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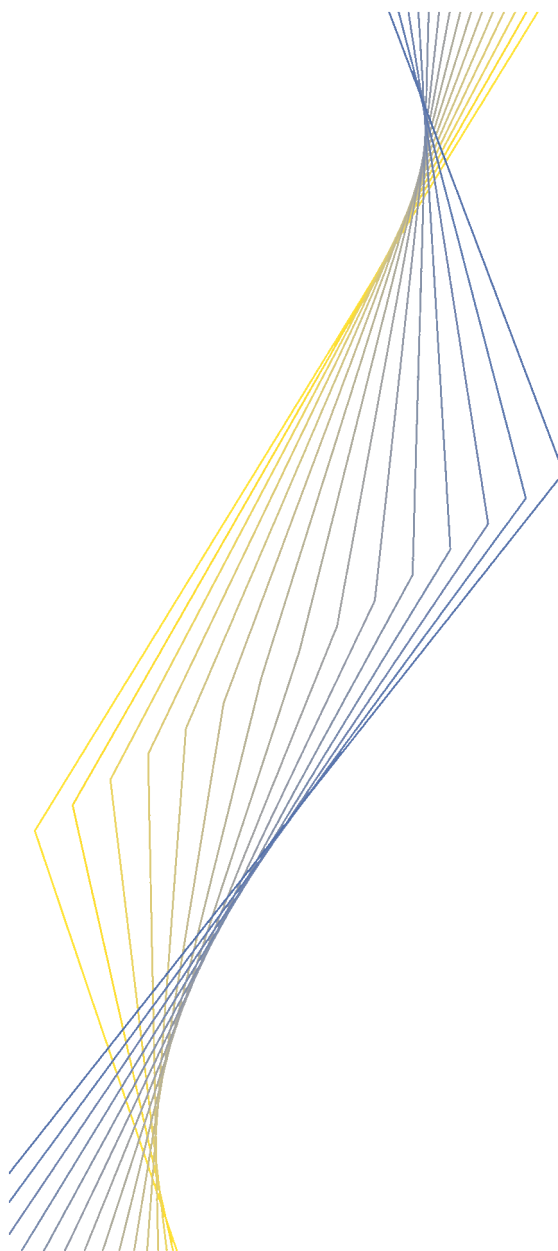
WORKING PAPER NO. 227

**MONETARY POLICY IN A
LOW PASS-THROUGH
ENVIRONMENT**

BY TOMMASO MONACELLI

April 2003

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BY TOMMASO MONACELLI²

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Abstract

We study the effects on the optimal monetary policy design problem of allowing for deviations from the law of one price in import goods prices. We reach three basic results. First, incomplete pass-through renders the analysis of monetary policy of an open economy fundamentally different from the one of a closed economy, unlike canonical models with perfect pass-through which emphasize a type of isomorphism. Second, and in response to efficient productivity shocks, incomplete pass-through has the effect of generating endogenously a short-run tradeoff between the stabilization of inflation and of the output gap. Third, in studying the optimal program under commitment relative to discretion, we show that the former entails a smoothing of the deviations from the law of one price, in stark contrast with the established empirical evidence. In addition, an optimal commitment policy always requires, relative to discretion, more stable nominal and real exchange rates.

Keywords: deviations from the law of one price, policy trade-off, gains from commitment, exchange rate channel.

JEL Classification Number: E52, E32, F41

Non-technical Summary

This paper studies the effects on the optimal monetary policy design problem of allowing for deviations from the law of one price in import goods prices. Recently we have witnessed a growing interest in macroeconomics for the development of small-scale models applied to the analysis of monetary policy. The so-called New-Keynesian synthesis, exemplified by the work of Clarida, Gali and Gertler (1999) and Woodford (2002), bears the attractive feature of preserving tractability within the rigor of a dynamic optimizing general equilibrium setup.

The goal of a realistic representation of how in practice monetary policy is conducted in open economies has motivated the work of Benigno and Benigno (2002), Gali and Monacelli (2002), McCallum and Nelson (2001), Clarida, Gali and Gertler (2001), Ghironi (2000). Yet a limitation shared by all these models is the assumption that the pass-through of exchange rates to (import) prices is complete. This lies in stark contrast with the well-established empirical evidence that deviations from the law of one price for traded goods prices are large and pervasive.

This paper argues that allowing for incomplete pass-through bears important implications for the design of the optimal monetary policy problem.

First, incomplete pass-through alters the form of the canonical small-scale sticky-price model that has become the hallmark of the recent literature on the analysis of monetary policy. This framework typically reduces to a tractable two-equation dynamical system for inflation and output gap, consisting of a new Keynesian Phillips curve and of a dynamic IS-type equation. The paper shows that, unlike Clarida, Gali and Gertler (2001) who argue that the closed and the open economy version of the "canonical model" can be considered isomorphic to one another, the introduction of incomplete pass-through renders the analysis of monetary policy of an open economy fundamentally different from the one of a closed economy.

Second, allowing for deviations from the law of one price has the effect of generating endogenously a short-run trade-off between the stabilization of inflation and of the output gap. This has two consequences. On the one hand it renders the problem of optimal monetary policy non-trivial as well as realistic. On the other it marks a distinction from some of the recent literature (based on the prototype Calvo sticky-price model with perfect pass-through) that, in order to generate a meaningful policy trade-off, has typically resorted to ad-hoc (inefficient) cost-push shocks as exogenous shifters of the Phillips curve (Clarida et al, 1999, 2001). In our framework with incomplete pass-through a trade-off between policy objectives emerges in response to efficient productivity shocks and, furthermore, independently of the measure of inflation (CPI or producer price) featured in the loss criterion adopted by the Central Bank.

Third, the presence of such a real policy trade-off allows, within a fully forward-looking setup, to contrast the features of the optimal policy program under commitment to the one under discretion. As emphasized by Woodford (2002) there is

a fundamental reason why a discretionary behavior results in suboptimal outcomes in forward-looking models. Namely that discretion does not allow to design an efficient response to unexpected temporary shocks. This generates a source of gains from commitment which differs from the one outlined in the traditional analysis and related to the presence of an average inflation bias (see e.g., Kydland and Prescott 1977). More importantly, the study of this dimension of monetary policy is unfeasible within a large class of NOEM models that assumes one-period predetermined prices (or wages). For such an assumption typically gives rise to a Lucas-type aggregate supply curve in which the forward-looking nature of inflation is neglected, and along with it the channel through which the anticipation of future policy conduct comes to play a role. In our setting, to the contrary, a critical channel to the optimal commitment policy (relative to discretion) is the possibility, through the exchange rate (which is a forward-looking variable), to affect the expected future path of the deviations from the law of one price, and in turn the equilibrium path of inflation and output gap. A key contribution is to show that the optimal program, relative to the case with discretion, entails a partial, though not a complete, stabilization of the deviations from the law of one price. This is suggestive of a puzzle in the light of the established empirical evidence that deviations from the law of one price are rather large and persistent.

1 Introduction

Recently we have witnessed a growing interest in macroeconomics for the development of small-scale models applied to the analysis of monetary policy. The so-called New-Keynesian synthesis, exemplified by the work of Clarida, Gali and Gertler (1999) and Woodford (2002), bears the attractive feature of preserving tractability within the rigor of a dynamic optimizing general equilibrium setup. This has provided an ideal ground for the study of the optimal conduct of monetary policy, the design and implementation of simple interest rate rules, and for a direct exploration of the data. Surprisingly much less attention has been devoted to the development of a similar paradigm in an open economy context. Several recent contributions within the so-called New Open Economy Macroeconomics (NOEM) literature have taken the form of elegant but highly stylized models in which the analysis of monetary policy is often still confined to inspecting the effects of money supply shocks.¹ The goal of a realistic representation of how in practice monetary policy is conducted in open economies has motivated the work of Benigno and Benigno (2002), Gali and Monacelli (2002), McCallum and Nelson (2001), Clarida, Gali and Gertler (2001), Ghironi (2000). Yet a limitation shared by all these models is the assumption that the pass-through of exchange rates to (import) prices is complete. This lies in stark contrast with the well-established empirical evidence that deviations from the law of one price for traded goods prices are large and pervasive.²

The goal of this paper is to emphasize that allowing for incomplete pass-through bears important implications for the design of the optimal monetary policy problem. First, incomplete pass-through alters the form of the canonical small-scale sticky-price model that has become the hallmark of the recent literature on the analysis of monetary policy. This framework typically reduces to a tractable two-equation dynamical system for inflation and output gap, consisting of a new Keynesian Phillips curve and of a dynamic IS-type equation. Our first contribution is to show that, unlike Clarida, Gali and Gertler (2001) who argue that the closed and the open economy version of the "canonical model" can be considered isomorphic to one another, the

¹As of now this literature is extremely rich. See Lane (2000) for a survey and contributions listed under Bryan Doyle's New Open Economy Macroeconomics Homepage at http://www.geocities.com/brian_m_doyle/open.html

²See Rogoff (1996) and Goldberg and Knetter (1997) for extensive theoretical and empirical surveys. The work by Engel (1993, 1999, 2002), Rogers and Jenkins (1995) strongly documents deviations from the law of one price also for consumer prices at a high level of disaggregation.

introduction of incomplete pass-through renders the analysis of monetary policy of an open economy fundamentally different from the one of a closed economy.

Second, allowing for deviations from the law of one price has the effect of generating endogenously a short-run trade-off between the stabilization of inflation and of the output gap. This has two consequences. On the one hand it renders the problem of optimal monetary policy non-trivial as well as realistic. On the other it marks a distinction from some of the recent literature (based on the prototype Calvo sticky-price model with perfect pass-through) that, in order to generate a meaningful policy trade-off, has typically resorted to ad-hoc (inefficient) cost-push shocks as exogenous shifters of the Phillips curve (Clarida et al, 1999, 2001). In our framework with incomplete pass-through a trade-off between policy objectives emerges in response to efficient productivity shocks and, furthermore, independently of the measure of inflation (CPI or producer price) featured in the loss criterion adopted by the Central Bank.

Third, the presence of such a real policy trade-off allows, within a fully forward-looking setup, to contrast the features of the optimal policy program under commitment to the one under discretion. As emphasized by Woodford (2002) there is a fundamental reason why a discretionary behavior results in suboptimal outcomes in forward-looking models. Namely that discretion does not allow to design an efficient response to unexpected temporary shocks. This generates a source of gains from commitment which differs from the one outlined in the traditional analysis and related to the presence of an average inflation bias (see e.g., Kydland and Prescott 1977). More importantly, the study of this dimension of monetary policy is unfeasible within a large class of NOEM models that assumes one-period predetermined prices (or wages).³ For such an assumption typically gives rise to a Lucas-type aggregate supply curve in which the forward-looking nature of inflation is neglected, and along with it the channel through which the anticipation of future policy conduct comes to play a role. In our setting, to the contrary, a critical channel to the optimal commitment policy (relative to discretion) is the possibility, through the exchange rate (which is a forward-looking variable), to affect the expected future path of the deviations from the law of one price, and in turn the equilibrium path of inflation and output gap. A key contribution is to show that the optimal program, relative to the case with discretion, entails a partial, though not a complete, stabilization of the deviations from the law of one price. This is suggestive of a puzzle in the light of the

³See, e.g, Obstfeld and Rogoff (1995), Corsetti and Pesenti (2002), Sutherland(2002).

established empirical evidence that deviations from the law of one price are rather large and persistent.

Turning to the recent literature, Devereux and Engle (2002b), Corsetti and Pesenti (2002) and Sutherland (2002) also study the impact of incomplete pass-through on the optimal conduct of monetary policy. Their framework differs from the one of the present paper for it features one-period predetermined prices and hence does not lend it self to the analysis of the dynamic gains from commitment undertaken here.⁴ Adolfson (2002) and Smets and Wouters (2002) are contributions more in line with the present paper. They differ in three dimensions. First, their setting cannot be reduced to a tractable compact form easily comparable to the small-scale canonical sticky-price model previously adopted by the literature. Second, they focus only on the optimal policy under discretion, and hence neglect the crucial role played under commitment by the expectational channel of the exchange rate to inflation. This is a critical dimension of the monetary policy problem explored in detail in this paper. Third, they do not focus on the comparison of alternative policy rules.

The rest of the paper is organized as follows. Section 2 presents a model of the world economy in which two asymmetric countries, a small open economy and a large approximately closed one, coexist. Section 3 analyzes the basic trade-offs implied by the introduction of incomplete pass-through, while Section 4 discusses the details of the optimal monetary policy program. Section 5 compares the performance of alternative simple rules for monetary policy. Section 6 concludes.

2 The Model

2.1 Domestic Households

The domestic economy is populated by infinitely-lived households, consuming Dixit-Stiglitz aggregates of domestic (C_H) and imported (C_F) goods, by domestic firms producing a differentiated good, and by a continuum of importing firms that operate as price setters in the local market. All goods are tradeable. In the following, lower case letters indicate log deviations from respective steady-state values while capital letters indicate levels. Let's define C as a composite consumption index:

$$C_t \equiv \left[(1 - \gamma)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (1)$$

⁴Furthermore Burstein, Eichenbaum and Rebelo (2002) obtain incomplete pass-through as a consequence of the presence of distribution costs.

with C_H and C_F being indexes of consumption of domestic and foreign goods respectively.⁵ Notice that under this specification η measures the elasticity of substitution between domestic and foreign goods. The optimal allocation of expenditures between domestic and foreign goods implies:

$$C_{H,t} = (1 - \gamma) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \quad ; \quad C_{F,t} = \gamma \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \quad (2)$$

where $P_t \equiv [(1 - \gamma)P_{H,t}^{1-\eta} + \gamma P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}}$ is the consumer price index (CPI).

We assume the existence of complete markets for state-contingent money claims expressed in units of domestic currency. Under this assumption the first order conditions of the consumer's problem are standard and can be written in a convenient log-linearized form as:

$$w_t - p_t = \sigma c_t + \varphi n_t \quad (3)$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} (r_t - E_t\{\pi_{t+1}\}) \quad (4)$$

where w_t is the nominal wage, n_t is labor hours, r_t is the log nominal interest rate, and π_t is the *CPI inflation rate*.⁶

In the rest of the world a representative household faces a problem identical to the one outlined above. Hence a set of analogous optimality conditions characterize the solution to the consumer's problem in the world economy. As in Galí and Monacelli (2002), however, the size of the small open economy is negligible relative to the rest of the world, an assumption that allows to treat the latter as if it was a closed economy.⁷

⁵Such indexes are in turn given by CES aggregators of the quantities consumed of each type of good. The optimal allocation of any given expenditure within each category of goods yields the demand functions:

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} \quad ; \quad C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\varepsilon} C_{F,t}$$

for all $i \in [0, 1]$, where $P_{H,t} \equiv (\int_0^1 P_{H,t}(i)^{1-\varepsilon} di)^{\frac{1}{1-\varepsilon}}$ and $P_{F,t} \equiv (\int_0^1 P_{F,t}(i)^{1-\varepsilon} di)^{\frac{1}{1-\varepsilon}}$ are the price indexes for domestic and imported goods respectively, both expressed in home currency. The elasticity of substitution between goods within each category is given by $\varepsilon > 1$.

⁶This follows from maximizing a separable utility function of the form $\frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\varphi} N_t^{1+\varphi}$ under a standard sequence of budget constraints. Hence σ denotes the inverse of the intertemporal elasticity of consumption and φ the inverse of the elasticity of labor supply.

⁷Notice that, more precisely, this is a world of two asymmetric countries in which one is small relative to the other (whose equilibrium is in the limit taken as exogenous). This kind of setup allows to model explicitly the role of financial markets and risk sharing and to overcome a typical problem of unit-root in consumption that characterizes traditional small open economy models with

2.1.1 Pass-through, the Real Exchange Rate, and Deviations from PPP

Log-linearization of the CPI expression around a steady-state yields:

$$p_t = (1 - \gamma) p_{H,t} + \gamma p_{F,t} \quad (5)$$

Domestic producer inflation (defined as the rate of change in the index of domestic goods prices), and CPI-inflation are linked according to

$$\begin{aligned} \pi_t &= (1 - \gamma)\pi_{H,t} + \gamma\pi_{F,t} \\ &= \pi_{H,t} + \gamma \Delta s_t \end{aligned} \quad (6)$$

where

$$s_t \equiv p_{F,t} - p_{H,t} \quad (7)$$

denotes the (log) terms of trade, i.e., the *domestic currency relative price of imports*. Notice that the equation above holds independently of the degree of pass-through. The change in this price can be written in terms of relative inflation rates as:

$$\Delta s_t = \pi_{F,t} - \pi_{H,t} \quad (8)$$

The treatment of the rest of the world as an (approximately) closed economy (with goods produced in the small economy representing a negligible fraction of the world's consumption basket) implies that $p_t^* = p_{F,t}^*$, and $\pi_t^* = \pi_{F,t}^*$, for all t , i.e., an equivalence between domestic and CPI inflation holds in the world economy.

Under incomplete pass-through the *law of one price* does *not* hold. This has implications for the relationship between the real exchange rate and the terms of trade. Let's define $e_t \equiv \log \mathcal{E}_t$ as the (log) nominal exchange rate (i.e., the domestic currency price of one unit of foreign currency). In particular, by using equation (5), one can write:

$$\begin{aligned} q_t &= e_t + p_t^* - p_t \\ &= (e_t + p_t^* - p_{F,t}) + (1 - \gamma)s_t \\ &= \psi_{F,t} + (1 - \gamma)s_t \end{aligned} \quad (9)$$

incomplete markets. See Schmitt-Grohe and Uribe (2002) for a discussion on how to "close small open economy models".

where

$$\psi_{F,t} \equiv (e_t + p_t^*) - p_{F,t} \quad (10)$$

denotes the deviation of the *world* price from the *domestic* currency price of imports, a measure of the deviations from the law of one price. In what follows we will define this measure as the *law-of-one price gap* (l.o.p gap henceforth).

Equation (9) deserves some comments. It stands clear that two are the sources of deviation from aggregate PPP in this framework. The first one is due to the heterogeneity of consumption baskets between the small economy and the rest of the world, an effect captured by the term $(1 - \gamma)s_t$, as long as $\gamma < 1$. For $\gamma \rightarrow 1$, in fact, the two aggregate consumption baskets coincide and relative price variations are not required in equilibrium. This will become more clear below when I illustrate risk sharing. The second source of deviation from PPP is due to the deviation from the law of one price, captured by movements in $\psi_{F,t}$. With incomplete pass-through the l.o.p gap contributes to the volatility of the real exchange rate. It will stand clear later that the term $\psi_{F,t}$ plays a key role in determining the dynamics of imports inflation.

2.2 Domestic Producers

In the market of the domestic goods, there is a continuum of monopolistic competitive firms (owned by consumers), indexed by $i \in [0, 1]$. They operate a CRS technology: $Y_t(i) = Z_t N_t(i)$, where Z is a total factor productivity shifter. Cost minimization typically leads to the following efficiency condition for the choice of labor input :

$$mc_t = (w_t - p_{H,t}) - z_t \quad (11)$$

where mc indicates the real marginal cost which is common across producers. In the following, domestic (log) productivity is assumed to follow a simple stochastic autoregressive process:

$$z_t = \rho z_{t-1} \xi_{z,t} \quad (12)$$

where $0 \leq \rho \leq 1$ is a persistence parameter and $\xi_{z,t}$ is an i.i.d shock.

Domestic firms are allowed to reset their price according to a standard Calvo-Yun rule, which implies receiving a price signal at a constant random rate θ_H . Let then θ_H^k be the probability that the price set at time t will still hold at time $t + k$. Firm i faces

domestic and foreign demand. For simplicity we assume that the export price of the domestic good, $P_H^*(i)$, is flexible and determined by the law of one price. This kind of pricing technology leads typically to the following log-linear equation for equilibrium newly set prices:

$$p_{H,t}^{new} = (1 - \beta\theta_H) E_t \left\{ \sum_{k=0}^{\infty} (\beta\theta_H)^k (mc_{t+k} + p_{H,t+k}) \right\} \quad (13)$$

The domestic aggregate price index evolves according to:

$$P_{H,t} = [\theta_H (P_{H,t-1})^{1-\varepsilon} + (1 - \theta_H) (P_{H,t}^{new})^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \quad (14)$$

By log-linearizing (14) and combining with (13) one can derive a typical forward-looking Phillips curve:

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \lambda_H mc_t \quad (15)$$

where $\lambda_H \equiv \left(\frac{(1-\theta_H)(1-\beta\theta_H)}{\theta_H} \right)$. An aggregate supply relation of this kind has become a basic ingredient of recent optimizing models of the so-called New Keynesian Synthesis.⁸

2.3 Incomplete Pass-Through and Imports Pricing

We now turn to discuss the dynamic of import pricing, which is the central modelling novelty of the paper. In recent work Campa and Goldberg (2002) estimate import pass-through elasticities for a range of OECD countries. They find that the degree of pass-through is partial in the short-run and that it becomes gradually complete only in the long-run. Their results imply a rejection of both the extreme assumptions on import pricing that characterize a wide array of papers in the NOEM literature: *local* vs. *producer* currency pricing.⁹ According to the first view domestic currency prices of imports are totally unresponsive to exchange rate movements in the short run, while the opposite is true in the latter case. What this evidence suggests is that a setup featuring incomplete exchange rate pass-through should allow the deviations from the law of one price to be, as well as large, gradual and persistent.

⁸See Woodford (1999a), Clarida, Gali and Gertler (2000).

⁹The original Obstfeld and Rogoff (1995) paper assumes PCP, while in the LCP category fall, among many others, papers by Betts and Devereux (2000), Chari, Kehoe and McGrattan (2002), Devereux and Engel (2001).

In this section we develop the model in the direction of accounting for these facts. We assume that the domestic market is populated by local retailers who import differentiated goods for which the law of one price holds "at the dock". In setting the *domestic currency* price of these goods the importers solve an optimal (dynamic) markup problem. This generates deviations from the law of one price in the short run, while complete pass-through is reached only asymptotically, implying a long-run holding of the law of one price. This feature is more in line with the empirical patterns described above and critically distinguishes our modelling of incomplete pass-through from the one of other recent papers (see e.g., Corsetti and Pesenti, 2002).

Consider a local retailer importing good j at a cost (i.e., price paid in the world market) $\mathcal{E}_t P_{F,t}^*(j)$, where \mathcal{E} is the level of the nominal exchange rate. Like the local producers, the same retailer faces a downward sloping demand for such good and therefore chooses a price $P_{F,t}(j)$, expressed in units of domestic currency, to maximize:

$$E_t \left\{ \sum_{k=0}^{\infty} \beta^k \Lambda_{t,t+k} \theta_F^k (P_{F,t}(j) - \mathcal{E}_{t+k} P_{F,t+k}^*(j)) C_{F,t+k}(j) \right\}$$

$$s.t. C_{F,t}(j) = \left(\frac{P_{F,t}(j)}{P_{F,t}} \right)^{-\varepsilon} C_{F,t}$$

where $P_{F,t}^*(j)$ is the foreign-currency price of the imported good, θ_F^k is the probability that the price $P_{F,t}(j)$ set for good j at time t still holds k periods ahead, and $\beta^k \Lambda_{t,t+k}$ is a relevant stochastic discount factor. In general, the degree of stickiness in the adjustment of domestic prices θ_H is allowed to differ from the one of import prices expressed in local currency θ_F .

The FOC of this problem yields:

$$P_{F,t}^{new}(j) = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{E_t \{ \sum_{k=0}^{\infty} \beta^k \Lambda_{t,t+k} \theta_F^k (\mathcal{E}_{t+k} P_{F,t+k}^* C_{F,t+k}(j)) \}}{E_t \{ \sum_{k=0}^{\infty} \beta^k \Lambda_{t,t+k} \theta_F^k C_{F,t+k}(j) \}} \quad (16)$$

The log-linear aggregate imports price evolves according to:

$$p_{F,t} = \theta_F p_{F,t-1} + (1 - \theta_F) p_{F,t}^{new} \quad (17)$$

The log-linear version of (16) yields:

$$p_{F,t}^{new} = (1 - \beta \theta_F) E_t \sum_{k=0}^{\infty} (\beta \theta_F)^k (\psi_{F,t} + p_{F,t+k}) \quad (18)$$

By combining (17) with (18), one can obtain an aggregate supply curve for imports goods:

$$\pi_{F,t} = \beta E_t \pi_{F,t+1} + \lambda_F \psi_{F,t} \quad (19)$$

where $\lambda_F \equiv \frac{(1-\theta_F)(1-\beta\theta_F)}{\theta_F}$. Therefore import price inflation rises as the world price of imports exceeds the local currency price of the same good. In other words, a nominal depreciation determines a wedge between the price paid by the importers in the world market and the local currency price applied in the domestic market. This wedge acts as an increase in her real marginal cost and therefore increases foreign goods inflation. The parameter θ_F governs the degree of pass-through.¹⁰ Notice that in the case $\theta_F = 0$ equation (18) reduces to a simple law-of-one price equation $p_{F,t} = e_t + p_t^*$. Notice also that equation (19) can be written, integrating forward, as

$$\pi_{F,t} = E_t \left\{ \sum_{k=0}^{\infty} \beta^k \lambda_F \psi_{F,t+k} \right\} \quad (20)$$

which shows that imports price inflation is a purely forward-looking variable, for its current behavior depends on the current and expected future deviations from the law of one price.

2.3.1 Risk Sharing and Uncovered Interest Parity

The existence of complete markets for nominal state contingent securities has implications for consumption risk sharing. Formally movements in the ratio of the marginal utilities of consumption must imply, in equilibrium, movements in the real exchange rate. This typically implies a log-linearized condition¹¹:

$$c_t = c_t^* + \frac{1}{\sigma} q_t \quad (21)$$

which can be rewritten as

$$c_t = c_t^* + \frac{1}{\sigma} ((1-\gamma)s_t + \psi_{F,t}) \quad (22)$$

¹⁰In fact the textbook definition of exchange rate pass-through is the percentage change in the local currency import price resulting from a one percent change in the exchange rate between importing and exporting country (see Goldberg and Knetter, 1997).

¹¹See Chari, Kehoe and McGrattan (2002), Galí and Monacelli (2002) for a formal derivation of this condition.

where σ is the intertemporal elasticity of substitution in consumption. Hence deviations from the law of one price, by affecting the movements of the real exchange rate, affect the movements of the relative consumption baskets as well.

Under complete international asset markets it is also possible to derive a standard log-linear version of an uncovered interest parity condition

$$r_t - r_t^* = E_t\{\Delta e_{t+1}\} \quad (23)$$

It is easy to show that such an equation results from combining efficiency conditions for an optimal portfolio of bonds by both domestic and foreign residents.

2.3.2 Decomposition of the Real Marginal Cost

By combining (4), (11) and (22) one obtains, after aggregation, an equilibrium equation for the domestic real marginal cost (or inverse of the domestic markup), which also expresses the equilibrium in the labor market:

$$\begin{aligned} mc_t &= (w_t - p_{H,t}) - z_t \\ &= (w_t - p_t) + \gamma s_t - z_t \\ &= \sigma c_t + \varphi y_t + \gamma s_t - (1 + \varphi)z_t \\ &= \varphi y_t - (1 + \varphi)z_t + \sigma y_t^* + s_t + \psi_{F,t} \end{aligned} \quad (24)$$

Equation (24) shows that the domestic real marginal cost is increasing in domestic output (through its effect on employment and therefore the real wage) and decreasing in domestic technology (through its direct effect on labor productivity). However, open economy factors as well affect the real marginal cost: world output (through its effect on labor supply via risk sharing) and a "relative price effect" captured by s_t and $\psi_{F,t}$.

2.4 Goods Market Equilibrium

To describe the equilibrium in the domestic goods market it is first useful to consider log-linearized versions of the isoelastic demand functions. In particular local and foreign demand for domestic goods can be written respectively:

$$\begin{aligned} c_{H,t} &= -\eta(p_{H,t} - p_t) + c_t \\ &= \eta\gamma s_t + c_t \end{aligned} \quad (25)$$

$$\begin{aligned}
c_{H,t}^* &= -\eta(p_{H,t}^* - p_t^*) + c_t^* \\
&= \eta((p_{F,t} + \psi_{F,t}) - p_{H,t}) + c_t^* \\
&= \eta(s_t + \psi_{F,t}) + c_t^*
\end{aligned} \tag{26}$$

Hence foreign demand for domestic goods (i.e., exports) *rises* both when the terms of trade depreciate (i.e., the price p_H falls relative to p_F) and when the domestic currency price of foreign goods p_F falls relative to the world price (i.e., ψ_F rises).

Finally, the demand for imports will read

$$\begin{aligned}
c_{F,t} &= -\eta(p_{F,t} - p_t) + c_t \\
&= -\eta(1 - \gamma)s_t + c_t
\end{aligned} \tag{27}$$

Goods market clearing implies $y_t(i) = (1 - \gamma) c_{H,t}(i) + \gamma c_{H,t}^*(i)$ for all goods i . After aggregating, substituting the above demand functions and rearranging the by using (22) one obtains a simple proportionality relation between domestic and foreign output which is as well affected by the existence of incomplete pass-through:

$$y_t - y_t^* = \frac{1}{\sigma}[\omega_s s_t + \omega_\psi \psi_{F,t}] \tag{28}$$

where $\omega_s \equiv 1 + \gamma(2 - \gamma)(\sigma\eta - 1) > 0$ and $\omega_\psi \equiv 1 + \gamma(\sigma\eta - 1) > 0$ are the elasticities of relative output to the domestic currency relative price of imports and the l.o.p gap respectively, with $\omega_s \geq \omega_\psi$.

The expression in (28) makes clear that any movement in relative output requires, in equilibrium, an adjustment in relative prices, summarized by the right hand side of the above equation. Consider the case, for instance, of a rise in domestic output relative to the rest of the world. Equilibrium requires a real depreciation, which in turn can be achieved in two ways: either a fall in the *domestic currency* price of domestic goods (relative to foreign goods, i.e., a rise in s_t) or a nominal depreciation triggering a deviation from the law of one price for imports (i.e., a rise in $\psi_{F,t}$).

2.5 Policy Target in the Rest of the World

Let's first describe how the equilibrium looks like in the rest of the world. The equilibrium real marginal cost is given by:

$$mc_t^* = (\sigma + \varphi)y_t^* - (1 + \varphi)z_t^* \quad (29)$$

which is simply the closed economy (i.e., obtained for $\gamma = 0$) version of equation (24). Therefore the natural (flexible-price) level of output in the world economy easily obtains by imposing $mc_t^* = 0$ (which implies $\pi_t^* = 0$):

$$\bar{y}_t^* = \left(\frac{1 + \varphi}{\sigma + \varphi} \right) z_t^* \quad (30)$$

As in a canonical sticky-price model with Calvo price staggering, under fully flexible prices the output gap will be completely stabilized, i.e.,

$$\tilde{y}_t^* = y_t^* - \bar{y}_t^* = 0 \quad (31)$$

Throughout it is assumed that the monetary authority in the rest of the world aims at replicating the flexible price allocation by simultaneously stabilizing inflation and the output gap. It is well known that such a policy also coincides with the first best outcome.¹²

2.6 Flexible Domestic Prices

In this section we describe the equilibrium dynamics in the small economy under the assumption that *domestic producer* prices are flexible. This is useful to formally derive two results. First, that nominal exchange rate volatility is linked to the degree of pass-through. Second, that for a sufficiently low degree of pass-through the l.o.p gap must respond positively to a (relative) productivity shock.

In the case of flexible domestic prices the pricing equation (13) yields a constant markup. Therefore we can assume, without loss of generality, that domestic prices remain fixed at their optimal level, as firms would have no incentive to deviate from such a state of affairs. By imposing a constant markup in equation (24) and substituting equation (28) an expression for the domestic flexible price level of output reads:

$$\bar{y}_t = \bar{y}_t^n - \left(\frac{\omega_s - \omega_\psi}{\sigma + \varphi\omega_s} \right) \bar{\psi}_{F,t} \quad (32)$$

¹²Goodfriend and King (1997). Woodford (2002) discusses under which conditions such a policy corresponds also to maximizing a second order approximation of households' welfare.

where $\bar{y}_t \equiv \left(\frac{\omega_s(1+\varphi)}{\sigma+\varphi\omega_s}\right) z_t + \left(\frac{\sigma(1-\omega_s)}{\sigma+\varphi\omega_s}\right) y_t^*$ denotes the *natural* level of output, i.e., the one that would obtain in the case of both flexible domestic prices *and* complete pass-through. Below we show how to obtain a reduced form expression for $\bar{\psi}_{F,t}$. Notice also that the two measures of output gap exactly coincide in the special case $\omega_s = \omega_\psi$.

The l.o.p gap can then be written

$$\bar{\psi}_{F,t} = \bar{e}_t - \bar{p}_{F,t} \quad (33)$$

and the terms of trade

$$\bar{s}_t = \bar{e}_t - \bar{\psi}_{F,t}$$

By using equation (28) and noticing that $\bar{s}_t = \frac{\sigma}{\omega_s}(\bar{y}_t - y_t^*) - \frac{\omega_\psi}{\omega_s}\bar{\psi}_{F,t}$ the nominal exchange rate can be written as

$$\bar{e}_t = \frac{\sigma}{\omega_s}(\bar{y}_t - y_t^*) + \left(1 - \frac{\omega_\psi}{\omega_s}\right) \bar{\psi}_{F,t} \quad (34)$$

which can be rearranged, using (32), to obtain

$$\bar{e}_t = \bar{e}_t^n + \left(\frac{\varphi(\omega_s - \omega_\psi)}{\sigma + \varphi\omega_s}\right) \bar{\psi}_{F,t} \quad (35)$$

where $\bar{e}_t^n \equiv \frac{\sigma(1+\varphi)}{\sigma+\varphi\omega_s} (z_t - z_t^*)$ is the natural nominal exchange rate. Hence notice that, as long as $\omega_s \neq \omega_\psi$, deviations from the law of one price contribute to the volatility of the nominal exchange rate beyond the one implied by its natural level. Therefore the model seems consistent with the view that a lower degree of pass-through is associated with higher exchange rate volatility.¹³ Intuitively, the lower the pass-through the larger will be the nominal exchange rate variation required to achieve a given adjustment in real relative prices along the transition to the equilibrium.

Next it is instructive to derive a reduced-form expression for the l.o.p gap as a function of relative productivity. In Appendix A we show that the dynamic of the l.o.p gap can be written:

$$\bar{\psi}_{F,t} = \Gamma(z_t - z_t^*) - \frac{\mu_1(\sigma + \varphi\omega_s)}{\sigma + \varphi\omega_\psi} \bar{p}_{F,t-1} \quad (36)$$

¹³See e.g., Betts and Devereux (2000). However Devereux and Engle (2002) show that a low pass-through is a necessary but not sufficient condition for generating both an exchange rate volatility in line with the data and to be consistent with the so-called Baxter-Stockman disconnect puzzle (according to which movements in relative prices seem delinked from the ones of real quantities).

where $\mu_1 < 1$ and $\Gamma \equiv \frac{\sigma(1+\varphi)}{\sigma+\varphi\omega_\psi} \left(1 - \frac{(\sigma+\varphi\omega_s)\beta\mu_1\lambda_F}{(\sigma+\varphi\omega_\psi)(1-\rho\beta\mu_1)} \right)$. One can easily show that $\Gamma > 0$ for a sufficiently low degree of pass-through, which in turn implies that the l.o.p gap *must rise* in response to a rise in domestic productivity.¹⁴ This result, which indeed depends on importers feeding nominal exchange rate movements on domestic currency import prices only gradually, will be useful below in our analysis of inflation dynamics in response to productivity shocks.

2.7 The Supply Block

We now proceed by illustrating how the introduction of incomplete pass-through affects the supply side relationships of the model. Let's define the *output gap* as the percentage deviation of current output from the natural level of output, i.e.,

$$\tilde{y}_t \equiv y_t - \bar{y}_t^n \quad (37)$$

where again it is important to recall that the natural level of output is the one that would obtain under both flexible prices and complete pass-through. Equation (28), in turn, implies that the output gap is proportional to both the (domestic) terms of trade gap and the l.o.p gap:

$$\tilde{y}_t = \frac{\omega_s}{\sigma} \tilde{s}_t + \frac{\omega_\psi}{\sigma} \psi_{F,t} \quad (38)$$

Therefore the equilibrium real marginal cost (24) can be written, after combining with (38), as

$$mc_t = \left(\varphi + \frac{\sigma}{\omega_s} \right) \tilde{y}_t + \left(1 - \frac{\omega_\psi}{\omega_s} \right) \psi_{F,t} \quad (39)$$

Hence the presence of incomplete pass-through breaks down the proportionality relationship between the real marginal cost and the output gap which typically characterizes the canonical sticky-price model with imperfectly competitive markets. With incomplete pass-through, in fact, the real marginal cost is proportional to both the deviations of current output from its natural level and to the deviations from the law of one price. In response to productivity shocks the potentially contrasting equilibrium behavior of these two determinants of the real marginal cost will be the key

¹⁴In particular $\Gamma > 0$ is satisfied for $\lambda_F < \frac{(\sigma+\varphi\omega_\psi)(1-\rho\beta\mu_1)}{(\sigma+\varphi\omega_s)\beta\mu_1}$, which in turn requires a sufficiently low degree of pass-through, i.e., a sufficiently high θ_F .

to understand the policy trade-off faced by the monetary authority.¹⁵ The analysis below will further elaborate on this point.

Notice that the expression for the equilibrium real marginal cost in (39) allows an interesting interpretation of the deviations from the law of one price as *endogenous supply shocks*. In fact, by replacing (39) in (15) one obtains

$$\pi_{H,t} = \beta \{E_t \pi_{H,t+1}\} + \kappa_y \tilde{y}_t + \kappa_\psi \psi_{F,t} \quad (40)$$

where $\kappa_y \equiv \lambda_H \left(\varphi + \frac{\sigma}{\omega_s} \right)$ and $\kappa_\psi \equiv \lambda_H \left(1 - \frac{\omega_\psi}{\omega_s} \right)$. The result in equation (36) establishes that the term $\psi_{F,t}$ will rise in response to a rise in domestic productivity (for a sufficiently low degree of pass-through). Hence (for any given output gap) positive movements in inflation can result from endogenous movements in the l.o.p gap which can in turn be induced by (efficient) positive variations in productivity. This contrasts with a practice that has become common in models of the New Keynesian Phillips curve of "appending" (inefficient) cost-push terms to the right hand side of (40) as a proxy for supply shocks.

By solving equation (40) forward it yields:

$$\pi_{H,t} = E_t \left\{ \sum_{k=0}^{\infty} \beta^k (\kappa_y \tilde{y}_{t+k} + \kappa_\psi \psi_{F,t+k}) \right\} \quad (41)$$

which implies that domestic inflation is entirely forward-looking, depending on current and expected future values of the output gap and of the l.o.p gap.

2.7.1 CPI-based Aggregate Supply

In this section we show that the interpretation of the deviations from the law of one price as (endogenous) supply shocks continues to hold also when the broader CPI measure of inflation is considered. Recall that, up to a log-linear approximation, CPI inflation can be written as a convex combination of both domestic and import price inflation (as from equation (6)). It is again natural to express the equilibrium in terms of deviations from the frictionless allocation (where domestic flexible prices and complete pass-through both hold).

¹⁵One can also notice that the theory-based measure of the output gap implied by this setup is one in which the same output gap is proportional not only to the labor share (via the real marginal cost) but also to the l.o.p gap, which is another observable variable. The same would not hold in the case of complete pass-through, where one would recover the same proportionality between labor share and real marginal cost that characterizes prototypical closed economy models.

By combining (24), (15), (19) and (38) one obtains the following expression for a CPI-based aggregate supply curve:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_y^c \tilde{y}_t + \kappa_\psi^c \psi_{F,t} \quad (42)$$

where $\kappa_y^c \equiv (1 - \gamma)\kappa_y$ and $\kappa_\psi^c \equiv (1 - \gamma)\kappa_\psi + \gamma\lambda_F$.

Therefore, like domestic producer inflation, CPI inflation as well features a Phillips curve forward-looking representation. The novelty of the framework with incomplete pass-through is the second term on the right hand side. A rise in the l.o.p gap, for a given output gap, causes a rise in CPI inflation. A full stabilization of inflation, then, would require a fall in the output gap. Furthermore, notice that $\sigma = \eta = 1$ implies $\kappa_\psi^c = \gamma\lambda_F > 0$. Hence deviations from the law of one price continue to affect CPI inflation even in the special case of $\sigma = \eta = 1$. This will be important below to qualify an additional existing trade-off between the stabilization of the output gap and of the CPI measure of inflation.

2.8 The Demand Block

To complete the description of the model it is useful to rewrite in a more compact form the aggregate demand equations as well. Notice, first, that by using (22) one can rewrite the market clearing condition as:

$$y_t = \left(\frac{\omega_s}{1 - \gamma} \right) c_t + \left(1 - \frac{\omega_s}{1 - \gamma} \right) c_t^* - \left(\frac{\gamma\eta}{1 - \gamma} \right) \psi_{F,t} \quad (43)$$

By substituting (43) into (4) and making use of the definition of the output gap and of equation (6) one can write the following aggregate demand equation:

$$\tilde{y}_t = E_t \{ \tilde{y}_{t+1} \} - \frac{\omega_s}{\sigma} (r_t - E_t \{ \pi_{H,t+1} \} - \bar{r}r_t) + \Gamma_y E_t \{ \Delta \psi_{F,t+1} \} \quad (44)$$

where $\Gamma_y \equiv \left(\frac{\gamma(1-\gamma)(\sigma\eta-1)}{\sigma} \right)$ and $\bar{r}r_t \equiv \sigma \left(\frac{\varphi(\omega_s-1)}{\sigma+\varphi\omega_s} \right) E_t \{ \Delta y_{t+1}^* \} - \left(\frac{\sigma(1-\rho)(1+\varphi)}{\sigma+\varphi\omega_s} \right) z_t$ is the *natural real interest rate*. Notice that the natural real rate depends not only on domestic productivity, but also on the expected growth in world output.

Equation (44) shows that, to the extent that $\sigma\eta > 1$, expected changes in the output gap are negatively related to expected future changes in the l.o.p gap. By using (38), an equivalent way of rewriting equation (44) emphasizes the direct link between the output gap and the terms of trade gap \tilde{s}_t :

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{\omega_s \Omega_y}{\sigma} (r_t - E_t\{\pi_{H,t+1}\} - \bar{r}r_t) - \frac{\omega_s \Omega_y}{\omega_\psi} \Gamma_y E_t\{\Delta \tilde{s}_{t+1}\} \quad (45)$$

where $\Omega_y \equiv \frac{\omega_\psi}{\sigma(\omega_\psi + \gamma(1-\gamma))(\sigma\eta - 1)} > 0$. Hence expected changes in the output gap are positively related to expected future changes in the terms of trade *gap*.

2.9 Breaking the Canonical Representation

One result of the recent open economy New Keynesian optimizing framework is that the model's equilibrium dynamics can be represented in a output gap-inflation space (a so called *canonical representation*) which is isomorphic to its closed economy counterpart. The effect of adding the openness dimension would result only in the *slope* coefficients of the standard optimizing aggregate demand and supply relationships being modified. By considering the joint system described by the supply equation (40) and the demand equation (44) it stands clear that the introduction of incomplete pass-through has the effect of breaking the isomorphism between the closed and the open economy version of the canonical sticky-price model.¹⁶ To better understand this it is useful to analyze the monetary policy channels to inflation in the present model. First, there is a typical *aggregate demand channel*. This is common to both a closed and an open economy. Namely, changes in the nominal interest rate affect the real rate and the output gap via equation (44) and in turn inflation via both (40) and (42). In an open economy this channel is strengthened by the expenditure switching effect that works through changes in the terms of trade and in turn in the trade balance. With complete pass-through this channel affects only the sensitivity of output gap movements to the real interest rate. In that case, as from equation (38) under $\psi_{F,t} = 0$, the terms of trade are simply proportional to the output gap, and their effect on demand simply feeds in via a modification of the slope of the aggregate demand equation. With incomplete pass-through there is an independent (aggregate demand) *l.o.p channel* to inflation that works via equation (44). This is the first factor that contributes to breaking the isomorphism between closed and open economy representations of the standard optimizing sticky-price model.¹⁷

The second channel to inflation in the model summarizes a series of *aggregate*

¹⁶See Gali and Monacelli (2002) and Clarida, Gali and Gertler (2002) for open economy models in which the isomorphism still holds due to the presence of complete pass-through.

¹⁷However notice that such isomorphism continues to hold in the extreme cases of $\gamma = 0$ (closed economy) and $\gamma = 1$ (consumption basket of the small economy coinciding with the one of the foreign economy), as well as in the special case of $\sigma = \eta = 1$

supply effects. First, the nominal exchange rate affects CPI inflation directly. This effect is obviously de-emphasized with incomplete pass-through. Second, the nominal exchange rate affects the terms of trade, the product wage and the real marginal cost via equation (24). We have already shown above that, via equation (39), this results in an independent supply-side channel linking the l.o.p gap to inflation (both producer and CPI), and hence in a second channel that alters the result of isomorphism.

3 Policy Trade-offs in the Small Economy

We now turn to the illustration of how the introduction of incomplete pass-through can crucially shape the range of trade-offs faced by the monetary authority of the small economy. We first have the following result:

- *Under incomplete pass-through, and under the assumption that $\sigma\eta > 1$, the domestic producer flexible price allocation is no longer feasible. Therefore the monetary authority faces a trade-off between stabilizing producer inflation variability and stabilizing either the output gap or the l.o.p gap:*

$$\text{var}(\pi_{H,t}) = 0 \longrightarrow \text{var}(\tilde{y}_t) > 0, \text{var}(\psi_{F,t}) > 0$$

The intuition follows directly from the real marginal cost equation (39). Consider, for instance, a rise in the relative productivity of the domestic economy. This, *ceteris paribus*, tends to lower the output gap and to exert a downward pressure on the real marginal cost. However it also implies a nominal depreciation and, considering the result in equation (36), also a rise in the l.o.p gap for a sufficiently low degree of pass-through. Any attempt to stabilize the output gap by lowering interest rates would then boost the nominal depreciation and therefore imply a further rise in the l.o.p gap. Therefore the monetary authority *cannot simultaneously stabilize the domestic markup and target the law of one price*. The novel aspect of this result is that this trade-off arises endogenously in response to (efficient) productivity shocks.¹⁸ The derivation of a CPI-based aggregate supply curve in equation (42) is useful to understand that the monetary authority not only faces a trade-off between stabilizing domestic inflation and the output gap but also between stabilizing the CPI measure of inflation and the output gap. We have the following result:

¹⁸Recall, however, that in the special case in which $\omega_s = \omega_\psi$, movements in the l.o.p gap do not affect the domestic real marginal cost, and therefore the trade-off between domestic inflation and output gap variability is not binding, exactly like in the case of complete pass-through.

- *Under incomplete pass-through, and regardless of the values assumed by the parameters σ and η , it is unfeasible for the monetary authority to simultaneously stabilize CPI inflation and the output gap:*

$$\text{var}(\pi_t) = 0 \longrightarrow \text{var}(\tilde{y}_t) > 0, \text{var}(\psi_{F,t}) > 0$$

The intuition follows naturally from the CPI-based aggregate supply curve (42). However, in this case, it is interesting to notice that the trade-off persists even in the case of $\sigma\eta = 1$, given that $\Theta_\psi > 0$. To understand this, notice that two are the channels through which incomplete pass-through has an effect on CPI inflation. First, by affecting the domestic real marginal cost through equation (39). Second, by rising imports inflation through equation (19). In the case $\sigma\eta = 1$ the first channel is neutralized, while the second effect continues to hold given that $\kappa_\psi^c = \gamma\lambda_F$.

4 Optimal Monetary Policy Design

In this section we characterize the optimal monetary policy design problem. The focus of attention, similar to Woodford (2002) for the case of a closed economy, is on the nature of the optimal dynamic program for the monetary authority in the presence of households and firms adopting forward-looking decisions. The possibility of affecting future private sector's expectations gives rise to gains from commitment relative to a regime in which only discretionary optimization is feasible. This is a central insight of the recent analysis of optimal monetary policy in sticky-price models.¹⁹ The open economy dimension, along with the presence of incomplete pass-through, adds further wrinkles to the analysis. The presence of a l.o.p channel to inflation, as it stands clear from inspecting equation (42), calls for an optimal management of the deviations from the law of one price and therefore of both the nominal and the real exchange rate along the optimal path. Furthermore, and in order to affect future inflation expectations, such deviations from the law of one price must be optimally managed against the output gap path, with these two variables further interacting through the aggregate demand relationship summarized by equation (44).

The (endogenous) conflict between policy objectives discussed in the previous sections motivates the choice of our loss criterion. We assume that the domestic authority sets policy in order to minimize a quadratic loss function which penalizes the

¹⁹Woodford (2000), Clarida, Gali and Gertler (2000).

variability of CPI inflation and output gap around some target values. In particular we assume that such targets are zero for both variables.²⁰ The choice of including CPI inflation in the loss function appears the most natural. As pointed out in Svensson (2000) all small economies that have adopted regimes of inflation targeting have chosen to target a CPI measure of inflation, rather than a producer price or GDP measure, which would correspond to the index π_H in this analysis.²¹

Before turning to the analysis of the optimal policy design problem, let me characterize the rational expectations equilibrium in the small economy.

Definition 1 *Conditional on the definition of an appropriate monetary policy rule, and under the assumption that the world economy pursues a policy of strict price stability, a rational expectations equilibrium for the small economy can be computed as a set of processes $\{\pi_t, \pi_{H,t}, \tilde{y}_t, \pi_{F,t}, \psi_{F,t}, e_t, r_t\}_{t=0}^{\infty}$ that solves the system of equations (10), (23), (40), (42), (19), (44), (6) for any given set of processes for the exogenous variables $\{\bar{r}_t, z_t^*, \pi_t^*, r_t^*, \tilde{y}_t^*\}_{t=0}^{\infty}$.*

4.1 Time Consistent Policy

We begin by assuming, for the sake of exposition, that the monetary authority lacks a device that allows a commitment to a once-and-for-all plan at time 0, and therefore

²⁰The assumption that the target value for the output gap is zero implies that there is no bias in the *average* inflation rate resulting from discretionary optimization. However, and stemming from the forward looking nature of both measures of inflation, the response to technology shocks under discretionary optimization will still result in an inefficient outcome. See Woodford (1999a) for a detailed analysis in the context of a closed economy model.

²¹An obvious alternative would be to assume that the monetary authority tries to maximize the welfare of domestic households. Woodford (2000) shows how to obtain, within a closed economy model, a second order *accurate* approximation of households' utility and use it to solve a tractable quadratic control problem. In open economy forward-looking models with Calvo pricing this has been shown to be a much more complicated task, as argued in Benigno and Benigno (2002) and Gali and Monacelli (2002). In particular, in such models an accurate quadratic approximation of households' welfare can be obtained only under very specific assumptions on preferences and on the value of the international elasticity of substitution. The issue of how computing welfare maximizing policies in fully dynamic open economy models still remains a subject of research. See Faia and Monacelli (2003) for an alternative approach based on the direct solution of the Ramsey problem and on the explicit consideration of all the distortions characterizing the equilibrium of the economy. However, notice that by choosing to target CPI inflation the Central Bank is implicitly targeting a weighted average of both sources of nominal rigidities in the economy, namely stickiness in domestic prices and stickiness in the domestic currency prices of imported goods. This is likely to approximate very closely the underlying welfare maximizing policy, as argued, for instance, in Corsetti and Pesenti (2002). Having said that, the current paper's scope remains the one of providing a tractable way of accounting for incomplete pass-through in small-scale optimizing monetary policy models.

reoptimizes discretionally at each point in time. Let's define by $b_w > 0$ the relative weight attached to output gap variability in the loss criterion. Hence the problem becomes the one of minimizing

$$\frac{1}{2}[\pi_{t+k}^2 + b_w \tilde{y}_{t+k}^2] \quad (46)$$

in each given date t , subject to the constraints given by equations (10), (23), (42), (19), (44), (6), (40). In this problem, and stemming from the assumed impossibility for the monetary authority to affect private sector's expectations, terms involving future expectations are treated parametrically. In *Appendix B* we show that this problem leads to the following simple optimality condition linking inflation and the output gap in every period t :

$$\tilde{y}_t = -\Theta_c \pi_t, \text{ all } t \quad (47)$$

where $\Theta_c \equiv \frac{\kappa_y^c + \frac{\kappa_\psi^c}{a}}{b_w} > 0$ and $a \equiv \frac{1+\gamma(\sigma\eta-1)}{\sigma} > 0$. Condition (47) typically suggests that the monetary authority contracts real activity in response to a rise of CPI inflation above the target. The parameter Θ_c measures the magnitude of the implied optimal adjustment of the output gap. Notice that Θ_c is decreasing in the degree of pass-through θ_F . In fact, the lower the pass-through (the higher θ_F) the smaller the slope λ_F of the import price equation (19). Notice also that by substituting (47) into (42) it yields

$$\pi_t = \frac{\kappa_\psi^c}{1 + \Theta_c \kappa_y^c} E_t \left\{ \sum_{j=0}^{\infty} \left(\frac{\beta}{1 + \Theta_c \kappa_y} \right)^j \psi_{F,t+j} \right\} \quad (48)$$

The above expression shows that, under the optimal discretionary policy, CPI inflation must *rise* in response to both current and expected future deviations from the law of one price.

4.1.1 A Contractionary Bias

It is particularly interesting to notice that the higher κ_ψ^c , i.e., the higher the sensitivity of inflation to movements in the l.o.p gap, the larger will be the contraction in real activity associated to any given variation in inflation. This suggests the existence, as an effect of incomplete pass-through, of a policy *contractionary bias*. One way to analyze this issue more formally is by means of a thought experiment. Let's maintain that the monetary authority's problem is the one of minimizing (46). Yet let's assume

that the same policy authority treats the deviations from the law of one price simply as exogenous shifters of the short-run Phillips curve (42). This strategy allows an interesting parallel with recent models in which the presence of a policy trade-off depends simply on ad-hoc cost push shocks. Hence the problem is now to choose π_t and \tilde{y}_t period by period, given the vector of exogenous variables, $\{\overline{rr}_t, z_t^*, \pi_t^*, r_t^*\}$, to minimize (46) subject only to

$$\pi_t = F + \kappa_y^c \tilde{y}_t \quad (49)$$

where $F \equiv \beta E_t\{\pi_{t+1}\} + \kappa_\psi^c \psi_{F,t}$ is a composite term which is taken as given by the policy authority in her maximization problem. Notice that in so doing the monetary authority not only recognizes that future private sector's expectations cannot be manipulated, but treats movements in the l.o.p gap as exogenous as well. The first order condition of this problem reads:

$$\tilde{y}_t = -\Theta_x \pi_t, \text{ all } t \quad (50)$$

where $\Theta_x \equiv \frac{\kappa_y^c}{b_w} > 0$, with Θ_x measuring the sensitivity of the output gap to movements in inflation. Notice in particular that

$$\Theta_c > \Theta_x$$

The case just illustrated may be viewed as one in which the policy authority is treating deviations from the law of one price as exogenous cost push shocks. Or, alternatively, as one in which the policy authority does not face any trade-off between the aggregate demand and the exchange rate channel to inflation. In the more general case, though, when movements in the l.o.p gap are instead treated as endogenous, the monetary authority has to recognize that by lowering interest rates in response to a rise in productivity it will also trigger a nominal depreciation and therefore a rise in the l.o.p gap, which tends to rise inflation even further. Hence, and relative to the case in which deviations from the law of one price were treated as exogenous shocks, for any given initial rise in inflation the policymaker would have to contract the output gap more sharply. The former example also illustrates that the main effect of incomplete pass-through on the optimal policy design problem is that the aggregate demand-output gap channel and the exchange rate channel to inflation must be optimally managed one against the other.

4.2 Optimal Plan

In the case in which commitment is feasible as of time zero, the policy authority is assumed to choose a state-contingent plan $\{\pi_t, \pi_{H,t}, \tilde{y}_t, \pi_{F,t}, \psi_{F,t}, e_t, r_t\}_{t=0}^{\infty}$ to minimize the discounted sum of losses:

$$\frac{1}{2} E_t \left\{ \sum_{j=0}^{\infty} \beta^j [\pi_{t+j}^2 + b_w \tilde{y}_{t+j}^2] \right\} \quad (51)$$

subject to the constraints (10), (23), (42), (19), (44), (6), (40). For the sake of exposition the details of such problem are deferred to *Appendix B*. An evaluation of the equilibrium dynamics along the optimal program will require a numerical simulation of the model.

5 Dynamics under the Optimal Policy

In this section we compute numerically the equilibrium dynamics of selected variables conditional on the optimal policy program and in response to an unexpected *rise in domestic productivity* (relative to the rest of the world). The benchmark calibration employed is as follows. We assume $\beta = 0.99$, $\sigma = 1$, $\eta = 1.5$, $\varphi = 3$. The parameter θ_H , which governs the degree of stickiness of domestic prices, is set equal to 0.75, a value consistent with an average period of one year between price adjustments. The parameter θ_F , which governs the degree of pass-through, is also set to a benchmark value of 0.75, but will then vary depending on the sensitivity analysis conducted. The persistence of the productivity process is set $\rho = 0.9$ and its standard deviation is calibrated to take a unitary value. The relative weight attached to output gap volatility b_w in the monetary authority's loss function is set equal to a baseline value of 0.2 (although it will be varied in the analysis below).

All parameters describing the equilibrium in the foreign economy are assumed to take values identical to the ones in the small open economy. In addition, the small economy is characterized by an openness index $\gamma = 0.4$.

Figure 1 compares the response of selected variables under the optimal commitment policy (*solid* line) to the one under discretionary optimization (*dashed* line). Several aspects are worth emphasizing. First, under discretion and in response to the rise in relative productivity, CPI inflation and the l.o.p gap both tend to *rise* on impact, while the output gap falls below the target value. The important point to notice is that the impact effect of the positive productivity shock on inflation and

output gap resembles the one in response to a cost-push shock. The rise in the l.o.p gap, in particular, is the result of a nominal depreciation combined with a sluggish movement in the domestic currency price of imports.

Second, under commitment the Central Bank trades off some volatility in the output gap in order to achieve, relative to discretion, a stronger stabilization of the l.o.p gap and in turn a stronger stabilization of the variables of interest for her loss criterion. The key is that under commitment the Central Bank can manipulate the expectations about the future behavior of the exchange rate and therefore indirectly of the l.o.p gap. In this case the initial nominal exchange rate depreciation is strongly dampened. Expectations of a persistent nominal appreciation are then generated to smooth the current rise in the l.o.p gap and in turn induce a fall in the expected future l.o.p gap. This produces an overshooting in inflation which is observed to fall persistently below steady state after a few periods. Correspondingly, and given the trade-off between the stabilization of the output gap and of the l.o.p gap, the output gap rises above its long run value for several periods. It is important to notice that this entails a possibly larger volatility of the output gap under commitment relative to discretion (see the quantitative results below). Yet the larger instability in the output gap is traded off against a smoother path of the l.o.p gap, a strategy which yields a more stable path of inflation.

Third, a typical feature of the optimal commitment policy in forward looking models emerges here, namely that the (CPI) price level exhibits a stationary dynamic.²² This feature is the result of the possibility of the commitment plan to be history dependent. On the contrary, under discretion, any temporary shock affecting inflation at time t will have a permanent effect on the price level, for the policy authority cannot commit to a certain future path for both the output gap and the l.o.p gap that allows future policy to be conditional on past shocks, and therefore undo the deviations of the price level from a stationary path (or eventually from a trend in the case of a positive value for the inflation target). However, while the CPI level exhibits a stationary dynamic under the optimal program, the same does not hold for the producer domestic price level.

The results above already illustrate the gains from commitment that characterize the optimal policy design problem. It is worth recalling that such gains emerge endogenously in response to efficient domestic productivity shocks. The statistics reported in *Table 2* confirm this intuition. Second moments of selected variables

²²See Woodford (1999a) for a discussion in the context of a closed economy models.

under commitment are compared to the ones under discretionary optimization. Two scenarios are reported. The first is labelled low weight on the output gap and corresponds to a value of $b_w = 0.2$, while the second scenario features $b_w = 0.5$, which is typically considered a high value in the literature. Two observations are in order. First, it stands clear that the optimal strategy, relative to the discretionary policy, trades off a larger output gap volatility for smoother deviations from the law of one price. This is particularly evident when the weight on output gap volatility is high. Second, notice that implementing the optimal commitment policy entails much less volatile nominal and real exchange rates relative to discretion.²³ Hence the presence of incomplete pass-through builds a case for restricting exchange rate volatility but not for fixing exchange rates which would correspond to a suboptimal policy in our case.

6 Simple Policy Rules

It is almost conventional wisdom among researchers that, in practice, monetary policy is conducted according to targeting rules. Their virtues in terms of simplicity and transparency are often emphasized. In this section we investigate how the performance of three alternative simple rules is affected by the presence of incomplete pass-through: CPI targeting (CPIT), Domestic Producer Inflation Targeting (DIT) and Exchange Rate Peg (PEG). In the analysis we maintain the assumption that monetary policy in the rest of the world aims at replicating the flexible price allocation. In the case of CPIT the domestic authority follows a strategy which implies setting $\pi_t = 0$ for all t . In the case of *DIT* the policy authority simply aims at stabilizing the rate of domestic producer price inflation, namely $\pi_{H,t} = 0$ for all t . This outcome can be implemented if and only if, in turn, the domestic real marginal cost is stabilized, which implies $\tilde{y}_t = -\frac{\kappa_\psi}{\kappa_y}\psi_{F,t}$. Hence, under DIT, the output gap is proportionally and inversely related to the l.o.p gap. In a PEG regime, the monetary authority of the small economy permanently fixes the nominal exchange rate vis a vis the rest of the world by implementing $r_t = r_t^*$ all t .

It is interesting to analyze the impact of imperfect pass-through on the relative volatility of selected variables across alternative policy rules. This is the content of the four panels in *Figure 2*, which display the effect of varying the degree of pass-through

²³Notice that Table 1 reports second moments for nominal and real depreciation *rates*, as the corresponding levels are non-stationary in this context.

(measured between 0 and 1 on the horizontal axis) on the volatility of output gap, domestic inflation, l.o.p gap and real exchange rate across the three policy regimes: DIT, CPIT and PEG. Notice, at first, that the volatility of all variables is unaffected by the degree of pass-through under a PEG. In particular, notice that a PEG implies a complete stabilization of the l.o.p gap, but also generates a larger volatility in both producer inflation and the output gap relative to DIT and CPIT. For a sufficiently high degree of pass-through this holds for CPI inflation as well. Overall this implies not only that such a regime is quantitatively further away from the optimal outcome relatively to the other two cases, but also, and most importantly, that a complete targeting of the law of one price cannot coincide with the optimal program.

The same figure suggests that the volatility of the output gap is always larger under CPIT than under DIT. This has an obvious implication in terms of the loss criterion employed in our analysis, i.e., *DIT can be preferable to CPIT*, for any given degree of pass-through, when the weight attached to output gap volatility in the Central Bank's loss criterion is relatively high. However, it stands clear that the lower the degree of pass-through, the closer the resemblance between CPIT and DIT. In the limiting case of null pass-through (i.e., $\theta_F \rightarrow 1$), the two regimes tend to coincide. This is easy to understand. In such a case, in fact, domestic currency import prices are completely fixed, so that stabilizing the producer price level corresponds exactly to stabilizing the CPI level. Finally, it is worth noticing that varying the degree of pass-through has also a substantial effect on the volatility of the *real* exchange rate. Such volatility is larger under DIT relative to CPIT for any degree of pass-through (with the exception of the limiting case of null pass-through), while it is again unaffected by the pass-through under a PEG.

7 Conclusions

We have constructed a framework to analyse the impact of incomplete exchange-rate pass-through in a fully forward-looking model of monetary policy. We have shown that the presence of incomplete pass-through (on imports price) alters the optimal monetary policy design problem in a fundamental way, by generating endogenously a trade-off between the stabilization of inflation and the stabilization of the output gap. Most interestingly, such trade-off holds independently of the measure of inflation (CPI or domestic producer) being targeted by the policymaker of the small economy. The reason is that, with incomplete pass-through, the domestic real marginal cost is

proportional to both the output gap and the l.o.p gap, which is a measure, in our context, of the deviations from the law of one price. In equilibrium, it is unfeasible for the policy authority to simultaneously stabilize the output gap and the l.o.p gap. In fact, the same change in the interest rate (meant as the instrument of policy) has both an aggregate demand channel (affecting the output gap) and an exchange rate channel (affecting the l.o.p gap), with the sign of the former effect being opposite to the sign of the latter. A key result of the paper is that along the optimal commitment program the monetary authority has to optimally weigh one channel relative to the other. In particular, the optimal program involves a partial, yet not a complete, stabilization of the l.o.p gap. Another insight of our analysis is that the effectiveness of the exchange rate channel can be appreciated only within a comparison of the optimal commitment program to the discretionary policy. In the former case, the possibility of committing to a certain future path of the l.o.p gap (and therefore of the nominal exchange rate) is crucial for the minimization of the policy authority's loss criterion employed here. It is also worth noticing that, in general, an optimal commitment policy entails much smoother nominal and real exchange rates relative to discretion.

The framework developed in this paper, due to its tractability, lends itself to several possible extensions. For example, and given the particular form of the New Keynesian Phillips curve derived here, one could explore empirically the role of the l.o.p gap in the determination of the real marginal cost and therefore of the inflation dynamics. Furthermore, one may wish to extend this setup, along the lines of McCallum and Nelson (2001), to include a role for imported inputs of production along with imported consumer goods, and allow for possibly different degrees of pass-through on different types of goods prices. Finally, it would be particularly interesting to analyze, in our framework, the interaction between monetary policy regimes and degree of pass-through in a setup where the latter is determined endogenously. Important steps in this direction have been recently taken by Devereux, Engel and Storgaard (2003).

A Derivation of Equation (36)

In this Appendix we show how to obtain equation (36). By substituting in (33) one can write an expression for the domestic currency price of imports as a function of relative productivity and the l.o.p gap:

$$\bar{p}_{F,t} = \frac{\sigma(1+\varphi)}{\sigma+\varphi\omega_s}(z_t - z_t^*) - \left(\frac{\sigma+\varphi\omega_s}{\sigma+\varphi\omega_\psi}\right)\bar{\psi}_{F,t} \quad (52)$$

Notice, in particular, that under PPT the above expression reduces to:

$$\begin{aligned} \bar{p}_{F,t}^n &= \bar{e}_t^n = s_t^n \\ &= \frac{\sigma(1+\varphi)}{\sigma+\varphi\omega_s}(z_t - z_t^*) \end{aligned} \quad (53)$$

i.e., the domestic currency price of imports moves exactly in line with the nominal exchange rate and the terms of trade.

By combining (52) with (19) one can express the dynamic of the imports price $\bar{p}_{F,t}$ in terms of a second order stochastic difference equation:

$$\delta_F \bar{p}_{F,t} = \bar{p}_{F,t-1} + \beta E_t \{\bar{p}_{F,t+1}\} + \frac{\lambda_F \sigma(1+\varphi)}{\sigma+\varphi\omega_\psi}(z_t - z_t^*) \quad (54)$$

where $\delta_F \equiv 1 + \beta + \frac{\lambda_F(\sigma+\varphi\omega_\psi)}{\sigma+\varphi\omega_s} > 1$. Under the assumption, for the sake of simplicity, that $\rho = \rho^*$, the above equation has a unique stationary solution of the form

$$\bar{p}_{F,t} = \mu_1 \bar{p}_{F,t-1} + \Omega(z_t - z_t^*) \quad (55)$$

where $\mu_1 \equiv \frac{\delta_F}{2} \left(1 - \sqrt{1 - \frac{4\beta}{\delta_F^2}}\right) < 1$ and $\Omega \equiv \frac{\lambda_F \beta \mu_1 \sigma(1+\varphi)}{(\sigma+\varphi\omega_\psi)(1-\rho\beta\mu_1)} > 0$. Hence it stands clear that the domestic currency price of imports must rise in response to a rise in relative productivity. Among other things, the elasticity of $\bar{p}_{F,t}$ to relative productivity depends positively on λ_F (the slope of the imports price inflation equation (19)) and therefore on the degree of pass-through (with a low degree of pass-through implying a high θ_F and in turn a low λ_F).

Finally by substituting (35) and (19) into (33) one can derive an expression for the l.o.p gap which corresponds to equation (36) in the text.

B Deriving the Optimal Plan

When the monetary authority has the possibility of committing as of time zero her quadratic control problem consists in choosing a state contingent plan $\{\pi_t, \pi_{H,t}, \tilde{y}_t, \pi_{F,t}, \psi_{F,t}, e_t, r_t\}_{t=0}^{\infty}$ to minimize

$$\frac{1}{2}E_t\left\{\sum_{j=0}^{\infty}\beta^j(\pi_{t+j}^2 + b_w\tilde{y}_{t+j}^2)\right\}$$

subject to the sequence of constraints (10), (23), (42), (19), (44), (6), (40) holding in all periods $t + j$, $j \geq 0$.

After taking first differences of (10) and combining with (23) one can setup the Lagrangian:

$$\begin{aligned} \max & -\frac{1}{2}E_0\left\{\sum_{t=0}^{\infty}\beta^t\left\{((1-\gamma)\pi_{H,t} + \gamma\pi_{F,t})^2 + b_w\tilde{y}_t^2\right\}\right. \\ & + 2\phi_{1,t}[\pi_{H,t} - \beta\pi_{H,t+1} - \kappa_y\tilde{y}_t - \kappa_\psi\psi_{F,t}] \\ & + 2\phi_{2,t}[\tilde{y}_t - \tilde{y}_{t+1} + \frac{\omega_s}{\sigma}(r_t - \pi_{H,t+1} - \bar{r}\bar{r}_t) - \frac{\gamma(\sigma\eta - \omega_\psi)}{\sigma}(\psi_{F,t+1} - \psi_{F,t})] \\ & + 2\phi_{3,t}[\pi_{F,t} - \beta\pi_{F,t+1} - \lambda_F\psi_{F,t}] \\ & \left. + 2\phi_{4,t}[\psi_{F,t+1} - \psi_{F,t} - r_t + r_t^* - \pi_{t+1}^* + \pi_{F,t+1}]\right\} \end{aligned}$$

where $\phi_{1,t+j}$, $\phi_{2,t+j}$, $\phi_{3,t+j}$, $\phi_{4,t+j}$ are Lagrange multipliers associated with the constraints at time $t+j$. Notice that in this setup the constraint (6) has been substituted.

The first order conditions of this problem read:

$$\pi_t(1-\gamma) + (\phi_{1,t} - \phi_{1,t-1}) - \frac{\beta^{-1}\omega_s}{\sigma}\phi_{2,t-1} = 0 \quad (56)$$

$$b_w\tilde{y}_t - \kappa_y\phi_{1,t} + \phi_{2,t} - \beta^{-1}\phi_{2,t-1} = 0 \quad (57)$$

$$\frac{\omega_s}{\sigma}\phi_{2,t} + \phi_{4,t} = 0 \quad (58)$$

$$-\kappa_\psi\phi_{1,t} + \frac{\gamma(\sigma\eta - \omega_\psi)}{\sigma}(\phi_{2,t} - \beta^{-1}\phi_{2,t-1}) - \lambda_F\phi_{3,t} + \beta^{-1}\phi_{4,t-1} - \phi_{4,t} = 0 \quad (59)$$

$$\gamma\pi_t + \phi_{3,t} - \phi_{3,t-1} + \beta^{-1}\phi_{4,t-1} = 0 \quad (60)$$

Therefore an optimal plan is defined, for any given policy weight b_w , as a bounded solution $\{\pi_{H,t}, \tilde{y}_t, \pi_{F,t}, \phi_{1,t}, \phi_{2,t}, \phi_{3,t}, \phi_{4,t}\}_{t=0}^{\infty}$ to the system of equations (10), (23), (40), (19), (44) and (56)-(60), along with the initial conditions $\phi_{1,-1} = \phi_{2,-1} = \phi_{3,-1} = \phi_{4,-1} = 0$.

Optimal Policy under Discretion.

When the policymaker lacks a commitment device the problem will be to minimize $\pi_t^2 + b_w \tilde{y}_t^2$ period by period taking as given the private sector's expectation terms contained in (10), (23), (42), (19), (44), (6), (40). The first order conditions of such problem are:

$$\pi_t(1 - \gamma) + \phi_{1,t} = 0 \quad (61)$$

$$b_w \tilde{y}_t - \kappa_y \phi_{1,t} + \phi_{2,t} = 0 \quad (62)$$

$$\frac{\omega_s}{\sigma} \phi_{2,t} + \phi_{4,t} = 0 \quad (63)$$

$$-\kappa_\psi \phi_{1,t} + \frac{\gamma(\sigma\eta - \omega_\psi)}{\sigma} \phi_{2,t} - \lambda_F \phi_{3,t} - \phi_{4,t} = 0 \quad (64)$$

$$\gamma\pi_t + \phi_{3,t} = 0 \quad (65)$$

Therefore a Markov-perfect (time consistent) solution is a set of processes $\{\pi_{H,t}, \tilde{y}_t, \pi_{F,t}, \phi_{1,t}, \phi_{2,t}, \phi_{3,t}, \phi_{4,t}\}$ that satisfies (61)-(65) along with (10), (23), (40), (19), (44) at all dates for any given policy preference weight b_w .

Conditions (61)-(65) above can be easily rearranged to obtain the following condition linking CPI inflation and the output gap under the optimal policy:

$$\tilde{y}_t = -\Theta_c \pi_t \quad (66)$$

where $\Theta_c \equiv \frac{\kappa_y + \frac{\kappa_\psi}{a}}{b_w}$ and $a \equiv \frac{1 + \gamma(\sigma\eta - 1)}{\sigma} > 0$, which is condition (47) in the text.

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Table 1

Volatility and Central Bank Loss under Alternative Monetary Policy Arrangements					
	Low weight on Output Gap (bw=0.2)		High weight on Output Gap (bw=0.5)		
	Commitment	Discretion	Commitment	Discretion	
<i>CPI Inflation</i>	0.0002	0.0022	0.0004	0.0023	
<i>Output Gap</i>	0.0012	0.0008	0.0005	0.0001	
<i>Law of One Price Gap</i>	0.1101	0.5164	0.1452	0.5340	
<i>Producer Inflation</i>	0.0024	0.0001	0.0015	0.0001	
<i>Nominal Exchange Rate</i>	0.1472	0.5442	0.1865	0.5638	
<i>Real Exchange Rate</i>	0.1396	0.4995	0.1748	0.5153	
<i>CB Loss</i>	0.0004	0.0024	0.0006	0.0024	

Note: The standard deviation of domestic and foreign productivity shocks is 1. The cross-country correlation of the shocks is 0.7. Exchange rates statistics refer to first differences.

Figure 1. Domestic Productivity Shock: Commitment vs. Discretion

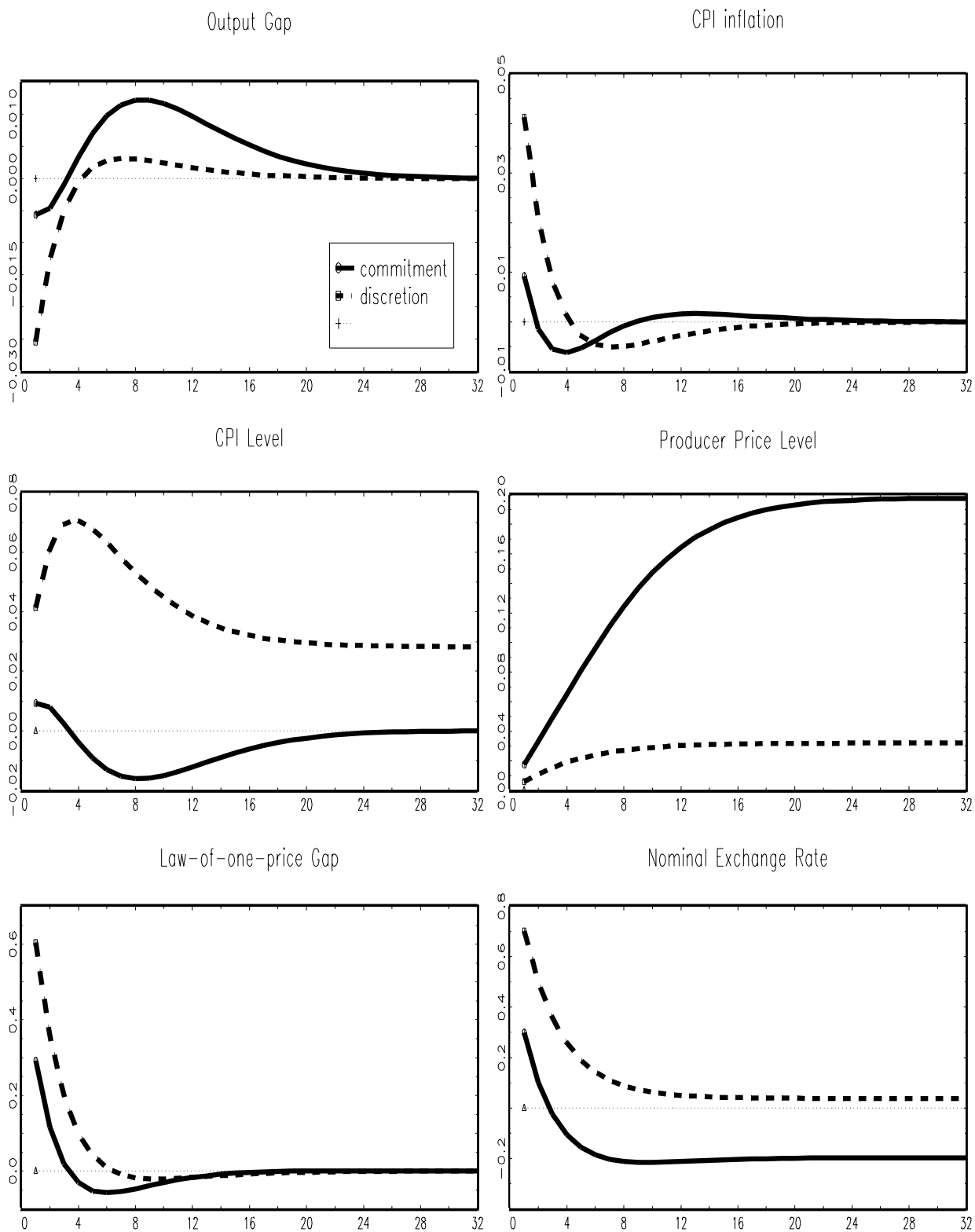
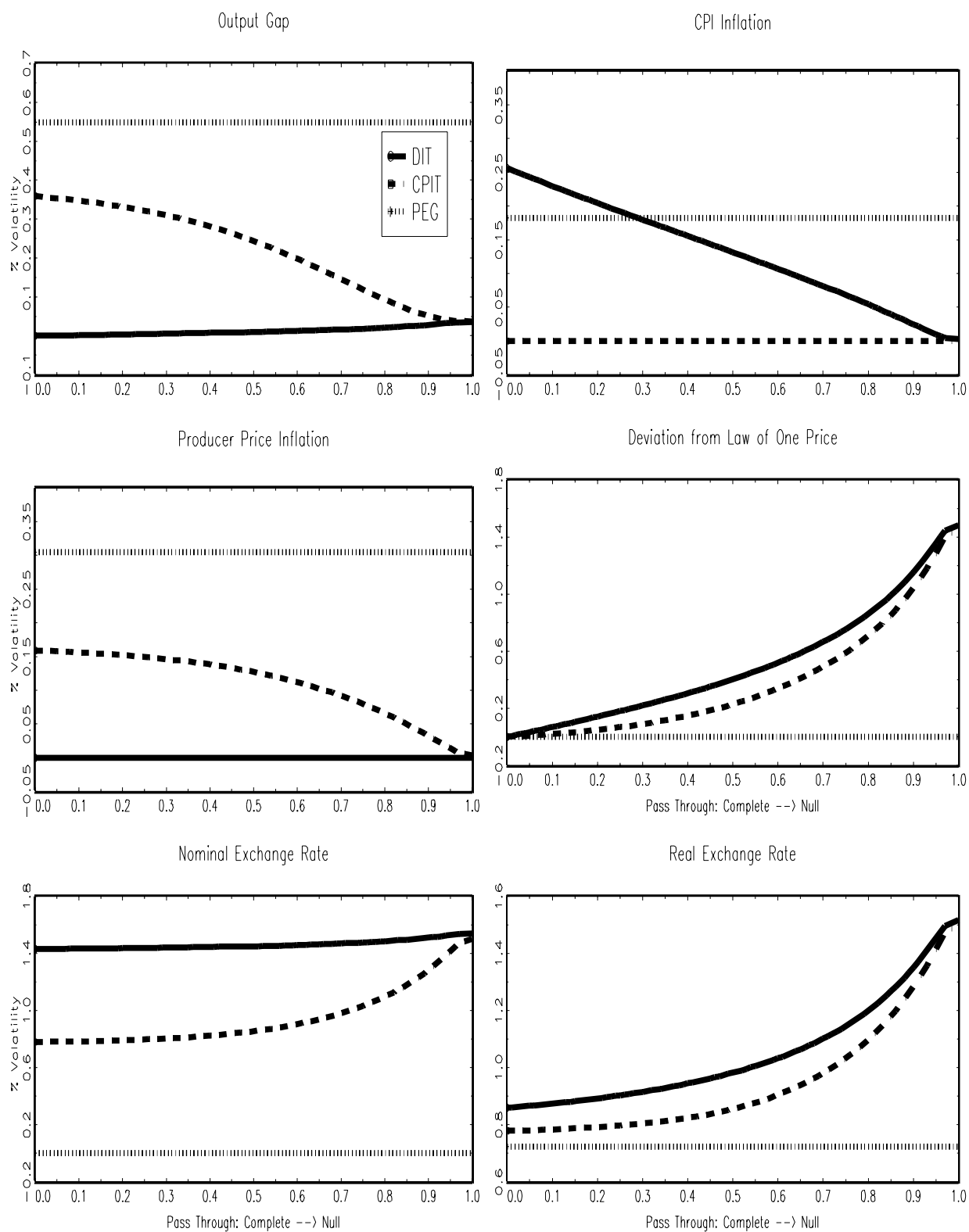


Figure 2. Pass-through and Volatility under Alternative Policy Rules



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