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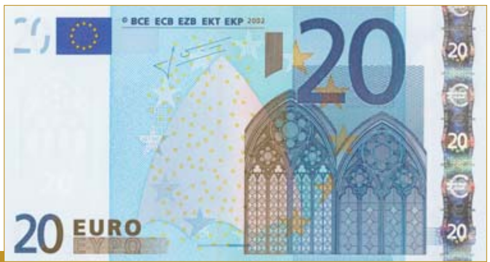

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CONTENTS

Abstract

Policymakers do not always follow a simple rule for setting policy rates for various reasons and thus their choices are co-driven by a decision to follow a rule or not. Consequently, some observations are censored and cause bias in conventional estimators of typical Taylor rules. To account for the censored and discrete process of policy rate setting, I devise a new method for monetary policy rule estimation and demonstrate its ability to outperform the existing conventional estimators using two examples.

J.E.L. Classification: E4, E5
Key words: Monetary policy; Policy rule; Bias in parameters

Non-technical summary

Recent literature has questioned the performance of the typical monetary policy rules of Taylor type and contrasted the poor predictability of future market rates by financial market participants (likely using an estimated Taylor rule) with the almost perfect explanatory power of Taylor rules fitted on historical data. It follows that the parameter estimates of policy rules are biased since this is the only way how to reconcile these contradictory observations.

The literature also suggests that the source of the bias in parameter estimates is due to an omitted persistent effect that also influences policy rate settings. I propose to associate such an effect with the effect of censoring rule in monetary policy rate decisions. Consequently, I develop an appropriate estimation technique that takes the effect of a censoring rule into account and thus delivers unbiased parameter estimates of a systematic policy that is based on Taylor type rules.

Empirical application considers policy rules estimation in two quite different counties. First, an explicit inflation targeting regime in a small and open economy (the Czech Republic) and second, an implicit inflation targeting in a large and relatively closed economy (the U.S.). Therefore it provides sufficient evidence for the conclusion that the parameters estimated through conventional methods, i.e., neglecting partially of in full the effect of censoring rule, are biased.

In particular, using the case of the Czech Republic, the central bank produces unconditional inflation projections, which contain a calibrated feedback rule, namely the Taylor rule with smoothing. Since the policymaker uses the resulting endogenous policy rate trajectory as a base for the actual policy rate settings, the difference between the calibrated rule and the conventionally estimated parameters of the same rule is likely to represent the effect of the censoring (bias in parameters). Indeed, it turns out that by accounting for the effects of censoring rule in estimation, the values of calibrated parameters in the policy rule are confirmed by the estimates.

The importance of the new method in evaluating the systematic part of the monetary policy is also confirmed by an additional application to the U.S. data. The estimation using the new method helped in reconciling some of the unintuitive or imprecise results of parameter estimation in the literature by conventional methods.

1. Introduction

Conventional estimators applied to typical monetary policy rules neglect the discrete and censored nature of the policy rate changes and thus yield biased parameter estimates. Macroeconomists tend to focus on a set of variables including output gap, inflation forecast and its target, and neutral real interest rates. However, as noted by Rudebusch (2002, 2006) and Soderlind et al (2005), such policy rules fit the historical data well, but fail to predict the future and therefore the conventionally estimated policy rule parameters are biased.

Even though the issue of biased parameter estimates in policy rules has often been neglected in the literature, the recent widespread usage of estimated policy rules for macroeconomic models and policy advice makes the unbiased estimation of parameters in policy rules increasingly relevant. A policy recommendation from staff to policymakers ought to be based on correctly estimated policy sensitivities to the fundamentals since biased simulations would distort the relevant policy tradeoffs that policymakers face and could lead to suboptimal decisions. In turn, such decisions could raise doubts about the abilities of the central bank and adversely affect its credibility. Therefore, in this paper I propose an unbiased estimator for policy rule estimation and provide two applications.

For seminal papers in the empirical literature devoted to studying policy rules we go back to Rosett (1959), who suggested applying an ordered probit to address the discrete nature of discount rate moves by the Federal Reserve (FED). A sequence of papers applying alternative discrete dependent variable models followed, including Feinman (1993) and Hakkio and Pearce (1992). Most recently, Choi (1999) derived a two-sided type II tobit that accounts not only for the discrete nature of the discount rate but also for its partial censoring. It is apparent that zero policy rate changes have the potential to be censored, which is Choi's conjecture; however, he also assumes that the non-zero policy rate changes are uncensored. The latter assumption is, however, not entirely correct. The monetary authority adjusts its policy rate usually by a quarter of a percentage point to avoid sudden policy rate reversals, i.e., it aims at avoiding instability in financial markets (advocated by Cukierman, 1989; Goodfriend, 1991; and Rudebusch, 1995) and limits the number of large policy rate changes that could lead to a loss in credibility (see Goodhart, 1997).

Thus, the outcome of a monetary policy decision meeting would most often be a quarter of a percentage point increase (decrease) in the policy rate even if the selected fundamentals (usually specified in the Taylor rule with smoothing) would justify an adjustment in the rates by half a percentage point or more. This implies that the *non-zero* policy rate changes are also potentially censored due to the presence of some kind of selection (or censoring) rule determining by how much to change the policy rates.

Since all policy rate decisions are potentially censored, estimation of the typical Taylor rules by conventional methods is an unsatisfactory approximation. Thus, depending on the nature of the approximation errors, special estimation methods may be necessary to produce unbiased and consistent estimates. In order to account for possible censoring of all policy rate changes, I devise a two-stage estimation procedure that combines an ordered probit and a censored regression. Since the ordered probit delivers unbiased parameter estimates, I suggest using these for deriving a censoring indicator (including non-censored observations) that I subsequently use in the censored regression. This procedure accounts for generally unknown censoring rules and thus delivers unbiased coefficients without loss in efficiency of estimates.

In addition, since the resulting marginal effects are constant, they are directly comparable to the calibrated linear policy rules. Therefore it is advantageous to use the method for initial calibration, verification, and update of linear policy rules in policy practice.

The empirical analysis addresses two aspects. First, I empirically explore the issue of biasedness of estimated parameters in policy rule by least squares using the example of the Czech National Bank's (CNB) policy rule. I show that while the Taylor type rule fits the past data almost perfectly, the future policy rate variation remains unpredicted by the market and by the policy rule of the central bank's staff. In this way I empirically confirm the issue of biased least squares estimates of Taylor type rules.

Second, I use two country examples to demonstrate the advantage of the developed estimation method. Firstly, I estimate the policy rule of the CNB. I chose the case of the CNB, since it is an inflation targeting central bank that uses an unconditional inflation forecast. And also because I had access to the real-time data that determined the endogenous trajectory of the policy rate, based on which the bank board decides on policy rates. Secondly, I apply the method to the U.S. data set used by Choi (1999) and discuss the improvements in the new estimator compared to the two-sided type II tobit and least squares.

The developed estimator proved to be superior over ordinary least squares as well as over the two-sided type II tobit in both empirical applications. In the case of the CNB's rule, the new estimator revealed that the underlying policy rule (after accounting for censoring rule) was the one that was used by staff for making recommendations to the bank board. In this way I found the true parameters of the latent policy rule followed by the board and which was subject to censoring rule. In the case of the FED's rule, the estimates derived using the new method also helped to reconcile some of the unintuitive estimation results by conventional methods.

The rest of the paper is organized as follows: in Chapter 2 I explore the biasedness of the typical Taylor type rules on the example of the Czech National Bank. In Chapter 3 I describe a model of the policy rate decisions and in Chapter 4 I present the new policy rule estimation procedure. Chapter 5 contains estimation results of the Czech National Bank's policy rule and Chapter 6 presents the results for the FED. Chapter 7 concludes.

2. The bias in conventional policy rule estimates

Under the assumption of rational expectations of the financial market participants, the future policy rate changes of the monetary authority should be more predictable in a distant future, the more the policy maker applies policy rate smoothing. Rudebusch (2002) provides evidence of a low portion of forecastable variability in future policy rates by the financial market expectations in the U.S. (as many other authors, for instance Fuhrer and Moore, 1995, or Mankiw and Miron, 1986) and displays his evidence as proof of, in fact, the non-inertial policy rule (claiming that shocks are correlated and monetary authority is free of inertia). This argument, however, stands in contrast to significant portion of the current literature, for instance Goodhart (1999), McCallum and Nelson (1999), or Clarida et al. (2000), who find high policy rate inertia in empirical investigations using various policy rule specifications.

In this paper we present the evidence from the term structure implications for monetary policy inertia in the Czech Republic and document that failure of the rational financial market participants to predict policy rate changes in distant future might not be a clear proof of non-inertial behavior of the monetary institution. We especially put forward the observation of low forecastable variation in future policy rates by the monetary authority (staff) itself, by using the endogenous policy rate trajectory produced by the Czech National Bank's staff for predicting future policy rate changes. Consequently, we face the following contradictory observations. On one side, neither the market nor the central bank (staff) itself can predict the future policy rates (see subsection 2.1). On the other side, the bank applies very high degree of smoothing, which is embedded in the endogenous policy rate trajectory. In addition, it turns out to be even higher in empirical estimates (see subsection 2.2) and such policy rule seems to perfectly explain the policy actions in the past.

As a result, the only way how to reconcile these contradictory observations is that the typical Taylor type policy estimates are biased. In this way we provide an empirical support for in some sense 'misspecification issue' raised by Soderlind et al. (2005) based on simulations. In addition, it leads to the similar conclusion as proposed by Rudebusch (2006), namely that the 'misspecification' in Taylor type inertial rule might not be in the dynamics but stems from some persistent omitted factors that also influence policy. In the next chapters we associate this effect with censoring rule.

2.1 Marginal regressions

We start with evaluating the forecastable variance of the future changes in policy rate by the market participants. We take the term structure of the forward rate agreements and test the predictability of the policy rate changes in a variety of forecast horizons. The following relation was tested using quarterly data covering the unconditional inflation targeting in the Czech Republic from October 2003 throughout January 2006:

$$i_{t+j}-i_t = \alpha_j+\beta_j\,(i_{t,t+j}^{FRA} - i_t)+\varepsilon_t\,. \qquad (2.1)$$

The letter j stands for quarters and runs from one to four. The three month (interbank three month rate – 3M PRIBOR) interest rate from *forward rate agreements* (FRA) set at time t for the period starting in j quarters is denoted as $i_{t,t+j}^{FRA}$. The inter-bank spot rate is denoted by i_t, and further α_j represents the average term premium for the respective period $t+j$ and β_j is the coefficient representing the relation between the realized and j-*th* horizon expected change in the rate. The error term ε_t is assumed to be i.i.d.

I opted for estimating the slope of the yield curve at every particular horizon j rather than tangency to it, since in this specification we can minimize the influence of time varying term premia embedded in the forward contracts. In all regressions, there is only one average term premium, which is captured by α_j. Such a specification is thus advantageous for the purpose of evaluating the predictability of the future interest rates.

In order to perform a complementary test for the central hypothesis that if the central bank smoothes its policy rates, a large share of the variability of the policy rates at more distant horizons should also be forecastable, we collected data for endogenous trajectories of the policy rate at each quarterly staff's inflation-forecast round in the Czech National Bank and evaluated the forecastable variance in the realized policy rate changes. The endogenous trajectory is based on the policy rule with a smoothing coefficient of .75. The smoothing in the policy rule seems to be rather close to the maximum smoothing of 0.8 that is justified by reasonable calibration of theoretical models.[1] Therefore, provided that the Taylor rule with smoothing is a correct description of the reality, a small portion of the future policy rate variability explained by the endogenous policy rate trajectory would be contradictory evidence leading to rejection of the central hypothesis that high policy rate smoothing implies high future rates predictability. Hence, we estimate the following equation for the central bank:

$$i_{t+j}-i_t = \alpha_j + \beta_j \, (i^*_{t,t+j} - i_t) + \varepsilon_{t,}, \tag{2.2}$$

where $i^*_{t,t+j}$ represents the future policy rate from the endogenous policy rate trajectory[2] (mapping the three months interbank rate[3]) set at time t for j quarters ahead.

And finally, we also tested whether the central bank has sufficient credibility among market participants, i.e., whether the market successfully anticipates the endogenous policy rate trajectory of the central bank. For this purpose, we estimate another similar equation:

$$i^*_{t,t+j} - i_t = \alpha_j + \beta_j \, (i_{t,t+j}^{FRA} - i_t) + \varepsilon_t \, . \tag{2.3}$$

2.2 Data and estimation results

Making use of the data from the internal documents of the bank board of the Czech National Bank about macroeconomic unconditional projections (containing the endogenous policy rate trajectory for j quarters ahead), which are being made public with a delay of six years, and data from the Bloomberg database about the forward rate agreements at corresponding frequency to match the quarterly projections, we estimated the relations (2.1) through (2.3).

The first result that follows from the regression (2.1), as displayed in Table 1, is that the interest rate at distant horizons is rather unpredictable by the market. In particular, we found a relatively large portion of the explained variability of the future realized policy rate development only at horizons up to two quarters ahead. An exclusively high portion of explained variability was found in the first quarter and somewhat lower in the second; however, as we move towards more distant quarters the share of explained variability drops literally to zero. Also, the slope coefficient is declining from unity rather rapidly, considering its insignificance already in the third quarter.

[1] Rudebusch (2002) provides an interval 0-0.8 for optimal smoothing, which is also consistent with the findings by Woodford (1999) or Levin et al. (1999), for instance.

[2] For time t it is derived as $i^*_t = 0.75 i_{t-1} + (1-0.75)(r_t^{eq} + p_t^e + 1.2(p_t^e - p_t^{tar}) + 0.4 gap_t),$ and similarly for time $t+1$, etc. by moving the explanatory variables into the future.

[3] Since there is very tight relation between policy rate, i.e. the two-week repo rate, and the three months PRIBOR.

Table 1: Forecasting actual policy rate

Quarters j ahead	$i_{t+j}-i_t = \alpha_j + \beta_j (i_{t,t+j}^{FRA} - i_t)$				$i_{t+j}-i_t = \alpha_j + \beta_j (i^*_{t,t+j} - i_t)$			
	α_j	β_j	R^2-adj.	Obs	α_j	β_j	R^2-adj.	Obs
1Q	0.016(0.02)	0.894***(0.074)	0.95	9	-0.001(0.058)	0.732***(0.216)	0.57	9
2Q	-0.092(0.088)	0.978***(0.264)	0.61	9	-0.018(0.10)	0.696**(0.227)	0.51	9
3Q	-0.21(0.17)	0.526(0.34)	0.17	8	-0.114(0.166)	0.363(0.27)	0.10	8
4Q	-0.231(0.289)	0.271(0.41)	0.001	7	-0.123(0.234)	0.093(0.327)	0.001	7

Note: The stars denote significance as follows: *** 1%, ** 5% and * 10%. Standard errors are given in parentheses.

The second result follows from the estimation of regression (2.2), presented also in Table 1. It stipulates that the endogenous policy rate trajectory is not predicting the variability of the future policy rate any better than the market. The proportion of explained variability in total variability in the policy rate plummets to zero relatively quickly, similarly to the case of financial market forecasts. The slope coefficient diverges from unity relatively quickly as well.

Table 2: Forecasting endogenous trajectory

Quarters j ahead	$i^*_{t+j}-i_t = \alpha_j + \beta_j (i_{t,t+j}^{FRA} - i_t)$			
	α_j	β_j	R^2-adj.	Obs
1Q	0.016(0.06)	0.787***(0.222)	0.59	9
2Q	-0.10(0.089)	1.016***(0.23)	0.66	9
3Q	-0.174(0.127)	1.06***(0.237)	0.67	9
4Q	-0.212(0.154)	1.113***(0.22)	0.73	9

Note: The stars denote significance as follows: *** 1%, ** 5% and * 10%.
Standard errors are given in parentheses.

The results for the last equation (2.3) that are displayed in Table 2 show, that the predictability of the endogenous trajectory by the market is very high along the entire considered horizon. The portion of explained variability reaches 65-75 percent. In addition, the slope is rather close to unity and statistically significant, in all the horizons.

This suggests relatively effective communication of the governing council in directing the market regarding the endogenous trajectory, considering that the implicit policy trajectory is not directly shared by the central bank with the market, and speaks for high credibility of the Czech National Bank.

Consequently, the results imply that the hypothesis that poor performance of the market in predicting variability in the future policy rate is a sign of low smoothing in the policy rate is not supported in general. This follows from the observation that we find low forecasted variation of the future rates even though we know for certain that the endogenous policy rate trajectory contains a very high smoothing coefficient of 0.75.

2.3 The fit of the inertial Taylor rule

As it has been disseminated in the Forecasting and Policy Analysis System (CNB 2003), the Czech policy rate obeys the following forward looking Taylor rule:

$$i_t = \beta_0 i_{t-1} + (1-\beta_0)(r_t^{eq} + p_t^e + \beta_1(p_t^e - p_t^{tar}) + \beta_2 gap_t) + e_t, \qquad (2.4)$$

where β_0, β_1, and β_2 are calibrated parameters, i_t denotes the quarterly average of the actual policy rate (the two-week repo rate) and i_{t-1} denotes its one period (quarter) lag. r_t^{eq} stands for the real equilibrium interest rate, p_t^e labels the forecasted inflation in one year ahead and p_t^{tar} denotes the corresponding inflation target. The output gap is denoted by gap_t. The variables r_t^{eq}, $(p_t^e - p_t^{tar})$, and gap_t have been taken from the unconditional quarterly forecast rounds carried out by the Czech National Bank's staff. As such these variables together with the calibrated parameters β's in the model define the model's quarterly average of the policy rate (i.e., the endogenous trajectory).

Nevertheless, the policy rate decision meetings take place on monthly frequency and thus the quarterly averages of policy rate in reality do not match the model's policy rate. Thus, when estimated on real data (quarterly averages of policy rate) there is a discrepancy that is represented in (2.4) by e_t i.e., the error term.[4]

The estimate through ordinary least squares of the equation (2.5) resulted as follows:

$$\underset{}{i_t} = \underset{(0.05)}{0.83^{***}}\, i_{t-1} + \underset{(0.05)}{0.17^{***}}(r_t^{eq} + p_t^e) + \underset{(0.08)}{0.24^{**}}(p_t^e - p_t^{tar}) + \underset{(0.07)}{0.09}\, gap_t \tag{2.5}$$

(R^2-adjusted = 0.99; Obs = 12, s.e. in parentheses, stars denote significance: *10%, **5%, and ***1 %).

It follows that the policy rule (2.5) describes almost entirely the variation of the policy rate in the past, since the R^2 – adjusted equals 0.99. However, the future rate is predicted neither by the forward rates (containing the communication of the monetary policy and is likely based on an estimated rule) nor by the model's policy rate, i.e. i^*_t,[5] which means that the parameters in the estimated rule are biased. In other words, there might be a problem with the specification (or estimation) of the empirical policy rule as was pointed out and verified by Soderlind et al. (2005) using simulations. Indeed, since the Czech bank board gets the advice for policy rate adjustment i_t based on i^*_t and then applies some selection (censoring) rule and defines the final outcome i_t, the specification explaining i_t has to take into account the possibility of existence of selection rule that is not orthogonal to the rest of the explanatory variables in (2.4). In the following section I analyze the selection (censoring) rule in more detail.

3. The policy rate model

The decision about setting the key policy rate is a result of a complex process. At every monetary policy decision meeting, the policymakers assess the current and forecasted macroeconomic conditions (such as output gap, inflation, and equilibrium interest rate), which define a set of the core indicators (which usually enter a typical estimated rule), and

[4] Denoting the model's policy rate (the endogenous trajectory) as i^*_t then the actual policy rate is given as $i_t = i^*_t + e_t$. The structure of the error term is discussed in the subsequent chapters; the accent is placed on the difference between rounding error and an effect of selection (censoring) rule.

[5] The calibration in the model is the following: $i^*_t = 0.75i_{t-1} + (1-0.75)(r_t^{eq} + p_t^e + 1.2(p_t^e - p_t^{tar}) + 0.4gap_t)$. And since the model's policy rate is performing equally poorly in predicting the future market rate (policy rate) then we can consider that the bank board communication is based on the model's policy rate.

considering all other relevant information (hard data as well as soft arguments), they decide whether to adjust or to keep the current policy rates setting.

Observed changes in policy rate are characterized by lumpiness (induced by limited number of policy meetings in a year and discrete changes in policy rate) and as such they fall into the category of discrete and potentially censored data. However, the discreteness and potential censoring is man-made, i.e. it is generated by the policymakers and thus there exists a censoring rule together with its determinants.

Let us define $\Delta i_t^* = i_t^* - i_{t-1}$, which represents the uncensored change in policy rate that would correspond to the typical Taylor rule. Hence, the changes in the observed policy rate settings Δi_t might only partially coincide with the unobserved Δi_t^* due to an impact of the censoring rule on Δi_t. It follows that

$$\Delta i_t = \Delta i_t^* + \xi(Z_t'\delta) + \eta_t = X_t'\beta + \xi(Z_t'\delta) + \eta_t \qquad (3.1)$$

where the term η_t represents a random discretion – i.i.d. random error $N(0, \sigma^2)$ and its size falls into the interval of ±12.5 basis points (b.p.) – the effect of rounding up or down to the entire multiples of 25 b.p. This represents the obvious source of lumpiness in policy rate. The second part of the error term is the effect of censoring rule, denoted by $\xi(Z_t'\delta)$, which is defined as the difference between the unobserved policy rate change (Δi_t^*) and the observed policy rate change (Δi_t). The effect of censoring rule $\xi(.)$ is derived from a set of variables Z_t, which may also contain some or all of the variables in X_t.

Thus, not accounting for the censoring rule biases the estimates of β in the least squares regression if $X_t'\xi(Z_t'\delta) \neq 0$, which is likely the case since the censoring rule might be correlated with the explanatory variables in X_t.[6]

If we denote by β the coefficients pertaining to the explanatory variables in X_t, which can be thought of as variables in the typical Taylor type rules, we can model the partially observed policy rate (Δi_t^*)

$$\Delta i_t^* = X_t'\beta \qquad (3.2)$$

by using the following formalization of the observation-by-observation censored model. Observations are said to be censored from the right, uncensored, and censored from the left as follows:

$$\begin{aligned} \Delta i_t - T_l \leq \Delta i_t^* \quad & if \quad & \xi(Z_t'\delta) + \eta_t \leq T_l \\ \Delta i_t \approx \Delta i_t^* \quad & if \quad & T_l < \xi(Z_t'\delta) + \eta_t \leq T_u \\ \Delta i_t - T_u > \Delta i_t^* \quad & if \quad & \xi(Z_t'\delta) + \eta_t > T_u , \end{aligned} \qquad (3.3)$$

where the thresholds T_u and T_l are equal to the size of 12.5 and -12.5 basis points (b.p.), respectively. If $\xi(Z_t'\delta) = 0$, then it holds for all t that $\Delta i_t^* + \eta_t = \Delta i_t$ and the estimation can proceed with a linear estimator since $E(\Delta i_t^*) = E(\Delta i_t)$. Since in practice we neither know the uncensored continuous policy rate Δi_t^* nor the censoring rule and its determinants, we ought to

[6] The estimate of β is equal to $\hat{\beta} = (X_t'X_t)^{-1}X_t'\Delta i_t - (X_t'X_t)^{-1} X_t'\xi(Z_t'\delta)$, while not accounting for $\xi(Z_t'\delta)$ leads to $\hat{\beta}^* = (X_t'X_t)^{-1}X_t'\Delta i_t$. It follows that $\hat{\beta}^* \neq \hat{\beta}$ if $X_t'\xi(Z_t'\delta) \neq 0$, see Greene (2003).

devise an appropriate estimation method that would deliver unbiased parameter estimates in widely used Taylor rules. In the next section I present such an estimation method.

4. The estimation procedure: 2S-CNREG

I design the following two-stage estimation procedure. The first stage is an ordered probit.[7] Let Δi_t be an observed discrete ordered policy rate response taking values $\{m_1, m_2, \ldots, m_n\}$, where m_j denotes a particular magnitude of observed change in policy rate; there are n such distinct sizes of policy rate changes. The change in the implicit policy rate Δi_t^*, defined as $\Delta i_t^* = i_t^* - i_{t-1}$, is determined by the following identity:

$$\Delta i_t^* = X_t' \alpha , \tag{4.1}$$

where α denotes the vector of coefficients corresponding to the explanatory variables in X_t. The estimation of α is based on the variability of difference between the implicit policy rate (4.1) and the observed policy rate as in (3.1). We can express the relation between the latent (implicit policy rate) variable Δi_t^* and the observed variable Δi_t as follows:

$$\begin{aligned} \Delta i_t &= m_1 && \text{if} && \Delta i_t^* \leq Tm_1 \\ &= m_2 && \text{if} && Tm_1 < \Delta i_t^* \leq Tm_2 \\ &\ldots \\ &= m_n && \text{if} && \Delta i_t^* > Tm_n , \end{aligned} \tag{4.2}$$

which means that at each of the m_j thresholds, denoted as $Tm_1 < Tm_2 < \ldots < Tm_n$, the magnitude of policy rate change m_j in observed policy rate discretely switches to a different one in an ordered manner. There are n such thresholds in the sample.

The maximum likelihood for the ordered probit is:

$$L = \Pi_{t=1,\ldots,n}\{[1-\Phi(X_t'\alpha - Tm_1)]^{I(\Delta it=m1)} [\Phi(X_t'\alpha - Tm_1) - \Phi(X_t'\alpha - Tm_2)]^{I(\Delta it=m2)} \ldots [\Phi(X_t'\alpha - Tm_n)]^{I(\Delta it=mn)}\}.$$

In the case that the data contains multiple sizes of changes (n is large), the ordered probit will deliver consistent but inefficient parameter estimates. Besides, the inconstancy (non-linearity) of the marginal effects of exogenous variables in ordered probit complicates their direct use for policy rule calibration. Therefore, I suggest using the consistently estimated parameters α from ordered probit (see White, 1982) for evaluating the censoring indicator and subsequently perform a censored regression.

Since the estimated sizes of thresholds in ordered probit $Tm_1, Tm_2 \ldots, Tm_n$ will depend on the direction and frequency of censoring, and since the policy rate is usually adjusted by entire multiples of 25 b.p., the true thresholds take the values of entire multiples of 12.5ρ b.p.

The term ρ represents a normalization of the generally rescaled thresholds in ordered probit, which has the unique function to convert the size of thresholds to the ones directly comparable

[7] Similarly to the frictions model by Rosett (1959).

with the policy rate values: $\rho = \sigma_{Xt'\alpha}/\sigma_{\Delta it}$ and $\sigma_{Xt'\alpha}$ denotes the standard deviation of $X_t'\alpha$, while $\sigma_{\Delta it}$ stands for the standard deviation of Δi_t.

Thus, for the evaluation of the censoring indicator are used the values of multiples of 12.5ρ b.p., since the underlying idea is to compare what the policymakers would have done – conditional on $X_t'\alpha$, given that they adjust the policy rate by multiples of a quarter of a percentage point – with what they actually did.

In particular, in order to classify observed policy rate changes into censored from the left, from the right, and uncensored, I need to evaluate for each single observation the conditional probability[8]: (1) that the size of the implied policy rate change corresponds to the observed change up to the ±12.5 b.p., i.e. $P(\Delta i_t \approx \Delta i_t^* | X_t'\alpha) = P(-0.125 < \Delta i_t - (1/\rho)X_t'\alpha \leq 0.125)$, (2) that the size of the implied rate change is higher than observed by more than the rounding up error, i.e., more than 12.5 b.p., i.e. $P(\Delta i_t + 0.125 \leq \Delta i_t^* | X_t'\alpha) = P(\Delta i_t - (1/\rho)X_t'\alpha \leq -0.125)$, and finally (3) that the size of the implied rate change is lower than the observed one by more than 12.5 b.p., i.e. $P(\Delta i_t - 0.125 > \Delta i_t^* | X_t'\alpha) = P(\Delta i_t - (1/\rho)X_t'\alpha > 0.125)$.[9]

Observations are then said to be censored from the left, right, and uncensored as follows:

$$\Delta i_t - 0.125 > \Delta i_t^* \text{ if } P(\Delta i_t - 0.125 > \Delta i_t^* \mid X_t'\alpha) = \max_{\psi \in \{\Delta i_t \approx \Delta i_t^*, \Delta i_t + 0.125 \leq \Delta i_t^*, \Delta i_t - 0.125 > \Delta i_t^*\}} \{P(\psi \mid X_t'\alpha)\} \quad (4.3)$$

$$\Delta i_t + 0.125 \leq \Delta i_t^* \text{ if } P(\Delta i_t + 0.125 \leq \Delta i_t^* \mid X_t'\alpha) = \max_{\psi \in \{\Delta i_t \approx \Delta i_t^*, \Delta i_t + 0.125 \leq \Delta i_t^*, \Delta i_t - 0.125 > \Delta i_t^*\}} \{P(\psi \mid X_t'\alpha)\}$$

$$\Delta i_t \approx \Delta i_t^* \text{ if } P(\Delta i_t \approx \Delta i_t^* \mid X_t'\alpha) = \max_{\psi \in \{\Delta i_t \approx \Delta i_t^*, \Delta i_t + 0.125 \leq \Delta i_t^*, \Delta i_t - 0.125 > \Delta i_t^*\}} \{P(\psi \mid X_t'\alpha)\}$$

and since $\sum_{\psi \in \{\Delta i_t \approx \Delta i_t^*, \Delta i_t + 0.125 \leq \Delta i_t^*, \Delta i_t - 0.125 > \Delta i_t^*\}} P(\psi \mid X_t'\alpha) = 1$, the observation are uniquely classified.

In other words, the first relation in (4.3) with the highest probability states that while observing a change in the announced policy rate Δi_t, the implied policy rate by variables X_t and parameter estimated α suggests a significantly (by more than 12.5 b.p.) greater decrease in policy rate (Δi_t^*) then observed (Δi_t) and thus we speak about a censored observation from the left.

Similarly, the second relation in (4.3) with the highest probability states that a greater increase in the policy rate would have occurred if the decision would have been based only on the

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[8] I suggest using the median rule for classification of observation into censored and uncensored and perform an observation-by-observation censored regression. Such an approach to classification of observations into outliers is not uncommon in robust estimation, where the probability of being outlier is also used for identification of outliers, for instance see Rousseeuw and Leroy (1987). In our case, however, it is not a just general outlier classification; there is a strong rationale to consider observations to be potentially censored.

[9] $P(\Delta i_t - (1/\rho)X_t'\alpha \leq -0.125 \mid X_t'\alpha) = 1 - \Phi(\Delta i_t - (1/\rho)X_t'\alpha + 0.125)$; $P(\Delta i_t - (1/\rho)X_t'\alpha > 0.125 \mid X_t'\alpha) = \Phi(\Delta i_t - (1/\rho)X_t'\alpha - 0.125)$; and $P(-0.125 < \Delta i_t - (1/\rho)X_t'\alpha \leq 0.125 \mid X_t'\alpha) = \Phi(\Delta i_t - (1/\rho)X_t'\alpha + 0.125) - \Phi((\Delta i_t - (1/\rho)X_t'\alpha - 0.125)$. The presented evaluation of the probabilities uses the difference between the observed and implied policy rate change, which is measured against the thresholds. Nevertheless, it is equivalent to the notation, where thresholds take various sizes, not just ± 0.125, but entire multiples of 0.125. This follows from the fact that, for instance the probability $P(\Delta i_t - (1/\rho)X_t'\alpha \leq -0.125 \mid X_t'\alpha)$ can be rewritten as $P(-X_t'\alpha \leq -\rho\ (0.125+\Delta i_t) \mid X_t'\alpha)$, which states that the fitted values are compared to the threshold $\rho\ (0.125+\Delta i_t)$, which is dependent on the size of observed policy rate change (an entire multiple of 0.25 p.p.). In the empirical application I use a set of (so called "discretion") thresholds instead of computing the difference between the actual and implied policy rate change. Nevertheless, both ways lead to identical results.

variables contained in X_t and parameter estimated α – thus we observe censoring from the right. And finally, according to the last relation in (4.3), if the probability that the difference between the implicit rate change and the observed rate change is equal to the rounding error of ±12.5 b.p., is the maximum probability out of the three evaluated probabilities, such observation is declared as uncensored.

In the second stage we complement the censored regression model by using the indicator of censoring derived on the basis of the first stage estimation (as described above). Besides preserving the efficiency of estimates, in the presence of uncensored observations, the parameters will be constant and compatible with those calibrated in the linear policy rules.[10] The second stage of the model can be represented as follows:

$$\Delta i_t^* = X_t'\beta \tag{4.4}$$

The estimation of the censored regression follows the standard maximum likelihood method. The likelihood function for the observation-by-observation censored regression model can be written as follows:

$$L = \Pi_{t=1,\ldots,n}\{[1-\Phi(X_t'\beta-\Delta i_t)]^{I(CI=-1)}\,[\sigma^{-1}\,\phi\,[(\Delta i_t - X_t'\beta)/\sigma]]^{I(CI=0)}\,[\Phi(X_t'\beta-\Delta i_t)]^{I(CI=1)}\}.$$

Where observations censored from the left, right, and uncensored are in the censoring indicator (CI) assigned value -1, 1, and 0, respectively. Since some of the observations in the dependent variable have been adjusted in order to be closer to the median observations, the distribution of the errors has been changed and thus might exhibit heavier tales compared to normal. In order to account for this I suggest using bootstrap (Bradley, 1979) to derive the standard errors using the sampling distribution.

5. Estimating the CNB's policy rule

The verification of the proposed method is demonstrated using data for the policy rule of the Czech National Bank, which is one of the pioneers of explicit inflation targeting in the region of Central and Eastern Europe. The advantage of using the Czech example is mainly in the availability of unique data for the true (and real-time data)[11] determinants and calibrated coefficients of the change in the policy rate Δi_t^*:

$$\Delta i_t^* = X_t'\beta\,,$$

based on which the Czech bank board has been advised to adjust policy rate

$$\Delta i_t = X_t'\beta + \xi(Z_t'\,\delta) + \eta_t\,. \tag{5.1}$$

[10] The difference between parameters α and β is mainly such that the former is varying in variables, whereas the latter is constant. The conversion of the former into the latter is not straightforward, as the literature is not consensual on the issue. See Greene (2003).

[11] In this way we can avoid the argument of Lansing (2002) that estimated high policy rate inertia on revised data is misleading since estimations with real-time data on the output gap show much smaller policy rate inertia.

The term $\xi(Z_t'\ \delta)$ represents the censoring effect of the bank board due to variables in Z_t, that can contain some or all of the variables in X_t.

5.1 Specification and Data

Although the regime of inflation targeting was implemented at the beginning of 1998, the Czech National Bank transited to an unconditional inflation forecast in early 2003. Since then, besides previously producing and publishing the inflation forecasts and announcing inflation targets, the policy rule became an integral part of the policy framework. As it was disseminated in the Forecasting and Policy Analysis System (CNB 2003), the model's policy rate Δi_t* obeys the following forward looking Taylor rule:

$$\Delta i_t^* = (\beta_0\text{-}1)it_{-1} + (1\text{-}\ \beta_0)(r_t^{eq} + p_t^e + \beta_1(p_t^e\text{-}p_t^{tar}) + \beta_2 gap_t), \qquad (5.2)$$

that is also the main input into the board decision about the policy rate setting Δi_t:

$$\Delta i_t = (1\text{-}\ \beta_0)(r_t^{eq} + p_t^e\text{-}\ i_{t\text{-}1}) + (1\text{-}\ \beta_0)\beta_1(p_t^e\text{-}p_t^{tar}) + (1\text{-}\ \beta_0)\beta_2 gap_t + \xi(Z_t'\ \delta) + \eta_t, \qquad (5.3)$$

where $\xi(Z_t'\ \delta)$ and η_t represent the censoring and rounding effect, respectively. Further, β_0, β_1, and β_2 are calibrated parameters, and $i_{t\text{-}1}$ denotes one period (month) lagged policy rate. The real equilibrium interest rate is denoted by r_t^{eq}, further p_t^e labels the forecasted inflation in one year ahead, and p_t^{tar} denotes the corresponding inflation target. The output gap is denoted as gap_t.

Besides the monthly two-week repo rate (policy rate), the data further comprises the quarterly deviation of the forecasted inflation from its target, output gap, and equilibrium nominal policy rate that we collected from the internal CNB's baseline forecast database for each quarterly inflation forecast. For the sake of using monthly observations on policy rate changes, we have interpolated the quarterly explanatory variables into monthly frequency through quadratic match-average. The time of our sample spans from 2003 January throughout 2005 December, which is motivated by the fact that since early 2003, when a policy rule recalibration took place, the calibration of the policy rule has not been changed. Descriptive statistics of the data used in the analysis are shown in Table 3.

Table 3: Sample descriptive statistics (in %)

	Mean	Std. Dev.	Max.	Min.
Two-week repo rate	2.14	0.27	1.75	2.5
Policy neutral rate	3.62	0.46	2.66	4.35
Inflation forecast deviation from target (p.p.)	-.86	0.47	-1.63	-0.12
Output gap	-1.17	0.73	-2.44	-0.39

The sample period is characterized by a negative output gap, inflation forecast under the target, and policy rates below their neutral level. As for the statistics on policy rate changes, the rate has been changed nine times out of 36 monthly meetings of the council. Three times the council decided to increase and six times to decrease the rate. All changes in the two-week repo rate were of the size of 25 b.p. At twenty seven meetings the rates remained unchanged.

5.2 Estimation results

I present three regressions. First, I estimated the equation (5.3) using the ordinary least squares, i.e., ignoring possible policy rate censoring issues. Then, I estimated the two-sided-type II tobit,[12] allowing only zero policy rate changes to be potentially censored. And finally, I applied the two-stage procedure 2S-CNREG that consists of an ordered probit in the first stage and the observation-by-observation censored regression in the second stage.

The results of parameter estimates are summarized in Table 4, along with the statistics pertaining to them. In all estimated equations, the Durbin's *h* statistics confirm no autocorrelation of the first degree at 5% significance level. In the 2S-CNREG are reported two standard deviations, one pertains to the imposed normality assumption on residuals and the other to the sampling distribution (using bootstrap with 50 sample replications). The bootstrapped standard errors are slightly higher, thus somewhat lowering statistical significance of estimated parameters.

As it appears in the Table 4, the parameter on smoothing term (β_0-1) by OLS results biased. There is a statistically significant difference between mean estimates by OLS and the 2S-CNREG, which amounts to 0.04. Similarly, the remaining coefficients by OLS are accordingly lower. In addition, the parameter of the output gap $\beta_2(1-\beta_0)$ in OLS regression results statistically insignificant.

A test based on comparing parameter estimates can be easily devised, for instance, on the platform of the Hausman specification test (Hausman, 1978). One can construct the Hausman *m*-statistic and test the following standard hypothesis. Under the H0: both the OLS (two-sided type II tobit) and 2S-CNREG estimates are consistent and asymptotically efficient, while under H1: only the estimates from the 2S-CNREG procedure are consistent.

Thus, using the Hausman test[13] (see Table 4) for systematic difference in estimates by 2S-CNREG vs. OLS, where 2S-CNREG is always consistent and OLS is possibly consistent and more efficient, I confirmed that the OLS estimates are systematically biased at 10 % significance level.

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[12] The modified two-step Heckman's procedure for the two-sided type II tobit (Choi, 1999). The first step is the sign determining ordered probit; the likelihood function follows: $L = \Pi_{t=1,\dots,n}\{[1-\Phi(X_t'\beta_o - To_1)]^{I(\Delta it=-1)} [\Phi(X_t'\beta_o - To_1) - \Phi(X_t'\beta_o - To_2)]^{I(\Delta it=0)} \dots [\Phi(X_t'\beta_o - To_n)]^{I(\Delta it=1)}\}$, where To_i denotes the tolerance ancillary parameters. The secons step proceeds with the ordinary least squares with inverse Mill's ratio (λ_t): $\Delta i_t = X_t'\beta + \gamma\hat{\lambda}_t + \varepsilon_t + \eta_{H,t}$.
Where ε_t denotes the model error and ε_t stands for the Heckman's approximation error, $\eta_{H,t} = \lambda_t - \hat{\lambda}_t$. The estimate of λ_t is denoted by $\hat{\lambda}_t$ and $\hat{\lambda}_t = I_{(\Delta it=-1)}[-\phi(X_t'b_o - To_1/\Phi(X_t'b_o - To_1)] + I_{(\Delta it=1)}[\phi(X_t'b_o - To_2/\Phi(X_t'b_o - To_2)]$. The vector of parameters b_1 is the estimate of β_o. I applied White's (1980) approach to derive consistent standard errors using the second step residuals e_i as $(K_t'K_t)^{-1}K_t'Var(\varepsilon_t)K_t(K_t'K_t)^{-1}$, where $K_t'Var(\varepsilon_t)K_t = \Sigma_{i=1,2,\dots,n}\, e_ik_ik_i'$. By k_i I denote the element of $K_t = (X_t : \hat{\lambda}_t)$.

[13] The *m*-statistics, for OLS vs. 2S-CNREG, reads: $m = q'(V_{OLS}-V_{2S\text{-}CNREG})^{-1}q$, where V_{OLS} and $V_{2S\text{-}CNREG}$ represent consistent estimates of the asymptotic covariance matrices of β_{OLS} and $\beta_{2S\text{-}CNREG}$, and $q = \beta_{OLS} - \beta_{2S\text{-}CNREG}$. The *m*-statistic is then distributed χ^2_k with k degrees of freedom, where k is the rank of the matrix $(V_{OLS}-V_{OP\text{-}Cenreg})$. A generalized inverse is used, as recommended by Hausman (1978).

Table 4: Estimation results of the CNB's policy rule

	Heckman's proc.	2S-CNREG	OLS	MODEL
First step				
i^{eq}-i^{t-1} $[1-\beta_o]$	5.6***(1.9)	5.6***(1.9)		
p^{e}-p^{tar} $[(1-\beta_o)\beta_1]$	-2.2*(1.3)	-2.2*(1.3)		
ygap $[(1-\beta_o)\beta_2]$	1.5**(0.7)	1.5**(0.7)		
LL	-14.32	-14.32		
T_{m1}	6.2***(2.8)	6.2***(2.8)		
T_{m2}	11.5***(4.1)	11.5***(4.1)		
T_{m0*}		-5.3		
T_{m1*}		5.3		
T_{m2*}		10.6		
T_{m3*}		26.3		
Second step				
i^{eq}-i^{t-1} $[1-\beta_o]$	0.08***(0.03; 0.04)	0.09***(0.02; 0.05)	0.06***(0.02)	0.09
p^{e}-p^{tar} $[(1-\beta_o)\beta_1]$	0.12***(0.04; 0.06)	0.09***(0.035; 0.05)	0.07**(0.03)	0.11
ygap $[(1-\beta_o)\beta_2]$	0.05(0.03; 0.06)	0.05* (0.03; 0.04)	0.04(0.03)	0.04
LL		23.29		
σ		0.11***(0.01; 0.02)		
IMR	0.07*(0.03; 0.03)			
(ps)-R^2	0.78	(truncated at) 1	0.3	
Hausman test	$\chi2_{(3)}[-4.76]$~N/A	consistent	$\chi2_{(3)}[6.93] = 0.07$	calibrated
DW	1.64	1.86	2.28	
Durbin's *h*	1.09	0.42	-0.85	

Note: Standard errors in parenthesis; In Heckman's procedure the second s.e. is computed using White's (1980) approach. In the second step of the 2S-CNREG, the second standard deviation in parenthesis is derived using bootstrap with 50 replications. The discretion thersholds T_{m0*}, T_{m1*}, T_{m2*}, and T_{m3*} are computed using $\sigma_{(Xt'\alpha)} = 4.66$ and $\sigma_{(\Delta it)} = 0.11$.

These results confirm the importance of the effect that is not taken into account by conventional estimators applied to typical Taylor rule with smoothing.

Importantly, as we see from the comparison between the 2S-CNREG and the CNB's policy rule calibration (see MODEL in Table 4), the parameter estimates are very close to the calibrated rule[14]. This suggests that the policy makers in their decisions attach a significant weight to the staff's advice based on the calibrated policy rule. Nevertheless, they also follow some censoring rule that causes a biased parameter estimates in OLS. Thus, in the end accounting for the effect of censoring, the parameter estimates unveil the underlying policy rule – the systematic base for decision, in this case the model's policy rule calibration.

[14] The displayed numbers in the Table 4 in the column MODEL represent *monthly frequency equivalents of the original quarterly frequency* calibration of the policy rule. The conversion is based on the following relation: β_0 *– quarterly* = $(\beta_0 - monthly)^3$.

The results for the two-sided type II tobit estimated through Heckman's procedure (see Table 4) are partially insignificant due to the small sample of non-zero rate changes. The small sample is a general problem for this method since the second stage is performed on a subsample of non-zero policy rate changes that is often substantially smaller.
Nevertheless, since the model assumes only zero policy rate changes being potentially censored, i.e. it omits the possibility of censored non-zero changes which might often prove important, the coefficients might be biased. Nevertheless, two out of the three coefficients in Heckman's proc. do not appear to be statistically significantly different from those of 2S-CNREG. Nevertheless, the estimates are not more efficient and thus asymptotical assumptions imposed in Hausman test are not satisfied.

In addition, the two-sided tobit type II relies on the estimated selection rule and thus requires knowledge of its determinants. This is another and likely largest drawback of the method since some of the determinants of the selection (censoring) rule are often not directly measurable.

6. Estimating the FED's policy rule

Another example of policy rule estimation is intended to provide more evidence of the performance of the new estimation technique, especially in a larger data smaple. I follow the Benchmark specifications of the discount rate estimations as formulated by Choi (1999), since his model appears to be, to my knowledge, the most advanced model to date. Hence the Benchmark regression I (which corresponds to the 'Model I: equation (1a)' in Choi, 1999) specification is

$$\Delta i_t = \beta_1 + \beta_2\Delta i_{t-1} + \beta_3 i_{t-1} + \beta_4 y_{t-1} + \beta_5\Delta y_t + \beta_6\pi_{t-1} + \beta_7\Delta\pi_t + u_{t,} \tag{6.1}$$

and the extended specification for some additional potential objectives, denoted as Benchmark regression II (which corresponds to the 'Model I: equation (1b)' in Choi, 1999), is written as follows

$$\Delta i_t = \beta_1 + \beta_2\Delta i_{t-1} + \beta_3 i_{t-1} + \beta_4 y_{t-1} + \beta_5\Delta y_t + \beta_6\pi_{t-1} + \beta_7\Delta\pi_t + \beta_8 m_t + \beta_9 s_t + v_t \tag{6.2}$$

where $\Delta i_t = i_t - i_{t-1}$ and $\Delta i_t^* \approx \Delta i_t$. In both specifications (6.1) and (6.2) might arise an identification problem stemming from censoring rule effect (contained in residuals u_t and v_t) as described in Chapter 3 such that $\Delta i_t^* \neq \Delta i_t$ and hence the policy rule parameter estimates in OLS and potentially also in the two-sided type II tobit are biased.

The lagged official discount rate as the last day rate is denoted as i_{t-1} and the (lagged) difference of the official discount rate as Δi_t (Δi_{t-1}).[15] The lagged percentage deviation of the industrial production index (1987=100) from its trend is denoted as y_{t-1}, where the trend is derived as a geometric interpolation of benchmark rates (see Choi 1999). Similarly, Δy_t is the first difference of the gap in industrial production. Further, π_{t-1} is the lagged deviation of the y-o-y inflation from the assumed implicit inflation target of 2 % and $\Delta\pi_t$ is its first difference. And finally, m_t stands for the y-o-y monetary aggregate M1 growth as a deviation from its Hodrick-

[15] Nevertheless, more appropriate would likely be to use the U.S. Federal funds rate target as the dependent variable instead. However, since I present the benefits of the new estimator I preserve the original specifications and variable definitions as in Choi (1999).

Prescott trend and s_t stands for the difference of the lagged official discount rate from the Federal funds rate target set prior to the discount rate announcement (for further details, see Choi 1999).

The descriptive statistics of the data sample, spanning from September 1974 to March 1995, used in the analysis are presented in Table 5.

Table 5: Sample descriptive statistics (in %)

	Mean	Std. Dev.	Max.	Min.
Discount rate (last day rate)	7.16	2.76	14	3
Inflation deviation from an implicit target (p.p.)	3.73	3.38	12.65	-1.26
Industrial production gap	-0.24	3.77	6.62	-10.67
Misalignment (discount rate vs. market rate) (p.p.)	-0.28	1.16	1.25	-5.55
M1 gap (HP trend)	0.13	1.58	5.37	-4.13

As we can see from the Table 5, the investigated period was characterized by quite a substantial variation in policy rate (attaining maximum at 14% and minimum at 3%) as well as in the difference of inflation from the implicit inflation rate target (peaking at 12.6% and reaching minimum at -1.26%). The inflation seems to be on average in excess compared to the implicit target (the mean of the difference is 3.73%). Similarly, quite a large portion of variation can be seen in the case of the gaps of industrial production and monetary aggregate M1. Nevertheless, the gaps seem to be well stabilized over the sample period, which follows from the nearly zero mean in both variables. And finally, the misalignment of the discount rate with the market rate is also rather small, on average.

Employing the data set, I first present the replication of the results for Benchmark regressions by Choi (1999) and then apply 2S-CNREG method to the same data set and specification and interpret the differences. In addition, I present a simple ordinary least squares as the most conventionally used policy rule estimation method. Table 6 contains the results.

In the case of the 2S-CNREG method, the second step uses a transformed dependent variable and thus the errors might be following a different sample distribution. Therefore, I provide an alternative standard deviation that is a result from a bootstrap of 50 replications of the re-sampling and as such it better corresponds to the new sample distribution. The standard errors are however very robust to number of replications and exhibit similarity to the estimate using the assumption of normality. As such the significances of the results are not changed by using the sample distribution instead of the normal one.

Table 6: Estimation results for the FED's policy rate

	Benchmark regression I [a]			Benchmark regression II [b]		
	Heckman's proc.	**2S-CNREG**	**OLS**	**Heckman's proc.**	**2S-CNREG**	**OLS**
First step						
Δi_{t-1}	0.270	0.523*		-0.37	-0.09	
	(0.301)	(0.286)		(0.35)	(0.32)	
i_{t-1}	-0.077*	-0.09**		-0.28***	-0.29***	
	(0.044)	(0.041)		(0.06)	(0.06)	
y_{t-1}	0.137***	0.109***		0.11***	0.09***	
	(0.03)	(0.028)		(0.04)	(0.03)	
π_{t-1}	0.097***	0.102***		0.18***	0.17***	
	(0.067)	(0.035)		(0.04)	(0.04)	
Δy_t	0.789***	0.69***		0.73***	0.62***	
	(0.137)	(0.119)		(0.15)	(0.13)	
$\Delta \pi_t$	0.306	0.434*		0.53	0.67***	
	(0.252)	(0.239)		(0.29)	(0.26)	
m_t				0.14*	0.15*	
				(0.08)	(0.07)	
s_t				-0.75***	-0.73***	
				(0.13)	(0.12)	
T_l	-1.799***	-3.8/-2.9/-1.9/-1.8		-2.69***	-5.3/-4.1/-2.8/-2.7	
	(0.296)			(0.39)		
T_u	1.39***	1.3/1.4/2/2.1/2.9		1.17***	1.1/1.2/1.9/2.1/3.3	
	(0.279)			(0.32)		
T_{l*}		-2.8/-1.9/-0.9/-0.3			-4.7/-3.1/-1.6/-0.5	
T_{u*}		0.3/0.9/1.6/2.2/2.8			0.5/1.6/2.6/3.6/4.7	
Log-L	-129.55	-189.27		-105.69	-160.76	
Second step						
intercept	0.129	0.047	0.048	0.11	0.02	0.14***
	(0.11, 0.11)	(0.046; 0.056)	(0.052)	(0.11;0.1)	(0.04; 0.04)	(0.05)
Δi_{t-1}	0.424**	0.347***	0.141**	0.17	0.16***	0.01
	(0.12, 0.11)	(0.068; 0.084)	(0.063)	(0.12;0.11)	(0.05; 0.062)	(0.06)
i_{t-1}	-0.03	-0.045***	-0.017**	-0.07***	-0.06***	-0.05***
	(0.02, 0.02)	(0.008; 0.013)	(0.008)	(0.02;0.02)	(0.01; 0.011)	(0.01)
y_{t-1}	-0.001	0.033***	0.016***	-0.01*	0.004	0.003
	(0.01, 0.01)	(0.005; 0.0049)	(0.006)	(0.01;0.01)	(0.005; 0.048)	(0.01)
π_{t-1}	0.033*	0.039***	0.021***	0.04***	0.03***	0.03***
	(0.01, 0.01)	(0.007; 0.011)	(0.007)	(0.01;0.01)	(0.01; 0.01)	(0.01)
Δy_t	0.153***	0.208***	0.141***	0.1***	0.08***	0.1***
	(0.04, 0.04)	(0.021; 0.008)	(0.025)	(0.04;0.04)	(0.02; 0.02)	(0.02)
$\Delta \pi_t$	0.211*	0.197***	0.096*	0.26***	0.14***	0.13***
	(0.1, 0.12)	(0.046; 0.036)	(0.06)	(0.09;0.1)	(0.04; 0.02)	(0.05)
m_t				0.06***	0.04***	0.02
				(0.03;0.03)	(0.01; 0.01)	(0.01)
s_t				-0.21***	-0.17***	-0.14***
				(0.03;0.03)	(0.02; 0.042)	(0.02)
IMR/σ	0.296***	0.183***		0.25***	0.15***	
	(0.03, 0.03)	(0.013; 0.022)		(0.04;0.04)	(0.01; 0.024)	
Hausman test [c]	$\chi2(6)[11.57] = 0.07$	consistent	$\chi2(6)[31.42] = 0.00$	$\chi2(7)[42.45] = 0.00$	consistent	$\chi2(7)[183.3] = 0.00$
Durbin's *h*	11.41	0.25	4.96	9.9	1.02	3.26
R^2/Nob/DW	0.87/57/0.72	0.78/247/1.98	0.26/247/1.79	0.89/57/0.89	0.78/247/1.92	0.43/247/1.59

Note: Standard errors are reported in parentheses. Stars denote significance level as follows: *10%, **5%, ***1%
Two standard errors are reported for Heckman's proc., the first pertains to the OLS estimate and the second is the adjusted standard error through White's (1980) procedure. In 2S-CNREG, the second standard deviation in the parentesis is computed using bootstrap of 50 replications. [a] Benchmark regression I corresponds to the Model I: equation (1a) in Choi (1999). [b] Benchmark regression II corresponds to the Model I: equation (1b) in Choi (1999), however the results differ from those in Choi (1999): Table I, since none of the provided monetray aggregates were yielding replication. Thus I opted for the closest estimates that resulted when using the gap in M1. [c] In Hausman test for the Benchmark regression II, the insignificant variable y_{t-1} was dropped to meet asymptotic assumptions for Hausman test.

As we can see from the table, the column titled Heckman's procedure (two-sided type II tobit) denotes the replicated regression of Choi (1999). Restating his findings in the second step of the estimation procedure applied to Benchmark regression I, all coefficients except for y_{t-1} have the correct sign ($\beta_3 < 0$ and $\beta_4, \beta_5, \beta_6,$ and $\beta_7 > 0$) and all variables except for Δi_{t-1} and y_{t-1} are statistically significant. Turning attention to the 2S-CNREG, the results in the second column reveal that by permitting for all observations to be potentially censored, all coefficients, including β_4 (y_{t-1}), preserve their correct sign and all variables appear statistically significant at the 1 percent significance level. Besides, there are number of coefficients that are statistically different in magnitude from Choi's estimates (testing whether Choi's parameter point estimate falls into an interval estimate of ordered probit and censored regression): y_{t-1}, i_{t-1}, and Δy_t, suggesting a bias in parameters of the two-sided type II tobit, due to ignoring the censoring of the non-zero observations. Also, based on the Hausman test, I could reject the consistency of the estimates of two-sided type II tobit at 10 % significance level (p-value = 0.07).

The Benchmark regression II includes two additional explanatory variables, i.e., the money gap m_t and the measure of the misalignment of the discount rate and the market rate, s_t. Heckman's procedure delivers coefficients that all have the correct sign except for y_{t-1} and all variables appear significant, except for Δi_{t-1}. In the case of the estimates derived through 2S-CNREG, all coefficients have a correct sign and all coefficients are statistically significant, except for y_{t-1}. In addition, the point estimates are statistically different from the two-sided type II tobit in the following three variables: $\Delta \pi_t$, m_t, and s_t, which again points at the biasedness of parameter estimates in the two-sided type II tobit. Similarly to the Benchmark regression I, the Hausman test shows that the parameters in two-sided type II tobit are inconsistent (at 1% significance level, see Table 6).

The problem of biased estimates can also be seen by comparing the parameters of the OLS with those of the 2S-CNREG procedure. In both Benchmark regressions, the Hausman test suggests misspecification in OLS estimates: p-values = 0.00 in both regressions.[16] The tests confirm the issue of biased parameter estimates in Taylor type rules when estimated by available conventional estimation methods.

The findings, in addition, are supported by the first order autocorrelation statistics (see Table 6). The Durbin's h statistic for Benchmark regression I and II, respectively takes values of 11.41 and 9.9 for two-sided type II tobit and value 4.96 and 3.26 for OLS. These values suggest autocorrelation at 5% significance level, which contrast with the statistics for 2S-CNREG, where the same statistics are 0.25 and 1.02, respectively, implying no autocorrelation of the first degree. These results promote the 2S-CNREG procedure to be more favorable estimator than the conventional ones.

[16] In the Benchmark regression II, the statistically insignificant variable y_{t-1} was dropped for evaluation of the Hausman test statistics.

7. Conclusion

In this paper I aim to contribute to the debate on unbiased policy rule estimation methods by pointing at the bias in the conventional estimators applied to policy rules of Taylor type. In particular, I provide an empirical documentation, using the data for the Czech inflation targeting episode, of the inconsistency between the perfect fit of Taylor rule on historical data and at the same time, a failure of the financial markets to predict future short term market rates. Such an inconsistency can only be reconciled if one accepts that estimates of Taylor rule are biased.

However, the bias in parameter estimates might stem from the conduct of the monetary policy. Namely, if the policymakers apply some selection – censoring – rule (having its determinants) in the process of setting policy rate (i.e., do not always follow the prescription of a rule), which is correlated with the standard variables in Taylor rules, the bias can appear. This is likely the case as we show that the policy rate trajectory of the CNB's staff (resulting from the unconditional inflation forecasts of the CNB), as the major underlying input into the policymakers decision on key rates setting, does not predict policy rates in the future better then the financial markets.

In order to fully account for the effects of censoring rule I develop an estimation procedure (combining ordered probit and censored regression) that produces unbiased parameter estimates of the standard Taylor type rule.

I provide two empirical applications. Firstly, I use the case of the Czech Republic and show that aside of the censoring effect, the systematic policy rule that is used by the bank board is identical with the calibrated policy rule embedded in the staff's model used for unconditional inflation forecasts. Secondly, I present an empirical application by estimating the FED's policy rule. Using the same rule's specification and data set as by Choi (1999) I contrast the biases caused by neglecting censoring in general (in the case of ordinary least squares) and by neglect of non-zero policy rate changes (two-sided type II tobit). As a result, the new estimation procedure proves to be superior for producing unbiased estimates of policy rules.

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