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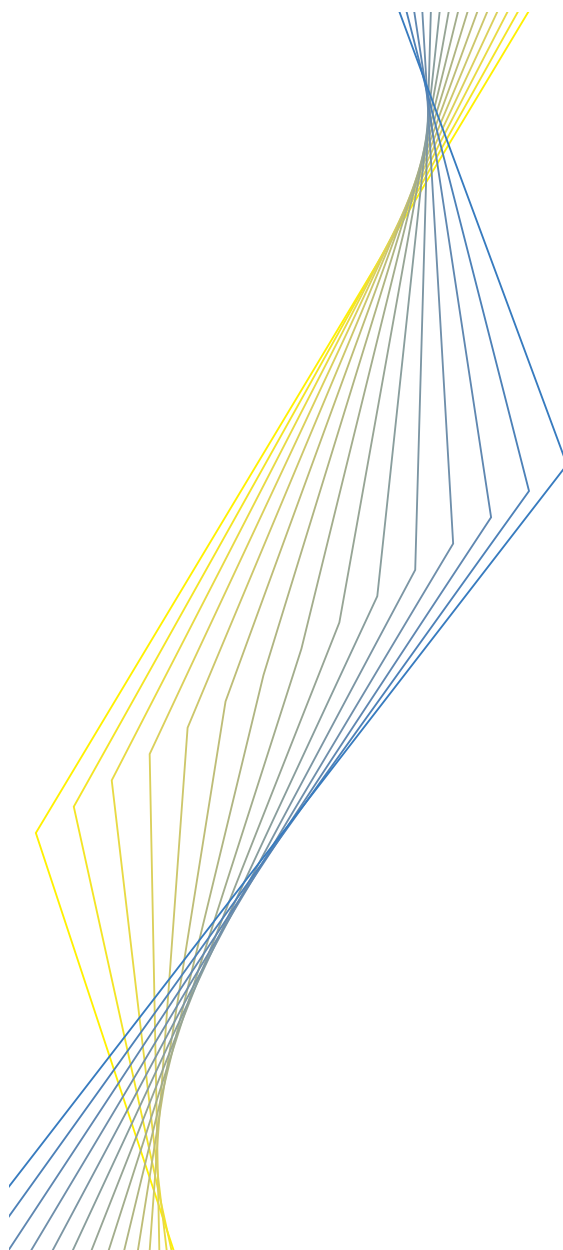
**WORKING PAPER NO. 271**

**INDETERMINACY  
AND SEARCH THEORY**

**NICOLA GIAMMARIOLI**

**SEPTEMBER 2003**

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AND SEARCH THEORY<sup>1</sup>**

**NICOLA GIAMMARIOLI<sup>2</sup>**

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### **Abstract**

This paper investigates a dynamic general equilibrium model with search. In particular, search externalities are reflected by an increasing returns to scale matching function, which may imply an indeterminate equilibrium. Hence, the model is capable to generate business fluctuations, driven by self-fulfilling belief, characterised by unemployment persistence. A numerical simulation shows that the degree of externalities needed for indeterminacy is not too far from existing empirical estimates and the implied dynamics of employment is richer than that of standard RBC models with search.

*Keywords:* Search Theory, Matching Function, Indeterminacy, General Equilibrium.

*JEL Classification:* E10, E24, J64

## Non-technical summary

The macroeconomic literature on dynamic stochastic general equilibrium models displaying indeterminacy relies on some assumptions that are not widely recognized as plausible. In particular, the production function is assumed to display large increasing returns to scale, due to either monopolistic competition in the intermediate goods market or externalities in the production process. Moreover, the labor market is considered to be competitive (among other features unemployment is ignored) and characterized by both labor supply and demand schedules either upward sloping or downward sloping. However, empirical evidence shows economies with high and persistent unemployment rates coupled with rigid wages: labor costs do not move enough to restore the equilibrium and unemployment is the natural consequence. One way to cope with this theoretical puzzle is to consider departures from the Walrasian model of the labor market. Among the different non-Walrasian models, search theory has attracted great attention in the profession

This paper analyses the possibility of indeterminacy in a general equilibrium model, once the labor market is described according to search theory. In details, the model presented can be associated with indeterminate dynamics, when labor market externalities are explicitly taken into account and eventually determine an increasing returns to scale matching function. The source of indeterminacy, however, is different from existing studies: it is not related to distortions introduced by institutions but, rather, to peculiar features of the labor market. In particular, if the probability of a representative firm finding an agent of the opposite side is different from the “social” probability of a vacancy being filled, the model may display indeterminacy for different values of the externality parameters.

The model derived in the paper is rather simple. Nevertheless, it is able to produce a series of interesting results and insights. In particular, using a matching function which incorporates externalities, displaying increasing returns to scale, and solving a (pseudo) social planner problem, the model is capable to generate indeterminacy for plausible values in the parameter space. Moreover, such a result can be derived by looking at a decentralized version of the model in which the representative household runs a firm hiring workers and at the same time supplies jobs to other firms. In this case the wage equation is determined outside the maximization problem using a Nash Bargaining solution. Finally, a simple simulation of the model shows that the increasing returns in the matching function matter for the dynamics of employment when the steady state equilibrium is perturbed by a shock: the underlined dynamics is richer and displays more persistency than that of standard RBC models with search.

# 1 Introduction

A number of economists have studied economies characterized by a multiplicity of equilibria in which fundamentals are unable to pin down a determinate equilibrium<sup>1</sup> : a path toward the stable steady state could be found only taking into account “animal spirits” or self-fulfilling belief. Once this class of models has been calibrated to match US data, it is able to explain not only the contemporaneous correlations of output, consumption, and investment (as standard RBC models) but is also able to successfully capture the dynamics of the data (Farmer and Guo, 1994), exhibiting a realistic degree of persistence.

However, the literature of indeterminacy relies on some assumptions that are not widely recognized as plausible. First, the production function is assumed to display increasing returns to scale, due to either monopolistic competition in the intermediate goods market or externalities in the production process. In particular, the degree of increasing returns needed for indeterminacy seems to be too high compared to microeconometrics studies. Second, in order to reconcile increasing returns with competitive behavior, the “private” production function is assumed to be homogeneous of degree one while the “social” production function is characterized by increasing returns. Third, the labor market is considered to be competitive (among other features unemployment is ignored) and characterized by both labor supply and demand schedules either upward sloping or downward sloping.

With regard to the labor market, empirical evidence shows economies with high and persistent unemployment rates coupled with rigid wages: labor costs do not move enough to restore the equilibrium and unemployment is the natural consequence. One way to cope with this theoretical puzzle is to consider departures from the Walrasian model of the labor market. Among the different non-Walrasian models, search theory has attracted great attention in the profession.

Despite the fact that search theory has generated many partial equilibrium models, few at-

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<sup>1</sup> See among others Howitt and McAfee (1988, 1992), Kehoe et al. (1992), Farmer (1993), Benhabib and Farmer (1994).



tempts have been made to embody this view of the labor market into the standard Kydland and Prescott real business cycle model. Monika Merz (1995) and David Andolfatto (1996) were among the first to successfully integrate search theory into otherwise standard dynamic stochastic general equilibrium models. In particular, they were able to reproduce standard macro variable fluctuations as well as previously neglected movements in employment, unemployment and vacancies. Moreover, differences between real wages and labor productivity are clearly taken into account and the dynamics of the variables considered is richer than in standard RBC models with a competitive labour market.

In a recent paper, Burda and Weder (2000) brought together both indeterminacy and real business cycle models with a non-Walrasian labor market. They showed how a dynamic stochastic general equilibrium model with search exhibits more output persistence than standard RBC models and may display indeterminacy of rational expectation paths for plausible parameter values. Although their findings can be derived assuming constant returns to scale for both the production function and the matching function, the results depend critically on distortions in the labor market introduced by institutions (e.g. payroll taxation and unemployment benefits).

The present paper analyses the possibility of indeterminacy in a general equilibrium model, once the labor market is described according to search theory. In detail, it is my intention to show that models like those of Andolfatto and Merz can be associated with indeterminate dynamics, when labor market externalities are explicitly taken into account and eventually determine an increasing returns to scale matching function. The source of indeterminacy is different from that of Burda and Weder(2000): while their results hinge on distortions introduced by institutions, my analysis focusses on peculiar features of the labor market. In particular, if the probability of a representative firm finding an agent of the opposite side is different from the “social” probability of a vacancy being filled, the model may display indeterminacy for different values of the externality parameters.

The paper is organized as follows: in the next section, I briefly present the achievements and

shortcomings of the indeterminacy literature. In Section 3, the main tool of search theory, the matching function, is introduced and discussed. Section 4 is devoted to showing the conditions under which a simple dynamic general equilibrium model displays indeterminacy with special emphasis on the labor market features necessary to obtain such a result. In particular, using a matching function with increasing returns to scale a theoretical condition for indeterminacy is found. The same result is demonstrated to arise in the decentralized version of the model in which firms and workers decisions are taken separately (section 5). In the following section a numerical exercise is performed and the theoretical condition for indeterminacy is verified by data simulation. Finally, a review of empirical studies of the matching function and a discussion on the plausibility of the parameter values chosen characterizes section 7. Section 8 concludes, discussing some issues for future research.

## 2 Related Literature

In this section, I introduce the main features and shortcomings of the indeterminacy literature with a particular emphasis on the labor market characteristics implied by this class of models. Moreover, it is my intention to show that the assumptions concerning the labor market are crucial in obtaining an indeterminate equilibrium path in the studies reviewed below.

In a standard dynamic stochastic general equilibrium model, Benhabib and Farmer (1994) find that the equilibrium is indeterminate once the production function is assumed to display increasing returns to scale and its parameters are chosen in a particular range of values. Notably, they interpret the theoretical condition for indeterminacy as a peculiar parametrization of labor demand and labor supply. For instance, this condition can be read as a requirement for the slope of the labor demand curve to be positive and steeper than labor supply (Picture 1). Although this eclectic parametrization of labor demand and supply generates indeterminacy and it is able to replicate US data features better than a standard RBC model (Farmer and Guo, 1994), it has

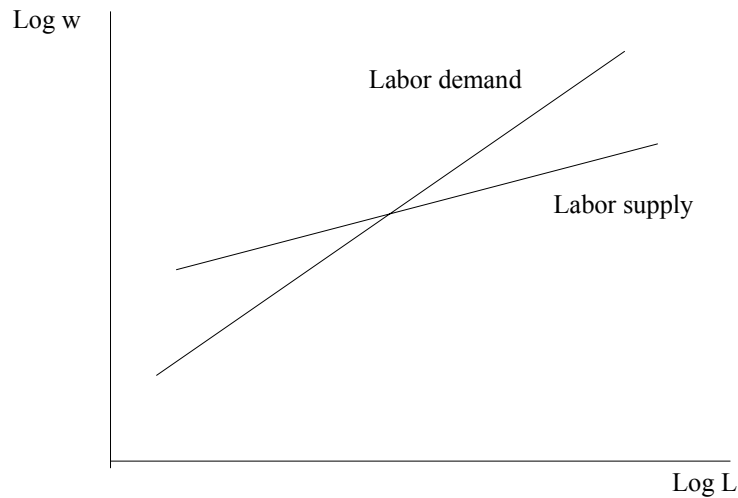


Figure 1: Labor demand and labor supply slope up

been criticized as being implausible by a number of authors.<sup>2</sup> Moreover, the required degree of increasing returns to scale in production has been found at odds with recent empirical studies (Basu and Fernald, 1997)

In a more recent paper, Benhabib and Farmer (2000) try to explain the real effects of money (the so-called monetary transmission mechanism) by using a dynamic stochastic general equilibrium model in which real balances have been introduced into a standard production function. In particular, they show that rational expectations are not sufficient to pin down a particular equilibrium, indeterminacy arises and, therefore, the monetary transmission mechanism can be explained by agents' self-fulfilling expectations. In details, agents in the real world resolve the indeterminacy problem by coordinating on a particular equilibrium that has the property that prices are predetermined one period in advance. Empirically, the model is able to fit real data for particular values of the preference parameters. However, the parametrization chosen implies a labor supply curve with a negative slope, albeit derived from a market equilibrium condition

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<sup>2</sup> See for example Aiyagari (1995).

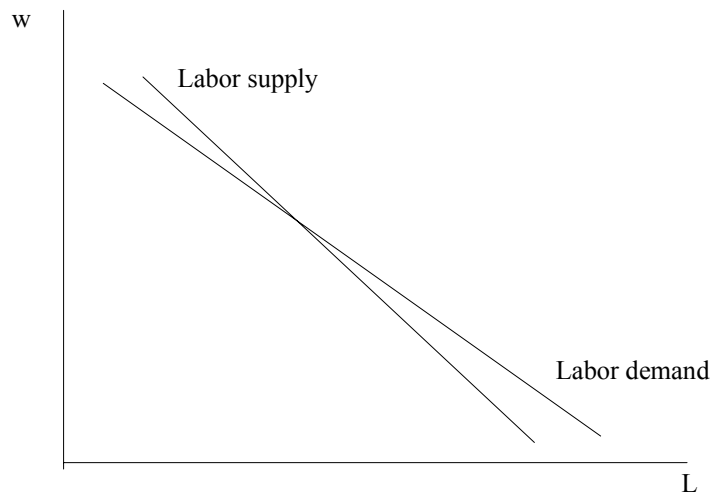


Figure 2: Labor demand and labor supply slope down

(Figure 2). As in the previous case (upward sloping labor demand and supply), the implications of the parameters choice appear too eclectic and not widely accepted. In conclusion, although downward sloping labour demand and supply schedules are able to reproduce aggregate phenomenon such as the procyclicality of employment, their foundation are at odds with microeconomics studies of the labor market.

In order to overcome the non standard implications of the indeterminacy condition on labor supply and demand, Bennett and Farmer (2000) generalize the results found in Benhabib and Farmer (1994) by using a model with a Walrasian labor market and preferences that are non separable in consumption and leisure. Their main findings are that the degree of increasing returns to scale in production needed for indeterminacy is lower than that of previous studies, and that the condition for indeterminacy found does not imply labor demand and supply schedules with “wrong” slopes. In particular, labor demand slopes down and the (constant-consumption) labor supply slopes up. However, the parametrization of their model triggers a difference between the constant-consumption and the Frisch labor supply curves, where the Frisch labor supply curve

is a function of the real wage holding constant the marginal utility of consumption. Therefore, even if the constant-consumption labor supply slopes up, Frisch labor supply curve is downward sloping and crosses labour demand with the “wrong” slope.

The studies reviewed above reveal that general equilibrium models displaying indeterminacy are able to capture some key features of actual data such as their dynamics and degree of persistency. However, indeterminacy conditions hinge on peculiar features of the labor market which are not always supported by empirical evidence. Moreover, a common characteristic of the models presented is the assumption of a competitive labor market which is always maintained. In the rest of the paper I will relax such an assumption. In particular, in order to provide a plausible foundation for indeterminacy in simple general equilibrium models, it is my intention to take into account frictions and externalities arising in the labor market.

### **3 Search Theory and the Matching Technology**

In the standard neo-classical theory, all markets are competitive and agents’ decisions are coordinated by prices. In particular, wages move instantaneously in order to keep labor demand and labor supply in balance. Therefore unemployment, if exists, it is only temporary. Empirical evidence, however, shows economies with high unemployment rates and rigid wages. In other words, wages do not move enough to restore the equilibrium and unemployment is the natural consequence.

One way to take these features into account is to consider departures from the Walrasian model of the labor market. Among the different non-Walrasian models, I consider here the so called search theory. Originated by the Phelps volume (1970), search theory has been developed by many economists in the 1980s and in the 1990s.<sup>3</sup>

Unlike the frictionless labor market, workers and firms are heterogeneous and meet in a one-to-one fashion, in a decentralized labor market. The process of matching up workers and firms,

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<sup>3</sup> In particular see Blanchard and Diamond (1989), Pissarides (1990), Mortensen and Pissarides (1994).

summarized by a matching technology, is a complex, time consuming and costly process of search in which different kinds of externalities arise. Indeed, the number of new matches depends crucially on the relative number of agents in the two sides of the market (labor market tightness) as well as on their efforts in searching and recruiting. In particular, for a given agent, a positive externality arises if the number of the agents in the other side of the market increases. On the other hand, a negative externality is related to the increasing number of agents in the same side of the market (congestion).

The simplest matching function  $M = M(\cdot, \cdot)$  describes the job creation flow and depends on vacant jobs posted by firms ( $V$ ) and the number of unemployed workers ( $1 - L$ ):

$$M = M(V, 1 - L)$$

The matching technology above, like a production technology, describes a relationship between inputs (vacancies and unemployment) and output (flow rate at which workers and vacant jobs form new job-worker matches)<sup>4</sup>.

In the standard search literature the matching function  $M(\cdot, \cdot)$  is assumed homogeneous of degree one (constant return to scale)<sup>5</sup>, non decreasing and concave. In particular, given the matching function properties, the probability of a vacant job being filled and the probability of an unemployed worker finding a job are endogenous and depend on the labor market tightness  $X = \frac{V}{1-L}$ :

$$\frac{M(V, 1 - L)}{1 - L} = M\left(\frac{V}{1 - L}, 1\right) \equiv m(X)$$

and

---

<sup>4</sup> Note that the aggregate labor force has been normalized to 1.

<sup>5</sup> Blanchard and Diamond (1989) for the US and Pissarides (1986a) for the United Kingdom have estimated constant return to scale matching functions.

$$\frac{M(V, 1 - L)}{V} = \frac{m(X)}{X} \equiv q(X)$$

Note that, given the probabilities  $m(X)$  and  $q(X)$ , the duration of unemployment is equal to  $1/m(X)$  and the duration of a vacancy is equal to  $1/q(X)$ .

The probabilities above can help to understand the externalities displayed by a search model. The probability for an unemployed worker to find a job,  $m(X)$ , decreases with congestion caused by an increase in unemployment ( $1 - L$ ) and increases in market thickness represented by an increase in the number of posted vacancies. The reverse is true for  $q(X)$ , the probability of a match for a vacant job.

In what follows I will move from the standard assumptions above, by taking into account that the externalities implied by a matching technology may lead, eventually, to an increasing returns to scale matching function, an hypothesis which has gained a large empirical support in the literature<sup>6</sup>. Moreover it is worth analyzing the effect of this departure from the mainstream in looking at the dynamics and the propagation mechanism of a general equilibrium model in which those features are explicitly taken into account. One way to introduce an IRS matching function into a standard RBC model is by using arguments similar to those used in the indeterminacy literature for production technologies. In particular, a representative worker and a representative firm face a constant return to scale matching function of the following form:

$$M = AV^a(1 - L)^b \tag{1}$$

where  $a + b = 1$  and  $A > 0$  is a shifting parameter.

Representative agents consider the aggregate shift parameter  $A$  as given, exogenous to their decisions. However  $A$  is determined by the activities of other agents in the economy. In particular

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<sup>6</sup> The issue will be discussed in section 7.

the externalities produced by such activities can be modelled as follows:

$$A = \bar{V}^{\alpha-a} (1 - \bar{L})^{\beta-b} \quad (2)$$

where  $\alpha \geq a$ ,  $\beta \geq b$ ,  $\bar{V}$  and  $1 - \bar{L}$  represent the economic-wide levels of vacancies and unfilled jobs, and  $\alpha + \beta \geq 1$ .

The economy is assumed to be populated by an infinite number of firms and workers such that representative agents consider the two quantities above as given, however, recognizing that in equilibrium  $V = \bar{V}$  and  $1 - L = 1 - \bar{L}$ , the “private” matching function (1) can be transformed into the following “social” matching function:

$$M = V^\alpha (1 - L)^\beta \quad (3)$$

When  $\alpha + \beta > 1$ , the matching function described by (3) displays increasing returns to scale. In other words, representative agents face a CRS function (1) while in aggregate, the “social” matching function exhibits increasing returns to scale (3).

The economic intuition for introducing the externality in the search framework in the same way it has been done for production technologies elsewhere can be explained by using labour market segmentation arguments. Agents, in making their decisions, look at what surrounds them without taking into account the effects of labour market conditions prevailing in the overall economy. For instance, a firm operating in a defined geographical area would look at the labour market conditions in that area without considering neither the effect of its decision on neighboring areas nor the effects of labour market condition of other areas in the region it. However, what matters for the economy-wide matching technology are the conditions in the entire economy and not only those prevailing in a specific area.



## 4 The Benchmark Model

In this section, I briefly describe the preferences of the representative consumer, the production technology, the aggregate employment evolution and the maximization problem set up. In the next subsections the solution of the problem is found and the dynamics of the system around the steady state discussed.

The representative consumer derives his utility only from consumption ( $C$ ) and preferences are described by a logarithmic function such that the instantaneous utility is specified as:

$$U(C) = \log C \quad (4)$$

The production technology is assumed to display constant returns to scale and depending only on labor, in other words output is linear in labor:

$$Y = F(L) = L \quad (5)$$

In this economy capital plays no role and aggregate output is entirely consumed. Hence, the aggregate resource constraint of the economy reduces to:

$$Y = C$$

The standard frictionless Walrasian labor market is replaced by one that can be described using a simple search and matching model, in which externalities are explicitly taken to account and eventually increasing returns to scale in matching matter. The labor market considered here is characterized by a continuous flow of people from employment to unemployment ad *vice versa*.

Even in the steady state, when employment is constant, people moves in opposite directions (in this case inflows match exactly outflows).

Job destruction (outflows from employment) is an exogenous process determined by the separation rate  $\delta$ . On the other hand, the job creation process is summarised by the matching function (1) described above. In order to focus only on movements between employment and unemployment, and not in and out the labour force the assumption  $L = \bar{L} = 1$  is imposed. Therefore, the dynamics of employment is given by:<sup>7</sup>

$$\dot{L} = M - \delta L \tag{6}$$

The maximization problem of the economy described above can be solved using the social planner paradigm, even if it is not going to give the first best solution. As a matter of fact in what follows the social planner behaves myopically: he does not recognize the external effect of choosing a determinate level of vacancies (or the effect of choosing different amount of resources devoted to recruitment activities). In other words, he considers  $\bar{V}$  and  $1 - \bar{L}$  as given, as external to his problem.

Analytically, the social planner evaluates the stream of consumption services ( $C$ ) according to:

$$\int_0^{\infty} e^{-\rho t} [U(C)]$$

where  $\rho$  is the discount rate and  $U$  is the instantaneous utility function defined in (4). If the production function is linear in labor (5) and the entire output is consumed, the “pseudo” social planner problem can be expressed as follow:

$$\max_V \int_0^{\infty} e^{-\rho t} [\log L - V] \tag{7}$$

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<sup>7</sup> The variables depend on time. We do not write time dependence to avoid heavy notation.

subject to (6).

Note that the social planner takes into account the cost of posting a vacancy for the representative firm and such a cost (normalised to the number of vacancies) is expressed in utility units. The choice of introducing the cost of posting a vacancy as a utility cost simplifies the analysis and, given the capital is absent in the model, it does not influence the result.

#### 4.1 Problem's Solution

The present value Hamiltonian of the (pseudo) social planner problem can be summarized by the following expression:

$$H = \log L - V + \Lambda [M - \delta L] \quad (8)$$

where  $\Lambda$  represents the co-state variable.

The first order condition with respect to  $V$  is given by:

$$-1 + \Lambda \frac{aM}{V} = 0 \quad (9)$$

The derivative with respect the co-state gives the expression for the employment dynamics:

$$\dot{L} = M - \delta L \quad (10)$$

and along the optimal path the shadow value of labor has to satisfy the following rule:

$$\dot{\Lambda} = (\rho + \delta)\Lambda - \frac{1}{L} + \Lambda \frac{bM}{1-L} \quad (11)$$

Finally, the transversality condition of the problem is given by:

$$\lim_{t \rightarrow \infty} e^{-\rho t} \Lambda(t) = 0 \quad (12)$$

In order to analyze in a clearer way the dynamic of the solution, it is useful to divide expressions (10) and (11) by  $L$  and  $\Lambda$  respectively and make the following logarithmic transformations:  $l = \log L$ ,  $\lambda = \log \Lambda$ , and  $m = \log M$ .

The employment dynamics equation becomes:

$$\dot{l} = e^{m-l} - \delta \quad (13)$$

and the co-state equation:

$$\dot{\lambda} = \delta + \rho - e^{-l-\lambda} + b \frac{e^m}{1 - e^l} \quad (14)$$

Before analyzing the stability properties of this pair of differential equation around the steady state, it is necessary: first to prove the existence of the steady state, and second express  $m$  as a function of  $l$  and  $\lambda$ .

## 4.2 Existence of the Steady State

In the steady state  $\dot{\Lambda} = \dot{L} = 0$  such that from (10) and (11) it is possible to obtain:

$$\delta L = M \quad (10')$$

and

$$\rho + \delta = \frac{1}{L\Lambda} - \frac{b}{(1-L)} M \quad (11')$$

We can use the two static equations (3) and (9) to complete a four equation system in four unknowns:

$$M = V^\alpha (1 - L)^\beta \quad (3)$$

and

$$\Lambda = \frac{V}{aM} \quad (9)$$

Substituting  $M$  and  $\Lambda$  as in (3) and (9) into (10') and (11') and deriving  $V$  from (3), the resulting expression in  $L$  is:

$$a\delta^{\frac{\alpha-1}{\alpha}} L^{-\frac{1}{\alpha}} (1-L)^{\frac{\beta}{\alpha}} - b\delta L(1-L)^{-1} = \rho + \delta$$

It is clear that the function described by the left hand side of the expression above is monotone: the first derivative is negative in the domain of  $L$ , when  $L$  goes to 0 the function goes to infinity, while when  $L$  goes to 1 it goes to minus infinity. There is only one solution, therefore the steady state exists and is unique.

### 4.3 Local Dynamics

After having proved the existence and the uniqueness of the steady state, we have to express the dynamics around such a steady state. However, we need to express  $m$  as a function of  $l$  and  $\lambda$  first. In order to succeed in this task we use a log linearized version of the two static equations (3) and (9) that must be satisfied by  $M$  and  $V$  along their time paths. Hence expression (3) becomes:

$$m = \alpha v - \frac{L}{1-L} \beta l \quad (15)$$

and expression (9) is transformed in:

$$v = \tilde{a} + \lambda + m \quad (16)$$

where  $v = \log V$ , and,  $\tilde{a} = \log a$ .

The expression for  $m$  we looked for (demonstration in the appendix) is:

$$m = a_1 l + a_2 \lambda + K \quad (17)$$

where

$$a_1 = -u\beta \frac{1}{1-\alpha} \quad (18)$$

$$a_2 = \frac{\alpha}{1-\alpha} \quad (19)$$

$$K = \tilde{a} \frac{\alpha}{1-\alpha} \quad (20)$$

Substituting the expression for  $m$  into (13) and (14) we obtain the system governing the equilibrium dynamics of the state ( $l$ ) and co-state ( $\lambda$ ) variables :

$$\dot{l} = e^{(a_1-1)l+a_2\lambda+K} - \delta \quad (21)$$

$$\dot{\lambda} = \delta + \rho - e^{-l-\lambda} + b \frac{e^{a_1 l + a_2 \lambda + K}}{1 - e^l} \quad (22)$$

In looking at the dynamics of the system around the steady state, we linearize equation (21)-  
(22):

$$\begin{bmatrix} \dot{l} \\ \dot{\lambda} \end{bmatrix} = J \begin{bmatrix} \tilde{l} \\ \tilde{\lambda} \end{bmatrix}$$

where tildes denote deviations from the steady state and the elements of  $J$ , as shown in the appendix, are represented by:

$$J = \begin{bmatrix} (a_1 - 1)\delta & a_2\delta \\ (\rho + \delta + b\delta u) + ba_1u\delta + u^2b\delta & a_2ub\delta + (\rho + \delta + b\delta u) \end{bmatrix} \quad (23)$$

where:

$$u = \frac{L}{1-L} = \frac{e^l}{1-e^l}$$

At this stage of the analysis, it is crucial to check the stability of the system above. Technically, we must determine the sign of the two roots of the matrix  $J$ . In particular if the steady state  $(\lambda^*, l^*)$  is completely stable (two negative roots) the equilibrium will be indeterminate. In other words, all trajectories satisfying (21) and (22), in the neighborhood of  $(\lambda^*, l^*)$ , converge back to the steady state. On the other hand, if the two roots are of opposite signs only one trajectory converges back to the steady state while all the others diverge (saddle path).

The signs of the roots can be derived looking at the trace and the determinant of the matrix  $J$ , since they represent the sum and the product of the roots respectively.

$$Trace = (a_1 - 1)\delta + a_2ub\delta + (\rho + \delta + b\delta u) < 0 \quad (24)$$

$$Det = (\rho + \delta + b\delta u)[\delta(a_1 - 1) - a_2\delta] - a_2ub\delta^2(1 + u) > 0 \quad (25)$$

**Proposition 1** *The equilibrium is indeterminate if the following conditions are satisfied:  $\alpha > 1$  and  $\alpha > \frac{\beta}{b}$*

**Proof**

One can notice that the sign of the trace is changing according to the sign of  $a_1$  and  $a_2$ . In particular, when  $\alpha$  is in the neighborhood of 1, both  $a_1$  and  $a_2$  are close to infinity and their effect dominates in determining the signs of the trace and the determinant. Dealing with the trace first, if  $\alpha$  is close to one its sign will be determined by the following expression:

$$a_1\delta + a_2ub\delta$$

Substituting for  $a_1$  and  $a_2$  we obtain

$$\frac{1}{1-\alpha}\delta u(\alpha b - \beta)$$

For  $\alpha > 1$  the condition for negative trace is

$$\alpha > \frac{\beta}{b} \tag{26}$$

On the other hand for  $\alpha < 1$  the condition for a negative trace is

$$\alpha < \frac{\beta}{b}$$

However only in the first case, when  $\alpha > 1$ , the determinant is positive, supporting the indeterminacy hypothesis. Indeed with  $\alpha$  strictly greater than one all the three elements of the



determinant are positive, while in the second case, for  $\alpha < 1$ , the determinant is negative: the two roots are of opposite sign and the steady state is a saddle point.

The result that increasing returns in matching leads to multiple equilibria and indeterminate equilibrium paths is not new in the search literature (see for example Diamond, 1982). The source of such indeterminacy results depends on the positive feedback which works through search externalities: the increase in the number of potential searching partners makes matching easier. The positive feedback is that easier search makes matching more profitable. In the setting of the present paper, however, this mechanism works only for specific values of the externalities and for large increasing returns, not only in the overall matching technology but also for the vacancy input alone. In details, the parametrization needed to generate indeterminacy in the setting above implies that the duration of vacancies decrease with their number. Although this implication could seem unrealistic, it can be supported using the same arguments as those used to justify the introduction of search externalities in the matching function in section 3. In a world where labour markets are highly segmented and/or the economy is experiencing either a boom or a bust, posting a vacancy may determine large external effects, inducing more than proportional variation in labour supply and therefore a "perverse" implication for the overall matching process. Moreover, as it will be shown in section 7, recent empirical literature does not exclude the possibility of an elasticity of matching to vacancies greater than one.

## 5 Decentralized version of the model

In this section of the paper I will show that the results found above can be obtained looking at a representative household maximization problem. The representative household supplies workers to other families and receives a wage ( $w$ ), while, at the same time, it hires workers to run the family firm, paying a salary to them.

## 5.1 The Household Problem

The maximization problem of the representative household can be written as follows:

$$\max \int_0^{\infty} e^{-\rho t} [\log(L^D - wL^D + wL^S) - V] \quad (27)$$

where  $L^D$  is the labor used in the family firm,  $L^S$  is the labor supplied to the other firms and  $w$  is the wage rate, subject to:

$$\dot{L}^D = V \frac{M}{\bar{V}} - \delta L^D$$

$$\dot{L}^S = (1 - L^S) \frac{M}{1 - \bar{L}} - \delta L^S$$

It is worth noticing that in the two expressions above the probability of filling a vacancy  $M/\bar{V}$  and the probability of finding a job  $M/(1 - \bar{L})$  are considered as given by the representative household.

The present value Hamiltonian of the problem is:

$$H = \log[L^D - wL^D + wL^S] - V + \eta^D \left( V \frac{M}{\bar{V}} - \delta L^D \right) + \eta^S \left[ (1 - L^S) \frac{M}{1 - \bar{L}} - \delta L^S \right] \quad (28)$$

The resulting first order conditions of the problem are given by the following expressions:

$$-1 + \eta^D \frac{M}{\bar{V}} = 0 \quad (29)$$

$$\dot{L}^D = V \frac{M}{\bar{V}} - \delta L^D \quad (30)$$

$$\dot{L}^S = (1 - L^S) \frac{M}{1 - \bar{L}} - \delta L^S \quad (31)$$

$$\dot{\eta}^D = (\rho + \delta) \eta^D - \frac{1 - w}{L} \quad (32)$$

$$\dot{\eta}^S = (\rho + \delta) \eta^S - \frac{w}{L} + \frac{M}{1 - \bar{L}} \eta^S \quad (33)$$

## 5.2 Wage Equation

In order to solve the model we need an expression for the wage rate. As in the standard search literature, we use a Nash bargaining solution. In other words, in a labour market characterized by trade frictions, the real wage divides the rent generated by a match between the employee and the employer, according to a particular weight ( $\psi$ ), which represents the worker's bargaining power compared to that of the firm. In our dynamic settings the wage rate will be derived maximizing the following expression:

$$\max \left( \frac{\partial H}{\partial L^S} \right)^\psi \left( \frac{\partial H}{\partial L^D} \right)^{1-\psi} \quad (34)$$

where the two terms in brackets represent the worker's surplus and the producer's surplus respectively. The solution is:

$$\frac{\psi}{\frac{\partial H}{\partial L^S}} = \frac{1 - \psi}{\frac{\partial H}{\partial L^D}}$$

$$\frac{\psi}{\frac{w}{L} - \delta \eta^S - \eta^S \frac{M}{1 - \bar{L}}} = \frac{1 - \psi}{\frac{1 - w}{L} - \delta \eta^D}$$

$$\psi \left( \frac{1 - w}{L} - \delta \eta^D \right) = (1 - \psi) \left( \frac{w}{L} - \delta \eta^S - \eta^S \frac{M}{1 - \bar{L}} \right)$$

$$w = \psi + (1 - \psi) \frac{L}{1 - L} \eta^S M + \delta [(1 - \psi) \eta^S - \psi \eta^D] \quad (35)$$

However, as shown in the appendix, the last term of the expression above is zero:

$$\psi \eta^D = (1 - \psi) \eta^S \quad (36)$$

Hence the wage equation (35) becomes:

$$w = \psi + (1 - \psi) \frac{L}{1 - L} \eta^S M \quad (37)$$

### 5.3 Equivalence of the two approaches

The final step of demonstrating that the pseudo-social planner problem solved in the previous paragraph is consistent with the decentralized economy presented here, consists in showing that conditions (9)-(11) can be obtained from conditions (29)-(33).

In particular, the following proposition holds:

**Proposition 2** *If  $\psi = b = 1 - a$ , then the the pseudo social planner problem can be decentralized in a competitive equilibrium.*

#### Proof

Assuming  $\eta^D = \Lambda a$ , substituting for  $w$  according to (37), and rearranging, condition (32) can be transformed as follows:

$$a\dot{\Lambda} = (\rho + \delta) a\Lambda - \frac{1}{L} + \frac{\psi}{L} + \frac{\psi a \Lambda M}{1 - L} \quad (38)$$

$$\dot{\Lambda} = (\rho + \delta) \Lambda - \frac{1 - \psi}{aL} + \frac{\psi \Lambda M}{1 - L} \quad (39)$$

It is clear that given  $\psi = b = 1 - a$ , expression (39) is identical to (11).

Looking at (33) and considering  $\eta^S = \frac{\psi}{1-\psi}\eta^D = \frac{\psi}{1-\psi}a\Lambda = \frac{\psi}{1-\psi}(1-b)\Lambda$ , we obtain:

$$\frac{\psi}{1-\psi}(1-b)\dot{\Lambda} = (\rho + \delta) \frac{\psi}{1-\psi}(1-b)\Lambda - \frac{\psi}{L} + \psi \frac{M}{1-L} \frac{\psi}{1-\psi}(1-b)\Lambda \quad (40)$$

$$\dot{\Lambda} = (\rho + \delta) \Lambda - \frac{1-\psi}{1-b} \frac{1}{L} + \psi \frac{M}{1-L} \Lambda \quad (41)$$

Again for  $\psi = b$ , equation (41) is equal to expression (11).

Moreover, one can notice that the reparametrization of the co-state variable used above, namely  $\eta^S = \Lambda a$ , is enough to prove the equivalence between (29) and (9). Finally, recognizing that in equilibrium  $V = \bar{V}$ ,  $1 - L = 1 - \bar{L}$  and  $L^D = L^S$ , condition (10) can be obtained either from (30) or (31).

It is worth noticing that the proposition above is the continuous time version of the so-called Hosios condition<sup>8</sup>. The condition can be phrased as follows: when the wage bargaining parameter ( $\psi$ ) is equal to the elasticity of the matching function with respect unemployment (workers search effort), then the competitive equilibrium is equivalent to the solution of the social planner problem. In standard models with a constant returns to scale matching function, such a condition means that negative and positive externalities offset each other and the social planner solution is efficient. Here the condition is just technical and it has different welfare consequences.

In the context of the model presented above, I have demonstrated the equivalence between the competitive equilibrium and a pseudo social planner problem; where for pseudo social planner problem I mean a problem solved by a myopic dictator who does not recognize the external effect of aggregate searching and recruiting activities. Therefore, the solution of the problem does not represent the first best and policy intervention may play a role in increasing agents' welfare.

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<sup>8</sup> See Hosios (1990)

## 6 A Numerical Simulation

In this section the dynamics of the model above is simulated according to different parameter values. The discount rate parameter ( $\rho$ ) is assumed to be equal to 0.03, while the exogenous separation rate ( $\delta$ ) is set equal to 0.10.

$\alpha = a$	$\beta = b$	$r_1$	$r_2$	<i>Dynamics</i>
0.70	0.30	-1.7242	1.1084	<i>Saddle</i>
0.65	0.35	-1.4647	0.8539	<i>Saddle</i>
0.60	0.40	-1.2833	0.6720	<i>Saddle</i>
0.55	0.45	-1.1522	0.5358	<i>Saddle</i>
0.50	0.50	-1.0557	0.4297	<i>Saddle</i>
0.45	0.55	-0.9839	0.3443	<i>Saddle</i>
0.40	0.60	-0.9311	0.2732	<i>Saddle</i>
0.35	0.65	-0.8998	0.2240	<i>Saddle</i>
0.30	0.70	-0.8687	0.1593	<i>Saddle</i>

Table 1: Matching function with constant returns to scale

Table 3.1 above reports the sign and the size of the matrix  $J$  roots, assuming a constant returns to scale matching function ( $\alpha = a$  and  $\beta = b$ ), for different values of the elasticity parameters. The final column of the table summarises the implied dynamics of the simulated system (21)-(22). As in the standard model of search, the solution is a saddle path, i. e. the equilibrium is determinate and the fundamentals of the economy are able to pin down such an equilibrium.

Table 3.2 below considers the case of increasing returns in matching. In particular, the externality is assumed to arise from the vacancy side of the matching, while  $b$  is assumed to be equal to  $\beta$ .

It is clear that given  $\beta/b = 1$ , the condition for indeterminacy reduces to  $\alpha > 1$ . The theoretical results found in the paper are confirmed by the simulation: as soon as  $\alpha$  crosses one, the two roots become negative, leading to an indeterminate equilibrium. In this case the fundamentals are

$\beta = b$	$a$	$\alpha$	$r_1$	$r_2$	<i>Dynamics</i>
0.4	0.6	0.6	-1.2833	0.6720	Saddle
0.4	0.6	0.7	-1.2388	0.7211	Saddle
0.4	0.6	0.8	-1.2964	0.8437	Saddle
0.4	0.6	0.9	-1.5815	1.1756	Saddle
0.4	0.6	0.99	-4.2566	3.8827	Saddle
0.4	0.6	1.01	$-0.18395 + 4.0149i$	$-0.18395 - 4.0149i$	Sink
0.4	0.6	1.05	$-0.17836 + 1.75047i$	$-0.17836 - 1.75047i$	Sink
0.4	0.6	1.1	$-0.172149 + 1.20273i$	$-0.172149 - 1.20273i$	Sink
0.4	0.6	1.2	$-0.1618514 + 0.8103i$	$-0.1618514 - 0.8103i$	Sink

Table 2: Matching function with increasing returns to scale

unable to pin down uniquely the paths towards the steady state once the system is hit by a shock. Animal spirits may play a role in agents coordination toward one of the infinite possible equilibria. Moreover, the dynamics may be driven not only by real shocks as in the standard RBC models with search but also by the so called sunspots: agents self-fulfilling believes. Finally, it is worth noticing that the roots are complex, meaning richer dynamics. Not only is the persistence of shocks guaranteed by indeterminacy and the propagation mechanism influenced by search, but also cycles enrich the propagation mechanism by some features which, in principle, can better explain employment and unemployment movements in real world.

In order to better understand the propagation mechanism of a unitary shock to the equation governing the dynamics of labour, I have constructed three different impulse response functions according to different degrees of increasing returns to scale in the matching technology. Assuming a constant returns to scale case matching function ( $a = \alpha = 0.6$ ,  $b = \beta = 0.4$ ), picture 3 shows the dynamics of employment once a unitary perturbation has moved  $l$  from its equilibrium<sup>9</sup>.

The qualitative results are similar to those of Merz (1995) and Andolfatto (1996). When search theory is chosen to model the labour market in a general equilibrium model, the frictions and externalities, operating even under constant returns to scale, prevent an immediate restoration of the equilibrium: it takes some time before employment returns back to the long-run level. In other words, employment (and unemployment) displays a certain degree of persistence.

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<sup>9</sup> Note that  $\lambda$  is not perturbed by any shock. Eventually  $\lambda$  moves because the indirect influence of labour

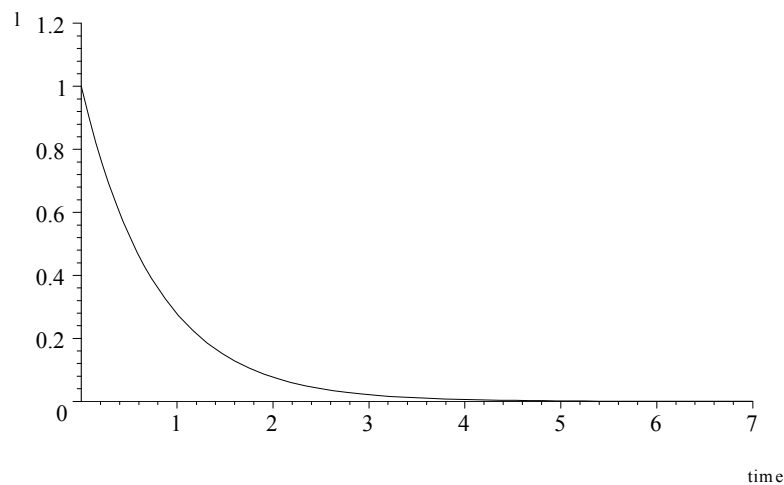


Figure 3: Impulse response of equilibrium employment to a unitary shock (constant returns to scale).

In picture 4 I reproduce the graphical results of a simulation of the model under the assumption of increasing returns in matching. The curve reproduces the dynamics of labour once, at time zero, it has been perturbed by a unitary shock. In particular the degree of returns to scale is equal to 1.5 with  $a = 0.6$ ,  $\alpha = 1.1$  and  $b = \beta = 0.4$ .

As one can notice the dynamics of employment is particularly complex. First it displays a sort of overshooting at the beginning of the time-period. In other words, the effect of the unitary shock to labour is magnified after it takes place: the curve shows an hump-shaped form. Second, the dynamics is characterized by cycles around the long-run equilibrium. This is peculiar to models of indeterminacy in general and to those exhibiting complex roots in particular. At this stage it is impossible to see clearly which phenomena of actual labour markets can be explained by this pattern of employment around the steady state. However, employment (and unemployment) seems to oscillate across time rather than showing an upward or downward linear trend. Third, the process of returning towards the steady state takes time, reproducing the persistence displayed

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dynamics.



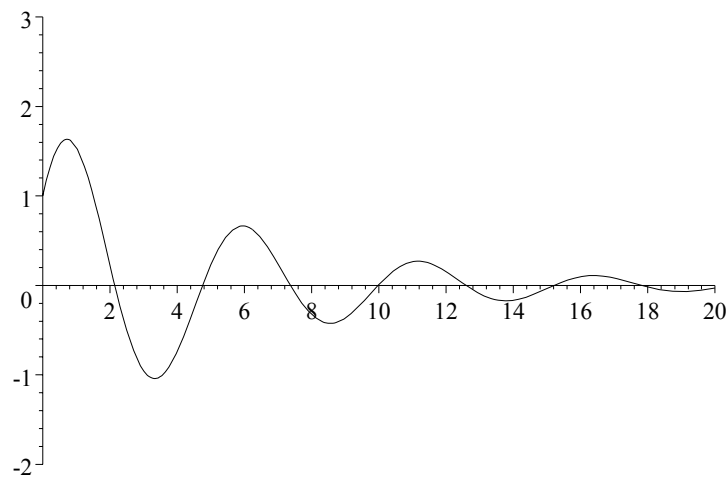


Figure 4: Impulse response of equilibrium employment to a unitary shock. (IRS=1.5)

by unemployment and not captured by standard RBC models in which labour market adjustments are immediate.

Picture 5 below reproduces the same exercise with a different degree of increasing returns to scale (1.6 instead of 1.5)<sup>10</sup>. The qualitative features of this second results are similar to the previous one: hump-shaped dynamics right after the shock, swings around the long-run equilibrium and persistent effect of the shock. However, it is worth analysing the quantitative result more in details. The peak effect of the shock is less pronounced than in the previous case, while the dynamics is less volatile with smoother and more persistent swings around the equilibrium.

Despite their simplicity, the preliminary and illustrative exercises carried on in the present section are instructive. First, the increasing returns in the matching function matter for the dynamics of employment when the steady state equilibrium is perturbed by a shock. The underlined

<sup>10</sup> In this simulation  $a = 0.6$ ,  $\alpha = 1.2$  and  $b = \beta = 0.4$ .

dynamics is richer than that of other RBC models with search not displaying indeterminacy. However, whether these features are able to capture real world characteristics should be checked in an empirical analysis. Second, not only the presence of increasing returns matter but also their degree is important.

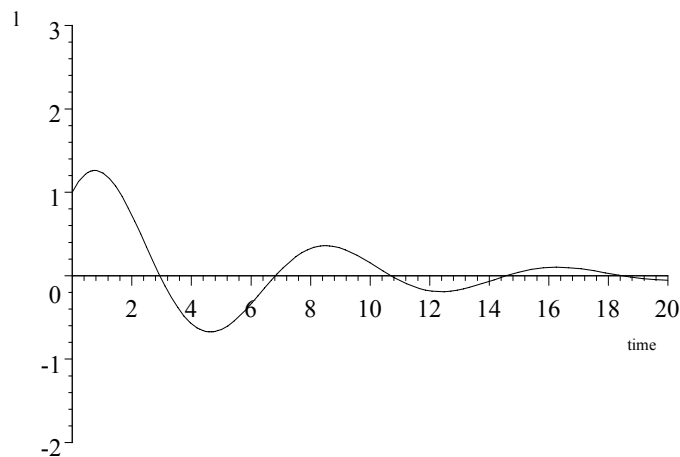


Figure 5: Impulse response of equilibrium employment to a unitary shock (IRS=1.6)

## 7 Increasing Returns in Matching

Regarding the possibility of empirically plausible increasing returns in matching, in this section I review several studies devoted to the estimation of the degree of returns to scale in matching. The key role played by increasing returns in matching is central in the search literature. It goes back to original ideas expressed by Diamond (1982) and subsequently developed by Howitt and McAfee (1987) and Howitt (1988). The possibility of “fragile equilibria”<sup>11</sup> (instability and indeterminacy) in a partial equilibrium setting has been extensively analyzed and besides the references above by Mortensen (1989) and Feve and Langot (1996), who clarify that in a dynamic search model with

<sup>11</sup> The concept of fragile equilibria was introduced by Blanchard and Summers (1986)

no-market clearing wage, the labour market equilibrium is indeterminate. In the model developed in the present paper we have shown that, under some conditions, the same results apply in a general equilibrium context.

However, while in the partial equilibrium environment increasing returns determine the multiplicity of equilibria regardless of their degree, in our model size matters. Indeed, proposition 1 says that in order to have a completely stable steady state we need large increasing returns and with the elasticity of matches to vacancies of at least greater than one. It means that in order to assess the plausibility of the results found, we must analyze whether the parameters calibration we have used is empirically supported.

Standard RBC models with search like those of Merz (1995) and Andolfatto (1996) assume constant returns to scale matching functions. This assumption allows them to solve the model, using the implied Hosios condition. The empirical support of a constant returns to scale matching function is found in Pissarides (1996a) and Blanchard and Diamond (1989). However several empirical investigations have found increasing instead of constant returns to scale. Feve and Langot (1986), using an international data set including France, Germany and the UK, estimate quite large increasing returns with the average sum of the matching function elasticities equal to 1.5. Munich et al. (1997) reach similar results for Eastern European countries, while Coles and Smith (1996), distinguishing between matching and contacts, conclude that the latter display a high degree of increasing returns in the UK labour market. Anderson and Burgess (2000), instead of using aggregate US data, analyse state-level data and show that the increasing returns to scale hypothesis cannot be rejected in most cases. Finally, Warren (1996), using a translog matching function finds increasing returns close to 1.4.

Although there is wide support for increasing returns, the more stringent condition imposed in the model above ( $\alpha > 1$ ) is more difficult to find in empirical studies. A value for the elasticity of matches with respect to vacancies greater than one implies that the duration of a vacancy decreases as the number of vacancies increases. However, even if this feature can be considered implausible

for an aggregate economy for an extended period of time, it might represent situations of high expansion when the opening of a vacancy induces more than proportional variations in labour supply. The same argument, *mutatis mutandis*, can be applied to periods of severe contraction. Moreover, in a highly segmented market populated by heterogeneous agents such a convexity of the matching function with respect to vacancies may well describe sectors or niches of the labour market.

Nevertheless, my theoretical result is supported by several empirical estimations of the matching function. Gross (1997) estimates the aggregate matching function for Germany over a long period, finding a value of  $\alpha = 1.267$  and  $\beta = 0.552$  in the period 1972-1983. Finally, Feve and Langot (1996) 's estimations, not only confirm degrees of returns to scale similar to the ones used in our calibration, but also an elasticity of matching with respect to unemployment in the range of 0.4 – 0.6, while the elasticity of vacancies is always close to one<sup>12</sup>.

## 8 Conclusions

Although the model presented above is rather simple, it is able to produce a series of interesting results and insights. In particular, using a matching function which incorporates externalities, displaying increasing returns to scale, and solving a (pseudo) social planner problem, the model is capable to generate indeterminacy for plausible values in the parameter space. Moreover, such a result can be derived by looking at a decentralized version of the model in which the representative household runs a firm hiring workers and at the same time supplies jobs to other firms. In this case the wage equation is determined outside the maximization problem using a Nash Bargaining solution.

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<sup>12</sup> However, they impose the restriction  $\alpha < 1$  in their estimation.

The preliminary results found in the present paper may be extended in several ways. First, future research should focuss on testing empirically the model, deriving an empirical matching function for the US or EU economy while also performing several simulations according to different parameters value. The closer to real data the results, the more relevant the model will be.

Second, capital should be introduced into the general framework. In particular, the production function should depend on both capital and labor, while an expression for the dynamics of capital accumulation should be added. The resulting model would be able to deal with not only sunspots but also productivity shocks.

Third, the separation rate determining the layoff from employment should be endogenized. In particular it is my intention to introduce money into the model. One possible way of doing so could be indeed to make  $\delta$  depend on money. In other words, a firm's layoff decisions would be influenced by monetary shocks.

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## 9 Appendix

In this section of the paper some of the equations used in the main text are explicitly derived.

### A. Deriving (17)

$$m = \alpha v - \frac{L}{1-L}\beta l \quad (\text{A.1})$$

$$v = \tilde{a} + \lambda + m \quad (\text{A.2})$$

$$\begin{bmatrix} 1 & -\alpha \\ 1 & -1 \end{bmatrix} \begin{bmatrix} m \\ v \end{bmatrix} = \begin{bmatrix} -\frac{L}{1-L}\beta & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} l \\ \lambda \end{bmatrix} + \begin{bmatrix} 0 \\ -\tilde{a} \end{bmatrix} \quad (\text{A.3})$$

$$\begin{bmatrix} m \\ v \end{bmatrix} = \begin{bmatrix} \frac{1}{1-\alpha} & \frac{-\alpha}{1-\alpha} \\ \frac{1}{1-\alpha} & \frac{-1}{1-\alpha} \end{bmatrix} \begin{bmatrix} -\frac{L}{1-L}\beta & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} l \\ \lambda \end{bmatrix} + \begin{bmatrix} \frac{1}{1-\alpha} & \frac{-\alpha}{1-\alpha} \\ \frac{1}{1-\alpha} & \frac{-1}{1-\alpha} \end{bmatrix} \begin{bmatrix} 0 \\ -\tilde{a} \end{bmatrix} \quad (\text{A.4})$$

$$\begin{bmatrix} m \\ v \end{bmatrix} = \begin{bmatrix} -\frac{L}{1-L}\beta\frac{1}{1-\alpha} & \frac{\alpha}{1-\alpha} \\ -\frac{L}{1-L}\beta\frac{1}{1-\alpha} & \frac{1}{1-\alpha} \end{bmatrix} \begin{bmatrix} l \\ \lambda \end{bmatrix} + \begin{bmatrix} \frac{\tilde{a}\alpha}{1-\alpha} \\ \frac{\tilde{a}}{1-\alpha} \end{bmatrix} \quad (\text{A.5})$$

$$m = a_1 l + a_2 \lambda + K \quad (\text{A.6})$$

$$v = a_1 l + \frac{1}{1-\alpha} \lambda + \frac{\tilde{a}}{1-\alpha} \quad (\text{A.7})$$

(A.6) corresponds to (3.17) in the text, where

$$a_1 = -\frac{L}{1-L} \beta \frac{1}{1-\alpha}$$

$$a_2 = \frac{\alpha}{1-\alpha}$$

$$K = \tilde{a} \frac{\alpha}{1-\alpha}$$

## B. The elements of $J$

Given the following system

$$\dot{l} = e^{(a_1-1)l+a_2\lambda+K} - \delta \quad (\text{A.8})$$

$$\dot{\lambda} = \delta + \rho - e^{-l-\lambda} + b \frac{e^{a_1l+a_2\lambda+K}}{1-e^l} \quad (\text{A.9})$$

we define  $J$  as the matrix of the derivatives with respect to  $l$  and  $\lambda$  of the two differential equations above, evaluated at the steady state  $(l^*, \lambda^*)$ :

$$\begin{bmatrix} (a_1 - 1) e^{(a_1-1)l^*+a_2\lambda^*+K} & a_2 e^{(a_1-1)l^*+a_2\lambda^*+K} \\ e^{-l^*\lambda^*} + \frac{ba_1 e^{a_1l^*+a_2\lambda^*+K}}{1-e^{l^*}} + \frac{be^{a_1l^*+a_2\lambda^*+K} e^{l^*}}{(1-e^{l^*})^2} & \frac{ba_2 e^{a_1l^*+a_2\lambda^*+K}}{1-e^{l^*}} + e^{-l^*\lambda^*} \end{bmatrix}$$

Substituting the following steady state conditions:

$$e^{(a_1-1)l+a_2\lambda+K} = \delta$$

$$e^{a_1l+a_2\lambda+K} = \delta e^l$$

$$e^{-l-\lambda} = \rho + \delta + b\delta u$$

where:

$$u = \frac{L}{1-L} = \frac{e^l}{1-e^l}$$

is it possible to obtain the expression (23) in the text:

$$J = \begin{bmatrix} (a_1 - 1) \delta & a_2 \delta \\ (\rho + \delta + b\delta u) + ba_1 u \delta + u^2 b \delta & a_2 u b \delta + (\rho + \delta + b\delta u) \end{bmatrix} \quad (23)$$

**C. Condition**  $\psi\eta^D = (1 - \psi)\eta^S$

**Lemma 3** For  $L \in (0, 1)$ , it follows:  $\psi\eta^D = (1 - \psi)\eta^S$

Conditions (4.6) and (4.7) can be rewritten as follows:

$$\dot{\eta}^D = \rho\eta^D - \frac{\partial H}{\partial L^D}$$

$$\dot{\eta}^S = \rho\eta^S - \frac{\partial H}{\partial L^S}$$

which can be rearranged as:

$$\frac{\partial H}{\partial L^D} = \rho\eta^D - \dot{\eta}^D$$

$$\frac{\partial H}{\partial L^S} = \rho\eta^S - \dot{\eta}^S$$

Using them into the result of the maximization of expression (4.8) we can write:

$$\psi \frac{\partial H}{\partial L^D} = (1 - \psi) \frac{\partial H}{\partial L^S}$$

$$\psi(\rho\eta^D - \dot{\eta}^D) = (1 - \psi) (\rho\eta^S - \dot{\eta}^S)$$

Substituting the expression for  $\dot{\eta}^D$  and  $\dot{\eta}^S$  as in (4.6) and (4.7) we obtain:

$$\psi \left[ \rho\eta^D - (\rho + \delta)\eta^D - \frac{1-w}{L} \right] = (1 - \psi) \left( \rho\eta^S - (\rho + \delta)\eta^S - \frac{w}{L} + \frac{M}{1-\bar{L}}\eta^S \right)$$

Substituting further for  $w$  as in (4.8) we are able to prove the proposition:

$$\psi\eta^D \left( 1 - \frac{1}{L} \right) = (1 - \psi)\eta^S \left( 1 - \frac{1}{L} \right)$$

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