regulatory tightening. The complete proof is in the Appendix A.3.

The mechanism behind this counter-intuitive result is simple: a regulatory tightening decreases the loan supply by intermediaries for whom the regulatory constraint is binding; with an inelastic demand, this invites other intermediaries to step in, choose an optimistic model and use a high leverage, which can ultimately lead to a higher average default risk in the banking sector.

The regulator's choice at $T = 0$. The regulator anticipates that intermediaries will choose either $L = 0$ or one of the models giving the lowest risk-weights. The regulator's choice thus boils down to choosing $\bar{\alpha}$, the minimum capital requirement that can be achieved by choosing the most favorable models. The equilibrium levels of r and μ depend both on $\bar{\alpha}$ and σ and the regulator's program is to maximize (1) in $\bar{\alpha}$:

$$\max_{\bar{\alpha}} \int_{\underline{\sigma}}^{\overline{\sigma}} \left[r_0(W + K - D(r^*(\bar{\alpha}, \sigma))) + \int_0^{D(r^*(\bar{\alpha},\sigma))} \rho(u) du \mathbb{E}_\sigma(1-t) \right] \psi(\sigma) d\sigma \tag{7}$$

The regulator needs to take into account that for certain realizations of σ the proportion of intermediaries choosing maximum leverage may be less than 1. Proposition 1 and the first-order condition of program (7) imply:

Proposition 3. *The optimal choice of $\bar{\alpha}$ by the regulator is such that the volume of loans is constant and equal to $K/\bar{\alpha}$ for $\sigma \leq \hat{\sigma}$, then decreasing for $\sigma > \hat{\sigma}$, with $\hat{\sigma} \in (\underline{\sigma}, \overline{\sigma}]$.*
The volume of loans is higher (resp. lower) than in the first-best when the actual model σ implies a high (resp. low) risk. In the particular case where $\hat{\sigma} = \overline{\sigma}$, the optimal $\bar{\alpha}^*$ satisfies:

$$r_0 = \rho(K/\bar{\alpha}^*)\mathbb{E}(\mathbb{E}_{\tilde{\sigma}}(1-t)) \tag{8}$$

There are two cases for the optimal regulation: it can be optimal to always impose high capital requirements, in which case in equilibrium all intermediaries will choose the most optimistic model and be constrained by the regulation, for any realization of σ. The best a regulator can do is then to set a floor on capital requirements so that when all intermediaries choose an optimistic model the volume of loans maximizes expected welfare when averaging over all possible models $\tilde{\sigma}$. As a result, the regulated volume of loans does not react at all to the intermediaries' information and to the riskiness of the loans: regulation is in effect neither model-based nor risk-sensitive. There is then too little lending when risk is low and too much when risk is high. See the Appendix A.4 for the complete proof.

When the highest realizations of σ are sufficiently unlikely, the optimal regulation is such that when risk is high not all banks are ready to choose a high leverage. The supply of loans is then lower for high values of σ and loan volume effectively reacts to risk. While in this case market forces help the regulator, there is still a distortion and loan volume remains excessive when risk is high.

3.3 Empirical and policy implications: market and regulation

Even when depositors are fully insured, the market still gives a counterweight to incentives to use optimistic models: when more banks adopt optimistic models and use a high leverage, the interest rate on loans goes down and increasing leverage is less profitable. Some banks choose not to use risk models strategically and stick to safe activities. Proposition 2 shows that the market and regulation are partial substitutes in limiting the use of over-optimistic models. A tighter regulation restricts the loan supply, inviting banks involved in more traditional activities to step in, thus offsetting regulatory tightenings.

Empirical predictions. Propositions 1 and 2 give new predictions about the use of internal risk models by regulated financial institutions. Getting data about what models are used in different institutions is challenging. Mariathasan and Merrouche (2012) identify "manipulation" of risk weights via proxies for the use of internal models such as the time at which a bank was approved for IRB, or the percentage of risk weights computed under IRB. Under the assumption that the selection of models is unaffected by incentives, this percentage should not be correlated with changes in regulatory or market conditions. The present paper on the contrary predicts the following:

Implication 1. *A more intensive use of internal models should be caused by:*
-1. A higher demand for loans.
-2. A less risky economic environment.
-3. A regulatory tightening, provided the elasticity of the demand for loans is lower than 1.

Points 1 and 2 give a cross-country implication: the imposition of the same regulatory floor on risk weights in different countries should lead to different choices of models, in a way that flattens the average default probabilities of banks across countries.

A concern with the regulation of banks is that a regulatory tightening will be neutralized by transfers from the regulated banks to unregulated entities (e.g. shadow banks). This can be introduced parsimoniously in the model by assuming there is an unregulated supply of loans $S(r, \sigma, c)$, where c is some measure of lending costs in the unregulated sector, so that $\partial S/\partial r \geq 0$, $\partial S/\partial \sigma \leq 0$ and $\partial S/\partial c \leq 0$. If r solving $D(r) = S(r, \sigma, c)$ is above $\bar{r}(\bar{\alpha}, \sigma)$ then in equilibrium the regulated sector is active, and section 3.2 can be adapted with a residual demand to the regulated sector $\bar{D}(r, \sigma, c) = D(r) - S(r, \sigma, c)$ instead of $D(r)$. Of particular interest here are equilibria with $0 < \mu < 1$. r is still equal to $\bar{r}(\bar{\alpha}, \sigma)$, independent of c and increasing in $\bar{\alpha}$. Then μ is determined by:

$$\mu(K/\bar{\alpha}) = D(r) - S(r, \sigma, c) \tag{9}$$

The derivatives $\partial r/\partial \bar{\alpha} \geq 0$ and $\partial S/\partial c \leq 0$ immediately give us the following:

Implication 2. *-1. A regulatory tightening causes an increase in the supply by unregulated intermediaries.*
-2. A negative shock on the costs c in the unregulated sector (e.g. higher funding costs) causes more intermediaries to adopt over-optimistic models.

Return to the example of section 3.1. Fig. 3 shows the expectation over σ of welfare[13], the volume of loans, the proportion of defaulting intermediaries, and the number of intermediaries with optimistic models, for different choices of $\bar{\alpha}$ by the regulator. Tightening the regulation leads more intermediaries to adopt the most optimistic model and for low levels of $\bar{\alpha}$ the

[13]More precisely, actual welfare minus what would be obtained if the credit supply were equal to K.

default probability increases when regulation tightens. The optimal $\bar{\alpha}$ for the regulator can be identified on the figure and is close to 9%. Assuming now that the regulator selects this value of $\bar{\alpha}$, Fig. 4 shows the same variables for the different realizations of $\tilde{\sigma}$, as well as the first-best level of loans. As expected from Proposition 1, when the true risk parameter is higher less intermediaries try to bypass the regulation.

[Insert Fig. 3 and 4 here.]

Policy implications. The model has important implications for current policy debates:
-1. *Without penalties, increasing the minimum capital requirements that can be achieved by reporting optimistic models can increase the risk of default in the banking sector*, in particular if interest rates on loans react strongly to the drop in supply.
-2. *The strategic choice of risk models reduces the effectiveness of counter-cyclical capital ratios.* This is a consequence of point 1: if for instance the demand $D(.)$ increases and the regulator increases capital requirements, both changes lead more banks to adopt optimistic models, so that aggregate risk may actually increase.

The regulatory community has started to react to the possibility that internal models are used by undercapitalized banks to bypass regulatory constraints (see e.g. BCBS (2013c), p. 15)[14]. The model allows to consider possible effects of current regulatory reforms. Many of the regulatory answers to the distrust in internal models can be seen as an increase in $\bar{\alpha}$: higher capital requirements under Basel III, floors based on Basel II's SA, the Collins amendment in the United States, provisions for model risk, higher regulatory multipliers and leverage ratios, for instance.

While such measures can be useful, they are not a natural response to an asymmetric information problem: if regulators fear that many banks use internal models to bypass regulatory constraints then such floors will be binding for many intermediaries, so that in the end the situation is similar to reverting to the previous state of the regulation when internal

[14] This somewhat delayed recognition may simply be due to the difficulty to assess the quality of a risk model, especially for credit risk. Alternative explanations were suggested in the literature such as a possible "capture by sophistication" of the regulators (Hakenes and Schnabel (2012)), or discretionary approval of optimistic models by supervisors to favor national banks (Rochet (2010)).

models were not used, with the same problems[15]. If on the contrary only a few banks are suspected to engage in this form of regulatory arbitrage (low μ in the model), then this is precisely the case in which higher floors encourage other institutions to also adopt optimistic models and increase their leverage, possibly increasing total risk as a result.

The strategic adoption of risk models is not a secondary problem requiring a fix but a serious issue that could damage current regulatory reforms. While not using any internal information may seem tempting, a more ambitious solution would be to give incentives to banks to use the models they find the most plausible.

4 Optimal regulation with hidden model

I now analyze under which conditions the regulator can use ex post penalties, and show specific difficulties associated with this mechanism in the presence of tail risk. Alternative mechanisms can be studied in this framework and are discussed in section 5.

As explained in section 2, an intermediary learns the realized σ, reports some σ', is allowed to leverage up to $L = K/\alpha(\sigma')$, then suffers t defaults in his portfolio and pays $T(\sigma', t)$ to the regulator. The correct σ can be thought of as the type of the intermediary, and the goal is to make the intermediary reveal his true type. In the presence of transfers, it is customary in the principal-agent literature to assume that the agents, here intermediaries, have a weight $\lambda < 1$ in the welfare criterion used by the principal (the regulator). This assumption ensures that the regulator minimizes the cost of the mechanism, without changing the first-best solution derived in section 2.[16] It is useful to denote $u(\alpha, r, t)$ the profit before transfers of an intermediary facing capital requirements α when the interest rate is r and t defaults realize:

$$u(\alpha, r, t) = (K/\alpha)\left(r(1-t) - r_0(1-\alpha)\right) \tag{10}$$

[15] See Kim and Santomero (1988) and Rochet (1992).

[16] In section 3 λ was implicitly assumed close to 1. Otherwise the regulator would have had an incentive to set capital requirements too high, as this is the only way to transfer money from intermediaries to consumers if explicit transfers are not allowed.

The regulator's program is then to maximize in $\alpha(.)$ and $T(.,.)$ the following:

$$\mathbb{E}\left(\mathbb{E}_\sigma(1-t)\int_0^{K/\alpha(\sigma)}\rho(u)du - r_0\frac{K}{\alpha(\sigma)} - (1-\lambda)\left[\int_0^{\theta(\alpha(\sigma),r(\alpha(\sigma)))} u(\alpha(\sigma),r(\alpha(\sigma)),t)f(t,\sigma)dt - \mathbb{E}_\sigma(T(\sigma,t))\right]\right) \tag{11}$$

where the first expectation is taken over all values of σ. Note that by choosing $\alpha(\sigma)$ the regulator determines how much is lent by the intermediaries and thus the interest rate $r(\alpha(\sigma))$ that prevails when a given σ realizes, with $r(\alpha(\sigma)) = \rho(K/\alpha(\sigma))$. If she could observe σ, the regulator would simply choose $\alpha(\sigma) = \alpha^*(\sigma)$ such that $r(\alpha^*(\sigma)) = r^e(\sigma)$, thus implement the first-best volume of loans and extract the intermediaries' surplus through transfers.

When the regulator cannot observe σ, a number of constraints have to be taken into account: (i) incentive compatibility (IC) - a bank must be better off telling the truth about the model; (ii) limited liability (LL) - the regulator cannot tax more than what the intermediary has earned; (iii) individual rationality (IR) - an intermediary must get more than his outside option. In this context the outside option would typically be to opt for Basel's SA, not use any internal model and earn a profit that still depends on the true state of the economy and is higher if loans are less risky (low σ). The outside option is then type-dependent (Jullien (2000)) and denoted as $\bar{\pi}(\sigma)$, with $\bar{\pi}' \leq 0$. Finally, the profit before transfers of an intermediary reporting σ' when the true model is σ is denoted $\pi(\sigma', \sigma)$. Formally, we have:

$$\forall \sigma, \sigma',\ \pi(\sigma,\sigma) - \mathbb{E}_\sigma(T(\sigma,t)) \geq \pi(\sigma',\sigma) - \mathbb{E}_\sigma(T(\sigma',t)) \tag{IC}$$

$$\forall \sigma,\ \pi(\sigma,\sigma) - \mathbb{E}_\sigma(T(\sigma,t)) \geq \bar{\pi}(\sigma) \tag{IR}$$

$$\forall \sigma, t,\ u(\alpha(\sigma), r(\alpha(\sigma)), t) \geq T(\sigma,t) \tag{LL}$$

The spirit of such a regulation is easy to understand: the regulator offers a profile of transfers $T(\sigma,t)$ such that an intermediary reporting σ is heavily taxed if the realized level of defaults was relatively unlikely given the model announced, or maybe rewarded if the realized level of defaults was likely. I will first show under which conditions the first-best can be achieved when the regulator faces no additional constraints on the transfers she implements. I will then introduce the additional constraint that the regulator does not have the capacity to bail out defaulting intermediaries, and show that it can cause important distortions when model uncertainty concentrates on tail risks.

4.1 Reaching the first-best when models are easy to distinguish

Consider first a simple example with only two types $\sigma_1, \sigma_2 > \sigma_1$, two possible realizations of defaults $\underline{t}, \bar{t} > \underline{t}$, and $\Pr(t = \underline{t}|\sigma_i) = p_i$, $p_1 > p_2$. To satisfy (IC) and bind (IR) the regulator can choose the following transfers: a type reporting the optimistic model σ_1 gets $\bar{\pi}(\sigma_1)/p_1$ if the low losses $\underline{t}$ realize, but 0 in the case of high losses; a type reporting the more conservative model σ_2 gets $\bar{\pi}(\sigma_2)$ irrespective of the realization. By definition, (LL) is met. (IC) for type σ_2 gives $\bar{\pi}(\sigma_2)/\bar{\pi}(\sigma_1) \geq p_2/p_1$, which is a necessary and sufficient condition on the parameters for the first-best to be achievable. It is impossible to reach the first-best if the outside option of type σ_1 is much higher than that of type σ_2 and the likelihood ratios of the two states under both models are not different enough.

Put differently, when profit decreases quickly in σ the two models must give very different predictions, otherwise a rent has to be left to the regulated. The following assumption of "distinguishable models" generalizes this idea to a continuum of types:

$$\forall t \in [0,1],\ \forall \sigma \in [\underline{\sigma}, \overline{\sigma}],\ \frac{d^2 \ln F(t,\sigma)}{d\sigma^2} \leq \frac{d^2 \ln \bar{\pi}(\sigma)}{d\sigma^2} \text{ and } \lim_{t \to 0} \frac{d \ln F(t, \underline{\sigma})}{d\sigma} \leq \frac{d \ln \bar{\pi}(\underline{\sigma})}{d\sigma} \quad \text{(DM)}$$

(DM) means that $F(.,\sigma)$ is more log-concave in σ than $\bar{\pi}$, i.e. it decreases more quickly in σ. Moreover, even close to the most optimistic model, the probability to have less than t defaults must be more sensitive to the model chosen than the outside option, at least for small values of t. When this assumption holds, we have:

Proposition 4. *With distinguishable models (DM), for any menu of capital requirements $\alpha(.)$ the transfers $T(.,.)$ below satisfy (IC) and (LL), and bind (IR) for every σ:*

$$T(\sigma,t) = \begin{cases} \max(0, u(\alpha(\sigma), r(\alpha(\sigma)), t)) & \text{if } t > a(\sigma) \\ u(\alpha(\sigma), r(\alpha(\sigma)), t) - \frac{\bar{\pi}(\sigma)}{F(a(\sigma),\sigma)} & \text{if } t \leq a(\sigma) \end{cases}$$

with $a(\sigma)$ increasing and such that:

$$\frac{F_2'(a(\sigma),\sigma)}{F(a(\sigma),\sigma)} = \frac{\bar{\pi}'(\sigma)}{\bar{\pi}(\sigma)} \tag{12}$$

With the proposed menu, an intermediary reporting model σ gets a constant payoff $\frac{\bar{\pi}(\sigma)}{F(a(\sigma),\sigma)}$ as long as the realized level of defaults is less than $a(\sigma)$, and zero otherwise. By definition such a mechanism satisfies (LL). Moreover, if he reports truthfully the intermediary gets exactly $\bar{\pi}(\sigma)$ in expectation, thus (IR) is binding. The Appendix A.5 shows that under (DM) it is possible to find a function $a(.)$ satisfying (12), which ensures (IC). It is thus possible to implement the first-best $\alpha^*(.)$ without leaving rents to intermediaries. Moreover, due to (MLRP), $a(.)$ is increasing: intermediaries announcing a low σ get a high payoff if the level of defaults is low, intermediaries announcing a higher σ get a lower payoff more often.

Fig. 5 gives an example. The parameters are the same as in section 3.3.[17] On the left panel I plot the expected payoff an intermediary gets if the true parameter is σ and he reports σ' for different values of σ' and σ. The mechanism is designed such that the maximum payoff is obtained for $\sigma' = \sigma$. On the right panel I show how this is achieved by plotting the payoff an intermediary gets when he reports the true σ and t defaults realize.

[Insert Fig. 5 here.]

4.2 Uncertainty on tail risk, bail-outs and second-best regulation

Bail-outs and incentives. The optimal menu just derived may include transfers for levels of default above those at which a bank itself defaults. Due to limited liability, the regulator cannot impose penalties for high levels of default, it may thus be necessary to *subsidize* defaulting banks who had announced high risk measures.

This is the case with the mechanism of Proposition 4 when $a(\sigma) > \theta(\alpha(\sigma), r(\alpha(\sigma)))$, but can happen more generally with any revealing mechanism, in particular when model uncertainty is focused on tail risk. For low levels of risk there is a lot of historical data to calibrate different models, such that they tend to deliver similar predictions, while for extreme levels data is much more sparse. This is at the same time the reason why the regulator would

[17] For simplicity $\bar{\pi}(\sigma)$ is assumed to be proportional to $\mathbb{E}_\sigma(1-t)$.

like to use the bank's expertise. We can model this situation in a stylized way by assuming that the different models are perfectly equivalent up to a given level of defaults:

$$\exists \tau \in [0,1] \; s.t. \forall (\sigma,\sigma') \in [\underline{\sigma},\overline{\sigma}]^2, \; \forall t < \tau, \; f(t,\sigma) = f(t,\sigma') \tag{UM}$$

Assumption (UM) is a violation of assumption (DM): instead of being easy to distinguish, the different models are **undistinguishable below** τ, that is differ only in the tail. The Appendix A.6 proves the following:

Proposition 5. *Under (UM), if $\theta(\alpha^*(\underline{\sigma}), r^e(\underline{\sigma})) < \tau$, then any revealing mechanism implementing the first-best, respecting limited liability and leaving no rents to intermediaries involves bail-outs: for some t, σ we have $u(\alpha^*(\sigma), r^e(\sigma), t) < 0$ and $T(\sigma, t) < 0$.*

Imagine that the correct model is $\underline{\sigma}$, the most optimistic one. If the regulator implements the first-best capital requirements, an intermediary will default for $t > \theta(\alpha^*(\underline{\sigma}), r^e(\underline{\sigma}))$. α^* is increasing and r^e decreasing in σ, so that this is the lowest θ implemented in the first-best. If it is below τ, an intermediary reporting $\underline{\sigma}$ is already in default when the realized t gives the regulator information about which models are more likely. It is then impossible to "punish" the use of such an optimistic model ex post. Instead, one needs to "reward" intermediaries who suffer high losses when they reported high risk measures, which automatically involves bailing out truthful but unlucky intermediaries.

Second-best with a no bail-out constraint. The use of bank bail-outs to ensure the truthful revelation of risk models may conflict with other unmodeled regulatory objectives. Proposition 5 implies that a constraint not to bail-out defaulting banks, while potentially necessary to foster market discipline for instance, comes at a cost: either rents will have to be left to intermediaries, or capital requirements will be different from their first-best values. The proof of the proposition actually shows the following:

Corollary 2. *When models are undistinguishable below τ (UM), if the regulator implements capital requirements $\alpha(.)$ with $\alpha' \geq 0$ and $\theta(\alpha(\underline{\sigma}), r(\alpha(\underline{\sigma})) < \tau$ but cannot bail out defaulting banks, then a mechanism satisfying (IC), (IR) and (LL) leaves an informational rent of at least $\bar{\pi}(\underline{\sigma}) - \bar{\pi}(\sigma)$ to any type σ.*

All intermediaries thus necessarily get a payoff at least equal to the highest outside option among all types, because this payoff can always be obtained by reporting the most optimistic risk model. As it is impossible to achieve first-best capital requirements without leaving rents to intermediaries, the regulator needs to find a second-best solution trading off rents and efficiency. Deriving the second-best solution is intractable with a continuum of types and default levels, but we can consider the special case of two types $\underline{\sigma}$, $\overline{\sigma}$ realizing with prior probabilities ψ and $1-\psi$. Assume that $\tau \in (\theta(\alpha^*(\underline{\sigma}), r^e(\underline{\sigma})), \theta(\alpha^*(\overline{\sigma}), r^e(\overline{\sigma})))$. It is straightforward to adapt the objective function (11) to this special case. The second-best solution maximizes (11) under (IC), (IR), (LL) and the no bail-out constraint:

$$\forall \sigma \in \{\underline{\sigma}, \overline{\sigma}\},\ u(\alpha(\sigma), r(\alpha(\sigma)), t) < 0 \Rightarrow T(\sigma, t) = 0 \qquad \text{(NBO)}$$

Proposition 6. *The second-best capital requirements $\alpha^{**}(.)$ when models are undistinguishable below τ (UM) and no bail-outs are possible (NBO) are of two types:*

1. High capital requirements. *$\alpha^{**}(\underline{\sigma})$ is such that $\theta(\alpha^{**}(\underline{\sigma}), r(\alpha^{**}(\underline{\sigma})) \geq \tau$. No intermediary defaults for $t < \tau$.*

2. Less risk-sensitive capital requirements. *$\alpha^{**}(\underline{\sigma})$ is such that $\theta(\alpha^{**}(\underline{\sigma}), r(\alpha^{**}(\underline{\sigma})) < \tau$. Capital requirements are lower than the first-best for $\overline{\sigma}$, and higher than the first-best for $\underline{\sigma}$ if $f(t, \underline{\sigma})/F(t, \underline{\sigma}) \leq 1$ for $t = \theta(\alpha^{**}(\underline{\sigma}), r(\alpha^{**}(\underline{\sigma})))$ and $E_{\overline{\sigma}}(1-t) \geq 0.5$.*

Solution 2. is optimal for λ, τ or ψ high enough, and is favored by a low $(\bar{\pi}(\underline{\sigma}) - \bar{\pi}(\overline{\sigma}))$.

The proof is in the Appendix A.7.[18] The first option is to increase $\alpha(\underline{\sigma})$ so much that no intermediary defaults for $t < \tau$. It may then be possible to find a mechanism similar to the one of Proposition 4 and leave no rents to the agent, at the cost of capital requirements higher than necessary for the low-risk type. The second option is to choose a capital requirement $\alpha^{**}(\underline{\sigma})$ closer to the first-best level but not allowing to distinguish the different models below the intermediary's default point, thus leaving a rent to the high-risk type.

The second solution involves a trade-off between rents and efficiency because interest rates

[18] Without the no bail-out constraint, assuming that the regulator cannot act on K or condition transfers on r is not a restrictive assumption as the first-best can be implemented. If this constraint is imposed however, these additional restrictions on the mechanism may matter. Proposition 6 gives the optimal regulation obtained under realistic constraints, but allowing the regulator to use more tools could lead to better outcomes (see section 5 for examples).

react to the regulation as they did in section 3: higher capital requirements for $\overline{\sigma}$ imply lower profits and higher interest rates. The incentives to increase leverage by reporting an optimistic model are higher, so that type $\overline{\sigma}$ must get a higher rent to report truthfully. To reduce these rents, second-best capital requirements are: (i) higher than in the firs-best for low-risk banks, which decreases how much a bank can lend by misreporting an optimistic model, and (ii) lower than in the first-best for high-risk banks, which decreases the interest rate when risk is actually high and thus reduces the incentives to increase leverage by misreporting.[19] There is an additional effect when capital requirements are increased for the low risk type, but this effect is always dominated under the mild assumption that $f(t,\underline{\sigma})/F(t,\underline{\sigma}) \leq 1$ near the default point, which should be true if we are considering a tail risk level, and $E_{\overline{\sigma}}(1-t) \geq 0.5$, which means that according to the pessimistic model loans have an average default probability lower than 50%.

Under solution 1 a higher bound on welfare is reached if transfers can be found that leave no rents even with $\alpha^{**}(\overline{\sigma}) = \alpha^{*}(\overline{\sigma})$. This may not always be possible however, in which case the regulator faces a similar trade-off, with an additional incentive to increase $\alpha^{**}(\underline{\sigma})$ further above τ to have more opportunities to punish a misreporting intermediary.

Solution 2 is surely preferred if it yields a higher welfare than the higher bound derived for solution 1. The main drawback of solution 1 is that capital requirements are inefficiently high for low-risk banks, an inefficiency that increases with τ. In the limit case where $\tau = 1$ no bank is allowed any leverage, which is surely dominated by solution 2. Conversely, the drawback of solution 2 is that high-risk banks get a rent. If λ is high the regulator is not very averse to leaving rents to the agent, and if ψ is high the probability to have a high-risk bank is low. In both cases the first-best is implemented in the limit, which is surely better than solution 1.

[19] The usual result of "no distortion at the top" thus does not obtain. This is because when an intermediary misreports he does not affect the equilibrium interest rate, which is determined by how much credit is supplied by all the other intermediaries.

4.3 Policy implications: regulatory options

A number of policy options to improve the credibility of Basel's pillar I are currently discussed.[20] Section 3 analyzed two of them: (i) simplifying the Basel framework and relying less on internal models, if at all, at the cost of capital requirements that are too high for low risk banks and too low for high risk banks; (ii) using non risk-based constraints as a *complement* to the current regulation, for instance a leverage ratio[21], at the cost of capital requirements too high for low-risk banks and possible counterproductive effects as shown in Proposition 2.

This section develops other options that aim at restoring the credibility of the IRB approach while directly tackling the associated asymmetric information problem:

Generalization of penalty mechanisms. A powerful tool to punish the use of over-optimistic models is to apply penalties to banks reporting low risk measures when high losses realize. A low-risk bank is ready to pay high penalties if high default levels realize, because this event is unlikely. A high-risk bank prefers paying transfers even when few defaults realize but without further penalties for high defaults. Similar transfers are already used for market risk models. They could be applied more generally at the level of a banking entity, with penalties, levies or restrictions on dividends when banks with low risk-weighted assets suffer high losses, using for instance Basel III's capital conservation buffer.

Proposition 4 shows a simple implementation of such a mechanism, as well as a limitation due to the possibility for banks to use Basel's standardized approach instead of reporting any model. The SA thus acts as a constraint on the IRB approach. The design of the latter was consistent with this idea, as it was supposed to encourage banks to use internal risk models by giving low capital requirements. However, not only the level, but also the risk-sensitivity of the SA matters: if standardized capital requirements react more to a bank's riskiness, a low-risk bank must be promised a higher payoff if it uses an internal model, which makes it more profitable for a high-risk bank to misreport. There may thus be a conflict between trying to achieve simultaneously a finer SA and a more truthful IRB approach.

[20] Lesle and Avramova (2012) give a useful overview of the policy options and associated trade-offs.

[21] See "The dog and the frisbee", speech given by Andrew Haldane at the Federal Reserve Bank of Kansas City's 36th economic policy symposium in Jackson Hole.

Penalties and selective bail-outs. Proposition 5 raises a second problem: in the presence of tail risk, one may learn that a bank was too optimistic regarding risk only when it is in default and cannot be punished anymore. A good example would be Dexia, which had good risk-weighted Tier 1 capital ratios before suffering the losses that led to its nationalization. How to impose penalties on a dead bank?

The first-best can still be reached by keeping the bank alive with a bail-out, and adjust the payoff for shareholders to the risk estimates previously reported. The only rationale for a bail-out here is giving incentives to truthfully report high risk measures: shareholders of a defaulting bank that warned the regulator about risks in advance should get some residual payment, nothing otherwise (see Harris and Raviv (2012) for a similar argument). The bail-outs can be financed by a tax when the bank does not default. More generally, this solution consists in adjusting the type of resolution of a bank to the conservativeness of its self-reported risk measures. Even without a bail-out, ensuring that shareholders or senior management still have some skin in the game after a default can help solving the asymmetric information problem.

High capital requirements or less risk-sensitive capital requirements. Bail-outs may not be credible if the sovereign is in a weak fiscal position, or they may have other undesirable consequences, such as encouraging moral hazard. Proposition 6 studies second-best options if the bank cannot be kept alive. The first possibility is to impose capital requirements that are so high for all types of banks that a misreporting bank always faces the threat of being punished while alive. It may then be possible to leave no informational rent, at the cost of inefficiently reducing the credit supply of low-risk banks. The second possibility is to compensate banks reporting high risk measures instead of punishing misreporting banks. In order to decrease the magnitude of this compensation, compared to the first best capital requirements are lower for high risk banks and higher for low risk banks, and are thus less risk-sensitive.

Imposing high capital requirements on all banks can be achieved by using a leverage ratio, but only in combination with an IRB approach and a penalty mechanism. However, such a policy will be preferred to imposing less risk-sensitive capital requirements only in particular cases. Fig. 6 gives an example using the same parameters as previously.[22] The right panel of the figure shows that first-best capital requirements are 7% (low-risk model) and 26% (high-risk model). While the distributions chosen do not strictly speaking satisfy (UM), a striking difference between them is that observing $t > 0.25$ has a 0.0001 probability with the former, against 0.30 with the latter. Assume $\tau = 0.25$. If the second-best option of high capital requirements is chosen, capital requirements are first-best for the high risk model, but equal to 14% i.e. twice too high for the low risk model, so that the associated default point is exactly equal to τ (left panel of the figure). The other solution is less risk-sensitive capital requirements, equal to 11% and 19%, giving less distortion to low risk banks but more to high risk banks. If λ is high, the regulator cares less about leaving rents to the banks, capital requirements are 8% and 22% and the total distortion is reduced. As shown in the proposition, if λ, τ or ψ are sufficiently high the solution of less risk-sensitive capital requirements will be preferred, even under the very optimistic assumption that no rents need to be left when choosing the high capital requirements solution. Conversely, high capital requirements can be optimal if the regulator thinks that optimistic models are very unlikely, if she puts a low weight on banks' profits, or if models are easier to distinguish.

[Insert Fig. 6 here.]

Choosing the best policy option involves a complicated trade-off, whose components are not all studied in this section.[23] Other possibilities also exist, such as relying less on ex ante balance sheet requirements and more on ex post interventions by bank supervisors, as studied by Decamps, Rochet, and Roger (2004), the challenge being then to ensure that supervisors are not too forebearant. It is sometimes argued however that internal models are simply not sufficiently reliable. This section suggests some solutions to the strategic selection of risk models, other ones are discussed in section 5. Proposals to move away from the regulatory use of internal models should be traded off against more ambitious reforms trying to restore their credibility by dealing with the asymmetric information problem. A key parameter of this trade-off is the size of banks' informational advantage over supervisors

[22] $\underline{\sigma}$ is a Beta distribution with $a = 3.5, b = 50$ while $b = 13.5$ for $\overline{\sigma}$. $\psi = 0.5$ and $\lambda = 0.8$ (low lambda scenario) or 0.95 (high λ scenario).

[23] See BCBS (2013c) for a discussion.

(linked to the distribution ψ in the model), which depends on supervisory expertise and is thus country-specific.

5 Robustness and extensions

5.1 Other incentives to select models

This paper develops a simple model to study incentives to adopt optimistic models and various policy options. While this problem is now recognized and supported by empirical evidence, the model developed in section 3 abstracts from a number of elements that also affect strategic model selection. This section discusses the main elements and how they can be integrated into the framework of this paper.

Deposit insurance. Banks in the model do not pay a risk-based deposit insurance premium. In the relatively few countries in which premia are risk-based, they depend negatively on risk-weighted assets and thus strengthen incentives to use optimistic models. They could still be redesigned as a revealing mechanism, as discussed in section 5.2.

All of the bank's creditors are insured in the model. It is possible to include depositors not covered by deposit insurance, or to consider banks relying on wholesale funding. Even assuming uninsured creditors can detect over-optimistic risk estimates, in a competitive equilibrium banks will maximize the total payoff of shareholders and uninsured creditors and still neglect the losses to the deposit insurance fund. As a result, there is still an incentive to take too much risk in this case, and use optimistic models to bypass regulatory constraints.

Endogenous capital. Bank capital K is exogenously fixed in the model. The assumption underlying the Basel approach is that it is socially optimal to allow banks to have a relatively high leverage, higher for less risky banks. There is a wider debate on this assumption, see for instance Admati *et al.* (2011) and DeAngelo and Stulz (2013) for two different views. In this paper, if investors were ready to provide capital at a fair price it would be optimal for the regulator to ask 100% capital requirements. The assumption of a fixed K simply ensures that there is a social cost of high capital requirements, but the results are qualitatively unchanged if capital can be raised endogenously at some cost due to asymmetric information, as shown in the Internet Appendix. In particular, it is still true in that case that banks have an incentive to minimize their reported risk weights.

No double accounting. An intermediary is assumed to use a certain model σ' and compute expected profit according to the true model σ, thus avoiding to base his decisions on a mistaken model. A simple justification for this assumption is that the intermediary knows that the model σ' is biased and takes this into account. This assumption is not even necessary if the two models are undistinguishable below some threshold τ, higher than the bank's default point θ. The intermediary then loses nothing by evaluating his payoffs based on the wrong model. Indeed, forecasting mistakes concern only states of the world in which he is in default, and are thus privately irrelevant. This also explains why the regulatory requirement that a model must have been used several years by the bank before its regulatory approval may not be a sufficient safeguard.

Heterogeneous model choices. Finally, the use of optimistic risk models can also be seen as the outcome of a process in which new models are developed, with a competitive advantage for more "useful" models. Either their users tend to favor them, or their "suppliers", often specialized firms, realize that models both plausible and not too pessimistic attract more customers. The Internet Appendix sketches an evolutionary process where banks adopt over time the models that have been widely adopted and profitable in the past. The volume of loans converges to its equilibrium level derived in section 3, but interestingly we obtain some heterogeneity in the risk models chosen by banks in the steady state, with a bias towards optimistic models that increases over time. This is because the interest rate decreases over time as more intermediaries bypass the regulation. The interest rate converges to the level that makes intermediaries indifferent between the most optimistic risk model and no leverage, and early adopters of intermediate models keep them without using them to increase leverage.

Finally, it is of course possible to take into account traditional countervailing forces to risk-shifting such as random deposit withdrawals, risk aversion or a dynamic charter-value effect for instance. The choice of banks would be made smoother, at the cost of additional complexity of the model, but the main economic mechanisms would not be affected. The only assumption needed in this paper is that these forces are not strong enough to make the regulation non-binding for all banks.

5.2 Other regulatory tools

Section 4 focused on purpose on a particular mechanism to solve the asymmetric information problem between a bank and its regulator. This mechanism represents a small departure from the current regulatory framework based on the Basel accords, and is similar in spirit to what is applied for market risk models. A number of other mechanisms can be studied in the same framework, as shown in the Internet Appendix and briefly discussed here.

Model-based premia. Several papers in the literature have proposed to use deposit insurance premia to solve asymmetric information issues between banks and regulators, for instance Chan, Greenbaum, and Thakor (1992). Demand for loans is inelastic however in that paper, so that higher capital ratios do not imply fewer loans, a concern that seems currently important. It is in principle possible to do better with a mechanism relying on ex post observations as in section 4. Giammarino, Lewis, and Sappington (1993) study a moral hazard problem where the regulator can check the quality of a bank's assets ex post at no cost, an assumption that applies less well to the issue of internal risk models. The Internet Appendix sketches an extension with moral hazard, showing how intermediaries have incentives to focus on complex assets for which there is model uncertainty, even when they are on average riskier than simple assets.

Market prices. The regulator could in principle use market prices to detect misreporting banks. In the framework of section 3, she can actually infer ex post the value of σ from the realized interest rate r. It is thus tempting to base capital requirements on such observations. However, this will be anticipated by market participants in equilibrium, which can make market signals less informative or even destroy the existence of an equilibrium (Bond, Goldstein, and Prescott (2010)). I show in the Internet Appendix that if the regulator tries to use the market price of some junior claims to infer a bank's type then no equilibrium exists when model uncertainty is too high.

Benchmarking. Another solution for the regulator is to use reports from different banks, or benchmarking, as part of the Pillar 2 processes. I show in the Internet Appendix how to use Nash implementation in this context. However, the assumption that all intermediaries have the same information is to be interpreted as a simplifying assumption that allows to consider a representative bank. I also show how the model can be rewritten to feature banks with heterogeneous "correct" models without changing the main conclusions of the paper.

Auditing. A last possibility is to have the regulator inspecting the bank's model and look for suspicious "tweaks". Auditing models is costly, and is done just enough to prevent intermediaries from misreporting. The Internet Appendix studies the trade-off between implementing finely model-based capital ratios and reducing auditing costs with coarser ratios.

6 Conclusion

A model-based regulation exploits banks' better information about their own risks to compute capital ratios. This information however is private, and financial intermediaries might consider using biased models or misreport if they face incentives to do so. The process of adoption of new models may then be biased towards more "profitable" models.

This paper gives new predictions on how model selection reacts to market and regulatory changes. In particular, some reforms currently discussed such as risk-weight floors or leverage ratios tend to restrict the loan supply by banks for which these constraints are binding. As a result, other banks have more incentives to also adopt similar models and reach the new regulatory constraints, a substitution which may offset the intended effect.

Moreover, such instruments make capital requirements less risk-sensitive, thus penalizing banks whose risk is genuinely low. A more ambitious avenue would be to keep using internal models while giving the regulators tools to solve the associated asymmetric information problem. The framework of this paper allows to study several policy options under a variety of realistic constraints, such as restrictions on bail-outs and uncertainty on tail risks.

Finally, the strategic use of models can take place in other instances. The Solvency II regulation for insurance companies is comparable, with model uncertainty perhaps even more severe. Internal models are also used to measure the performance of employees, convey information inside firms, or to rating agencies and shareholders. In many cases "hidden model" problems may be as challenging as model risk and call for different solutions.

A Appendix - Proofs and Figures

A.1 Proof of Lemma 1

Differentiating (4) twice with respect to L, we get:

$$\pi_1'(L,\sigma) = \frac{d\theta(r,K/L)}{dL}\times 0 + \int_0^{\theta(r,K/L)} [r(1-t)-r_0]f(t,\sigma)dt \tag{A.1}$$

$$\pi_{11}''(L,\sigma) = \frac{d\theta(r,K/L)}{dL}(r(1-\theta(r,K/L))-r_0)f(\theta(r,K/L),\sigma) \tag{A.2}$$

The definition of $\theta(r,K/L)$ in (3) implies that π_{11}'' is positive, so that profit is convex in L as a result of limited liability. This leaves us with three possible choices: (i) $L=K, M=0$, (ii) $L=M=0$, (iii) $K/L=\alpha, M=K(1-\alpha)/\alpha$. Notice first that solution (i) is never optimal: if (i) is preferred to (ii) then we must have $r\mathbb{E}_\sigma(1-t)\geq r_0$, in which case $\pi_1'(K,\sigma)=r\mathbb{E}_\sigma(1-t)-r_0$ is positive and (iii) yields an even greater profit. The optimal choice thus depends on a comparison between (ii) yielding r_0K and (iii) yielding $\pi(K/\alpha,\sigma)$. $\pi(K/\alpha,\sigma)$ is strictly increasing in r, lower than r_0K when $r=r_0$ and goes to infinity for $r\to\infty$, which directly implies the lemma.

A.2 Proof of Proposition 1

To complete the proof, it is enough to show that the right-hand side of equation (5) is decreasing in σ for a given $\bar{r}$. This expression can be rewritten as:

$$G(\bar{r},\sigma) = \frac{F(\theta(\alpha,\bar{r}),\sigma)}{\alpha}\left[\bar{r}\mathbb{E}_\sigma(1-t|t\leq\theta)-r_0(1-\alpha)\right] \tag{A.3}$$

Differentiating with respect to σ gives:

$$\frac{\partial G}{\partial\sigma} = \frac{F_2(\theta(\alpha,\bar{r}),\sigma)}{\alpha}\left[\bar{r}\mathbb{E}_\sigma(1-t|t\leq\theta(\alpha,\bar{r}))-r_0(1-\alpha)\right] + \bar{r}\frac{F(\theta(\alpha,\bar{r}),\sigma)}{\alpha}\frac{\partial\mathbb{E}_\sigma(1-t|t\leq\theta(\alpha,\bar{r}))}{\partial\sigma} \tag{A.4}$$

Assumption (MLRP) implies that a distribution with a higher σ dominates a distribution with a lower one in the sense of first-order stochastic dominance. As a result $F(\theta(\alpha,\bar{r}),\sigma)$ and $\mathbb{E}_\sigma(1-t|t\leq\theta(\alpha,\bar{r}))$ decrease in σ, which shows that $\frac{\partial G}{\partial\sigma}$ is negative.

A.3 Proof of Proposition 2

Using the market equilibrium condition, μ is defined by $\mu=\bar{\alpha}D(\bar{r})/K$, where $\bar{r}$ is defined by equation (5). We can thus write:

$$\frac{d\mu}{d\bar{\alpha}} = \frac{D(\bar{r})}{K} + \frac{d\bar{r}}{d\bar{\alpha}}D'(\bar{r})\frac{\bar{\alpha}}{K} = \frac{D(\bar{r})}{K}\left[1-\frac{\bar{\alpha}\eta(\bar{r})}{\bar{r}}\frac{d\bar{r}}{d\bar{\alpha}}\right] \tag{A.5}$$

where $\eta(\bar{r}) = -D'(\bar{r})\bar{r}/D(\bar{r})$ is the elasticity of the demand for loans. Multiplying both sides of (5) by $\bar{\alpha}$ and using implicit differentiation:

$$\frac{d\bar{r}}{d\bar{\alpha}} = \frac{r_0(1 - F(\theta(\bar{\alpha}, \bar{r}), \sigma))}{\int_0^{\theta(\bar{\alpha},\bar{r})} (1-t) f(t,\sigma) dt} \geq 0 \tag{A.6}$$

Plugging (A.6) into (A.5) gives:

$$\frac{d\mu}{d\bar{\alpha}} = \frac{D(\bar{r})}{d\bar{\alpha}} \left[1 - \eta(\bar{r}) \frac{\bar{\alpha} r_0 (1 - F(\theta(\bar{\alpha}, \bar{r}), \sigma))}{\bar{r} \int_0^{\theta(\bar{\alpha},\bar{r})} (1-t) f(t,\sigma) dt} \right] \tag{A.7}$$

Equation (5) implies that the fraction after $\eta(\bar{r})$ is lower than 1, which shows that $\frac{d\mu}{d\bar{\alpha}} \geq 0$ when $\eta(\bar{r}) \leq 1$. Finally, using (6), when $\mu(\bar{\alpha}, \sigma)$ is low the effect of $\bar{\alpha}$ on p_d through $F(\theta(\bar{\alpha}, \bar{r}), \sigma)$ can be made infinitesimally small compared to the effect through $\mu(\bar{\alpha}, \sigma)$, so that the positive impact of $\bar{\alpha}$ on μ will translate into a positive impact on p_d.

A.4 Proof of Proposition 3

A.2 proved that $\mu(\bar{\alpha}, \sigma)$ increases in σ. For a given σ the regulator aims at an interest rate close to the fair interest rate $r^e(\sigma)$. As the actual interest rate is increasing in $\bar{\alpha}$ (as shown in A.3), for some values of σ we must have $r(\bar{\alpha}, \sigma) > r^e(\sigma)$ at an optimum, otherwise it would be welfare-improving to increase $\bar{\alpha}$. Moreover, Lemma 1 implies that when $r(\bar{\alpha}, \sigma) > r^e(\sigma)$ all intermediaries choose maximum leverage, so that $\mu(\bar{\alpha}, \sigma) = 1$. Thus all intermediaries choose to report an optimistic model for low values of σ. Denoting $\hat{\sigma}(\bar{\alpha})$ the highest σ for which $\mu(\bar{\alpha}, \sigma) = 1$ and taking out the constant term $r_0(W + K)$, the regulator's objective can be written as:

$$\max_{\bar{\alpha}} \int_{\underline{\sigma}}^{\hat{\sigma}(\bar{\alpha})} \left[\int_0^{\frac{K}{\alpha}} \rho(u) du \mathbb{E}_\sigma(1-t) - r_0 \frac{K}{\alpha} \right] \psi(\sigma) d\sigma + \int_{\hat{\sigma}(\bar{\alpha})}^{\overline{\sigma}} \left[\int_0^{D(r(\bar{\alpha},\sigma))} \rho(u) du \mathbb{E}_\sigma(1-t) - r_0 D(r(\bar{\alpha}, \sigma)) \right] \psi(\sigma) d\sigma \tag{A.8}$$

Differentiating with respect to $\bar{\alpha}$, the first-order condition for this program is:

$$\int_{\underline{\sigma}}^{\hat{\sigma}(\bar{\alpha})} \frac{K}{\alpha^2} \left(r_0 - \rho\left(\frac{K}{\bar{\alpha}}\right) \mathbb{E}_\sigma(1-t) \right) \psi(\sigma) d\sigma = \int_{\hat{\sigma}}^{\overline{\sigma}} \frac{dr(\bar{\alpha}, \sigma)}{d\bar{\alpha}} D'(r(\bar{\alpha}, \sigma)) \left[r_0 - r(\bar{\alpha}, \sigma) \mathbb{E}_\sigma(1-t) \right] \psi(\sigma) d\sigma \tag{A.9}$$

Inside the second integral we have $r_0 > r(\bar{\alpha}, \sigma)\mathbb{E}_\sigma(1-t)$, otherwise $r(\bar{\alpha}, \sigma) \geq r^e(\sigma)$ and all intermediaries would choose maximum leverage. For $\sigma > \hat{\sigma}$ the interest rate is thus lower than in the first-best, and moan volume is too high. As $dr/d\bar{\alpha}$ is positive, increasing $\bar{\alpha}$ would thus increase welfare for such realizations of σ. This property must still hold when $\sigma = \hat{\sigma}$ and $r(\bar{\alpha}, \hat{\sigma}) = \rho(K/\bar{\alpha})$, as intermediaries are still indifferent between maximum leverage and safe activities. Then as σ gets lower we reach a point where $r_0 < \rho(K/\bar{\alpha})\mathbb{E}_\sigma(1-t)$: the realized interest rate is too high and lowering $\bar{\alpha}$ would increase welfare. The optimal $\bar{\alpha}$ strikes a balance between these two regions. In the particular case where the optimal $\bar{\alpha}$ is such that $\hat{\sigma} = \overline{\sigma}$, rewriting (A.9) gives (8).

A.5 Proof of Proposition 4

Denoting $U(\sigma', \sigma)$ the payoff of an intermediary reporting σ' when the true model is σ, we have:

$$\begin{aligned} U(\sigma', \sigma) &= F(a(\sigma'), \sigma)\frac{\bar{\pi}(\sigma')}{F(a(\sigma'), \sigma')} \\ U_1'(\sigma', \sigma) &= \frac{(\bar{\pi}'(\sigma')F(a(\sigma'), \sigma) + a'(\sigma')f(a(\sigma'), \sigma)\bar{\pi}(\sigma'))F(a(\sigma'), \sigma')}{F(a(\sigma'), \sigma')^2} \\ &- \frac{(a'(\sigma')f(a(\sigma'), \sigma') + F_2'(a(\sigma'), \sigma'))\bar{\pi}(\sigma')F(a(\sigma'), \sigma)}{F(a(\sigma'), \sigma')^2} \end{aligned} \tag{A.10}$$

Incentive compatibility requires for every σ:

$$U_1'(\sigma, \sigma) = 0 \Leftrightarrow \frac{F_2'(a(\sigma), \sigma)}{F(a(\sigma), \sigma)} = \frac{\bar{\pi}'(\sigma)}{\bar{\pi}(\sigma)} \tag{A.11}$$

Is it possible to find such an $a(\sigma)$ for any σ? Under (DM) we have, for any σ, $F_2'(t, \sigma)/F_t(\sigma) < \bar{\pi}'(\sigma)/\bar{\pi}(\sigma)$ when $t \to 0$. Conversely, $F(1, \sigma) = 1$ so that when $t \to 1$ we have the opposite inequality. Under (MLRP), $F_2'(t, \sigma)/F(t, \sigma)$ is increasing in t, so that there exists a unique value $a(\sigma)$ satisfying (A.11). Moreover, (DM) implies that the left-hand side of (A.11) decreases more in σ than the right-hand side, while the left-hand side increases in a, so that $a(\sigma)$ is increasing.

We finally have to show that the second-order condition is met. We can use (A.11) to replace $F_2'(a(\sigma'), \sigma')$ in (A.10). After some rearrangements this gives us:

$$U_1'(\sigma', \sigma) \geq 0 \Leftrightarrow \frac{f(a(\sigma'), \sigma)}{f(a(\sigma'), \sigma')} \geq \frac{F(a(\sigma'), \sigma)}{F(a(\sigma'), \sigma')} \tag{A.12}$$

by (MLRP) we have $U_1'(\sigma', \sigma) \geq 0 \Leftrightarrow \sigma' \leq \sigma$, hence reporting σ globally maximizes $U(., \sigma)$.

A.6 Proof of Proposition 5

To reduce the notational burden, denote $\alpha = \alpha(\sigma)$, $r = r(\alpha)$, $\theta = \theta(\alpha, r)$, and use the same notations underlined for $\underline{\sigma}$. By contradiction, assume that the regulator wants to implement an increasing $\alpha(.)$ such that $\underline{\theta} < \tau$ without resorting to any bail-out. Assume that the true model is σ. If an intermediary of type σ reports $\underline{\sigma}$, he gets:

$$\int_0^{\theta(\underline{\alpha}, r)} \max[u(\underline{\alpha}, r, t) - T(\underline{\sigma}, t), 0]f(t, \sigma)dt$$

Notice that $r > \underline{r}$ so that $\theta(\underline{\alpha}, r) > \underline{\theta}$. This payoff is thus larger than if the integral was taken up to $\underline{\theta}$. Since for $t < \underline{\theta}$ we have $f(t, \sigma) = f(t, \underline{\sigma})$, a misreporting intermediary gets at least:

$$\int_0^{\underline{\theta}} [u(\underline{\alpha}, r, t) - T(\underline{\sigma}, t)]f(t, \underline{\sigma})dt \tag{A.13}$$

(IR) for type $\underline{\sigma}$ requires the following inequality to hold:

$$\int_0^{\underline{\theta}} [u(\underline{\alpha}, \underline{r}, t) - T(\underline{\sigma}, t)] f(t, \underline{\sigma}) dt \geq \bar{\pi}(\underline{\sigma}) \tag{A.14}$$

Using this inequality to replace transfers in (A.13), a misreporting intermediary thus gets at least:

$$\bar{\pi}(\underline{\sigma}) + \int_0^{\underline{\theta}} [u(\underline{\alpha}, r, t) - u(\underline{\alpha}, \underline{r}, t)] f(t, \underline{\sigma}) dt \tag{A.15}$$

An intermediary with type σ will report truthfully only if he gets a payoff at least as large as (A.15), which is greater than $\bar{\pi}(\underline{\sigma})$ as $r > \underline{r}$. Type σ thus gets an informational rent of at least $\bar{\pi}(\underline{\sigma}) - \bar{\pi}(\overline{\sigma})$.

A.7 Proof of Proposition 6

For brevity denote $\overline{\alpha} = \alpha(\overline{\sigma})$, $\overline{r} = r(\overline{\alpha})$, $\overline{\theta} = \theta(\overline{\alpha}, \overline{r})$, and use the same notations underlined for $\underline{\sigma}$.

Solution 1: if the regulator does not want agents to have rents she has to implement $\underline{\alpha}$ so that $\underline{\theta} \geq \tau$, otherwise Corollary 2 applies. A higher bound on welfare with this solution is obtained for $\underline{\alpha} = \underline{\alpha}_\tau$ such that $\underline{\theta} = \tau$, $\overline{\alpha} = \alpha^*(\overline{\sigma})$ and no rents left to any type. This higher bound is:

$$\begin{aligned} \bar{\mathcal{V}}_1 &= \psi \left[\mathbb{E}_{\underline{\sigma}}(1-t) \int_0^{K/\underline{\alpha}_\tau} \rho(u) du - r_0(K/\underline{\alpha}_\tau) \right] + (1-\psi) \left[\mathbb{E}_{\overline{\sigma}}(1-t) \int_0^{K/\alpha^*(\overline{\sigma})} \rho(u) du - r_0(K/\alpha^*(\overline{\sigma})) \right] \\ &- (1-\lambda)(\psi \bar{\pi}(\underline{\sigma}) + (1-\psi) \bar{\pi}(\overline{\sigma})) \end{aligned} \tag{A.16}$$

Solution 2: in the first-best we have $\alpha^*(\overline{\sigma}) > \alpha^*(\underline{\sigma})$. As the regulator can always choose $\overline{\alpha} = \underline{\alpha}$ and then leave the minimum possible rents to agents, we must have $\overline{\alpha} \geq \underline{\alpha}$ in the second-best. This implies that $\overline{r} \geq \underline{r}$ and $\overline{\theta} \geq \underline{\theta}$. As the problem here is to prevent type $\overline{\sigma}$ from misreporting $\underline{\sigma}$ and get lower capital requirements, (IR) is binding for $\underline{\sigma}$ and (IC) for $\overline{\sigma}$, which gives:

$$\int_0^{\underline{\theta}} [u(\underline{\alpha}, \underline{r}, t) - T(\underline{\sigma}, t)] f(t, \underline{\sigma}) dt = \overline{\pi}(\underline{\sigma}) \tag{A.17}$$

$$\int_0^{\overline{\theta}} [u(\overline{\alpha}, \overline{r}, t) - T(\overline{\sigma}, t)] f(t, \overline{\sigma}) dt = \int_0^{\theta(\underline{\alpha}, \overline{r})} [u(\underline{\alpha}, \overline{r}, t) - T(\underline{\sigma}, t)] f(t, \overline{\sigma}) dt \tag{A.18}$$

Notice that $\theta(\underline{\alpha}, \overline{r}) \geq \underline{\theta}$, so that for t in this interval an intermediary who reported $\underline{\sigma}$ and does not default must have misreported. It is then optimal to set $T(\underline{\sigma}, t) = u(\underline{\alpha}, \overline{r}, t)$ for t in this interval: an intermediary with type $\underline{\sigma}$ is already in default and thus unaffected by these penalties, while a misreporting intermediary with type $\overline{\sigma}$ gets a payoff of zero and is thus maximally discouraged from misreporting. We can then take the integral on the right-hand side of (A.18) only up to $\underline{\theta}$. Since for $t \in [0, \underline{\theta}]$ we have $f(t, \underline{\sigma}) = f(t, \overline{\sigma})$, (A.18) can be rewritten as:

$$\int_0^{\overline{\theta}} [u(\overline{\alpha}, \overline{r}, t) - T(\overline{\sigma}, t)] f(t, \overline{\sigma}) dt = \int_0^{\underline{\theta}} [u(\underline{\alpha}, \overline{r}, t) - T(\underline{\sigma}, t)] f(t, \underline{\sigma}) dt \tag{A.19}$$

(A.17) and (A.19) allow us to express the expected transfers as functions of $\overline{\alpha}$ and $\underline{\alpha}$ only, so that the regulator's objective (11) can be written as:

$$\begin{aligned}\mathcal{V}(\underline{\alpha},\overline{\alpha}) &= \psi\left[\mathbb{E}_{\underline{\sigma}}(1-t)\int_0^{K/\underline{\alpha}}\rho(u)du - r_0(K/\underline{\alpha})\right] + (1-\psi)\left[\mathbb{E}_{\overline{\sigma}}(1-t)\int_0^{K/\overline{\alpha}}\rho(u)du - r_0(K/\overline{\alpha})\right] \\ &- (1-\psi)(1-\lambda)\left[(K/\underline{\alpha})\int_0^{\underline{\theta}}(\overline{r}-\underline{r})(1-t)f(t,\underline{\sigma})dt\right] - (1-\lambda)\bar{\pi}(\underline{\sigma}) \qquad \text{(A.20)}\end{aligned}$$

Rearranging the first-order conditions with respect to $\underline{\alpha}$ and $\overline{\alpha}$ gives:

$$\frac{\mathbb{E}_{\underline{\sigma}}(1-t)\underline{r}-r_0}{1-\lambda} = \frac{1-\psi}{\psi}\left[\left(\overline{r}-\underline{r}+\underline{\alpha}\frac{d\underline{r}}{d\underline{\alpha}}\right)\int_0^{\underline{\theta}}(1-t)f(t,\underline{\sigma})dt - \underline{\alpha}\frac{d\underline{\theta}}{d\underline{\alpha}}(\overline{r}-\underline{r})(1-\underline{\theta})f(\underline{\theta},\underline{\sigma})\right] \qquad \text{(A.21)}$$

$$\frac{\mathbb{E}_{\overline{\sigma}}(1-t)\overline{r}-r_0}{1-\lambda} = -\frac{\overline{\alpha}^2}{\underline{\alpha}}\frac{d\overline{r}}{d\overline{\alpha}}\int_0^{\underline{\theta}}(1-t)f(t,\underline{\sigma})dt \qquad \text{(A.22)}$$

Remember that in the first-best the left-hand sides of both (A.21) and (A.22) would be zero. As $d\overline{r}/d\overline{\alpha} \geq 0$ we deduce from (A.22) that $\overline{r}$ is too low in the second-best, as is $\overline{\alpha}$. We would have the opposite for $\underline{\alpha}$ but for the term $d\underline{\theta}/d\underline{\alpha}$, which reflects that increasing $\underline{\alpha}$ reduces the interval $[\underline{\theta},\theta(\underline{\alpha},\overline{r})]$ over which misreporting can be punished at no cost. This effect is dominated under the assumptions of the proposition. First, the integral on the right-hand side of (A.21) is greater than $(1-\underline{\theta})F(\underline{\theta},\underline{\sigma})$, which is greater than $(1-\underline{\theta})f(\underline{\theta},\underline{\sigma})$. The right-hand side is thus positive if:

$$\left(\overline{r}-\underline{r}+\underline{\alpha}\frac{d\underline{r}}{d\underline{\alpha}}\right) - \underline{\alpha}\frac{d\underline{\theta}}{d\underline{\alpha}}(\overline{r}-\underline{r}) \geq 0$$

Simple computations show that:

$$\frac{d\underline{r}}{d\underline{\alpha}} = \frac{\underline{r}}{\underline{\alpha}\underline{\eta}},\ \frac{d\underline{\theta}}{d\underline{\alpha}} = \frac{r_0}{\underline{\alpha}\underline{\eta}\underline{r}}(\underline{\alpha}\underline{\eta}+(1-\underline{\alpha}))$$

where $\underline{\eta}$ is the loan demand elasticity evaluated at $\underline{r}$. We thus need to show that:

$$(\overline{r}-\underline{r})(\underline{r}\underline{\eta}-r_0\underline{\alpha}\underline{\eta}-r_0(1-\underline{\alpha}))+\underline{r}^2 \geq 0 \qquad \text{(A.23)}$$

As $\underline{r} \geq r_0$ this expression is increasing in $\underline{\eta}$. For $\underline{\eta}=0$, (A.23) is equivalent to $\underline{r}^2 \geq r_0(1-\underline{\alpha})(\overline{r}-\underline{r})$. This is certainly true if $\overline{r} \leq 2r_0$. As we already know that $\overline{r} \leq r^e(\overline{\sigma})$ the condition $E_{\overline{\sigma}}(1-t) \geq 0.5$ yields the desired result. Finally, comparing (A.16) and (A.20) shows in which extreme cases solution 2 is surely preferred.

A.8 Figures

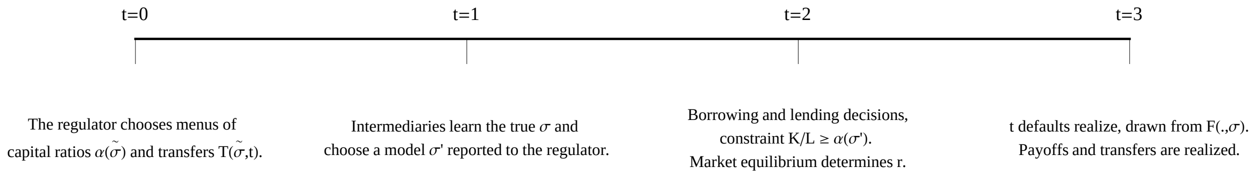

Figure 1: Timeline of the model.

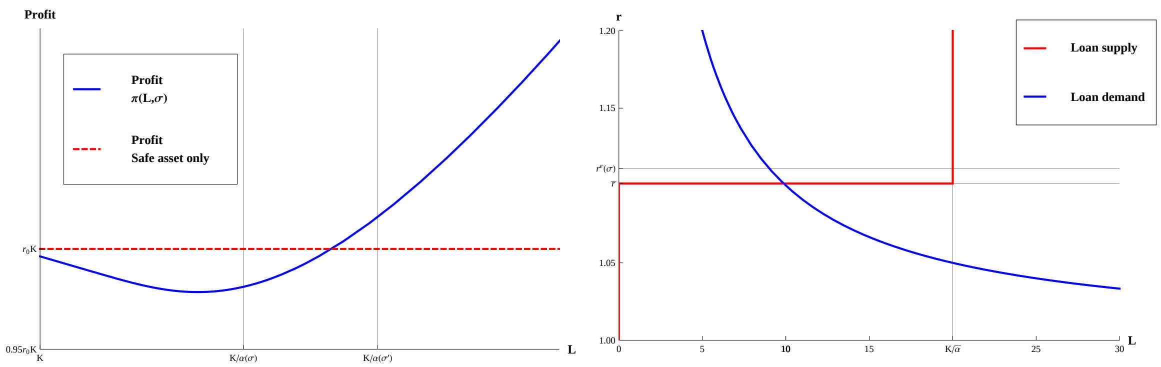

Figure 2: Profit as a function of loans (left), and market equilibrium (right).

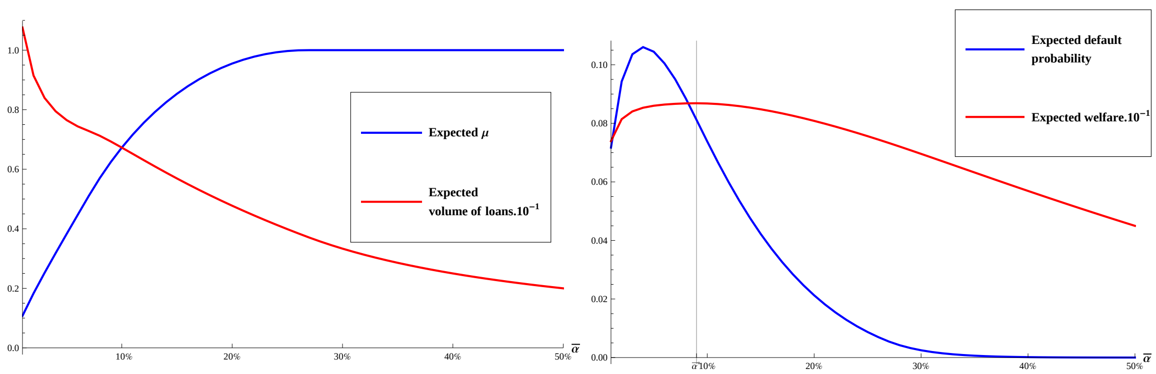

Figure 3: Proportion of users of the most optimistic model, loan volume, default probability and welfare, expectation over all realizations of $\tilde{\sigma}$ as $\bar{\alpha}$ increases.

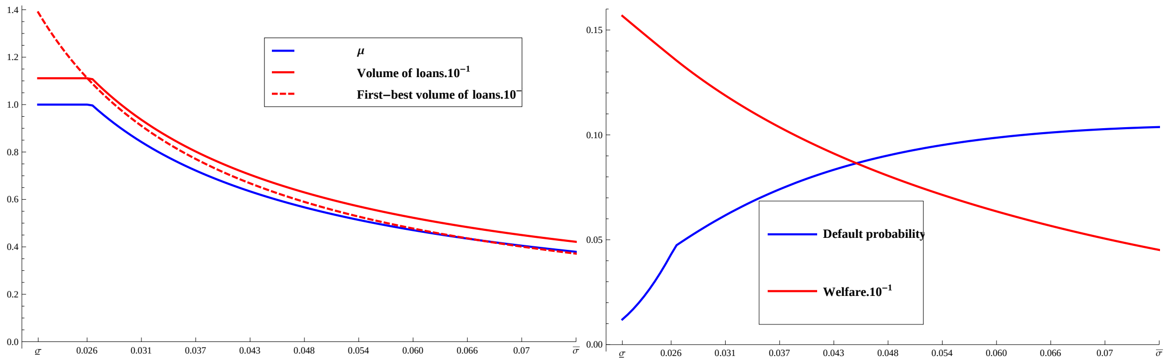

Figure 4: Proportion of users of the most optimistic model, actual and first-best loan volumes, default probability and expected welfare, for the optimal $\bar{\alpha}$ as σ increases.

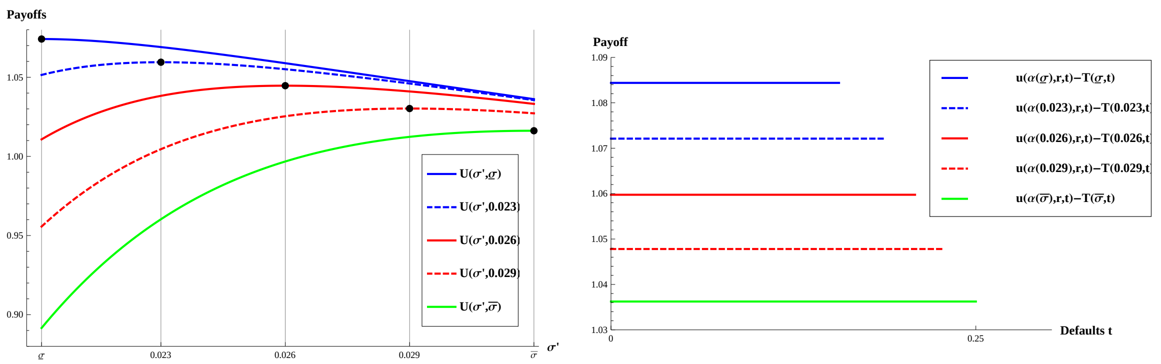

Figure 5: Expected payoff for a given σ to report σ' (left), and payoff from reporting the truth depending on the level of defaults (right).

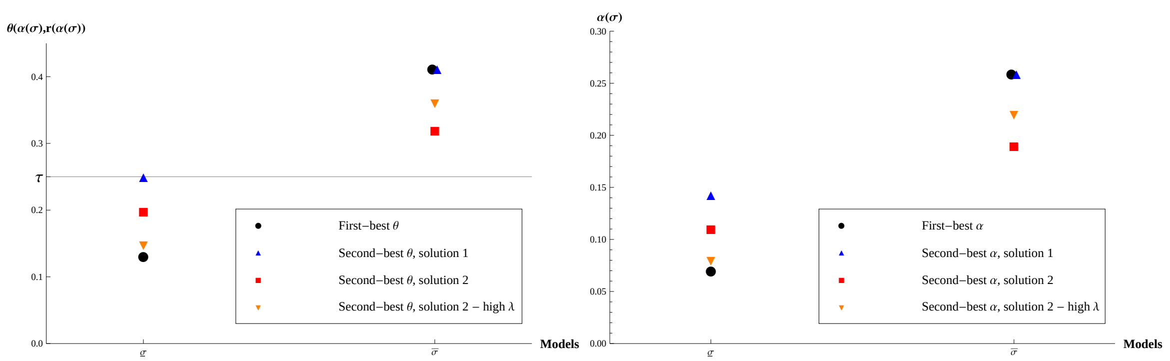

Figure 6: Default points (left) and capital requirements (right), in the first-best and several second-best scenarios.

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