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WORKING PAPER NO. 279

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BY GIANLUCA BENIGNO[2] AND PIERPAOLO BENIGNO[3]

October 2003

1 This is a substantial revision to "Implementing Monetary Cooperation through Inflation Targeting" which appeared as CEPR working paper no. 3226. This paper was produced in part whilst the second author visited the European Central Bank's Directorate General Research as part of their visitor programme. We are grateful to Efe Ok, Lars Svensson and Mike Woodford for helpful comments and suggestions. We thank seminar participants at the Bank of Italy, Bank of Japan, ECB, Federal Reserve Board of Governors, Princeton University, University of Exeter. We are grateful to Bianca de Paoli and Yasemin Gul for research assistance. All errors are our own. The opinions expressed herein are those of the author(s) and do not necessarily reflect those of the European Central Bank. This paper can be downloaded without charge from http://www.ecb.int or from the Social Science Research Network electronic library at http://ssrn.com/abstract_id=487419.
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ISSN 1561-0810 (print)
ISSN 1725-2806 (online)

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Abstract

This study analyzes international monetary policy cooperation in a two-country dynamic general equilibrium model with nominal rigidities, monopolistic competition and producer currency pricing. A quadratic approximation to the utility of the consumers is derived and assumed as the policy objective function of the policymakers.

It is shown that only under special conditions there are no gains from cooperation and moreover that the paths of the exchange rate and prices in the constrained-efficient solution depend on the kind of disturbance that affects the economy. It might be the case either for fixed or floating exchange rates. Despite this result, simple targeting rules that involve only targets for the growth of output and for both domestic GDP and CPI inflation rates can replicate the cooperative allocation.

Keywords: monetary policy cooperation, sticky prices, welfare analysis, targeting rules, inflation target.

JEL classification: E52, F41, F42.

Non technical summary

This study analyzes international monetary policy cooperation in a two-country dynamic general equilibrium model with nominal rigidities, monopolistic competition and producer currency pricing. We consider a model in which both the structure of the economy and the welfare criteria of the policymakers are derived from microfoundations. We revisit the scope for international monetary policy cooperation in a world in which goods and capital markets are perfectly integrated and where the disturbances that affect the economies are originated from productivity, demand, public expenditure and mark-up shocks.

This model aims to answer to some interesting questions in international macroeconomics. For example, what is the optimal choice of exchange rate regime in a perfectly integrated world? How is it possible to design monetary policy institutions that achieve the optimal cooperative outcome?

In general, in the optimal cooperative outcome, the behavior of the exchange rate depends on the kind of disturbance that hits the economy. Previous papers in the literature have shown that in the optimal cooperative allocation, the exchange rate moves in order to accommodate asymmetric productivity shocks as in the Friedman's case for flexible exchange rates while monetary policymakers are left with the domestic goal of price stability. We show that this result does not generalize to other shocks and to a more general model specification. When there are other disturbances such as mark-up and public expenditure shocks, a different behavior arises and the optimal cooperative outcome may imply a managed or sometimes a fixed exchange rate regime. On the other hand, prices and outputs move to accommodate the shocks.

At a first sight, this result would suggest that the task of designing institutions that can implement the cooperative solution is a difficult one, since it would require to specify some control of the exchange rate conditional on the type of disturbance that occurs. Indeed, policymakers that maximize in a non-cooperative game their own country's welfare are not in general able to replicate the cooperative allocation and gains from international monetary policy cooperation naturally arise in our model.

Despite this initial premise, we show that it is still possible to design simple targeting rules that implement the optimal cooperative outcome. In particular these targeting rules can be written as a combination of only domestic targets, both GDP and CPI inflation rates and the output growth, with no explicit reference to the exchange rate.

Targeting rules of the kind proposed here describe the optimizing behavior of central banks. They present some desirable properties. First, by committing to them, policymakers can implement the optimal cooperative allocation in a determinate equilibrium and moreover the rules are robust to different kind of shocks

and their properties. They are 'flexible' meaning that the desired levels for the target variables should not be achieved simultaneously but deviations are possible provided a special linear combination of target variables is kept equal to zero.

Another contribution of our analysis is the derivation of quadratic representations for the welfare of each country that can be directly compared to their closed-economy counterpart and to the ones that are instead just assumed in the previous literature on international monetary policy cooperation. Differently from both literatures, each country's utility approximation is quadratic in the deviations of the terms of trade, domestic and foreign outputs and GDP inflation rates from country-specific targets. These quadratic loss functions capture the different objectives that countries should aim to in formulating their stabilization policies.

"*The national economies that make up the world economy have become increasingly interdependent. Monetary policy in each country affects economic welfare both at home and abroad: the policymaker in each country generates externalities for the policymakers in the other countries. Therefore, the policymaker in each country must take account of the actions of policymakers in other countries.*"[1]

The previous quotation outlines the basic idea behind the literature on international monetary policy cooperation in the 80's and 90's. The existence of externalities, whether positive or negative, is the source of a need of international monetary cooperation when countries do not internalize the effects of their actions on other countries.

In this study, we depart from the previous literature, discussed among others in Canzoneri and Gray (1985), Canzoneri and Henderson (1991) and Persson and Tabellini (1995), by considering a two-country model in which both the structure of the economy and the welfare criteria of the policymakers are derived from microfoundations.[2] We revisit the scope for international monetary policy cooperation in a world in which goods and capital markets are perfectly integrated and where the disturbances that affect the economies are originated from productivity, demand, public expenditure and mark-up shocks.

This model aims to answer to some interesting questions in international macroeconomics. For example, what is the optimal choice of exchange rate regime in a perfectly integrated world? How is it possible to design monetary policy institutions that achieve the optimal cooperative outcome?

In general, in the optimal cooperative outcome, the behavior of the exchange rate depends on the kind of disturbance that hits the economy. In a similar model under the assumption that consumer prices are fully responsive to exchange

[1] Canzoneri and Henderson (1991), pg. 1.

[2] Our approach follows recent contributions in the open-macro literature which have studied the analysis of international monetary cooperation with microfounded models and utility-based welfare criteria, as Benigno and Benigno (2003), Corsetti and Pesenti (2001), Devereux and Engel (2003), Obstfeld and Rogoff (2002), Sutherland (2002a, 2002b), Tille (2003). However, differently from these analyses, we characterize a dynamic model in which prices are sticky and staggered following the Calvo (1983) model and we allow for a more general structure of the economy, in terms of preferences and shocks. With the use of numerical methods, Kollman (2003), Tchakarov (2003) and Sutherland (2001) have evaluated optimal monetary policies in two-country dynamic general equilibrium models.

rate movements, Devereux and Engel (2003) and Obstfeld and Rogoff (2002) have shown that in the optimal cooperative allocation, the exchange rate moves in order to accommodate asymmetric productivity shocks as in the Friedman's case for flexible exchange rates while monetary policymakers are left with the domestic goal of price stability.[3] We show that this result does not generalize to other shocks and to a more general model specification. When there are other disturbances such as mark-up and public expenditure shocks, a different behavior arises and the optimal cooperative outcome may imply a managed or sometimes a fixed exchange rate regime. On the other hand, prices and outputs move to accommodate the shocks.

At a first sight, this result would suggest that the task of designing institutions that can implement the cooperative solution is a difficult one, since it would require to specify some control of the exchange rate conditional on the type of disturbance that occurs. Indeed, policymakers that maximize in a non-cooperative game their own country's welfare are not in general able to replicate the cooperative allocation and gains from international monetary cooperation naturally arise in our model.[4]

Despite this initial premise, we show that it is still possible to design simple targeting rules that implement the optimal cooperative outcome. In particular these targeting rules can be written as a combination of only domestic targets, both GDP and CPI inflation rates and the output growth, with no explicit reference to the exchange rate.

As first emphasized by Svensson (2002, 2003), targeting rules of the kind proposed here can be interpreted as Euler equations that describe the optimizing behavior of central banks. In our context, they are constructed using the first-order conditions of the optimal cooperative solutions following the principles of Giannoni and Woodford (2002). As in the latter work, the rules that we propose present some desirable properties. First, by committing to them, policymakers can implement the optimal cooperative allocation in a determinate equilibrium and moreover the rules are robust to different kind of shocks and their properties. They are 'flexible' meaning that the desired levels for the target variables should

[3] Friedman (1953).

[4] Only under special cases the non-cooperative and cooperative solutions coincide. Our analysis here nests the cases discussed in Benigno and Benigno (2003), Devereux and Engel (2003) and Obstfeld and Rogoff (2002).

not be achieved simultaneously but deviations are possible provided a special linear combination of target variables is kept equal to zero.

Another contribution of our analysis is the derivation of quadratic representations for the welfare of each country that can be directly compared to their closed-economy counterpart and to the ones that are instead just assumed in the previous literature on international monetary policy cooperation.[5] Differently from both literatures, each country's utility approximation is quadratic in the deviations of the terms of trade, domestic and foreign outputs and GDP inflation rates from country-specific targets. These quadratic loss functions capture the different objectives that countries should aim to in formulating their stabilization policies.

The paper is structured as it follows. Section 1 presents the structure of the model. Section 2 presents the quadratic approximation to the utility-based welfare criteria. Section 3 studies the optimal transmission mechanism in cooperative allocation. Section 4 analyzes the gains from cooperation. Section 5 shows how to design targeting rules that can implement the cooperative solution. Section 6 concludes.

1 Structure of the Model

Household behavior

We consider a world economy populated by a measure one of households. The population on the segment $[0, n)$ belongs to the Home country (H) while the one on the segment $[n, 1]$ belongs to the Foreign country (F). Each individual maximizes the following utility function:

$$U_t^j = \mathrm{E}_t \left\{ \sum_{T=t}^{\infty} \beta^{T-t} \left[U(C_T^j, \xi_T^i) - V(y_T(j), \xi_T^i) \right] \right\},$$

where the index j denotes a variable that is specific to household j and the index i denotes a variable specific to the country H or F in which j resides. To clarify the notation that follows i will be replaced by a star when referring to country F and will be suppressed when referring to country H; E_t denotes the expectation

[5] Our model here may fill the lack of 'microeconomic underpinnings' in the literature of international monetary cooperation (see Persson and Tabellini, 1995).

conditional on the information set at date t and β is the intertemporal discount factor, with $0 < \beta < 1$. Households get utility from consumption and disutility from producing goods. The function U is increasing and concave in the consumption index C which is defined as a Dixit-Stiglitz aggregator of home and foreign bundles of goods as

$$C^j \equiv \left[n^{\frac{1}{\theta}}(C_H^j)^{\frac{\theta-1}{\theta}} + (1-n)^{\frac{1}{\theta}}\,(C_F^j)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$

where C_H^j and C_F^j are consumption sub-indexes of the continuum of differentiated goods produced respectively in country H and F

$$C_H^j \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}}\int_o^n c^j(h)^{\frac{\sigma-1}{\sigma}}dh\right]^{\frac{\sigma}{\sigma-1}}, \qquad C_F^j \equiv \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\sigma}}\int_n^1 c^j(f)^{\frac{\sigma-1}{\sigma}}df\right]^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ is the elasticity of substitution across goods produced within a country and θ is the elasticity of substitution between the bundles C_H and C_F. It is assumed that there is a continuum of goods produced in country H and F on the respective segments $[0, n)$ and $[n, 1]$. All the goods are traded across borders with no trade frictions. Here ξ^i denotes a generic vector of shocks (to be specified in the analysis that follows) which are specific to country i. The appropriate consumption-based price indices expressed in units of the currency of the respective country i are defined as

$$P^i \equiv \left[n(P_H^i)^{1-\theta} + (1-n)\,(P_F^i)^{1-\theta}\right]^{\frac{1}{1-\theta}},$$

with

$$P_H^i \equiv \left[\left(\frac{1}{n}\right)\int_o^n p^i(h)^{1-\sigma}dh\right]^{\frac{1}{1-\sigma}}, \qquad P_F^i \equiv \left[\left(\frac{1}{1-n}\right)\int_n^1 p^i(f)^{1-\sigma}df\right]^{\frac{1}{1-\sigma}},$$

where $p^i(h)$ and $p^i(f)$ are prices in units of domestic currency of the home-produced and foreign-produced goods, respectively. Prices are set in the currency of the producer and the law of one price holds: $p(h) = Sp^*(h)$ and $p(f) = Sp^*(f)$, where S is the nominal exchange rate (the price of foreign currency in terms of domestic currency). Given these assumptions and the structure of preferences, purchasing power parity holds, i.e. $P = SP^*$. Moreover relative prices are independent of the currency of denomination, which means that when writing relative-price variables we can suppress the index i. The terms of trade are defined as the relative price of foreign-produced goods in terms of home-produced goods, $T \equiv P_F/P_H$, so that we have:

$$\left(\frac{P_H}{P}\right)^{\theta-1} = n + (1-n)T^{1-\theta}, \qquad \left(\frac{P_F}{P}\right)^{\theta-1} = nT^{\theta-1} + (1-n). \tag{1.1}$$

Finally V is an increasing convex function of household j's supply of one of the differentiated good $y(j)$ produced in its country. The total demands of the generic good h, produced in country H, and of the good f, produced in country F, are respectively:

$$y^d(h) = \left(\frac{p(h)}{P_H}\right)^{-\sigma}\left[\left(\frac{P_H}{P}\right)^{-\theta} C^W + G\right], \qquad y^d(f) = \left(\frac{p(f)}{P_F}\right)^{-\sigma}\left[\left(\frac{P_F}{P}\right)^{-\theta} C^W + G^*\right], \tag{1.2}$$

where $C^W \equiv \int_0^1 C^j dj$ is aggregate consumption in the world economy and G and G^* are country-specific government purchase shocks. By applying the appropriate aggregators to the above total demands of the differentiated goods, we obtain a country index of total production as

$$Y_H = \left(\frac{P_H}{P}\right)^{-\theta} C^W + G, \qquad Y_F^* = \left(\frac{P_F}{P}\right)^{-\theta} C^W + G^*. \tag{1.3}$$

From (1.1) and (1.3), it follows that movements in the terms of trade divert demand across countries.

We assume that markets are complete both at domestic and international levels. Households can trade in a set of nominal state-contingent securities denominated in the currency of the home country and they all inherit initial state-contingent wealth at time 0 such that their lifetime budget constraints are identical. This complete-market assumption implies that consumption is perfectly risk-shared among households within a country. Across countries, marginal utilities of income are equalized as in

$$U_C(C_t, \xi_t) = U_C(C_t^*, \xi_t^*), \tag{1.4}$$

at all times and across all states of nature. Equation (1.4) is derived from the set of optimality conditions that characterize the optimal allocation of wealth among the state-contingent securities, having used the assumption on the initial level of wealth and the fact that purchasing power parity holds.[6] At each time t, there is one of these conditions for each of the states of nature at time $t+1$. The set of optimality conditions of the households' behavior is completed by the appropriate transversality conditions.

In the analysis that follow, we assume that preferences are isoelastic as in

$$U(C_t^j, \xi_t^i) \equiv (g_t^i)^\rho \frac{(C_t^j)^{1-\rho}}{1-\rho}, \qquad V(y_t(j), \xi_t^i) \equiv (a_t^i)^{-\eta} \frac{(y_t(j))^{1+\eta}}{1+\eta},$$

where g_t^i and a_t^i are country-specific preference shocks appropriately normalized. We interpret g_t^i as a country-specific demand shock and a_t^i as a country-specific productivity shock. Here ρ is the inverse of the intertemporal elasticity of substitution in consumption, with $\rho > 0$, and η is the inverse of the elasticity of goods production,with $\eta \geq 0$.

Price-setting mechanism

Each household acts as a monopolist in selling its differentiated good. The overall demand of its good (1.2) is affected by the price chosen while P, P_H, P_F and C^W are taken as given. The price setting behavior is modelled following a partial adjustment rule *á la* Calvo (1983) according to which each seller has the opportunity to change its price with a given probability $1-\alpha$. We allow for different α^i across countries. When a household in the home country has the opportunity to set a new price in period t, it does so in order to maximize the expected discounted value of its net profits. The price setting decision at t determines the net profits at $t+s$ only in states of nature in which the seller does not change the price from $t+1$ to $t+s$ inclusive: this occurs with probability α^s. The objective function is then[7]

$$\mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \left[\frac{U_C(C_T, \xi_T)}{P_T} (1-\tau_T) \tilde{p}_t(h) \tilde{y}_{t,T}(h) - V(\tilde{y}_{t,T}(h), \xi_T) \right],$$

[6] We do not report these conditions here since they will not be used in the analysis that follows. Nor we report the standard stochastic Euler equations that price the risk-free nominal interest rates which are implied by these conditions because they will be needed only to determine the optimal path of the interest rates in a residual way, once the optimal paths of inflation and consumption are derived. Our model can be interpreted as a cashless limiting model (as in Woodford, 1998).

[7] All households within a country that can modify their price at a certain time face the same discounted value of the streams of current and future marginal costs under the assumption that the new price is maintained. Thus they will set the same price.

where after-tax sales revenues are converted in units of utility through the marginal utility of nominal income, $U_C(C_T, \xi_T)/P_T$, which is the same for all households belonging to a country because of the complete-market assumption; τ_t denotes a time-varying tax on sales;[8] $\tilde{p}_t(h)$ denotes the price of the good h chosen at date t in the producer currency and $\tilde{y}_{t,T}(h)$ is the total demand of good h at time T conditional on the fact that the price $\tilde{p}_t(h)$ has not changed,

$$\tilde{y}_{t,T}(h) = \left(\frac{\tilde{p}_t(h)}{P_{H,T}}\right)^{-\sigma} \left[\left(\frac{P_{H,T}}{P_T}\right)^{-\theta} C_T^W + G_T\right]. \tag{1.5}$$

The optimal choice of $\tilde{p}_t(h)$ is

$$\tilde{p}_t(h) = \frac{\mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} V_y(\tilde{y}_{t,T}(h), \xi_T) \tilde{y}_{t,T}(h)}{\mathrm{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{1}{\mu_T} \frac{U_C(C_T, \xi_T)}{P_T} \tilde{y}_{t,T}(h)}. \tag{1.6}$$

where $1/\mu_t$ has been defined as

$$\frac{1}{\mu_t} \equiv \frac{(1-\tau_t)(\sigma-1)}{\sigma}.$$

In particular μ_t can be interpreted as the inefficient wedge in the marginal rate of substitution between consumption and goods production when prices are flexible. In what follows we will refer to fluctuations in this wedge as mark-up shocks. Given the Calvo's mechanism, the evolution of the price index P_H is described by the following law of motion

$$P_{H,t}^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1-\alpha)\tilde{p}_t(h)^{1-\sigma}. \tag{1.7}$$

Similar conditions hold for the producers in country F, with the appropriate modifications.

[8] We introduce a time-varying tax on sales only to obtain inefficient fluctuations in the wedge between the marginal rate of substitution between consumption and goods production. We could have obtained the same outcome by introducing an heterogenous labor market in each industry and having a time-varying monopoly power of wage setters as in Clarida et al. (2002) and Woodford (2003). Giannoni (2001) obtains the same outcome by introduction a time-varying elasticity of substitution σ. To complete the characterization of the model, we assume that there are lump-sum taxes so that the intertemporal budget constraint of the government is not a constraint to take care of.

2 Welfare functions

A micro-founded model delivers a natural measure of welfare based on households' utility. The welfare criterion for country H is defined as

$$W = \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[U(C_t, \xi_t) - n^{-1} \int_0^n V(y_t(h), \xi_t) dh \right] \right\},$$

while for country F is

$$W^* = \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[U(C_t^*, \xi_t^*) - (1-n)^{-1} \int_n^1 V(y_t(f), \xi_t^*) df \right] \right\}.$$

We interpret W and W^* as the policy objective function for the respective monetary policymakers H and F.

Our model is not solvable in a closed-form solution and we approximate it around a steady state in which the four pairs of exogenous variables $(a_t,\ a_t^*)$, $(g_t,\ g_t^*)$, $(G_t,\ G_t^*)$, $(\mu_t,\ \mu_t^*)$ all take constant values equal across countries and such that $\bar{a},\ \bar{g},\ \bar{G} > 0$ and $\bar{\mu} \geq 1$. We further focus on a steady-state in which $\Pi_{H,t} \equiv P_{H,t}/P_{H,t-1} = 1$ and $\Pi_{F,t}^* \equiv P_{F,t}^*/P_{F,t-1}^* = 1$.[9] In this steady-state $\bar{T} = 1$, $\bar{C} = \bar{C}^*$, $\bar{Y} = \bar{Y}^*$ and $\bar{U}_C(\bar{C}, 0) = \bar{\mu}\bar{V}_y(\bar{Y}, 0)$. Unless $\bar{\mu} = 1$, the steady-state output and consumption are inefficiently low.

In the technical appendix we show that a second-order approximation to these objective functions can be written as

$$\begin{aligned} W &= \bar{U}_C \overline{C} E_0 \{ \sum_{t=0}^{\infty} \beta^t [\hat{C}_t - s_c^{-1} \bar{\mu}^{-1} \hat{Y}_{H,t} + \frac{1}{2}(1-\rho)\hat{C}_t{}^2 + \rho \hat{g}_t \hat{C}_t \\ &\quad -\frac{1}{2} s_c^{-1} \bar{\mu}^{-1}(1+\eta)\hat{Y}_{H,t}^2 + s_c^{-1}\bar{\mu}^{-1}\eta \hat{a}_t \hat{Y}_{H,t} + \\ &\quad -\frac{1}{2} s_c^{-1}\bar{\mu}^{-1}\sigma k^{-1}\pi_{H,t}^2]\} + \text{t.i.p.} + \mathcal{O}(||\xi||^3), \end{aligned} \tag{2.8}$$

$$\begin{aligned} W^* &= \bar{U}_C \overline{C} E_0 \{ \sum_{t=0}^{\infty} \beta^t [\hat{C}_t^* - s_c^{-1} \bar{\mu}^{-1} \hat{Y}_{F,t}^* + \frac{1}{2}(1-\rho)(\hat{C}_t^*)^2 + \rho \hat{g}_t^* \hat{C}_t^* \\ &\quad -\frac{1}{2} s_c^{-1} \bar{\mu}^{-1}(1+\eta)(\hat{Y}_{F,t}^*)^2 + s_c^{-1}\bar{\mu}^{-1}\eta \hat{a}_t^* \hat{Y}_{F,t}^* + \\ &\quad -\frac{1}{2} s_c^{-1}\bar{\mu}^{-1}\sigma (k^*)^{-1}(\pi_{F,t}^*)^2]\} + \text{t.i.p.} + \mathcal{O}(||\xi||^3), \end{aligned} \tag{2.9}$$

[9] Following Benigno and Woodford (2003), it can be shown that this steady state is the solution of necessary conditions of a constrained-optimization problem.

where we have defined $s_c \equiv \overline{C}/\bar{Y}$ and $k^i \equiv (1-\alpha^i)(1-\alpha^i\beta)/[\alpha^i(1+\sigma\eta)]$ and hats denote log-deviations of the variables from the steady-state, while $\pi_{H,t} \equiv \ln P_{H,t}/P_{H,t-1}$ and $\pi^*_{F,t} \equiv \ln P^*_{F,t}/P^*_{F,t-1}$. With t.i.p. we denote terms that are independent of policy and with $\mathcal{O}(||\xi||^3)$ we denote terms that are of third order or higher in an appropriate bound on the amplitude of the shocks. Following the method of Benigno and Woodford (2003), we use a second-order approximation to the structural equilibrium conditions to solve for the linear terms in (2.8) and (2.9).[10] As shown in the technical appendix, we take a second-order approximation to the pair of equations in (1.1) and (1.3), to equation (1.4), to equations (1.6) and (1.7) and the respective country F's counterpart; we combine appropriately those second-order approximations to eliminate the linear terms in (2.8) and (2.9). In order to abstract from the time-inconsistent features of the solutions, since at time 0 some prices are fixed from previous periods, we assume that policymakers are committed to past promises following a 'timeless perspective' commitment, as discussed in Woodford (2003, ch. 7). Using this form of commitment, we obtain that the maximization of the welfare of each country is equivalent to minimize the following quadratic loss function

$$L = \frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t[\lambda_{y_h}(\hat{Y}_{H,t}-\tilde{Y}^h_{H,t})^2+\lambda_{y_f}(\hat{Y}^*_{F,t}-\tilde{Y}^h_{F,t})^2+\lambda_q(\hat{T}_t-\tilde{T}^h_t)^2+\lambda_{\pi_h}\pi^2_{H,t}+\lambda_{\pi_f}\pi^{*2}_{F,t}] \tag{2.10}$$

for country H and

$$L^* = \frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t[\lambda^*_{y_h}(\hat{Y}_{H,t}-\tilde{Y}^f_{H,t})^2+\lambda^*_{y_f}(\hat{Y}^*_{F,t}-\tilde{Y}^f_{F,t})^2+\lambda^*_q(\hat{T}_t-\tilde{T}^f_t)^2+\lambda^*_{\pi_h}\pi^2_{H,t}+\lambda^*_{\pi_f}\pi^{*2}_{F,t}] \tag{2.11}$$

for country F, respectively, where $\lambda^i_{y_h}$, $\lambda^i_{y_f}$, λ^i_q, $\lambda^i_{\pi_h}$, $\lambda^i_{\pi_f}$ are parameters, defined in the technical appendix, and $\tilde{Y}^h_{H,t}$, $\tilde{Y}^h_{F,t}$, $\tilde{T}^h_t$, $\tilde{Y}^f_{H,t}$, $\tilde{Y}^f_{F,t}$, $\tilde{T}^f_t$ are combinations of the shocks of the model, defined as well in the technical appendix, and have the interpretation of desired levels for the respective variables.

Our approach shows that a quadratic representation of a welfare-based loss function has a different form compared to the quadratic objective functions that

[10] In a two-country model, Sutherland (2002b) has first used second-order approximations to the structural equilibrium conditions to derive an analytical quadratic representation of the welfare for each country. However, he focuses on a static model in which all prices are fixed one-period in advance.

have been previously assumed in the literature on international monetary policy cooperation. In those papers, the loss functions of the policymakers were quadratic in the deviations of output (or unemployment) with respect to a 'desired' level and in the CPI inflation rate, as in Canzoneri and Gray (1985) and Canzoneri and Henderson (1991). Other studies, as Persson and Tabellini (1995, 1996) have included also a concern for terms of trade stabilization. First, we emphasize that the loss functions of country H and F present the same target variables but with different weights and different 'desired' targets. In particular each policymaker should be concerned about quadratic deviations of both domestic and foreign outputs, domestic and foreign GDP inflation rates and of the terms of trade from country-specific desired targets. These expressions differ sharply from their closed-economy correspondents, as in Rotemberg and Woodford (1997), Woodford (2003, ch. 6) and Benigno and Woodford (2003). In these studies the loss function is usually quadratic in the inflation rate and in the deviation of output with respect to a desired target. This should be less surprising result once we observe that the objective function captures the distortions existing in the economy and that the two countries are interdependent both in the consumption and in the production of goods. In a static model, with prices all fixed one-period in advance, Sutherland (2002b) has shown that home and foreign utility-based welfare criteria depend on foreign and domestic outputs as well as on the nominal exchange rate.

3 The cooperative allocation

We first analyze the cooperative allocation with particular interest on the efficient transmission mechanism of shocks and the implied path of prices and exchange rate. We assume that policymakers that enter a cooperative agreement maximize a weighted average of the countries' welfare function

$$W^W = nW + (1-n)W^*.$$

In the technical appendix, we show that the cooperative allocation in a 'timeless' perspective commitment can be equivalently described as the choice of the paths

$\{\pi_{H,t}, \pi_{H,t}\}_{t=0}^{\infty}$ that minimize the following loss function[11]

$$L^W = \frac{1}{2}E_0 \sum_{t=0}^{\infty} \beta^t [n\lambda_y^w(\hat{Y}_{H,t} - \tilde{Y}_{H,t}^w)^2 + (1-n)\lambda_y^w(\hat{Y}_{F,t}^* - \tilde{Y}_{F,t}^w)^2 +$$
$$+ n(1-n)\lambda_q^w(\hat{T}_t - \tilde{T}_t^w)^2 + n\lambda_{\pi_h}^w \pi_{H,t}^2 + (1-n)\lambda_{\pi_f}^w \pi_{F,t}^{*2}], \qquad (3.12)$$

where the parameters λ_y^w, λ_q^w, $\lambda_{\pi_h}^w$, $\lambda_{\pi_f}^w$ and the variables $\tilde{Y}_{H,t}^w$, $\tilde{Y}_{F,t}^w$, $\tilde{T}_t^w$ are defined in the technical appendix. Here the variables $\tilde{Y}_{H,t}^w$, $\tilde{Y}_{F,t}^w$, $\tilde{T}_t^w$ represent the desired targets for the respective variables when countries cooperate. Minimization of the loss function is subject to the following constraints: the log-linear approximations to the AS equations

$$\pi_{H,t} = \kappa[(\hat{Y}_{H,t} - \tilde{Y}_{H,t}^w) + (1-n)\psi(\hat{T}_t - \tilde{T}_t^w) + u_t] + \beta E_t \pi_{H,t+1}, \qquad (3.13)$$

$$\pi_{F,t}^* = \kappa^*[(\hat{Y}_{F,t}^* - \tilde{Y}_{F,t}^w) - n\psi(\hat{T}_t - \tilde{T}_t^w) + u_t^*] + \beta E_t \pi_{F,t+1}^*, \qquad (3.14)$$

for country H and F, respectively, and the relation between terms of trade and outputs obtained by combining the log-linear approximations of (1.1) and (1.3)

$$(\hat{T}_t - \tilde{T}_t^w) = \theta^{-1}s_c^{-1}[(\hat{Y}_{H,t} - \tilde{Y}_{H,t}^w) - (\hat{Y}_{F,t}^* - \tilde{Y}_{F,t}^w)] \qquad (3.15)$$

where we have defined $\kappa^i \equiv k^i(\rho s_c^{-1} + \eta)$ and $\psi \equiv (1-\rho\theta)/(\rho s_c^{-1} + \eta)$ and where u_t and u_t^* are combinations of the structural shocks of the model, as shown in the technical appendix. By specifying a path for $\pi_{H,t}$ and $\pi_{F,t}^*$, the variables $\hat{Y}_{H,t}$, $\hat{Y}_{F,t}^*$ and $\hat{T}_t$ can be determined by (3.13)-(3.15) and this is all that is needed to evaluate (2.10), (2.11) or (3.12). Finally we need to consider the constraints on the initial conditions of $\pi_{H,0}$ and $\pi_{F,0}^*$ implied by the 'timeless perspective' equilibrium and given by $\pi_{H,0} = \bar{\pi}_{H,0}$ and $\pi_{F,0}^* = \bar{\pi}_{F,0}^*$. [12]

Equations (3.13) and (3.14) represent the aggregate supply equations for countries H and F obtained by log-linearizing equations (1.6) and (1.7) for each country. As in the closed economy counterpart, e.g. Galí and Gertler (1999) and Sbordone

[11]Our cooperative welfare criterion is obtained from a quadratic approximation around the same deterministic steady state around which we have approximated our single country welfare criteria. Since $\bar{\mu} \geq 1$, we will refer to the cooperative outcome as constrained-efficient.

[12]Here $\bar{\pi}_{H,0}$ and $\bar{\pi}_{F,0}^*$ are functions of predetermined and exogenous variables that will be self-consistent in the equilibrium in the sense that they will be the same functions that will result in equilibrium at later dates.

(2002), GDP inflation depends on the present discounted value of the aggregate real marginal costs. However, in open economies, real marginal costs are not in general only proportional to the output gap but they also depend on relative prices, namely the terms of trade.(see Svensson, 2000) This dependence captures the expenditure-switching effect; only in the special case in which $\rho\theta = 1$ the terms of trade channel disappears. Equations (3.13) and (3.14) replace the traditional expectations-augmented Phillips-curve of the models of Canzoneri and Henderson (1991), and Persson and Tabellini (1995, 1996). Equation (3.15) captures the relation between terms of trade and output differential across countries. This relation is also familiar to the previous literature.

As shown in the technical appendix, the shocks u_t and u_t^* are combinations of all the exogenous shocks of the model (not necessarily mark-up shocks) and capture the deviations of the natural levels of output and terms of trade –the ones that would prevail under flexible prices– from their desired targets, as defined in (3.12). In general, for any kind of shock, the optimal cooperative solution may not aim at mimicking the allocation that would arise under flexible prices.

To study the optimal cooperative allocation, we write the following Lagrangian

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t [\frac{1}{2} n \lambda_y^w y_{H,t}^2 + \frac{1}{2}(1-n)\lambda_y^w y_{F,t}^{*2} + \frac{1}{2} n(1-n)\lambda_q^w q_t^2 + \frac{1}{2} n \lambda_{\pi_h}^w \pi_{H,t}^2 +$$
$$+ \frac{1}{2}(1-n)\lambda_{\pi_f}^w \pi_{F,t}^{*2}] + n\varphi_{1,t}[\kappa^{-1}\pi_{H,t} - y_{H,t} - (1-n)\psi q_t - \beta\kappa^{-1}\pi_{H,t+1}] +$$
$$+(1-n)\varphi_{2,t}[\kappa^{*-1}\pi_{F,t}^* - y_{F,t}^* + n\psi q_t - \beta\kappa^{*-1}\pi_{F,t+1}^*] + n(1-n)\varphi_{3,t}[q_t +$$
$$-\theta^{-1}s_c^{-1}y_{H,t} + \theta^{-1}s_c^{-1}y_{F,t}^*] - n\varphi_{1,-1}\kappa^{-1}\pi_{H,0} - (1-n)\varphi_{2,-1}\kappa^{*-1}\pi_{F,0}^*$$

where we have defined $y_{H,t} \equiv (\hat{Y}_{H,t} - \tilde{Y}_{H,t}^w)$, $y_{F,t}^* \equiv (\hat{Y}_{F,t}^* - \tilde{Y}_{F,t}^w)$ and $q_t \equiv (\hat{T}_t - \tilde{T}_t^w)$ and we have appropriately normalized the Lagrange multiplier in a way to obtain time-invariant first-order conditions. The first-order condition with respect to $y_{H,t}$, $y_{F,t}^*$ and q_t are

$$\lambda_y^w y_{H,t} = \varphi_{1,t} + (1-n)\theta^{-1}s_c^{-1}\varphi_{3,t}, \tag{3.16}$$

$$\lambda_y^w y_{F,t}^* = \varphi_{2,t} - n\theta^{-1}s_c^{-1}\varphi_{3,t}, \tag{3.17}$$

$$\lambda_q^w q_t = \psi\varphi_{1,t} - \psi\varphi_{2,t} - \varphi_{3,t}, \tag{3.18}$$

for each $t \geq 0$, while the ones with respect to $\pi_{H,t}$ and $\pi_{F,t}^*$ are

$$\kappa\lambda_{\pi_h}^w \pi_{H,t} = -(\varphi_{1,t} - \varphi_{1,t-1}), \tag{3.19}$$

$$\kappa^* \lambda^w_{\pi_f} \pi^*_{F,t} = -(\varphi_{2,t} - \varphi_{2,t-1}), \tag{3.20}$$

for each $t \geq 0$.

We show in the appendix that equations (3.16)–(3.20), combined with the structural equations (3.13)–(3.15) and the initial conditions $\varphi_{1,-1}$ and $\varphi_{2,-1}$ determine the equilibrium path of outputs, inflation rates, and terms of trade along with the Lagrangian multipliers.

Our first objective is to characterize the constrained-efficient response of prices and exchange rate, among other variables, to the various disturbances that affect the economy. In previous related works, Devereux and Engel (2003) and Obstfeld and Rogoff (2002) have shown that, under producer currency pricing, the adjustment to asymmetric productivity shocks should be brought about by exchange rate movements. Their argument revisits in a micro-founded model the Friedman's case for flexible exchange rates. When prices are sticky, relative price movements are obtained through changes in the nominal exchange rate. In this way demand can be distributed efficiently across countries.

Our framework allows for a direct analysis of the robustness of these findings in a much more general model. An alternative way to approach the Friedman's argument for flexible exchange rate is to ask when it is optimal to maintain nominal prices, the ones that are sticky, stable and let the exchange rate absorb all the adjustment. In our model, this is equivalent to study under which conditions the flexible-price allocation is optimal, since stability of producer inflation rates at all times, $\pi_{H,t} = \pi^*_{F,t} = 0$ for all t, replicates the flexible-price allocation.

Our results weaken the case for a floating exchange rate regime. Indeed, the flexible price allocation is the optimal cooperative outcome only under special cases (see the technical appendix). The intuition relies on the evaluation of distortions and externalities built into the model. Here there are two distortions. The existence of monopoly power in goods market produces an inefficient output level in both countries while the staggering price-setting mechanism creates dispersions of demand across goods produced with the same technology under non-zero producer inflation. The assumption of nominal price stickiness gives a role for monetary policy to correct these inefficiencies.

In a cooperative agreement, policymakers aim to commit to policies that raise the expected level of consumption and output in both countries, since they are inefficiently low. As in Henderson and Kim (1999) and Obstfeld and Rogoff (1998), the ability to precommit does not prevent this possibility because the expected values of variables depend on the expected value of first-order and second-order terms.[13] In general, when $\bar{\mu} > 1$, i.e. the steady-state level of output is inefficiently low, stabilization policy could be used to increase the expected level of output. Importantly this happens no matter what is the source of the disturbances (productivity, demand, mark-up or public expenditure) or their nature (symmetric or asymmetric).

On the other hand a floating exchange rate regime is optimal when the composite shocks u and u^* are always zero. Here we discuss the cases in which this happens. When the steady-state level of output is efficient, $\bar{\mu} = 1$, it follows that $u_t = \hat{\mu}_t$ and $u_t^* = \hat{\mu}_t^*$. Absent pure mark-up shocks, it is optimal to stabilize producer prices, while the nominal exchange rate follows

$$\ln S_t/\bar{S} = \frac{\eta}{1 + \theta\eta s_c}[\hat{a}_t - \hat{a}_t^* - (\hat{G}_t - \hat{G}_t^*)]. \tag{3.21}$$

When the home country has a favorable productivity shocks, the home currency depreciates so that the demand for the home-produced goods can increase. The same effect follows a decrease in home government purchases. Instead, a mark-up shock drives an inefficient wedge in the marginal rate of substitution between consumption and goods production; in this case monetary policymakers have a role in stabilizing those inefficiencies and move away from the flexible price allocation.

The optimality of the flexible exchange rate regime holds also in the special case in which $\bar{\mu} > 1$, the steady-state level of government expenditure is zero, i.e. $s_c = 1$, and there are no mark-up or government expenditure shocks, i.e. $\hat{\mu}_t = \hat{\mu}_t^* = \hat{G}_t = \hat{G}_t^* = 0$. Again following productivity shocks the exchange rate behaves as in (3.21).[14]

On the other side, our analysis would suggest the optimality of fixed exchange rate regime in the special case in which $\bar{\mu} = 1$, $\theta s_c = \sigma$ and the economy is subject only to mark-up and demand shocks (see the technical appendix). The general

[13]Loosely speaking, in a commitment equilibrium the expected value of first-order terms is equal to zero, while terms of order higher than the second are not relevant in a second-order approximation.

[14]This case can be interpreted as an 'isoelastic' case in which even if one wishes the expected level of output cannot be moved and the stabilization to the shocks is no longer distorted

message that we want to emphasize here is that, beyond the specific parametric restrictions, the type of disturbance might suggest a very different prescription in terms of exchange rate regime even in our very simple framework.

In order to study the optimal transmission mechanism of shocks, we calibrate a quarterly model for countries with equal size, i.e. $n = 1/2$. Following Rotemberg and Woodford (1997), we assume that $\beta = 0.99$ and $\eta = 0.47$. We assume $\alpha = 0.66$ and $\alpha^* = 0.75$ implying an average length of price contracts equal to 3 and 4, respectively. We assume that the elasticity σ across goods produced within a country is 10, while the steady-state tax rate is $\bar{\tau} = 0.2$, which imply a value for $\bar{\mu}$ equal to 1.38 and a steady-state mark up of 38%. The steady-state level of consumption over output is calibrated to $s_c = 0.8$. Finally, the risk aversion coefficient ρ is usually assumed to be in a range between 1 and 5, and we use 3, while following Obstfeld and Rogoff (2000), the intratemporal elasticity of substitution θ is in the range 3 to 6 and we choose 4.5. An important implication of this calibration is that $\theta > \frac{1}{\rho}$, i.e. the intratemporal elasticity of substitution is higher than the intertemporal elasticity of substitution,which means that the home and foreign bundles of goods are substitute in the utility. [15]

Figure 1 shows the impulse response functions of several variables to a positive temporary productivity shock in the home country. Although the calibration used would imply a departure from the flexible-price allocation, this departure is quantitatively negligible. The GDP inflation rates and the output gaps (the latter taken with respect to the cooperative desired levels) do not move. All the adjustment is brought about by the terms of trade through the exchange rate. As the home country is experiencing a favorable productivity shock, demand should be diverted to home-produced goods since they are produced more efficiently. This can be done by a depreciation of the home currency which improves the terms of trade of the country and increases the demand and production of the goods.

A different outcome arises following a temporary home mark-up shock, as shown in figure 2. In a similar way to the closed-economy model of Clarida et al. (1999) and Woodford (2003, ch. 3), a mark-up shock in the home country is absorbed by a temporary fall in the home output gap and by an initial jump in home GDP

[15] Two goods are substitute in the utility when the marginal utility of one good decreases as the consumption of the other good increases.

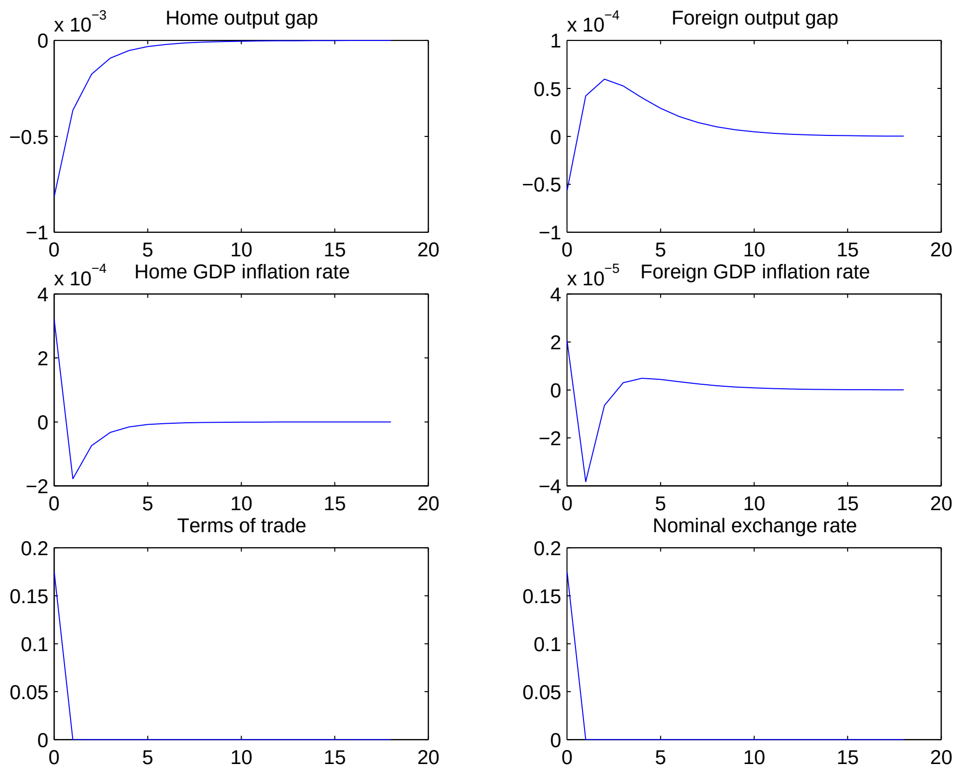

Figure 1: Impulse responses of home and foreign outputs, home and foreign GDP inflation rates, terms of trade and exchange rate to a home productivity shock.

inflation rate. After the shock, the output gap converges back to the initial steady state and the price level converges as well to the initial level through periods of deflation. The fall in the output worsen the home country terms of trade ($\hat{T}$ decreases) The key insight to understand the optimal transmission mechanism of the mark-up shock across countries is the link between foreign real marginal costs and the terms of trade. When goods are substitute in utility, an improvement in the foreign terms of trade ($\hat{T}$ decreases) reduces the real marginal costs for foreign producers, acting as a negative mark-up shock for them. Producer prices fall and the output gap rises. Home and foreign output gaps and the two GDP inflation rates commove in a negative way following the shock. Under a different parametrization, $\theta < \frac{1}{\rho}$, the commovement would be positive, while no spillover effect would occur if $\theta = \frac{1}{\rho}$. In the calibrated example the exchange rate appreciates but moves less than in the case the economy is hit by a productivity shock. There

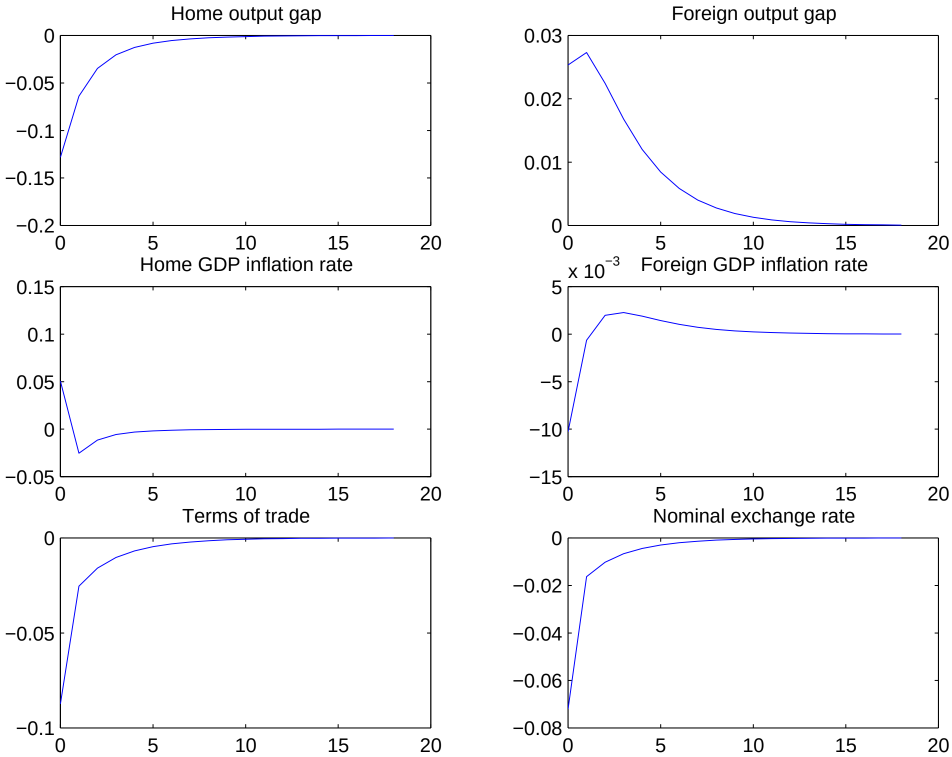

Figure 2: Impulse responses of home and foreign outputs, home and foreign GDP inflation rates, terms of trade and exchange rate to a home mark-up shock.

can be cases in which the exchange rate does not move at all following mark-up shocks. Most interesting, following any kind of stationary shock, the optimal cooperative solution requires both prices and exchange rate to revert back to their initial values.

4 Gains from cooperation

In the non-cooperative game each policymaker chooses its sequence of GDP inflation, $\{\pi_{H,t}\}_{t=0}^{+\infty}$ or $\{\pi_{F,t}^{*}\}_{t=0}^{+\infty}$, in order to minimize its loss function, (2.10) or (2.11), taking as given the strategy of the other policymaker. Policymakers have different incentives.

Each policymaker wishes to raise the expected utility of consumption and at the same time to lower the expected disutility of producing goods by diverting production to the other country. This can be done by a strategic use of the terms of trade. Indeed, in a non-cooperative equilibrium, each country can impose a negative externality on the other country in order to increase its own expected utility. It follows that the desired stabilization of the shocks, as perceived from a single country perspective, is different from the cooperative one. This is reflected by the differences among the loss functions (2.10), (2.11) and (3.12). Here, we note that there are two dimensions along which these differences arise: policymakers might target a different level for each variable or might put a different weight on the same component of the loss function.

However, there are some cases in which cooperative and non-cooperative equilibria coincide, as shown in the technical appendix. One simple case is when $L = L^* = L^W$. This occurs when at the same time $\theta = 1$, i.e. preferences are Cobb-Douglas across home and foreign produced goods, $s_c = 1$, i.e. there is no steady-state public expenditure, and moreover when there are only productivity shocks, $\hat{a}_t$ and $\hat{a}_t^*$, and symmetric demand shocks, i.e. $\hat{g}_t = \hat{g}_t^*$. We retrieve here the Obstfeld and Rogoff (2002) case. As discussed in Benigno and Benigno (2003), there is too much risk-sharing under these parameter restrictions. Indeed, the expected disutilities of goods production are equalized across countries along with the marginal utilities of consumption. In this case the terms of trade is ineffective in stabilizing shocks for its own country's utility, since the disutility of goods production is tied across countries. In a numerical example, Sutherland (2001, 2002b) and Tchakarov (2002) have quantified as important the gains from cooperation in the case that θ differs from 1.

Having observed that the nature of the negative externality lies directly in the use of the terms of trade, we can look at other cases of no gains from cooperation by focusing on the particular case in which the terms of trade channel is not effective. The previous literature on international monetary policy cooperation (see Sachs, 1988) has related the gains of cooperation to a parameter of interdependence that measures the importance of the terms of trade in the transmission mechanism across countries. In our context, the terms of trade interdependence is determined by the parameter ψ. When the intratemporal and intertemporal elasticity of substitution are equal, i.e. $\theta = 1/\rho$, then $\psi = 0$ and each policymaker can control its own output by manoeuvring its own GDP inflation rate. However, differently from the previous literature, this case does not necessarily imply the absence of gains from cooperation. Indeed, as clarified in Canzoneri et al. (2002), the case in which $\theta = 1/\rho$ describes economies that are independent of the terms of trade only in goods production, but they are still interrelated in goods consumption.

As shown in the technical appendix, the cooperative loss function (3.12), under this parameters restriction, simplifies to a quadratic form that displays only GDP inflations and output targets, since $\lambda_q^w = 0$, while the loss functions for each country simplify to

$$L = \frac{1}{2}\sum_{t=0}^{\infty}\beta^t[\lambda_{y_h}(\hat{Y}_{H,t} - \tilde{\tilde{Y}}^h_{H,t})^2 + \lambda_{\pi_h}\pi^2_{H,t}] + \text{t.o.c.} \tag{4.22}$$

for country H and

$$L^* = \frac{1}{2}\sum_{t=0}^{\infty}\beta^t[\lambda^*_{y_f}(\hat{Y}^*_{F,t} - \tilde{\tilde{Y}}^f_{F,t})^2 + \lambda^*_{\pi_f}\pi^{*2}_{F,t}] + \text{t.o.c.} \tag{4.23}$$

for country F, where t.o.c. denotes terms that are out of the control of the policymaker and include foreign GDP inflation and output. Note that these loss functions mirror the ones that arise in closed-economy models, as in Woodford (2003, ch. 6), since the objective function that can be controlled by each policymaker collapses to a standard quadratic function in an appropriately defined domestic output gap and GDP inflation. However, this result does not imply that cooperative and non cooperative solutions will necessarily coincide, since there are still spillover effects on consumption. Indeed the central planner weighs each country disutility of goods production less than what the single country does, since it recognizes that production in the country is absorbed by consumption in both economies, while the single country weighs more its disutility of goods production since it does not internalize the consumption and the utility of consumption of the other country. The optimal stabilization policies will be different between the cooperative and non-cooperative equilibria. Only when the desired targets between the pairs of loss functions L, L^W and L^*, L^W coincide, i.e. $\tilde{\tilde{Y}}^h_{H,t} = \tilde{Y}^w_{H,t}$ and $\tilde{\tilde{Y}}^f_{F,t} = \tilde{Y}^w_{F,t}$, then the cooperative and non-cooperative equilibria coincide and there are no gains from cooperation. In the technical appendix, we show that this happens when $s_c = 1$ and there are only productivity shocks and symmetric demand shocks.

The analysis of symmetric shocks is also an interesting source of comparisons with the previous literature. Models in the fashion of Canzoneri and Henderson

(1991) found that the gains from cooperation were arising even with symmetric disturbances. Obstfeld and Rogoff (2002) instead show that with symmetric productivity shocks there are no gains from coordination. Here, we find that this result holds both for symmetric productivity and demand shocks, provided $s_c = 1$. Otherwise, with other kind of disturbances, as for example mark-up and public expenditure shocks, or with $s_c < 1$, there are still gains from cooperation even when shocks are global.[16]

In general, the model analyzed here shows that the conditions under which there are no gains from cooperation are very restrictive. Although we do not quantify the magnitude of the gains from cooperation, it is worth mentioning that public expenditure shocks and mark-up shocks have been found to be important driving factors of the business cycle, as in Galí et al. (2003). Moreover, some simple numerical examples of Canzoneri et al. (2001) and Tchakarov (2002) have shown that this class of models which rely on microfounded loss functions can produce larger gains from cooperation than the previous literature did in the 80's and 90's

5 Designing targeting rules for international monetary cooperation

In the previous sections, we have considered policymakers that maximize the utility of the consumers in their respective countries. However, a policymaker that shares the preferences of the consumers or society does not internalize the negative externality that it may impose on other countries. How is then possible to design institutions, as central banks, with the 'right incentives'? There are several examples in the literature in which this issue is solved by delegating a new objective function to an independent agent, a central bank, as shown in the contributions of Rogoff (1985), Persson and Tabellini (1993, 1995, 1996), Walsh (1995), Svensson (1997), and Jensen (2000, 2002). As discussed in Svensson (2002, 2003), the design of institutions by imposing a commitment to a loss function can be interpreted as a 'general targeting rule', which is a general operational objective.

[16] Sutherland (2002b) shows that even symmetric productivity shocks may imply gains from cooperation. His framework is different from ours: indeed, he considers a structure in which contingent claims market open after policy makers have chosen their policy strategies.

Here we follow an alternative and perhaps more direct way to design institutions by the assignment of 'specific targeting rules' (as proposed in Svensson, 2003) that each policymaker should follow. These specific targeting rules represent Euler equations derived from the behavior of optimizing central banks.

Our goal is to design targeting rules that are optimal from the cooperative perspective. To this end we follow the method proposed by Giannoni and Woodford (2002). In a linear-quadratic model they show that optimal targeting rules can be obtained by eliminating the lagrange multipliers from the first-order conditions of the optimal policy problem. Targeting rules built on this principle present some desirable characteristics. First, by ensuring that these targeting rules hold at all time, a determinate rational expectations equilibrium can be achieved and this equilibrium coincides with the optimal policy from a timeless perspective. Second, these targeting rules are optimal regardless the statistical properties of the exogenous shocks. They depend on the shocks insofar as the targets specified in the loss function depend on them.

To derive the desirable targeting rules we use the first-order conditions (3.16) to (3.20). First, we take a weighted average with weights n and $(1-n)$ of (3.16) and (3.17), obtaining

$$\lambda_y^w[ny_{H,t} + (1-n)y_{F,t}^*] = n\varphi_{1,t} + (1-n)\varphi_{2,t}. \tag{5.24}$$

We take the difference of (3.16) and (3.17) and combine it with (3.18), obtaining

$$(\lambda_y^w + \theta^{-2}s_c^{-2}\lambda_q^w)(y_{H,t} - y_{F,t}^*) = (1 + \theta^{-1}s_c^{-1}\psi)(\varphi_{1,t} - \varphi_{2,t}), \tag{5.25}$$

where we have used the fact that $y_{H,t} - y_{F,t}^* = \theta s_c q_t$. We further note that we can write (5.25) as

$$\lambda_y^w(y_{H,t} - y_{F,t}^*) - \gamma q_t = (\varphi_{1,t} - \varphi_{2,t}) \tag{5.26}$$

where we have used the relation $\lambda_q^w = \theta s_c^{-1}\psi[s_c^2\lambda_y^w - \bar{\mu}^{-1}(\bar{\mu}-1)(1-s_c)s_c\eta(s_c\eta + \rho)^{-1}]$ and defined γ as $\gamma \equiv \psi\bar{\mu}^{-1}s_c^{-1}\eta(\bar{\mu}-1)(1-s_c)(s_c\eta + \theta^{-1})^{-1}$. By using (5.24) and (5.26), we can obtain

$$\lambda_y^w y_{H,t} - (1-n)\gamma q_t = \varphi_{1,t},$$

$$\lambda_y^w y_{F,t}^* + n\gamma q_t = \varphi_{2,t},$$

which combined with (3.19) and (3.20) yields the following relation

$$\kappa\lambda^w_{\pi_h}\pi_{H,t} + \lambda^w_y \Delta y_{H,t} - (1-n)\gamma\Delta q_t = 0, \tag{5.27}$$

$$\kappa^*\lambda^w_{\pi_f}\pi^*_{F,t} + \lambda^w_y \Delta y^*_{F,t} + n\gamma\Delta q_t = 0. \tag{5.28}$$

We now use the following price relations, the terms of trade identity in first difference

$$\hat{T}_t = \hat{T}_{t-1} + \Delta S_t + \pi^*_{F,t} - \pi_{H,t}, \tag{5.29}$$

and the PPP as well in first difference

$$\pi_t = n\pi_{H,t} + (1-n)(\Delta S_t + \pi^*_{F,t}) = \Delta S_t + \pi^*_t. \tag{5.30}$$

Using (5.29) and (5.30), we can rewrite (5.27) and (5.28) as[17]

$$(\kappa\lambda^w_{\pi_h} + \gamma)\pi_{H,t} + \lambda^w_y \Delta y_{H,t} - \gamma(\pi_t - \tilde{\pi}_t) = 0, \tag{5.31}$$

$$(\kappa^*\lambda^w_{\pi_f} + \gamma)\pi^*_{F,t} + \lambda^w_y \Delta y^*_{F,t} - \gamma(\pi^*_t - \tilde{\pi}^*_t) = 0, \tag{5.32}$$

where $\tilde{\pi}_t \equiv (1-n)(\tilde{T}^*_t - \tilde{T}^*_{t-1})$ and $\tilde{\pi}^*_t \equiv -n(\tilde{T}^*_t - \tilde{T}^*_{t-1})$.

We interpret conditions (5.31) and (5.32) as the targeting rules that the two central banks have to follow in order to implement the optimal cooperative allocation from a 'timeless perspective'. In particular, (5.31) should be assigned to the monetary policymaker in country H and (5.32) to the policymaker in country F. Surprisingly, each of these rules involves only a relation among domestic variables as the GDP inflation rate, the growth of domestic output and the CPI inflation rate. To support this assignment, each of these targeting rules can be derived as the result of a Nash game in which each policymaker chooses the sequence of its GPD inflation rate as a function of the shocks taking as given the strategy of the other policymaker and minimizing the common loss function (3.12).

As in the general approach of Giannoni and Woodford (2002), we show in the appendix that by committing to these rules a determinate rational expectations equilibrium can be achieved that implements the optimal cooperative solution from a 'timeless perspective'.

The above rules present some other interesting characteristics. For both policymakers, they involve the same set of target variables –GDP, CPI inflation and

[17]We thank Mike Woodford for suggesting this interpretation.

output– with the same combination of weights in the overall targeting rules. However, each of these target variables enters into the targeting rule in deviation to a 'desired' target which is instead country specific. For both countries, the 'desired' target for GDP inflation is zero, while the 'desired' targets for output and CPI inflation are in general different from zero and country specific. Most interesting, the 'desired' targets for CPI inflation rates move in the opposite direction when comparing the two countries and implicitly define a desired path for the exchange rate such that $\ln S_t/\bar{S} = \tilde{T}_t^*$.

These targeting rules are also flexible, in the words of Svensson, meaning that each 'desired' target for the target variable does not need to be necessarily achieved, but what matter is only an appropriate linear combination given by the parameters of the model of target variables with respect to 'desired' targets.

Differently from the closed-economy counterpart (see Giannoni and Woodford, 2003), our targeting rules should include a distinction between GDP and CPI inflation rates. Indeed, in the basic model by Giannoni and Woodford (2003), with only sticky prices and monopolistic competition, the optimal targeting rule is expressed as a combination of inflation rate and output growth with respect to a desired target. In their framework there is no distinction between GDP and CPI inflation rates.

The extent to which the CPI target is relevant depends on the parameters of the model and not on the kind of disturbance that affects the economy. The dependence on CPI target disappears when steady-state monopolistic distortions are completely offset, $\bar{\mu} = 1$, the two economies are independent, $\rho\theta = 1$, and when in the steady-state government purchase is equal to zero, $s_c = 1$. Interestingly, the case $\theta = 1$ does not appear as a case that exclude CPI from the target and in general the conditions that define the absence of gains from coordination do not necessarily coincide with the conditions that exclude CPI inflation from the target.

Figure 3 plots the impulse response function following a home productivity shock of the target variables that are part of the targeting rules and their 'desired' targets. Consistent with our previous result, all the gaps are quantitatively zero following a productivity shock. Home CPI inflation rate and output growth (and their desired levels) jump on impact and then turn negatively to restore the initial level of the variables. The opposite occurs in the foreign country. Most interesting,

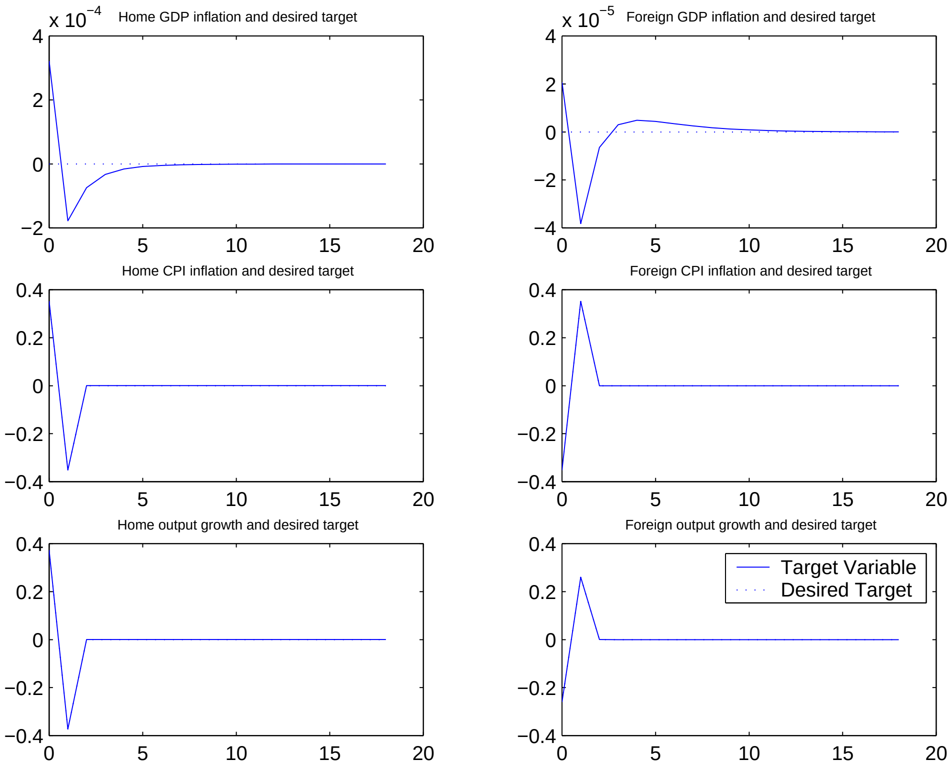

Figure 3: Impulse responses of the target variables and 'desired' targets in the targeting rules following a home productivity shock.

Figure 4 shows the impulse response functions following a home mark-up shock. This figure features the flexibility of adopting such targeting rules. While on impact GDP inflation overshoots its 'desired' value of zero, CPI inflation and output growth undershoot their 'desired' level. The opposite occurs in the subsequent periods. While the GDP price level should not move following the shock, the actual GDP price increases and then converges to the initial steady state. On the opposite, output and the CPI price level should fall and the converge monotonically to their initial value. Instead, their actual values fall more and then converge monotonically to the initial values at a faster speed.

The implementation of these targeting rules can be in principle solved as in Persson and Tabellini (1995) and Jensen (2000). Central banks are assumed to be risk neutral and their objective function is designed to be the loss function of the country plus a penalty determined by a contract which is written in terms of

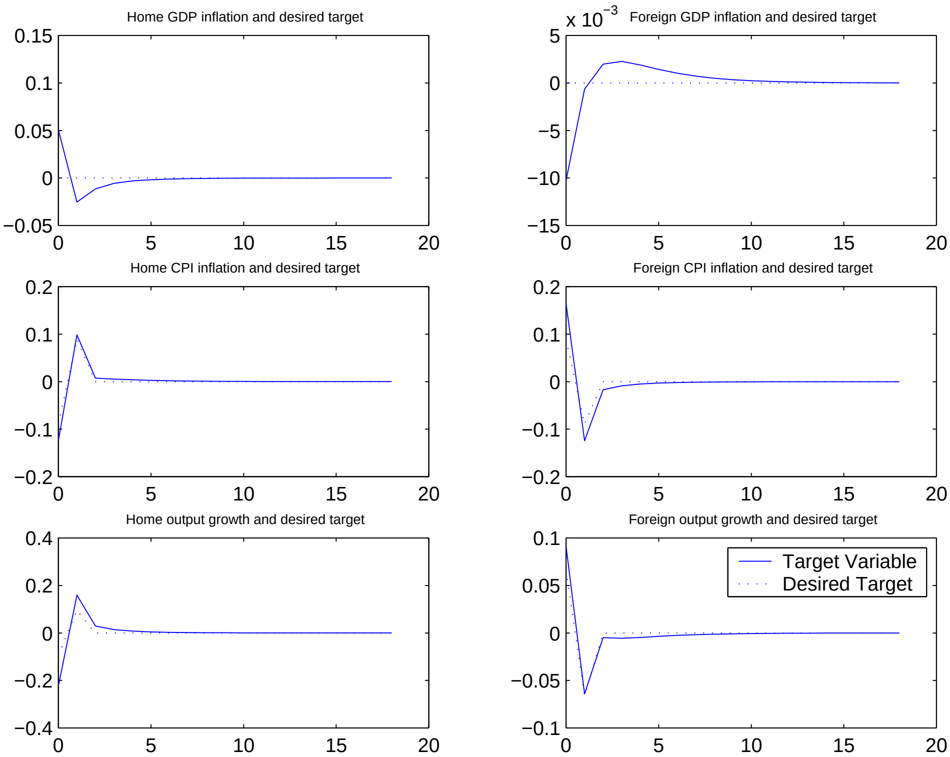

Figure 4: Impulse responses of the target variables and 'desired' targets in the targeting rules following a home mark-up shock.

observable variables.[18] Given these modified loss functions, the two central banks, acting in a non-cooperative equilibrium, implement the cooperative outcome. In our context, maintaining the assumption that central banks are risk neutral, we can design contracts of the form $\delta_0^i - \delta_1^i(\Lambda_t^i)^2$ for $i = H, F$ and given parameters δ_0^i and δ_1^i where Λ_t^i is defined as the country i targeting rule, e.g. $\Lambda_t^H \equiv (\kappa\lambda_{\pi_h}^w + \gamma)\pi_{H,t} + \lambda_y^w \Delta y_{H,t} - \gamma(\pi_t - \tilde{\pi}_t)$. Given these contracts, central banks are forced to follow the targeting rules. However, the restriction written in the contract is not stronger than the one implied by adjusting the objective functions of the central banks using contracts, in the fashion of Persson and Tabellini (1995) and Jensen (2000). As in these approaches, central banks maintain the flexibility implicit in

[18]In Persson and Tabellini (1995, 1996), the contract is restricted to be linear in observed variables but the optimal contract need to have state-contingent parameters. Jensen (2000) shows that by assigning quadratic contracts in observed variables the contract can be made non-state contingent.

the targeting rules and moreover the flexibility in choosing their instrument to meet their objectives. However, both our approach and theirs do not really solve the delegation problem and shift the cooperation problem at the delegation stage.[19]

6 Conclusions

We have shown that in a two-country general equilibrium model characterized by goods and financial markets integration, the efficient paths of the exchange rate and prices depend on the source of the disturbance that hits the economy. The interaction between the existing distortions and source of disturbance generates in general gains from cooperation so that policymakers that maximize their own welfare behave inefficiently in the non-cooperative allocation. This lack of coordination can be amended by assigning simple targeting rules to each policymaker so that the optimal cooperative outcome can be achieved. We have shown that surprisingly these rules depend only on domestic variables despite full goods and capital market integration.

Further research should investigate the robustness of these findings for economies in which asset markets are incomplete and when consumer prices are less responsive to exchange rate changes as in Devereux and Engel (2003) and possibly also the interdependence between monetary and fiscal policies that we have neglected here.

[19]Indeed, each country has no incentive to assign to its central bank that type of contract even in the case that the other country is behaving in that way. This is discussed in McCallum (1995) and extensively in Bilbiie (2002).

One solution would be to follow the folk theorem in delegation games of Fershtman et al. (1991), as in Persson and Tabellini (1995), and condition the parameters of the contracts, δ_0^i and δ_1^i, to the possible outcomes as in "take-it-or-leave-it" offers. However, this set of state-contingent non-linear contracts will be highly unrealistic since part of them is contingent on the payoffs, which can be difficult to observe. The other solution, as in Persson and Tabellini (1995), is to consider a delegation to a common supranational institutions endowed with the cooperative loss function with the task to design appropriately contracts for the single central banks. Since, the cooperative solution is a Pareto allocation and creates a surplus over the non-cooperative allocation, in principle it would be possible to exploit the surplus to design a game in which each country obtains at least the non-cooperative outcome and participate to the international agreement; at the same time all the other agents in the economy, the supranational institutions and the two central bank, have their individual rationality constraint satisfied.

Another important open issue is the enforcement of the proposed targeting rules. We have briefly addressed this issue acknowledging that, as in previous contributions in the literature, the cooperation problem is simply shift at the delegation stage to a supranational authority.

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A Appendix

Technical Appendix

The technical appendix is available under the webpage of the Authors.

Proof of determinacy of the optimal cooperative solution

We show that the first-order conditions (3.16)–(3.20) combined with the constraints (3.13)–(3.15) and the initial conditions $\varphi_{1,-1}$ and $\varphi_{2,-1}$ yield to a determinate equilibrium. First we use (3.16)–(3.20) and (3.15) to write (3.13) and (3.14) in terms of only the lagrangian multipliers and the shocks as it follows

$$E_t\varphi_{1,t+1} = \left(1+\frac{1}{\beta}+\frac{\vartheta_1\xi\kappa}{\beta}\right)\varphi_{1,t}+\frac{(1-n)\vartheta_2\xi\kappa}{\beta}\varphi_{2,t}-\frac{1}{\beta}\varphi_{1,t-1}+\frac{\xi\kappa}{\beta}u_t \quad \text{(A.1)}$$

$$E_t\varphi_{2,t+1} = \left(1+\frac{1}{\beta}+\frac{\vartheta_3\xi\kappa^*}{\beta}\right)\varphi_{2,t}+\frac{n\vartheta_2\xi\kappa^*}{\beta}\varphi_{1,t}-\frac{1}{\beta}\varphi_{2,t-1}+\frac{\xi\kappa^*}{\beta}u_t^* \quad \text{(A.2)}$$

where

$$\vartheta_1 \equiv n\lambda_y^{-1}+(1-n)\tilde{\lambda}_q^{-1}(\theta s_c+\psi)^2,$$

$$\vartheta_2 \equiv \lambda_y^{-1}-\tilde{\lambda}_q^{-1}(\theta s_c+\psi)^2,$$

$$\vartheta_3 \equiv (1-n)\lambda_y^{-1}+n\tilde{\lambda}_q^{-1}(\theta s_c+\psi)^2,$$

$$\xi \equiv \kappa\lambda_{\pi_h}^w = \kappa^*\lambda_{\pi_f}^w,$$

$$\tilde{\lambda}_q \equiv \theta^2 s_c^2\lambda_y+\lambda_q.$$

where $\lambda_{\pi_h}^w$, $\lambda_{\pi_f}^w$, λ_y, λ_q, $\tilde{\lambda}_q$ are defined in the technical appendix. In particular, under reasonable parameters' restriction, $\lambda_{\pi_h}^w > 0$, $\lambda_{\pi_f}^w > 0$, $\lambda_y > 0$, $\tilde{\lambda}_q > 0$ which imply that $\xi > 0$, $\vartheta_1 > 0$ and $\vartheta_3 > 0$. We can write (A.1) and (A.2) in the following form

$$E_t z_{t+1} = \begin{bmatrix} A_1 & A_2 \\ A_3 & 0 \end{bmatrix} z_t + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} \epsilon_t \quad \text{(A.3)}$$

where $z_t' \equiv [\varphi_t \ \varphi_{t-1}]$ and $\varphi_t \equiv [\varphi_{1,t} \ \varphi_{2,t}]$; $\epsilon_t' \equiv [u_t \ u_t^*]$, A_j with $j = 1, 2, 3$, and B_1 are two by two matrices. In particular

$$A \equiv \begin{bmatrix} A_1 & A_2 \\ A_3 & 0 \end{bmatrix}$$

$$A_1 \equiv \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad A_2 \equiv \begin{bmatrix} -\beta^{-1} & 0 \\ 0 & -\beta^{-1} \end{bmatrix} \qquad A_3 \equiv \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

with

$$\begin{aligned} a_{11} &\equiv \left(1 + \frac{1}{\beta} + \frac{\vartheta_1 \xi \kappa}{\beta}\right) > 0 \\ a_{12} &\equiv \frac{(1-n)\vartheta_2 \xi \kappa}{\beta} \\ a_{21} &\equiv \frac{n \vartheta_2 \xi \kappa^*}{\beta} \\ a_{22} &\equiv \left(1 + \frac{1}{\beta} + \frac{\vartheta_3 \xi \kappa^*}{\beta}\right) > 0 \end{aligned}$$

and B_1 is a block-diagonal matrix with elements $\xi\kappa$, $\xi\kappa^*$. In order to study determinacy, we need to inspect the roots of the characteristic polynomial associated with the matrix A which is

$$P(\psi) = \psi^4 - (a_{11} + a_{22})\psi^3 + (a_{11}a_{22} - a_{21}a_{12} + 2\beta^{-1})\psi^2 - (a_{11} + a_{22})\beta^{-1}\psi + \beta^{-2}.$$

First we note that

$$\psi_1\psi_2\psi_3\psi_4 = \beta^{-2}, \tag{A.4}$$

$$\psi_1 + \psi_2 + \psi_3 + \psi_4 = a_{11} + a_{22} > 2(1 + \beta^{-1}); \tag{A.5}$$

moreover if $P(\psi) = 0$ then $P(\psi^{-1}\beta^{-1}) = 0$ so that we can further conclude that

$$\psi_1\psi_2 = \beta^{-1} \qquad \psi_3\psi_4 = \beta^{-1}. \tag{A.6}$$

Moreover, by Descartes sign rule all the roots are positive. We note that

$$\begin{aligned} P(1) &= (1 + \beta^{-1})^2 - (1 + \beta^{-1})(a_{11} + a_{22}) + a_{11}a_{22} - a_{21}a_{12} \\ &= \xi \lambda_y^{-1} \tilde{\lambda}_q^{-1} > 0 \end{aligned}$$

$$P(0) = \beta^{-2} > 0$$

The fact that all the roots are positive and that $P(1) > 0, P(0) > 0$ imply that there are either 0 or 2 real or complex roots or 4 complex roots within the unit circle. Conditions (A.5) and (A.6) exclude the first and latter possibilities. From conditions (A.6), we can further conclude that the two roots are within the unit circle. The unique and stable solution of the system is obtained with the following steps. Let V the two by four matrix of left eigenvectors associated with the unstable roots. By pre-multiplying the system (A.3) with V we obtain

$$E_t k_{t+1} = \Lambda k_t + V B \epsilon_t \tag{A.7}$$

where Λ is a two by two diagonal matrix of the unstable eigenvalues on the diagonal and $k_t \equiv V z_t$. The unique and stable solution to (A.7) is given by

$$k_t = -\sum_{j=0}^{\infty} \Lambda^{-j} V B E_t \epsilon_{t+j}$$

which implies that

$$\varphi_t = -V_1^{-1} V_2 \varphi_{t-1} - V_1^{-1} \sum_{j=0}^{\infty} \Lambda^{-j} V B E_t \epsilon_{t+j} \tag{A.8}$$

where V_1 and V_2 are such that $V = [V_1 \; V_2]$. Equation (A.8) characterizes the optimal path of the vector φ_t given initial condition φ_{-1}; the paths for y_H, y_F^*, π_H, π_F^*, q_t can be derived using the conditions (3.16)–(3.20).

Proof of determinacy of the solution implemented by the targeting rules.

We now show that the targeting rules (5.31) and (5.32), combined with the conditions (5.29) and (5.30) and the constraints (3.13) to (3.15) yield to a determinate equilibrium that coincides with the optimal cooperative solution. We follow here an argument similar to Woodford (2003, ch. 6). It is easy to see that (5.31) and (5.32) combined with the conditions (5.29) and (5.30) imply (5.27) and (5.28). Let us define $\varphi_{1,t}$ and $\varphi_{2,t}$ for all $t \geq -1$ as

$$\varphi_{1,t} \equiv \lambda_y^w y_{H,t} - (1-n)\gamma q_t, \tag{A.9}$$

$$\varphi_{2,t} \equiv \lambda_y^w y_{F,t}^* + n\gamma q_t, \tag{A.10}$$

from which it follows that

$$\kappa \lambda_{\pi_h}^w \pi_{H,t} = -(\varphi_{1,t} - \varphi_{1,t-1}), \tag{A.11}$$

$$\kappa^* \lambda^w_{\pi_f} \pi^*_{F,t} = -(\varphi_{2,t} - \varphi_{2,t-1}). \tag{A.12}$$

Using (3.15) and (A.9)-(A.12), we can then retrieve the system of equations (A.1) and (A.2) which yields to a determinate equilibrium given the initial conditions

$$\varphi_{1,-1} \equiv \lambda^w_y y_{H,-1} - (1-n)\gamma q_{-1},$$

$$\varphi_{2,-1} \equiv \lambda^w_y y^*_{F,-1} + n\gamma q_{-1}.$$

Indeed the lagrangian multiplier $\varphi_{1,-1}$ and $\varphi_{2,-1}$ measure the commitment to expectations taken in periods before time 0. The timeless perspective optimal policy is the one that assigns a particular value to the commitment to expectations prior to period 0 such that the resulting optimal policy is time invariant.

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