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Abstract

We propose the method of eigenvalue filtering as a new tool to extract time series subcomponents (such as business-cycle or irregular) defined by properties of the underlying eigenvalues. We logically extend the Beveridge-Nelson decomposition of the VAR time-series models focusing on the transient component. We introduce the canonical state-space representation of the VAR models to facilitate this type of analysis. We illustrate the eigenvalue filtering by examining a stylized model of inflation determination estimated on the Czech data. We characterize the estimated components of CPI, WPI and import inflations, together with the real production wage and real output, survey their basic properties, and impose an identification scheme to calculate the structural innovations. We test the results in a simple bootstrap simulation experiment. We find two major areas for further research: first, verifying and improving the robustness of the method, and second, exploring the method's potential for empirical validation of structural economic models.

Key words: business cycle, inflation, eigenvalues, filtering, Beveridge-Nelson decomposition, time series analysis
JEL Classification: C32, E32

Non-technical Summary

This working paper develops a methodology for time series decomposition based on the Beveridge and Nelson (BN) framework, and applies it to the Czech data. It adds to the existing empirical filtering literature that has so far, especially as concerns the Czech experience, seemed to prefer the unobserved components (UC) methodology. The BN decompositions have the advantage of imposing less economic and identification restrictions on the underlying series; the uncovered cycles and trends are therefore more driven by the data themselves. This methodology then appears a useful complement to the UC techniques based usually on an a priori theoretical model. Cross-checking the results from the two types of methods can lead to a more robust assessment of the cyclical position of the economy to the benefit of monetary policy making. In addition, the application of the BN methodology can give additional insight into the workings of the economy at specific moments of its history. As a further extension of this research, the BN methodology can be more useful in empirical validation of DSGE models than more often used UC based approaches.

The standard BN methodology decomposes time series (or vectors of them) into their trend and transient components by identifying permanent and temporary effects of random innovations (shocks). The transient components may, however, contain (apart from the business cycle) also irregular or highly volatile fluctuations. In order to facilitate the economic interpretation, we therefore further decompose these transient elements into their business-cycle and irregular parts by imposing certain eigenvalue restrictions corresponding to some definition of the business cycle. The method can be easily generalized to define and extract any other desired types of time series components.

We apply the multivariate eigenvalue filtering to extract the trend, business-cycle and irregular components from a set of measures of Czech inflation and real economic activity in order to investigate the workings of the economy typified by stylized Phillips curve mechanics. For that purpose we combine net CPI inflation, industrial WPI inflation, import price inflation, the real production wage in the business sector, and real gross domestic product. The last two measures should capture movements in the real marginal cost of domestic value added. The robustness of the results is checked in a bootstrap simulation experiment.

Our empirical analysis yields several important conclusions. Overall, we find evidence favoring the hypothesis of a) faster import price and exchange rate pass-through into domestic producer prices than directly into those of domestic consumers, b) of the nominal wage being more sticky than the nominal price, as real wage changes are induced by movements in the underlying price indices. This is especially illuminating in the period of 2000 and 2001, when import prices were rapidly changing. The expe-

rience from this period hints at substantial technology or labor supply (intratemporal preference) shocks. Importantly, a Phillips curve type of relationship is detected, as the behavior of the permanent components fails to exhibit strong signs of super neutrality, with disinflation having negative effects on real variables. However, the real marginal cost concept hypothesized in the Phillips curve in our working paper yields counterintuitive results, namely an inverse effect of real output on the one hand and the real wage on the other hand into the CPI and WPI, and needs to be further refined.

Last, the overall statistical properties of the eigenvalue filtering need to be subject of further research, especially the robustness checks, as they tend to display a significant skewness.

1 Introduction

In this paper we develop the eigenvalue filtering methodology for decomposition of the VAR time-series models, and apply the method to the Czech data. By developing and applying this technique based on the original multivariate Beveridge-Nelson (BN) framework [1] we add to the existing empirical filtering literature that has so far preferred in the monetary policy context the unobserved component (UC) methodology (and especially so in applications on the Czech economy). As the BN decompositions are less restrictive and more driven by the data themselves, they appear a useful complement to the UC techniques based usually on an a priori theoretical model. [2] Cross-checking results from the two types of methods can lead to a more robust assessment of the cyclical position of the economy to the benefit of monetary policy making. In addition, the application of the BN methodology can give additional insight into the workings of the economy at specific moments of its history and, in turn, can be useful for empirical validation of dynamic stochastic general equilibrium models (DSGE), as claimed in Section 3.

Extracting the cyclical and trend components from observed time series has become an increasingly important activity at central banks in the last decade, especially as many of them have moved towards more or less explicit inflation targeting regimes. The medium-term focus of this type of monetary policy in particular raises demands for a good understanding of business cycle mechanisms and for robust knowledge of the current business cycle position of the economy, especially as regards the cyclical fluctuations and interactions between inflation and real economic activity. Correspondingly, many central bank researchers have recently been preoccupied with multivariate techniques that explore these economic relationships, but are simultaneously more immune to the common (e.g. end-point) problems of many mechanical filters. As a product, a suite of structural multivariate UC models have been created at many of these banks to support forecasting and policy analysis systems, particularly focused on identifying the cycles in real output, the real exchange rate, the real interest rate, and the real marginal costs of producers.

The relative neglect of the BN approaches is in this regard rather unfortunate, given the extra insight that can be learnt from comparing the two approaches noted above. Moreover, we argue here, this neglect is bound to change, as many of the central banks increasingly experiment with using DSGE models in their forecasting systems.

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1 See Beveridge and Nelson (1981) and Engle and Granger (1987).

2 Morley, Nelson, and Zivot (2003) show that the usually encountered differences in the results of the BN and UC decompositions mainly arise not because of some principal inconsistency of the two methods but rather because of substantial differences in the additional underlying identification assumptions.

Applying these models to data requires a very good distinction between real (supply) and nominal (demand) shocks. It is here where the flexibility of the BN techniques in describing the co-movements between trend and cyclical components is helpful. Whereas real shocks will in general be associated with their negative correlation (as long-run productivity fundamentals improve, the actual output catches up only gradually), no correlation should be expected for nominal shocks (that raise, say, output temporarily above the long-run trend). In this paper we further develop this flexibility by removing irregular or erratic fluctuations from the observed data, thereby focusing the decomposition only on the frequencies that are subject of the DSGE modeling. The recovered cycle and trend properties of data can then be used for empirical validation of these models.

In this paper we develop and apply a BN-based multivariate eigenvalue filtering method to extract the trend, business-cycle and irregular components from a set of measures of Czech inflation and real economic activity in order to investigate the workings of the economy typified by stylized Phillips curve mechanics in a small open economy. We thus illustrate the potential of the BN approach in understanding the nature of economic business cycle, both in theory and in application to the Czech data.

This approach is intended to complement the existing vast stock of experience in using UC filtering techniques in research and monetary policy inference in the Czech National Bank, which is documented *inter alia* by Coats, Laxton, and Rose (2003) and Beneš and N'Diaye (2004). Transition towards using more BN based techniques is taking place against the backdrop of introducing a DSGE model as the new core forecasting tool of the Bank. Given the same theoretical foundations of the two techniques, such a transition should be viewed as a smooth learning process, rather than a discontinuous new practice.

The paper is organized as follows. In Section 2 we review the Beveridge-Nelson permanent-transient decomposition in the vector moving-average and common-trends frameworks, and introduce the canonical state-space representation as a convenient device to deal with the VAR models. We extend the Beveridge-Nelson concept in Section 3 focusing on the filtering of the transient component based on the properties of the underlying eigenvalues. In Section 4 we discuss the structural identification in the canonical state-space framework based on long-run recursiveness. We apply the eigenvalue filtering method and the proposed structural identification scheme to an empirical model of the Czech business cycle with a set of measures of Czech inflation and real economic activity, examine their economic and stochastic properties, and verify the method in a simple bootstrap simulation experiment in Sections 5 and 6. Section 7 concludes.

2 Canonical state-space representation of VAR models

The subject of our interest is finite-order autoregressive models for vector processes (VAR models) in which the deterministic part has been removed prior to the analysis,

$$A(L)y_t = \varepsilon_t, \quad A(L) = \mathrm{I}_n - \textstyle\sum_{i=1}^{p} A_k L^i, \tag{1}$$

where y_t is an n-dimensional random vector, ε_t is an i.i.d. a vector of the linear forecast error with mean of zero and covariance matrix Ω, L is the lag operator, and $0 < p < \infty$. We furthermore restrict ourselves to vector processes whose elements are integrated of order 0 or 1 or cointegrated of order $(1,1)$ as defined by Johansen (1995). Finally, we only allow for the zero-frequency unit roots excluding hence the seasonal integration and cointegration for ease of the exposition. We therefore impose the following conditions adopted from Johansen (1995) and extended to cover the two limit cases [i.e. $\mathrm{I}(0)$ models, or $\mathrm{I}(1)$ models without cointegration]:

(i) $\operatorname{rank} A(1) = r$ where $0 \leq r \leq n$,

(ii) the inverse characteristic equation $|\mathrm{I}_n \lambda^p - \sum_{k=1}^{p} A_k \lambda^{p-k}| = 0$ has $n-r$ roots equal to 1 and all other roots inside the unit circle, $|\lambda| < 1$.

We use the term VAR to refer to such processes (1) that satisfy Conditions (i) and (ii) throughout the paper.

Our primary objective is to extend the concept of the Beveridge-Nelson decomposition[3] focusing on the properties of the transient component. More specifically, the VAR process has the Wold, or vector moving-average (VMA), representation,

$$\Delta y_t = C(L)\,\varepsilon_t,$$

which can be further re-written as

$$y_t = C(1) \textstyle\sum_{i=0}^{\infty} \varepsilon_{t-i} + C^{\star}(L)\,\varepsilon_t, \tag{2}$$

to establish the multivariate Beveridge-Nelson permanent-transient decomposition, see e.g. Engle and Granger (1987), where $C(1)$, the "factor loading" matrix, has rank $k = n - r$ corresponding to the number of independent stochastic trends that underlie permanent movements in y_t, whereas $C^{\star}(L) = \sum_{i=0}^{\infty} C_i^{\star} L^i$ is an absolutely summable generating function, $\sum_{i=0}^{\infty} |C_i^{\star}| < \infty$, which produces transient stationary fluctuations of the process around these trends. However, a more convenient representation to compare our work with is the common trends (CT) form introduced by Stock and Watson (1988), or alternatively by King, Plosser, Stock, and Watson (1987),

$$y_t = F\overline{y}_t + \widehat{y}_t \quad \text{such that} \quad (1-L)\overline{y}_t = G\,\eta_t, \quad \widehat{y}_t = D(L)\,\eta_t, \tag{3}$$

[3] See Beveridge and Nelson (1981) for the treatment of univariate models.

where $\bar{y}_t$ is an k-dimensional vector of common trends, $\widehat{y}_t$ is an n-dimensional vector of transient components, F is an $n \times k$ matrix of rank k, $D(L)$ is again an absolutely summable generating function, and some structural identification scheme is imposed on the reduced-form innovations such that $\eta_t = \Omega^{-1/2}\varepsilon_t$, $\mathrm{E}\eta_t\eta_t' = I_n$, with $\Omega^{1/2}\Omega^{1/2\prime} = \Omega$. The structural identification is discussed in detail in a separate section.

In our analysis, we convert the VAR model to its state-space representation based on the Jordan canonical factorization of the system transition matrix, and term this as the canonical state-space (CSS) form. We show that within the VAR models the CSS representation is useful as a starting point for calculating the two other ones above and convenient for further examination and decomposition of the process, $\widehat{y}_t$, that we later call the eigenvalue filtering. Note that when adopting the state-space representation we deal with the process in levels following therefore the approach of Casals, Jerez, and Sotoca (2002) rather than Proietti (1997). We start from the first-order companion form

$$y_t = HY_t, \quad Y_t = TY_{t-1} + H'\varepsilon_t, \tag{4}$$

with the vector of stacked observations $Y_t = [y_t' \ \ldots \ y_{t-p+1}']'$, the selection matrix $H = [\mathrm{I}_n \ 0_n \ \ldots \ 0_n]$, and the system transition matrix

$$T = \begin{bmatrix} A_1 \ldots A_p \\ \mathrm{I}_m \ 0_{m\times n} \end{bmatrix},$$

where $m = n(p-1)$, by calculating $T = P^{-1}\Lambda P$, where P is a matrix whose columns are T's eigenvectors, and Λ is a diagonal matrix of T's eigenvalues.[4] We then obtain the CSS representation for (1),

$$y_t = P_1 x_t, \quad x_t = \Lambda x_{t-1} + P^1\varepsilon_t, \tag{5}$$

with $Px_t = Y_t$, $P_1 = HP$, and $P^1 = P^{-1}H'$, which partitions P and P^{-1} conformably with H as $P = [P_1{}' \ P_2{}']'$ and $P^{-1} = [P^1 \ P^2]$ so that P_1 and P^1 are respectively $n \times np$ and $np \times n$ blocks.

Assuming without loss of generality that the eigenvalues in Λ occur in descending order by their moduli, partitioning Λ so that the leading $k \times k$ block is an identity matrix, $\overline{\Lambda} = \mathrm{I}_k$, whereas the remaining diagonal block, denoted by $\widehat{\Lambda}$, contains the other eigenvalues, and partitioning furthermore P_1 and P^1 conformably with these

[4] The eigenvalues associated with T are obviously identical to the roots of the inverse characteristic equation in Condition (ii), see e.g. Lütkepohl (1993) for a proof.

blocks so that $P_1 = [\overline{P}_1\ \widehat{P}_1]$ and $P^1 = [\overline{P}^{1\prime}\ \widehat{P}^{1\prime}]'$, we may easily calculate both the VMA representation (2),

$$C(1) = \overline{P}_1 \overline{P}^1, \quad C^\star(L) = \widehat{P}_1 (\mathrm{I}_{np-k} - \widehat{\Lambda} L)^{-1} \widehat{P}^1,$$

where the existence of the inverse in the latter term is guaranteed by construction, and the CT representation (3),

$$F = \overline{P}_1, \quad G = \overline{P}^1 \Omega^{1/2}, \quad D(L) = \widehat{P}_1 (\mathrm{I}_{np-k} - \widehat{\Lambda} L)^{-1} \widehat{P}^1 \Omega^{1/2}.$$

based again on a particular identification scheme $\Omega^{1/2}$.

Finally, we examine the implications of the existence of cointegrating and/or serial-correlation cofeature vectors for the CSS representation. First note that the definitions of cointegration and common features relate to truly $\mathrm{I}(1)$ models, in other words to those where at least one element of y_t is $\mathrm{I}(1)$.[5] We therefore impose such a restriction in the remainder of this section.

Following *inter alia* Vahid and Engle (1993), a vector β is called a cointegrating vector if $\beta' y_t$ is $\mathrm{I}(0)$; similarly, a vector γ is called a serial-correlation cofeature vector if $\gamma' \Delta y_t$ is orthogonal to all observed information prior to t (random walk). The latter is in fact only the strong-form reduced-rank structure as recognized by Hecq, Palm, and Urbain (2000), but we skip the weak form here. Clearly, β must lie in the left-null space of $\overline{P}_1 \overline{P}^1$, whereas γ in the overlap of the left-null spaces of the individual matrices defining the generating function $C^\star(L) = \widehat{P}_1 (\mathrm{I}_{np-k} - \widehat{\Lambda} L)^{-1} \widehat{P}^1$. However, noting that γ must be therefore necessarily orthogonal to $C^\star(0) = \widehat{P}_1 \widehat{P}^1$, and that by construction $\overline{P}_1 \overline{P}^1 + \widehat{P}_1 \widehat{P}^1 = \mathrm{I}_n$, we conclude that beacuse $\gamma' \overline{P}_1 \overline{P}^1 = \mathrm{I}_n$, γ lies in the space spanned by the eigenvectors associated with the unit eigenvalues of $\overline{P}_1 \overline{P}^1$.

3 Eigenvalue filtering

In the light of the CSS representation the Beveridge-Nelson decomposition can be indeed viewed as extraction of components that are defined by the eigenvalues, or by certain properties of the eigenvalues, that carry these components. More specifically, the permanent-transient decomposition in the VAR models is given by the following

[5] See Definitions 3.1–3.2 in Johansen (1995).

state-space system,

$$y_t = \overline{y}_t + \widehat{y}_t = \overline{P}_1\overline{x}_t + \widehat{P}_1\widehat{x}_t, \tag{6a}$$

$$\overline{x}_t = \overline{\Lambda}\overline{x}_{t-1} + \overline{P}^1\varepsilon_t = \overline{x}_{t-1} + \overline{P}^1\varepsilon_t, \tag{6b}$$

$$\widehat{x}_t = \widehat{\Lambda}x_{t-1} + \widehat{P}^1\varepsilon_t, \tag{6c}$$

where $\overline{x}_t$ and $\widehat{x}_t$ are defined by the properties of the eigenvalues on the diagonals of $\overline{\Lambda}$ and $\widehat{\Lambda}$, respectively: Denoting by λ_i the diagonal entries of the respective matrices, we impose $\overline{\lambda}_i = 1$ and $\widehat{\lambda}_i \neq 1$.[6]

However, the conditions by which we discriminate among the eigenvalues, and consequently include the associated states into $\overline{x}_t$ or $\widehat{x}_t$, can be obviously any. We can therefore extend this procedure to a more generalized *eigenvalue filtering*; the idea is in principal based on Casals, Jerez, and Sotoca (2002). We choose a qualification criterion, $S(\lambda)$, which is either true or false for any admissible eigenvalue in the VAR model. Having converted the model to its CSS representation we re-order and partition the eigenvalue matrix Λ, denoting its two diagonal blocks by Λ^* and $\Lambda^\dagger$, so that $S(\lambda_i^*)$ is true for $\forall i$, and $S(\lambda_i^\dagger)$ is false $\forall i$. The states and the other state-space system matrices are partitioned conformably with it; the transformed state subvectors, $x_t = [x_t^{*\prime}, x_t^{\dagger\prime}]'$, then determine the components of y_t, denoted by y_t^* and $y_t^\dagger$, respectively. The resulting representation is fully analogous to (6). There are four convenient features of the proposed eigenvalue filtering:

(i) The components of the examined process, y_t^* and $y_t^\dagger$, as well as the underlying states, x_t^* and $x_t^\dagger$, are obtained as linear projections on a finite set of observations, namely on the stacked vector Y_t. The projection matrices thereof depend solely on the polynomial $A(L)$ in (1), and an appropriate selection matrix. In particular,

$$x_t^* = H^*P^{-1}Y_t,$$
$$y_t^* = P_1^*H^*P^{-1}Y_t,$$

where P and P^{-1} are assumed to have been already re-ordered appropriately, and $H^* = [\mathrm{I}_s, 0_{s\times np}]$ is the selection matrix with s being the number of qualified eigenvalues, and P_1^* is created similarly as $\overline{P}_1$.

(ii) The components are always complementary, regardless of the qualification $S(\lambda)$ in that $y_t = y_t^* + y_t^\dagger$.

(iii) We may extend the eigenvalue filtering to a multi-component framework.

[6] We may alternatively think of $|\overline{\lambda}_i| = 1$ and $|\widehat{\lambda}_i| < 1$ in more general cases where unit eigenvalues at frequencies other than zero occur in the model.

(iv) Since the filtering is described by a state-space system we may easily draw inferences about the stochastic properties of the filtered components in both the time and frequency domains.

The eigenvalue qualification can have a potentially strong economic interpretation. The most striking example is a further filtering of the classical Beveridge-Nelson transient component aimed at extracting some more systematic, business-cycle, fluctuations from $\widehat{y}_t$ that are net of irregular or erratic noise. As there is no unique agreed definition of the business cycle (or, equivalently, of the irregular noise) we use the traditional one dating back to Burns and Mitchell (1946) and ascribe to the business cycle only those fluctuations that last between 6 and 32 quarters. In terms of the eigenvalue qualification we propose the following two criteria:

(i) [Magnitude qualification] The business-cycle eigenvalue induces an impulse response of which not less than 10 % survives in the 6^{th} quarter and not more than 90 % in the 32^{nd} quarter.

(ii) [Phase angle qualification] The business-cycle eigenvalue induces an impulse response with periodicity of 6 or more quarters. [7]

These two conditions need to hold simultaneously for an eigenvalue to qualify as a business-cycle one. The former one leads to a range of approximately $[0.68, 0.93]$ for the moduli of a quarterly model's eigenvalues while the latter to a range of $[-\frac{\pi}{3}, \frac{\pi}{3}]$ for their phase angles. The qualification region within the unit circle is depicted as a shadow area in Figure 1. [8]

We find the eigenvalue filtering framework potentially appealing especially in the area of empirical validation of structural, say DSGE, models. These models usually claim to explain only certain dimensions of the observed data, [9] or even their population moments. To this end, the eigenvalue filtering may provide an econometrically relevant empirical counterpart to the examined model properties, net of those factors in the data generating process that are clearly beyond the model's scope, such as highly volatile or erratic—say non-fundamental—fluctuations in prices or financial markets indicators.

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[7] There is no upper bound imposed on the periodicity because we want the real, appropriately scaled, eigenvalues to qualify as the business-cycle ones too.

[8] Note, however, that the properties of the business-cycle components obtained from the eigenvalue filtering differ essentially from those computed by the the band-pass, or frequency-selective, filters proposed recently in the macroeconomics literature by Baxter and King (1999) or Christiano and Fitzgerald (2003). An ideal band-pass filter would entirely cut off the higher and lower frequencies in the spectrum of the desired business-cycle component whereas the eigenvalue-filter business-cycle components will always display continuous non-zero spectra over the whole range $[0, \pi]$.

4 Structural identification

With an estimated VAR converted to its CSS form we need to impose additional restrictions to identify the structural innovations, denoted by η_t, from the forecast errors, ε_t, and hence to make inferences about the impulse responses of y_t or its eigenvalue-filtered components, and about the corresponding variance decomposition. For future reference we denote by Ψ^η the matrix of the permanent effect of η_t on y_t, which is $\Psi^\eta = \overline{P}_1 \overline{P}^1 S$, where $S\eta_t = \varepsilon_t$.[10]

Our scheme to identify S is similar to the one used by King, Plosser, Stock, and Watson (1991),[11] in that we make the following identifying assumptions:

(i) the structural innovations are mutually uncorrelated and have unit variances: $\mathrm{E}\eta_t\eta_t' = \mathrm{I}_n$,

(ii) the structural innovations lie in the space spanned by the current and lagged observations: $S\eta_t = \varepsilon_t$ where S is nonsingular,

(iii) the number of the structural innovations with a permanent effect on y_t equals the number of underlying stochastic trends, and their effect has a recursive structure: $\Psi^\eta = [\Psi_1^\eta \; 0_{n\times(n-k)}]$ where Ψ^η is an $n \times k$ lower triangular matrix.

Note that Assumptions (i) to (iii) properly identify only the permanent structural innovations. The transient innovations can be, however, identified using the otherwise standard results from the VAR literature, i.e. by assuming certain recursiveness of their instantaneous effect on y_t. This can be performed as an extra step in the procedure suggested herein.

Our calculation of S and η_t involves the QR factorizations; this has been originally proposed by Hoffmann (2001) for the vector error-correction representations. Accordingly we proceed in three steps:

1. We arbitrarily pre-orthonormalize the forecast errors, e.g. by the Cholesky factor of their covariance matrix, denoting these newly created innovations by ξ_t,

$$K\xi_t = \varepsilon_t, \quad KK' = \Omega,$$

which produces $\mathrm{E}\xi_t\xi_t' = \mathrm{I}_n$. In the subsequent steps we only rotate the innovations by unitary matrices, and preserve thus the covariance matrix I_n: $\mathrm{E}M\xi_t\xi_t'M' =$

[9] In this regard, Geweke (1999) provides formal definitions of the so-called weak and minimal econometric interpretations of the DSGE models.

[10] The permanent effect is of course transmitted through na instantaneous effect of η_t on $\bar{x}_t$ as the latter are random walks.

[11] Cf. Appendix therein.

$M\mathrm{I}_n M' = I_n$ for any $MM' = \mathrm{I}_n$.

2. We separate the permanent and transient innovations. Recall that an innovation is permanent if and only if it immediately affects $\bar{x}_t$, and is transient if and only if it fails to do so. Clearly, the instantaneous multipliers for the entire state vector x_t with respect to ξ_t are given by the matrix $\Phi^{\xi} = P^1 K$. We may now partition this as

$$\Phi^{\xi} = \begin{bmatrix} \Phi_1^{\xi} \\ \Phi_2^{\xi} \end{bmatrix}$$

, conformably with the number of stochastic trends k so that the upper block Φ_1^{ξ} is $k \times n$, and find the QR factors of its transpose,

$$\Phi_1^{\xi} = R'Q',$$

where Q is an $n \times n$ matrix such that $QQ' = \mathrm{I}_n$ and R is an $n \times k$ matrix such that R' is lower triangular with zeros in its last $n-k$ columns. The matrix Q is now a suitable basis for a transformation which gives rise to two subvectors of innovations, permanent and transient. Letting $Q\nu_t = \xi_t$ we compute the effect of ν_t on x_t, denoted by Φ^{ν}, as follows:

$$\Phi^{\nu} = \Phi^{\xi} Q = \begin{bmatrix} R'Q' \\ \Phi_2^{\xi} \end{bmatrix} Q = \begin{bmatrix} R' \\ \Phi_2^{\xi} Q \end{bmatrix},$$

so that the first k elements of x_t, i.e. the subvector $\bar{x}_t$, are only affected by the first k elements of ν_t, denoted hence by $\bar{\nu}_t$. In other words, $\bar{\nu}_t$ are permanent while the remainder of ν_t is only transient with respect to their respective impacts on y_t. Consequently,

$$\Psi^{\nu} = \bar{P}_1 R' = \left[\bar{\Psi}^{\nu},\ 0_{n \times (n-k)} \right].$$

Note that whenever the VAR model is not cointegrated, $k = n$, we simply skip Step 2 since permanent are all innovations by Assumption (iii).

3. Finally, we find the desired structural innovations, η_t, to achieve a recursive structure of their permanent effect. This is accomplished by another QR factorization of the transpose of $\bar{\Psi}^{\nu} = U'Z'$ which yields a unitary $k \times k$ matrix Z, and a $k \times n$ matrix U such that U' is lower triangular. We define η by a diagonal matrix com-

posed of Z and an identity I_{n-k},

$$\widetilde{Z}\eta_t = \nu_t, \qquad \widetilde{Z} = \begin{bmatrix} Z & 0_{k\times(n-k)} \\ 0_{(n-k)\times k} & I_{n-k} \end{bmatrix},$$

whose permanent effect is

$$\Psi^{\eta} = \Psi^{\nu}\widetilde{Z} = \left[U'Z',\, 0_{n\times(n-k)}\right] \begin{bmatrix} Z & 0_{k\times(n-k)} \\ 0_{(n-k)\times k} & I_{n-k} \end{bmatrix} = \left[U',\, 0_{n\times(n-k)}\right].$$

The resulting transformation,

$$KQ\widetilde{Z}\eta_t = \varepsilon_t,$$

is nonsingular by its construction, and can be therefore used to calculate η_t directly from the forecast errors ε_t.

5 Application to the Czech business cycle

We apply the proposed eigenvalue filtering to extract the trend, business-cycle and irregular components from a set of measures of Czech inflation and real economic activity, and to make conclusions about their economic and stochastic properties. The primary economic motivation for the particular model adopted here comes from a stylized Phillips curve based on the staggered price-setting theory where the optimal producers' or distributors' price decisions are mainly determined by the fluctuations of the real marginal costs around their hypothetical flexible-price level. More background theory is found, *inter alia*, in Gali and Gertler (1999) or Christiano, Eichenbaum, and Evans (2005).

We use the following seasonally adjusted time series to set up a VAR model:

- One-quarter net CPI inflation, i.e. CPI net of administered prices,
- One-quarter WPI inflation in manufacturing,
- One-quarter import price inflation in domestic-currency units,
- the real production wage in the profit sector, i.e. nominal wage divided by the WPI
- Real GDP in chained 1995 CZK.

The last two variables on the list are to capture movements in the real marginal cost of domestic value added [12], whereas the effect of import price inflation is twofold: first,

[12] Fluctuations in the real marginal cost are generally determined by movements in the factor prices and in the level of production.

a direct impact on the CPI via directly consumed imports, and second, an indirect impact via material and intermediate imports passed through to producer prices and consecutively consumer prices.

Next, we make the following assumptions:

- all the examined time series are assumed I(1),
- CPI inflation, WPI inflation, and import price inflation are assumed to share one common stochastic trend; this restriction is imposed by the following basis of the cointegrating space: $[1,-1,0]'$ and $[0,1,-1]'$.

We regard the imposed first-order integration of inflation series as a useful shortcut for reduced-form within-sample modeling of the disinflation episode of the Czech economy in the 1990s. However, it is evidently inconsistent with the existence of an inflation-targeting monetary authority in itself and hence utterly inappropriate for any out-of-sample conclusions. With these I(1) inflation processes, the respective price levels themselves follow I(2) but cointegration of them makes the relative prices I(1) again; in other words, we allow for stochastic trends in relative prices, and these might be thought of as results of e.g. Balassa-Samuelson effects (in the case of the relative import price) or systematic changes in producers' markups due to changes in the market structure (in the case of the relative consumer-producer price).

Regarding the identification of structural innovations, recall that our set of five I(1) variables with a two-dimensional cointegrating space is underlain by three stochastic trends. According to our identification scheme set forth in Section 4 we obtain three innovations with permanent effect and two other with only transient effect. Furthermore, keeping the ordering as described in this section the matrix of permanent effect, Ψ^{η}, has necessarily its 3×4 top-right block filled with zeros (since the permanent effect of any structural innovation must be identical for all inflation processes).

To impose conveniently the desired cointegrating space we estimate the model as a second-order VEC, using the multivariate LS on a quarterly sample 1995:1—2003:4, and convert it to its VAR and CSS representations, (1) and (5), respectively. We then compute the individual components and their stochastic properties. This is discussed in the subsequent section. Finally, we examine the robustness and small-sample properties of the estimator in a simple bootstrap experiment whose design is also described in the subsequent section.

6 Estimation and Simulation Results

In this section we review the estimation results and accompany them with a list of issues that remain open or unclear.

As illustrated in Figure 1, only 4 out of the 12 stable eigenvalues of the estimated VAR qualify as contributors to the business-cycle fluctuations. When interpreting these business-cycle fluctuations and the corresponding trend components, extracted by eigenvalue filtering and plotted in Figures 2 and 3, we must always read a cyclical peak as an indicator of an innovation or accumulation of innovations that cause an ultimate decline in the respective variable over the long-run horizon. In other words, a positive transitory deviation means that the observed variable will be pushed downwards over the long run, if not hit by other shocks. Figure 2 also provides a simple eyeballmetric for the inspection of the relative signal-to-noise ratio in the examined series. In line with our intuition both of the real variables (output and wage) together with CPI inflation are considerably less noisy then import price inflation or WPI inflation. This may be viewed as a basis for our selection of robust measures of the business-cycle position of the economy in our future research.

Considering the identification across wider economic evidence, we find an interesting period in late 2000 and early 2001. A sudden drop in inflation trends was accompanied by a simultaneous decline in real domestic output and a rise in the real wage. The underlying hypothesis is that a fall in the relative import price also occurred about this time [since the relative price trends themselves are considered I(1)] and intratemporal goods substitution took place, cutting the demand for domestic goods. This is confirmed by the bottom-right panel in Figure 3. As WPI inflation almost immediately followed the new trend we may furthermore hypothesize a sharper or faster development in material and intermediate imports rather than those for final consumption; the CPI then reacted fully only in 2002. On the production side, substitution occurred between domestic value added (mainly labor) and imports, potentially pushing the marginal product of labor up. The fundamental source for these movements may lie in a shift of households' marginal rate of substitution between consumption and leisure, or as an observational equivalent, in institutional or bargaining changes in the labor market.

The overall business-cycle autocorrelations are reported in Table 1. First, we can check that there is a faster pass-through of import inflation to WPI inflation than CPI inflation. This may indicate either a high content of material or intermediate imports in gross domestic production and a relatively high elasticity of substitution between these imports and domestic value added, i.e. real output (which is also indirectly supported by a rather large negative correlation of real output and current-dated and lagged import prices) or a fundamentally different nature of price contracts in the wholesale and retail sectors.[13] Next, the reported overall correlation patterns fail to support the sketched Phillips curve in the preceding section as a systematic tie of inflation and real economic activity. Opposite signs on the correlations between WPI inflation and the real wage versus real output over the whole reported range of lags leave room for future refinements of the current concept of measuring the real

[13] For instance, a significantly shorter average duration of typical wholesale contracts.

marginal cost. Finally, Table 1 gives strong evidence in favor of countercyclical behavior of real wages, which is congruent with the traditional Keynesian interpretation of the business cycle conditioned upon nominal wage stickiness: however we can only discover a higher degree of stickiness in the nominal wage relative to the CPI from the negative correlations between the real wage and CPI inflation, not relative to the WPI.

The impulse responses to structural innovations computed as subject to the long-run recursive property are summarized in Table 2 and Figures 4 and 5. Depending on the sign of the long-run effect of permanent innovations on our model's variables, Table 2, we may attempt to attach a more or less structural interpretation to them. The first permanent innovation is clearly (dis-)inflationary, simultaneously affecting also real output (negatively) and the real production wage (positively). This is a breakdown of monetary super neutrality, simply because we have not imposed it in the model at all, and arises evidently as a consequence of the disinflation being run at real costs in the 1990s. The second permanent innovation influences solely the real variables, namely in the same direction, whereas the third one does the same with an opposite effect on each variable. They are thus candidates for, respectively, technology (productivity) and intratemporal preference innovations.

The response profiles are then depicted in the subsequent Figures 4 and 5. As noted in the preceding paragraph the disinflationary innovation incurs permanent real cost, although primarily the drop in CPI inflation is led by faster and more pronounced changes in import inflation—obviously in the nominal exchange rate indeed—and in domestic WPI inflation, with both of them jumping below or to the new steady-state level instantaneously. In all the reported shocks we can again find evidence favoring the hypothesis of the nominal wage being more sticky than the nominal price, as the real wage changes are markedly induced by movements in the underlying price indices. This is especially the case with the second permanent shock (compare the immediate profile of WPI inflation and the real wage).

It remains to verify the sampling variability of our results, in particular that of the estimates of the business-cycle components. We use an algorithm based on resampling from the observed data known as the non-parametric Cholesky factor bootstrap; the method has been described by Diebold, Ohanian, and Berkowitz (1998). We draw 1,000 samples of time-series data, re-estimate the underlying VAR models, re-filter the time-series components and construct the confidence bands as the 0.10 and 0.90 percentiles.

To summarize the exercise, the point estimates of the business-cycle component tend to be rather indicators for the upper (if positive) or lower (if negative) bands of the empirical distribution. The simulated distributions are asymmetric and skewed towards zero (evidently seen particularly with import inflation). Moreover, we also detect bimodal distributions peaking at zero and near to the point estimate. This is documented by the 0.10 and 0.90 percentiles attached to the actual estimates of the business-cycle component in Figure 6, and by selected profiles of empirical distributions plotted at the points of maximum deviations of the respective variables from zero, as in Figure 7. The sources of these distortions are subject of further research: they may be attributed to the small sample bias, together with the discrete nature of the eigenvalue qualification, in that there are only two discrete states: an eigenvalue either qualifies or fails to qualify. For this, a certain sort of fuzziness in the qualification conditions might improve the robustness and overall properties of the estimator.

7 Concluding Remarks

In this working paper we extend the multivariate Beveridge-Nelson decomposition of time series focusing particularly on the transient component. We introduce the canonical state-space form as a convenient representation of VAR models permitting further intuitive decomposition of the Beveridge-Nelson transient component and defining e.g. business-cycle and irregular fluctuations in time series on the basis of the properties of the underlying eigenvalues.

We use this concept to examine a simple empirical model of the inflation determination based on a stylized composite indicator of the real marginal cost in a small open economy using the Czech data. We characterize the estimated components of inflation in the CPI, WPI and import prices, together with real output and the real production wage, surveying their basic stochastic properties and identifying long-run recursive structural innovations. We investigate the impulse responses, and test the sampling variability of the results in a bootstrap simulation. The conclusions we make on this basis regard the speed of import price and exchange rate pass-through, the basic pro- or counter-cyclicality with implications for the degree of stickiness in prices and wages, and the relevance of the real marginal cost measure used in the model.

Finally, we find that major room for further research lies in two areas. First, in further investigation of the robustness of the method, particularly in documenting and explaining the towards-zero skewness of the component estimator which may arise probably also because of the discrete (true–false) nature of the eigenvalue qualification. Second, in exploring the method's potential for empirical validation of truly structural, e.g. dynamic stochastic general equilibrium, models in which only certain dimensions of the data properties, such as the correlation patterns pertaining to the fluctuations at business-cycle frequencies, are addressed.

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Table 1.
Estimates of autocorrelations of business-cycle components

Cross-correlations

Net CPI inflation	1.00	0.56	-0.21	-0.49	0.43
WPI inflation	0.56	1.00	0.43	0.25	-0.26
Import inflation	-0.21	0.43	1.00	0.65	-0.55
Real wage	-0.49	0.25	0.65	1.00	-0.23
Real output	0.43	-0.26	-0.55	-0.23	1.00

Serial first-order cross-correlations

Net CPI inflation	0.66	0.57	0.32	-0.10	0.09
WPI inflation	0.24	0.79	0.74	0.54	-0.32
Import inflation	-0.66	0.08	0.83	0.78	-0.63
Real wage	-0.51	-0.11	0.37	0.68	-0.04
Real output	0.46	-0.25	-0.36	-0.38	0.78

Table 2.
Asymptotic effects of permanent structural innovations

	#1	#2	#3
Net CPI inflation	-1.07	0.00	0.00
WPI inflation	-1.07	0.00	0.00
Import inflation	-1.07	0.00	0.00
Real wage	0.55	-0.23	-0.98
Real output	-0.35	-0.87	0.10

Figure 1.
Business-cycle qualification of eigenvalues

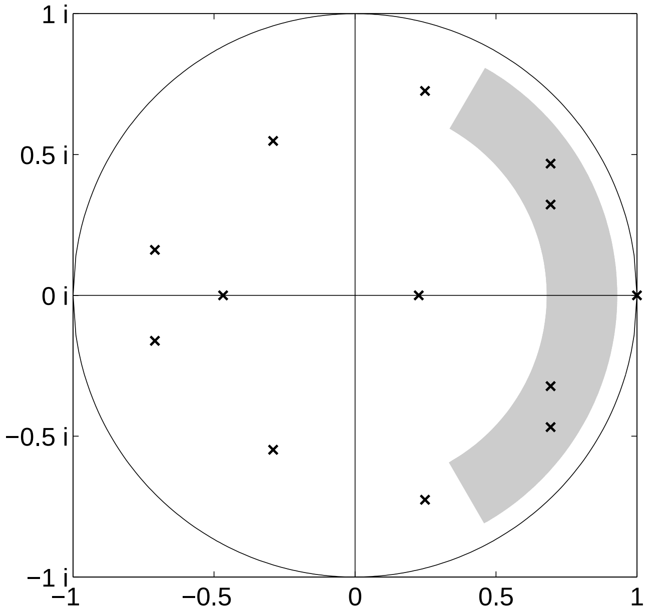


Figure 2.
Business-cycle [—] and irregular components [– –]

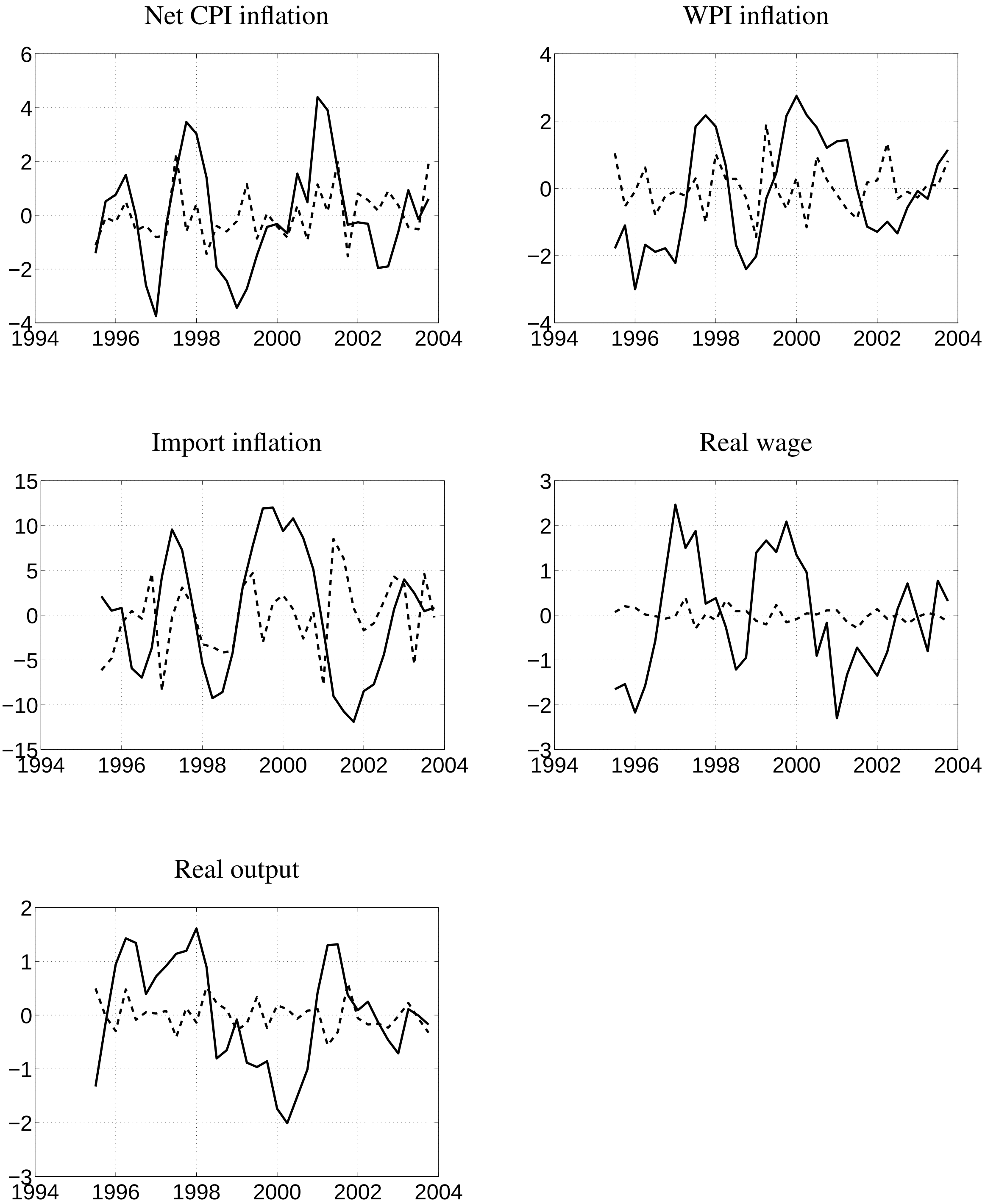


Figure 3.
Data [—] and trend components [– –]

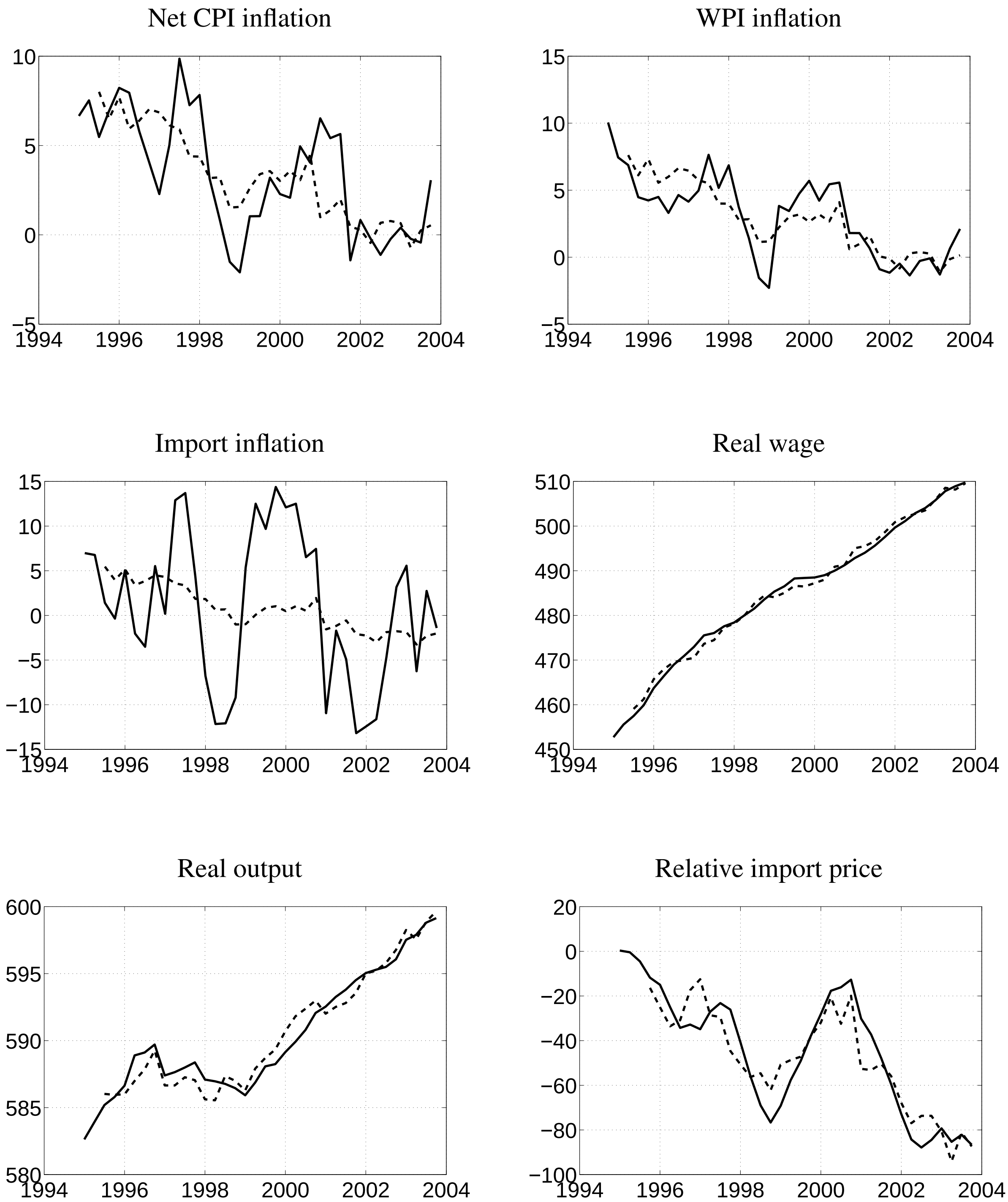


Figure 4.
Impulse responses of business-cycle + trend components

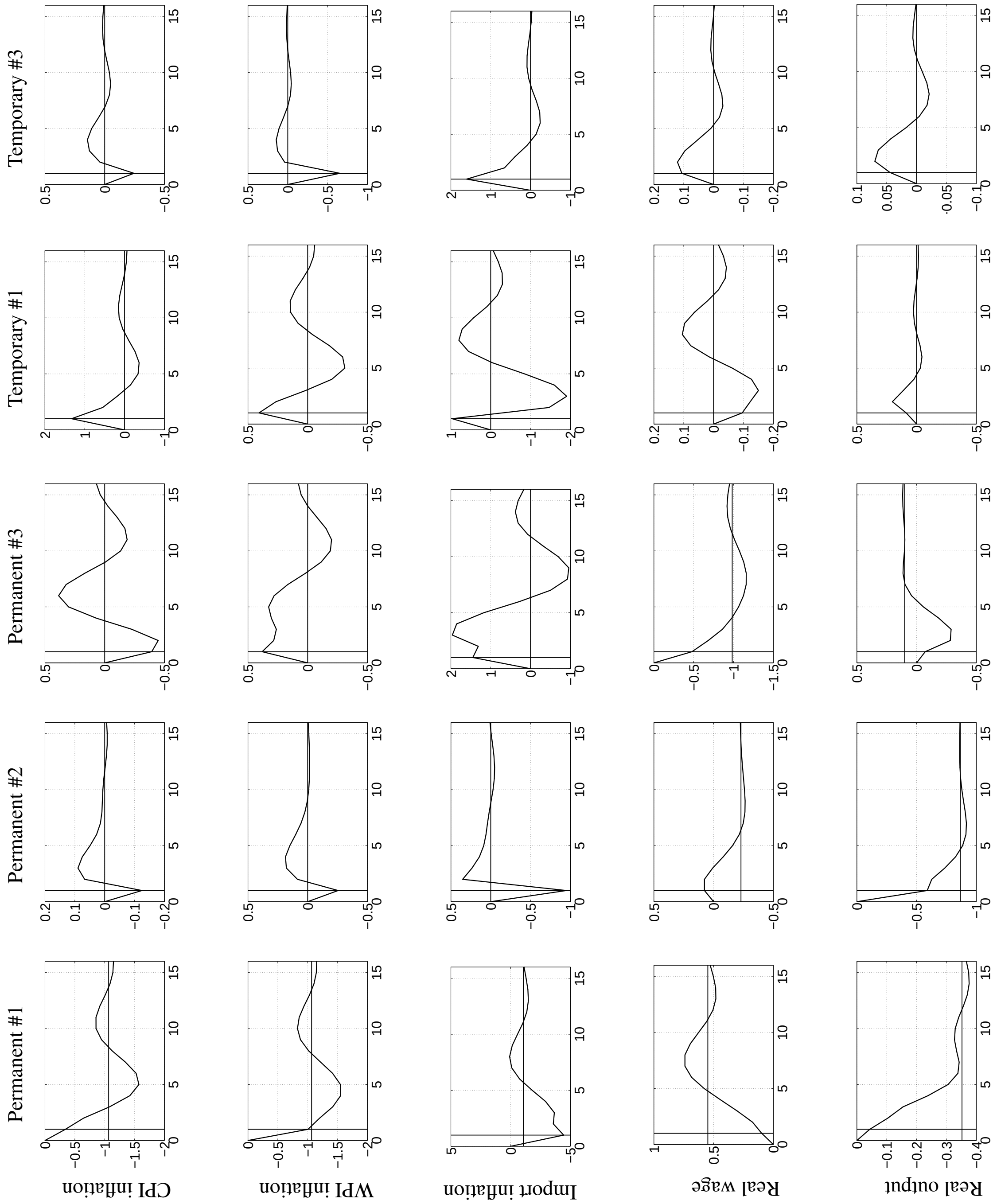


Figure 5.
Impulse responses of irregular high-frequency components

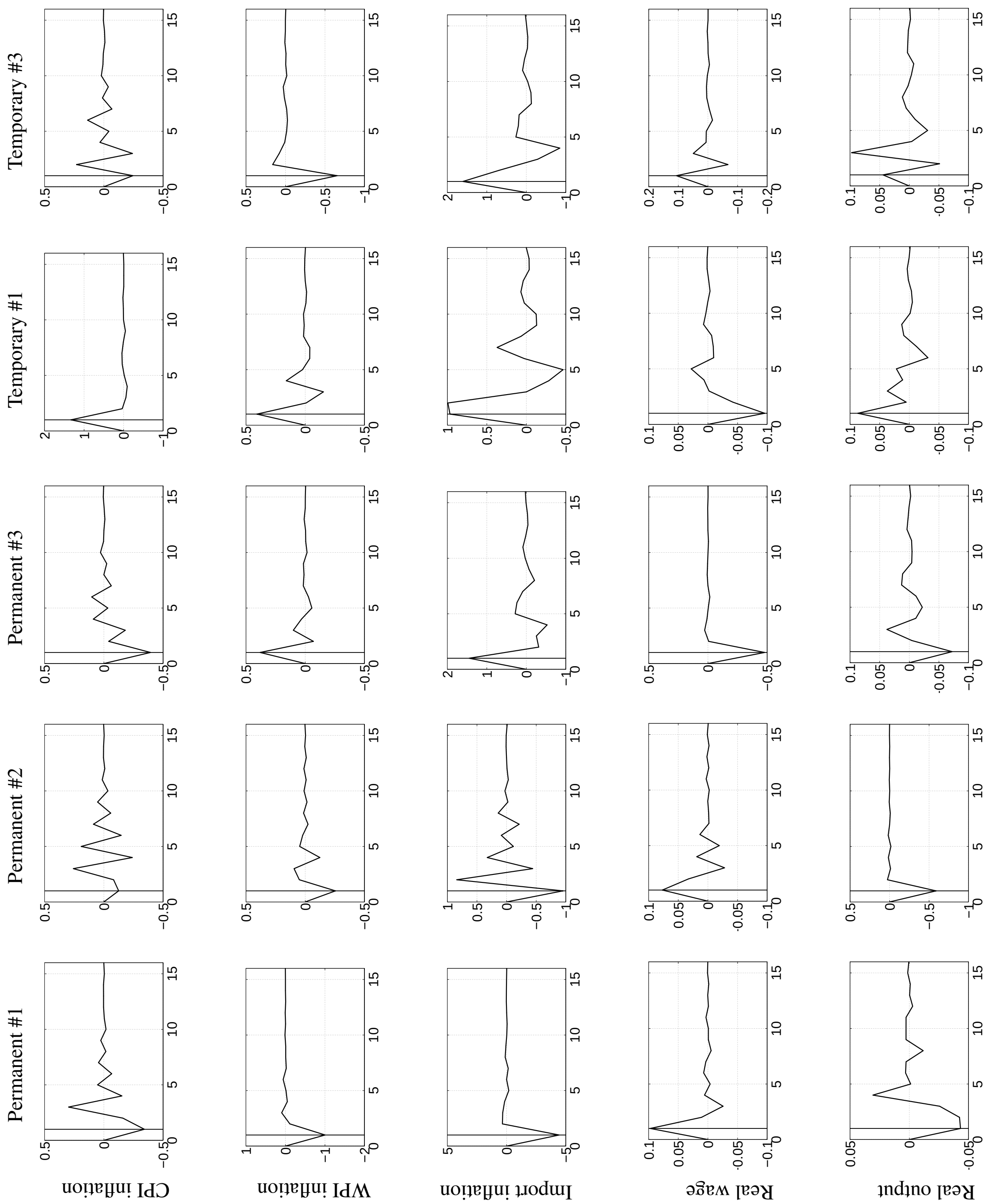


Figure 6.
Business-cycle components [0.10 and 0.90 bootstrapped percentiles]

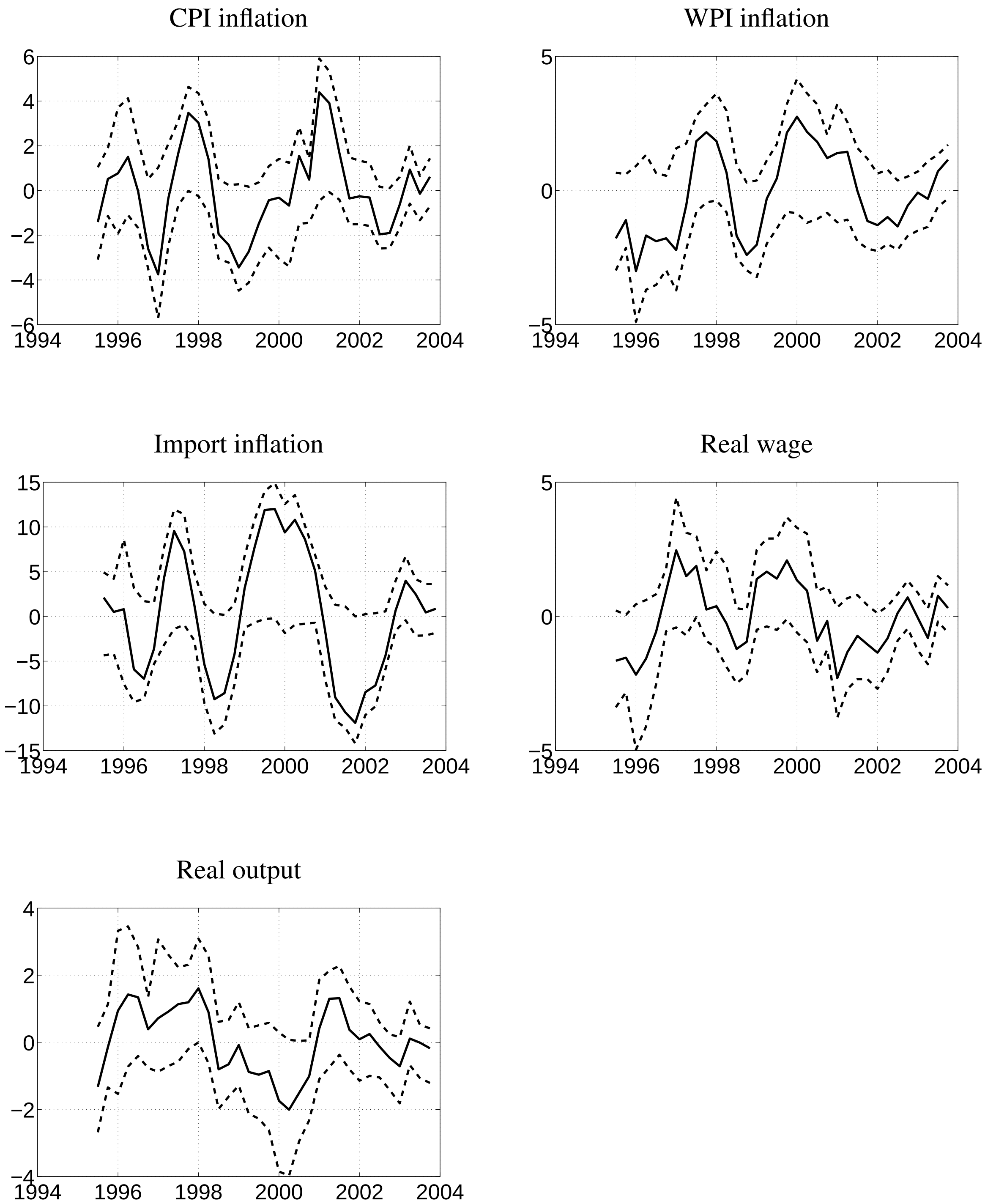
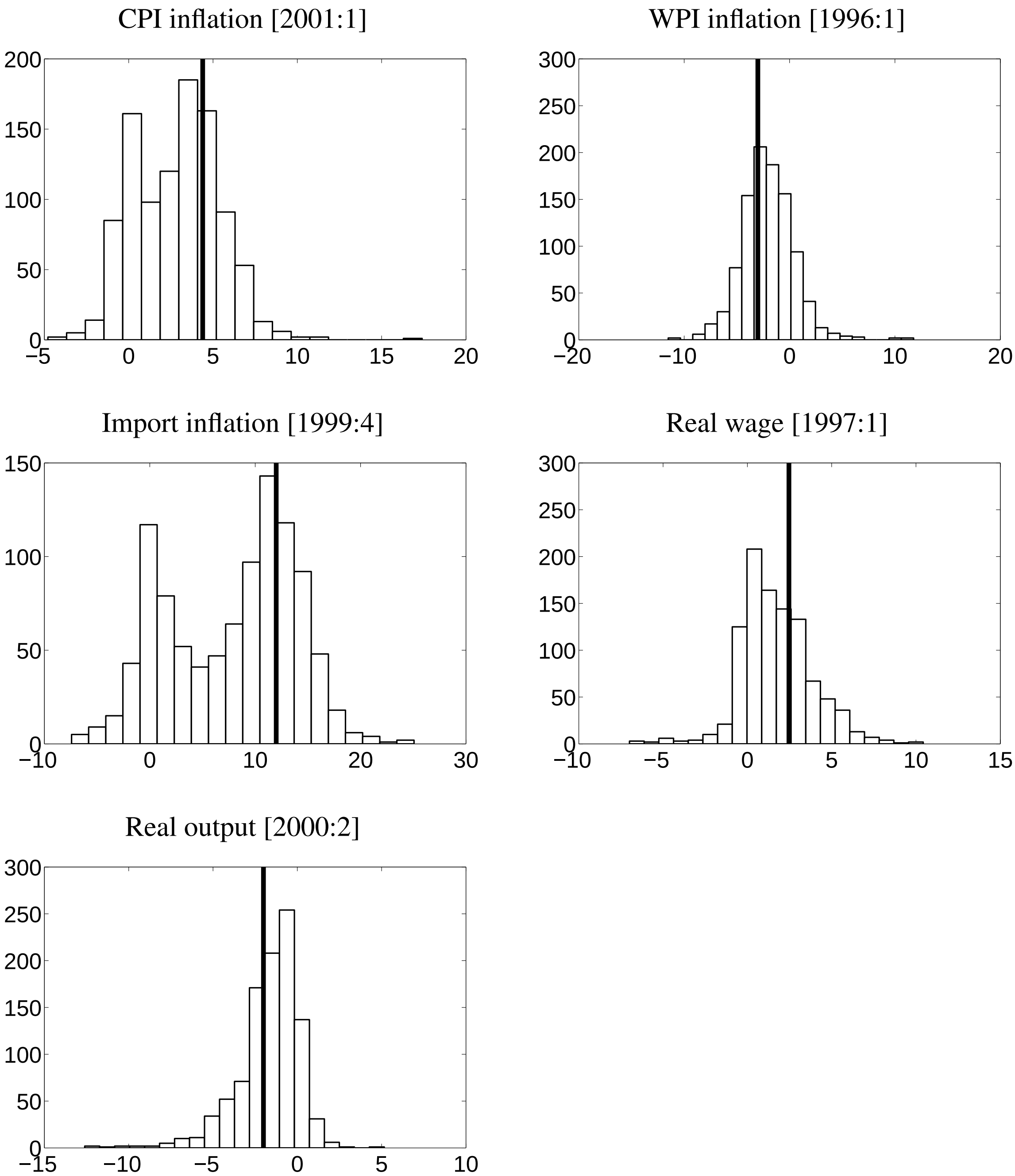


Figure 7.
Empirical bootstrapped distributions of business-cycle components



Profiles at the point of a maximum deviation from zero, vertical lines denote point estimates.

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