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AN APPLICATION OF INDEX NUMBERS THEORY TO INTEREST RATES

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and Lucia Steklacova

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Abstract

This paper uses index number theory to disentangle changes in aggregate retail interest rates due to changes in individual component rates ("interest rate effect") from those caused by changes in the weights of each component ("weight effect"), on the basis of the "difference" index numbers recently revisited by Diewert (2005). The paper, first discusses the optimal calculation of a binary index using axiomatic index number theory; on that basis, chain and direct indices are established; finally, the selected decomposition and indices are applied to monthly data on euro area interest rates on loans and deposits (MIR) for the period January 2003 – January 2008. It is concluded that relevant weight effects at euro area level are limited to a few indicators and periods of MIR, and that that the indices on interest rates can be a suitable tool in the analysis of variations in aggregate interest rates.

The euro area interest rates statistics (MIR statistics) are monthly statistics on the interest rates applied by Monetary Financial Institutions (MFI) on loans granted to and deposits received from Households (HHs) and Non-Financial Corporations (NFCs) resident in the euro area. MIR statistics are compiled by the Eurosystem[3] in three steps.[4] First, reporting institutions calculate weighted average interest rates of all their relevant loans and deposits and submit these weighted average interest rates and aggregated business volumes to their national central bank (NCB). Second, each NCB compiles the national average rates and the aggregate business volumes and submits these to the European Central Bank (ECB). In the third and final step, the ECB compiles euro area average rates and aggregate business volumes for each MIR category. At each of these steps, interest rates are calculated as the weighted average, by the corresponding amounts, of its components, in order to obtain the representative rate.

One of the features of this procedure is that, if the interest rate is different across credit institutions, across creditors/debtors or across countries, a change in the business volume may also have an impact on average interest rates. The effect of changes in volume of business across countries and its impact on the euro area MIR is monitored by the ECB. Similarly, the impact of changes in volume across institutions may give rise to changes in the national MIR, and could be of interest for the national central banks in order to explain the evolution and also to the ECB when analysing the development of national MIR.

This paper applies index number theory to disentangle changes in interest rates ("interest rate effects") from changes in volumes ("weight effects"). Most of the standard index number theory and applications are based on index number ratios. However, a change in interest is better understood and usually communicated in absolute values rather than in percentage values. For these reasons, this paper focuses on indices that keep the absolute values of the change rather than expressing it as a percentage. These indices, which are calculated in terms of differences, were first presented in the first part of the twentieth century and have recently been revisited by Erwin Diewert.

This paper is based on these "difference" (as opposed to "ratio") index numbers. Section 1 makes a comparison between interest rates and prices, and explains how index numbers can be applied to the former. Section 2 presents how a difference between two periods of an aggregate can be decomposed according to different formulas in two or three components representing the interest rate effect, the quantity of volume (weight) effect, and in some cases a mixed effect in the case of a third component, which add up to the total aggregate difference. In fact this decomposition is already a binary difference index, i.e. and index with only two periods, to which index number theory can be applied, and the alternative decompositions are defined and named after the corresponding indices in the standard ("ratio")

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index numbers. Section 3 defines a number of axioms or desirable properties of index numbers, following the adaptation of standard axiomatic approach of index numbers to difference index numbers as proposed by Erwin Diewert and taking into account certain particularities of the calculation of interest rates, in for example the fact that the changes in weights necessarily sum to zero, and adding some relevant considerations of a non-quantitative nature. Section 4 applies the axioms to the alternative decompositions and selects a so-called "Marshall-Edgeworth" decomposition, although it recognises that most of the other decomposition would provide also good results in terms of the axioms. On the basis of that selection, section 5 discusses how the selected decomposition obtained for a two period comparison must be used when applied to several periods, suggesting a chain index, i.e. an index that links the successive comparisons of two periods, as opposed to a direct index that compares each period with a fixed point in time. Section 6 discusses whether a notional interest rate, i.e. an interest rate from which weight effects have been excluded, can be calculated. The response is positive but it is advised not to use it as it could be confusing for users and public, and also taking into account that the notional rate does not provide any additional information to what is already contained in an index. Section 8 applies the "Marshall-Edgeworth" difference index, in the form of chain, direct and direct-rebased index to all euro area interest rate on deposits and loans statistics for the period January 2003 to January 2008. The results show that the impact of changes in weight are very limited in most of the cases and, in general, the indices do not deviates from the accumulated changes in the corresponding MIR original series, meaning that the changes in weights across countries are null or negligible. Furthermore, the three indices analysed behave very much in the same way in most of the MIR categories, with a few exceptions, in particular MIR indicator NB13 (New business. Loans to Households for consumption purposes, with floating rate and rate with initial period of fixation up to 1 year).

The results in the paper confirm that the regular calculation and publication of month-to-month decomposition could help analysts to interpret monthly changes in the euro area rates. This decomposition, might be accompanied by an index, which would accumulate all month-to-month decompositions from the starting point of MIR statistics (Jan03). In that sense, a chain index may have some advantages, it would permit to assess the evolution in longer periods in terms of changes in interest rates. Finally, it is noted that all the analysis contained in this paper is limited to Euro area rates and the impact of weight and rate effects caused by country aggregate figures, therefore not identifying the weight and rate effects at other aggregation levels. As a consequence, no confidential information would be disclosed with the publication of the decomposition and/or the index.

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[4]   For further information on the MIR categories, definitions and compilation refer to ECB (2001) and ECB (2002).

Introduction

The euro area Monetary Financial Institutions (MFIs) interest rates statistics (MIR) are monthly statistics on the interest rates applied by MFIs on loans granted to and deposits received from Households (HHs) and Non-Financial Corporations (NFCs) in the euro area. MIR statistics are compiled by the Eurosystem[5] in three steps.[6] First, reporting institutions calculate weighted average interest rates of all relevant loans and deposits and submit the average interest rate and aggregated business volume to their respective national central bank (NCB). Second, each NCB compiles the national average rates and the aggregate business volumes and submits these to the European Central Bank (ECB). In the third and final step, the ECB compiles euro area average rates and aggregate business volumes for each MIR category. At each of these steps, interest rates are calculated as the weighted average, by the corresponding amounts, of its components, in order to obtain the representative rate.

One of the features of this procedure is that, if the interest rate is different across credit institutions, across creditors/debtors or across countries, a change in the business volume may also have an impact on average interest rates. The effect of changes in volume of business across countries and its impact on the euro area MIR is monitored by the ECB. Similarly, the impact of changes in volume across institutions may give rise to changes in the national MIR, and may be of interest for the national central banks in order to explain the evolution and also to the ECB when analysing the development of national MIR.

The question has arisen on what tools are optimal to analyse the origin of changes in MIR and isolate, for example, changes in euro interest rates originating from changes in national interest rates from changes caused by variations in the relative weights of the countries. One possible approach to deal with this issue is to use the statistical tools developed in the field of index number theory; indeed, the traditional problem of distinguishing real from nominal growth rates in many macroeconomic variables or the calculation of price indices resembles very much the questions raised on MIR. In both cases, the objective is to distinguish changes in business volumes from changes in prices/rates. According to this view, in the words of Diewert (2002) "*the index number problem can be regarded as the problem of decomposing the change in a value aggregate, V^1/V^0, into the product of a part that is due to price change, $P(p^0, p^1, q^0, q^1)$ and a part that is due to quantity change, $Q(p^0, p^1, q^0, q^1)$*".

Nevertheless, an important difference is noted. While some economic variables for which indices are calculated do not have a relevant meaning when expressed in absolute values (e.g. price indices), interest rates certainly provide information when expressed in absolute values. Similarly a change in interest is better understood and usually communicated in absolute values rather than in percentage values. For these reasons, this paper focuses on indices that keep the absolute values of the change rather than expressing it

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[5]   The Eurosystem is the monetary authority of the Eurozone. It is a system of central banks consisting of the European Central Bank and the national central banks of the member states of the European Union whose currency is the euro

as a percentage. These indices were first presented in the first part of the twentieth century and have recently been revisited by Diewert (2005).

Another important methodological issue is the approach to obtain, examine and compare the alternative indices. Following the index number literature, three methodologies can be distinguished, the axiomatic approach, the economic approach, and the stochastic approach. In brief, the axiomatic approach compares the different indices on the basis of a number of mathematical features (axioms) with which the indices may comply or not; the economic approach obtains the index by using economic theory and maximisation techniques; the stochastic approach considers the evolution of individual prices as observations of the general inflation rate and includes stochastic factors. In this paper we limit ourselves to the axiomatic approach which, as shown below, seems very much applicable to this set of indices and to the concrete case of interest rate statistics. Regarding the economic approach, usually based on consumer utility functions and a basket of products and prices, it does not seem directly applicable to the case of aggregate interest rates. As for the stochastic approach, leaving apart the controversies on the issue (e.g. see Selvanathan and Prasada Rao (1994) and Diewert (1995)), it would require further study.

On the basis of the considerations above, this paper uses the work by Diewert (2005) as the starting point for the analysis. The paper is organised as follows: After the introduction, the first section compares the question at stake in MIR statistics with the price indices and familiarise the reader with the MIR notation, the second section presents a list of plausible decomposition of changes in MIR, of which the first component is already a binary "difference"[7] index of interest rates. The different alternatives are compared on the basis of a number of axioms presented in the third and fourth section; the fifth section goes a step further, applying the binary indices in the construction of a series; section six discusses notional interest rates, and section seven applies the selected index and indicator to all MIR categories available at euro area level since 2003.[8] Finally, section eight summarises the results and concludes with some recommendations. Annex 1 lists the different items in MIR statistics, annex 2 shows some summary results of the calculation of indices; annex 3 contains a chart for each MIR category, comparing the different indices; annex 4 presents the decomposition for selected MIR indicators and periods.

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[7] In this paper, the word "difference" denotes those decompositions of indices that show changes in absolute terms rather than as ratios, which is the usual approach in index numbers. Here we use the terminology proposed by Diewert (2005).

[8] Out of the 45 categories, 43 has been examined. The remaining two are the APRC categories, which has been excluded.

1. Index number theory and interest rates

Index number theory has been developed mainly in the context of price statistics. For that reason, a brief comparison between price statistics and MIR may help the analysis. The starting point in price statistics is the need to monitor the evolution of prices, isolating the price developments from changes in quantities. The most basic problem is to compare the price of a single commodity at two different points in time, say the price at time t, $(\mathbf{p}_t)$, with the price at a different point in time t-1, $(\mathbf{p}_{t-1})$. While prices for a single product can be directly compared in the case of one commodity, the joint analysis of a set of prices is done by composing the prices into a single indicator of transactions $\mathbf{Y_t}$ which is calculated as

$$\mathbf{Y_t} = \sum_k p(k)_t * v(k)_t \tag{1.1}$$

Where

Y_t	=	joint indicator of price at time t, which is the sum of each price at time t multiplied by each business volume at time t
$p(k)_t$	=	price of commodity k at point in time t
$v(k)_t$	=	volume of transactions of commodity k at point in time t

In basic terms, when comparing $\mathbf{Y_t}$ with $\mathbf{Y_{t-1}}$, it is required to find out what part of the difference corresponds to price developments and what part corresponds to movements in volumes. The impact of price developments is calculated by fixing a certain business volume for the two periods, e.g. using $v(k)_{t-1}$ for both periods (Laspeyres price index). From that we can obtain an index defined as:

$$P_L = \frac{\sum_k p(k)_t v(k)_{t-1}}{\sum_k p(k)_{t-1} v(k)_{t-1}} \tag{1.2}$$

Similarly if we calculate a 'quantity index' by fixing the same prices for both periods, e.g. $p(k)_t$ for both periods (Paasche quantity index), we would know the part of $\mathbf{Y_t}/\mathbf{Y_{t-1}}$ that is due to increases in volumes.

$$Q_P = \frac{\sum_k p(k)_t v(k)_t}{\sum_k p(k)_t v(k)_{t-1}} \tag{1.3}$$

Finally, for some price indices, it is possible to combine the above calculations in one as follows

$$\frac{Y_t}{Y_{t-1}} = \frac{\sum_k p(k)_t v(k)_{t-1}}{\sum_k p(k)_{t-1} v(k)_{t-1}} * \frac{\sum_k p(k)_t v(k)_t}{\sum_k p(k)_t v(k)_{t-1}} = P_L * Q_P \tag{1.4}$$

In other words, applying these price indices, an increase in transactions from one period to another can be, in principle, decomposed into an increase in price and an increase in volume.

Moving to MFI interest rate statistics, a number of similarities with price index theory exist. As in the case of prices, the comparison of an interest rate referring to a single loan or deposit at two different points in time i_t and i_{t-1} is straightforward. However, interest rates for an institution, for a country or for the euro area are the weighted average of the individual rates at each compilation level. From the point of view of euro area rates, these are the weighted averages of national interest rates, where the weight is the national business volume divided by the total euro area business volume.

$$\mathbf{I_t} = \sum_k i(k)_t * w(k)_t \qquad (1.5)$$

Where

I_t $\quad$ = $\quad$ euro area interest rate at time t

$i(k)_t$ $\quad$ = $\quad$ interest rate in country k at point in time t

$w(k)_t$ $\quad$ = $\quad$ weight of the business volume of country k (compared to total euro area business volume) at time t

$w(k)_t$ $\quad$ = $\quad$ $v(k)_t / \sum_k v(k)_t$ $\qquad\qquad$ (1.6)

Where $v(k)_t$ $\quad$ = business volume for country k at time t

There is a similarity between formula (1.1) and formula (1.5). In the first case, the total amount of transactions is obtained as the sum across products of each price multiplied by the corresponding transaction volumes; in the second case the euro area interest rate is the result of the sum across countries of each interest rate multiplied by the corresponding weight. It seems obvious that index theory can be easily applied to MIR by just substituting prices by interest rates and volume of transaction by weights.[9] In the case of MIR, when comparing euro area interest rates I_t with I_{t-1} it is required to find out what part of the change corresponds to developments in national rates and what part corresponds to changes in country weights, i.e. relative changes in business volumes. The change due to the development in national rates is calculated by fixing a certain weight for the two periods, e.g. using $w(k)_{t-1}$ for both periods (Laspeyres index). If we are interested in knowing this component of I_t / I_{t-1} , one possible approach is to calculate

$$P_L = \frac{\sum_k i(k)_t \, w(k)_{t-1}}{\sum_k i(k)_{t-1} \, w(k)_{t-1}} \qquad (1.7)$$

Similarly if we use a fixing the same prices for both periods, e.g. $p(k)_t$ in both periods (Paasche index), we would know the component of I_t / I_{t-1} that is due to changes in volumes.

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[9] It is noted that 'weights' in the context of interest rates has a slightly different meaning than 'weights' in the context of index theory. In the latter 'weight' is usually calculated as the division of transactions in one product (prices by quantities) by total transactions; in MIR it is simply the percentage of the value (in euro) of loans/deposits over total loans/deposits, therefore comparable to quantities in usual index theory but not to the usual meaning of 'weight' in index number theory.

$$Q_L = \frac{\sum_k i(k)_t \, w(k)_t}{\sum_k i(k)_t \, w(k)_{t-1}} \qquad (1.8)$$

Finally, as in the case of prices, it is possible to combine the above calculations in one, as follows

$$\frac{I_t}{I_{t-1}} = \frac{\sum_k p(k)_t \, v(k)_{t-1}}{\sum_k p(k)_{t-1} \, v(k)_{t-1}} * \frac{\sum_k p(k)_t \, v(k)_t}{\sum_k p(k)_t \, v(k)_{t-1}} = P_L * Q_P \qquad (1.9)$$

In other words, an increase in transactions from one period to another can in principle be decomposed into an increase in price and an increase in volume. Usually in the index theory this result is a sub-product of the development of an index. However, in the case of MIR the decomposition has a valid meaning on its own and can be considered as an intermediate step in the construction of possible indices.

2. Difference decompositions and binary difference index numbers

Difference index theory has been revisited and further investigated by Diewert (2005), on the basis of the work developed in the 1920s and 1930s by T. L. Benet and J. K. Montgomery. In essence, it consists in comparing $\mathbf{Y_t}$ with $\mathbf{Y_{t-1}}$ not by the means of a ratio $(\mathbf{Y_t} / \mathbf{Y_{t-1}})$ but as a difference, i.e. $\mathbf{Y_t} - \mathbf{Y_{t-1}}$. In terms of prices, this would be expressed as:

$$\mathbf{\Delta Y_{t, t-1}} = \mathbf{Y_t} - \mathbf{Y_{t-1}} = \Delta P(p_t, p_{t-1}, q_t, q_{t-1}) + \Delta Q(p_t, p_{t-1}, q_t, q_{t-1}) \qquad (2.1)$$

Here ΔP is a measure of the aggregate price change and ΔQ is a measure of the aggregate quantity or volume change.

Diewert (2005) simplifies the number of alternatives by restricting the functions to those which are composed as sums, and for which each summand only contains as variables the prices and quantities, for the two points in time, of a particular product. Furthermore, Diewert (2005) explains the aggregate price as the total value divided by the total quantity, or the unit value. Moreover, he notes that "*it is quite possible that one given decomposition would be useful for one purpose but not for another*", meaning that it can be useful to analyse prices but not quantities or the other way around.

This scheme can be directly applied to euro area MFI interest rates. The interest rate difference between two different periods is given by $\mathbf{\Delta I_{t, t-1}} = \mathbf{I_t} - \mathbf{I_{t-1}}$, and it can be decomposed into difference terms that represent the pure interest rate change $\mathbf{Int}$, the pure weight change $\mathbf{Wgh}$ and, in some cases, a mixed or composite effect $\mathbf{Mix}$.

$$\mathbf{\Delta I_{t, t-1}} = I_t - I_{t-1} = \mathbf{Int}\,(i_t, i_{t-1}, w_t, w_{t-1}) + \mathbf{Wgh}\,(i_t, i_{t-1}, w_t, w_{t-1}) + \mathbf{Mix}\,(i_t, i_{t-1}, w_t, w_{t-1}) \qquad (2.2)$$

As shown by Figure 1, if the change in euro area interest rate is only split into two terms (i.e. interest rate and weight effect), this implies that the mixed effect is distributed between them, either symmetrically or not.

Figure 1
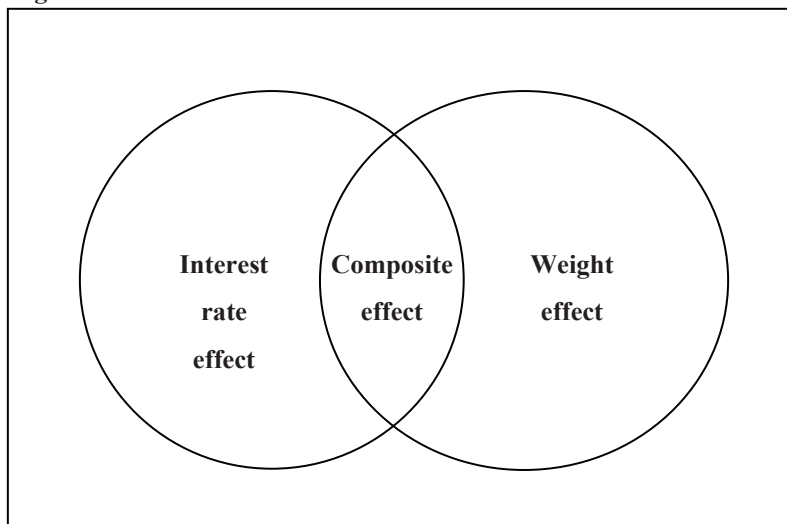
It is also noted that interest rates are actually prices per unit and therefore the construction for prices can be directly applied to MIR.

Now it is time to review the possible formulations of the decomposition where the first component is a binary index for the interest rates, in the sense that it compares two consecutive periods.[10] For that purpose we will use the usual terminology from ratio indices, adapting the denominations to the difference decompositions that are equivalent to the corresponding ratio index in terms of the use of prices and quantities (in our case rates and weights) from the different periods. For each group of decompositions, the interest rate term is the same, while changes in the other components are analysed in detail. The reason for this detailed analysis is that the decomposition in itself has analytical value, apart from serving as a building block for the indices.

2.1 Laspeyres-type decompositions

Laspeyres-type decomposition in 3 terms. In this and following decompositions, the first term is the interest rate effect or interest rate binary index, the second one is the weight effect and the third one (where it exists) is the mixed or composite effect.

$$\Delta I_{t,\,t-1} = \sum_k \Delta i(k)_{t,\,t-1} * w(k)_{t-1} + \sum_k \Delta w(k)_{t,\,t-1} * i(k)_{t-1} + \sum_k \Delta i(k)_{t,\,t-1} * \Delta w(k)_{t,\,t-1} \qquad (2.3)$$

where

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[10] The term "binary" is used for the comparisons just between two consecutive periods by Stuvel (1989).

$\Delta I_{t,\,t-1}$ = difference in level of euro area MIR between month t and month t-1; it can also be expressed as $\Delta I_{t,\,t-1} = I_t - I_{t-1}$, where I_t is the euro area interest rate in period t

$i(k)_t$ = national MIR level of euro area country k at month t

$\Delta i(k)_{t,\,t-1}$ = $i(k)_t - i(k)_{t-1}$, difference in national MIR level between month t and month t-1

$w(k)_t$ = national weight of euro area country k at month t

$\Delta w(k)_{t,\,t-1}$ = $w(k)_t - w(k)_{t-1}$, difference in national weight between month t and month t-1

We have used the term "Laspeyres" for this decomposition because the first component or interest binary index weighs the increase of each interest rate with the weight of the initial period t-1.

Laspeyres-type decomposition in 3 terms with extended weight effect. As opposed to the previous decomposition, the difference between country interest rate and aggregated interest rate is incorporated in the weight term instead of the country interest rate alone; this does not change the sum over all countries but allocates the effect at national level in a different way, possibly easier to interpret. In particular, the weight effect of each country is modified, now taking into account its relative position in respect to the total euro area rate.

$$\Delta I_{t,\,t-1} \;=\; \sum_k \Delta i(k)_{t,\,t-1} * w(k)_{t-1} + \sum_k \Delta w(k)_{t,\,t-1} * (i(k)_{t-1} - I_{t-1}) + \sum_k \Delta i(k)_{t,\,t-1} * \Delta w(k)_{t,\,t-1} \qquad (2.4)$$

As long as it is done equally across all countries, any other constant can be included in the weight effect without changing the weight effect at aggregate level. This is due to the fact that the sum of weight changes is always zero. This feature is demonstrated for the weight effect of the above Laspeyres decomposition with extended weight effect.

$$\sum_k \Delta w(k)_{t,\,t-1} * (i(k)_{t-1} - I_{t-1}) \;=\; \sum_k \Delta w(k)_{t,\,t-1} * i(k)_{t-1} - \sum_k \Delta w(k)_{t,\,t-1} * I_{t-1} =$$

$$= \sum_k \Delta w(k)_{t,\,t-1} * i(k)_{t-1} - I_{t-1} * \sum_k \Delta w(k)_{t,\,t-1} =$$

$$= \sum_k \Delta w(k)_{t,\,t-1} * i(k)_{t-1} - 0 =$$

$$= \sum_k \Delta w(k)_{t,\,t-1} * i(k)_{t-1} \qquad (2.5)$$

The extension in the aggregate component was initially proposed by Coene (2004).

Laspeyres-type decomposition in 2 terms. This is similar to the Laspeyres decomposition in three terms above (cf. equation (2.3)), but the last two terms are grouped, resulting in an interest rate effect and a weight effect. Please note that the interest rate of the second term now refers to the present period [t]. The two terms are obviously not symmetric, in the sense that rates and amounts refer to different periods in the two terms.

$$\Delta I_{t,\,t\text{-}1} \;=\; \sum_k \Delta i(k)_{t,\,t\text{-}1} * w(k)_{t\text{-}1} + \sum_k \Delta w(k)_{t,\,t\text{-}1} * i(k)_t \tag{2.6}$$

Laspeyres-type decomposition in 2 terms with extended weight effect. Similar to equation (2.4) the weight term here contains the difference between country interest rate and aggregated interest rate, with other parts of the formula being unchanged.

$$\Delta I_{t,\,t\text{-}1} \;=\; \sum_k \Delta i(k)_{t,\,t\text{-}1} * w(k)_{t\text{-}1} + \sum_k \Delta w(k)_{t,\,t\text{-}1} * (i(k)_t - I_t) \tag{2.7}$$

2.2 Paasche-type decompositions

Paasche-type decomposition in 3 terms. In this case, the interest rate effect is calculated in reference to the weight of the present period, rather than in reference to the previous period as in Laspeyres' decomposition (cf. equation (2.3)). Please note that the weight and interest rate in the first two terms now refer to the present period [t] and that this implies that the negative composite effect is subtracted instead of summed.

$$\Delta I_{t,\,t\text{-}1} \;=\; \sum_k \Delta i(k)_{t,\,t\text{-}1} * w(k)_t + \sum_k \Delta w(k)_{t,\,t\text{-}1} * i(k)_t - \sum_k \Delta i(k)_{t,\,t\text{-}1} * \Delta w(k)_{t,\,t\text{-}1} \tag{2.8}$$

Paasche-type decomposition in 3 terms with extended weight effect. As opposed to the previous decomposition, the difference between country interest rate and aggregated (euro area) interest rate, referring to the present period, is included in the weight term. This case is similar to the one analysed for the Laspeyres-type decomposition (cf. equation (2.4)).

$$\Delta I_{t,\,t\text{-}1} \;=\; \sum_k \Delta i(k)_{t,\,t\text{-}1} * w(k)_t + \sum_k \Delta w(k)_{t,\,t\text{-}1} * (i(k)_t - I_t) - \sum_k \Delta i(k)_{t,\,t\text{-}1} * \Delta w(k)_{t,\,t\text{-}1} \tag{2.9}$$

Paasche-type decomposition in 2 terms. Again, the difference to the previous Paasche case in three terms (cf. equation (2.8)) is that the two last terms are grouped, while the interest rate term remains the same. Please note that the interest rate of the second term now refers to the previous period [t-1]. As a result of this combination, the two terms are not symmetric in the sense that the reference periods in both terms are not identical.

$$\Delta I_{t,\,t\text{-}1} \;=\; \sum_k \Delta i(k)_{t,\,t\text{-}1} * w(k)_t + \sum_k \Delta w(k)_{t,\,t\text{-}1} * i(k)_{t\text{-}1} \tag{2.10}$$

Paasche-type decomposition in 2 terms with expanded weight term. Compared with the previous case, the difference between the country interest rate and the aggregated (euro area) interest rate is used in the second term instead of the country interest rate alone.

$$\Delta I_{t,\,t\text{-}1} \;=\; \sum_k \Delta i(k)_{t,\,t\text{-}1} * w(k)_t + \sum_k \Delta w(k)_{t,\,t\text{-}1} * (i(k)_{t\text{-}1} - I_{t\text{-}1}) \tag{2.11}$$

2.3 Marshall-Edgeworth-type decompositions

Marshall-Edgeworth-type decomposition.[11] This decomposition uses the simple average of the previous and present period weights to calculate the interest rate effect. The weight effect is calculated in the same way, resulting in a decomposition with only two terms. In this decomposition, the composite effect is distributed equally between the interest rate effect and the weight effect.

$$\Delta I_{t,t-1} = \sum_k \Delta i(k)_{t,t-1} \left(\frac{w(k)_t + w(k)_{t-1}}{2} \right) + \sum_k \Delta w(k)_{t,t-1} \left(\frac{i(k)_t + i(k)_{t-1}}{2} \right) \quad (2.12)$$

Marshall-Edgeworth-type decomposition with extended weight effect. Compared to the previous decomposition method, the weight term includes the difference of the country interest rate and the aggregated (euro area) interest rate instead of the country interest rate alone, for both subsequent periods. As explained above (cf. equation (2.4)), this expansion does not change the weight effect at the aggregated level, but offers other possibilities of interpretation at the detailed level.

$$\Delta I_{t,t-1} = \sum_k \Delta i(k)_{t,t-1} \left(\frac{w(k)_t + w(k)_{t-1}}{2} \right) + \sum_k \Delta w(k)_{t,t-1} \left(\frac{(i(k)_t - I_t)_t + (i(k)_{t-1} - I_{t-1})}{2} \right) \quad (2.13)$$

This decomposition was initially proposed by Coene (2004) on the basis of previous work by Berthier (2001).

2.4 Walsh- and Fisher-type decompositions

Walsh decomposition in three terms. This decomposition uses the geometric average of the previous and present period weights to calculate the rate effect. The weight effect is calculated symmetrically, resulting in a third term that is not easy to make explicit. Given the similarity to the Marshall-Edgeworth-type of decomposition, the "Rest" is presumably in most of the cases a figure not significantly different from zero.

$$\Delta I_{t,t-1} = \sum_k \Delta i(k)_{t,t-1} * \left(w(k)_t * w(k)_{t-1} \right)^{1/2} + \sum_k \Delta w(k)_{t,t-1} * \left(i(k)_t * i(k)_{t-1} \right)^{1/2} + Rest \quad (2.14)$$

Walsh decomposition in two terms is also possible, since we are mostly interested in the interest rate effect. Then the weight effect from the above stated three-term Walsh decomposition is included in the "Rest", which again cannot easily be made explicit. Given the similarity to the Marshall-Edgeworth-type decomposition, the "Rest" is assumed to contain mainly the weight effect.

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[11] Diewert (2005) calls this decomposition "Benet indicator", after his first presenter.

$$\Delta I_{t,\,t\text{-}1} = \sum_{k} \Delta i(k)_{t,\,t\text{-}1} * (w(k)_{t} * w(k)_{t\text{-}1})^{1/2} + Rest \qquad (2.15)$$

This decomposition is not further analysed.

Fisher decomposition in difference form cannot be used without implicitly using complex numbers, because it might in general lead to taking the square root of a negative number. Exploring this possibility goes beyond the aim of this paper. Only for illustration purposes, the formula for a two term decomposition is included but not further analysed.

$$\Delta I_{t,\,t\text{-}1} = \left(\sum_{k} \Delta i(k)_{t,\,t\text{-}1} * w(k)_{t\text{-}1} \right)^{1/2} * \left(\sum_{k} \Delta i(k)_{t,\,t\text{-}1} * w(k)_{t} \right)^{1/2} +$$

$$+ \left(\sum_{k} \Delta w(k)_{t,\,t\text{-}1} * i(k)_{t\text{-}1} \right)^{1/2} * \left(\sum_{k} \Delta w(k)_{t,\,t\text{-}1} * i(k)_{t} \right)^{1/2} + Rest \qquad (2.16)$$

2.5 Vartia-type decomposition

Vartia-type decomposition.[12] This decomposition is based on the logarithmic mean, which is the basis for the Vartia ratio index. In this indicator we deviate from the pattern followed up to now in which the increase in interest rates is presented in absolute terms. In this case the increase is the logarithm of the ratio of the interest rates in the two periods.

$$\Delta I_{t,t-1} = \sum_{k} \ln\left[\frac{i(k)_t}{i(k)_{t-1}}\right]\left(\frac{I(k)_t - I(k)_{t-1}}{\ln I(k)_t - \ln I(k)_{t-1}}\right) + \sum_{k} \ln\left[\frac{w(k)_t}{w(k)_{t-1}}\right]\left(\frac{I(k)_t - I(k)_{t-1}}{\ln I(k)_t - \ln I(k)_{t-1}}\right) \qquad (2.17)$$

where $I(k)_t = i(k)_t * w(k)_t$ is the contribution of country k to the euro area rate.

The Vartia-type decomposition has an important limitation in the context of MIR. Whenever the weight or the rate is zero for the previous period, the decomposition cannot be calculated for that point. A possible solution could be to use limits towards zero rather than proper zero values. This alternative is not further explored in this paper and therefore the use of the Vartia-type decomposition is not further analysed.

Other indices using logarithms, like the one proposed by Törnqvist (1936) do not seem to be easily expressed in an additive form.

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[12] Diewert (2005) calls this decomposition "Montgomery indicator" after his first presenter.

2.6 A generalisation of some decompositions

All the above decompositions work on the basis of two subsequent periods, where there are only two possible weights to be considered in the index. When making binary comparisons between two non-subsequent periods, t and t-h, the above formulas can also be used for direct comparisons. Nevertheless, new possibilities arise from using of intermediate weights. A few are shown below.

Generalised decomposition with single period weighting in the interest rate component (3 terms). If instead of using the weight corresponding to either the initial period (Laspeyres) or the final period (Paasche) an intermediate period weight or a different weight, e.g. from a previous period, is used, it is possible to construct the decomposition as follows:

$$\Delta I_{t,\,t-h} \;=\; \sum_k \Delta i(k)_{t,\,t-h} * w(k)_s + \sum_k \Delta w(k)_{s,\,t-h} * i(k)_{t-h} + \sum_k \Delta w(k)_{t,\,s} * i(k)_t \qquad (2.19)$$

where

$\Delta I_{t,\,t-h}$	=	difference in level of euro area MIR between month t and month t-h; it can also be expressed as $\Delta I_{t,\,t-h} = I_t - I_{t-h}$, where I_t is the euro area interest rate in period t
$i(k)_t$	=	national MIR level of euro area country k at month t
$\Delta i(k)_{t,\,t-h}$	=	$i(k)_t - i(k)_{t-h}$ difference in national MIR level between month t and month t-h
$w(k)_s$	=	national weight of euro area country k at month s
$\Delta w(k)_{s,\,t-h}$	=	$w(k)_s - w(k)_{t-h}$ difference in national weight between month s and month t-h
$\Delta w(k)_{t,\,s}$	=	$w(k)_t - w(k)_s$ difference in national weight between month t and month s

Generalised decomposition with single period weighting in the interest rate component (3 terms) and extended weight factor. As already repeated above, it is possible to include a constant in the weight terms, in this case the euro area aggregates for the respective periods.

$$\Delta I_{t,\,t-h} \;=\; \sum_k \Delta i(k)_{t,\,t-h} * w(k)_s + \sum_k \Delta w(k)_{s,\,t-h} * (i(k)_{t-h} - I_{t-h}) + \sum_k \Delta w(k)_{t,\,s} * (i(k)_t - I_t) \quad (2.20)$$

The interpretation of the second and third term if h<s<t runs along the lines of the change in weighting before and after the period used as weighting, in terms of the initial and final deviation of each national interest rate from the euro area interest rate.

Generalised decomposition with multiple weighting in the interest rate component (2 terms). Generalizing the Marshall-Edgeworth weighting, which corresponds to the semi-sum, where the weight is the simple average of the initial and final period, we can use all the intermediate periods, obtaining the following decomposition:

$$\Delta I_{t,\,t-h} \;=\; \sum_k \Delta i(k)_{t,\,t-h} * (\frac{1}{h+1} \sum_{t-h}^{t} w(k)_x) \; + \text{Rest}$$

3. Axiomatic properties of decompositions

In order to decide which decomposition, and consequently which index built on it is the most appropriate for MIR, following the axiomatic approach, a number of axiomatic properties can be defined. These properties are features that an ideal MIR index should comply with and are directly taken from or inspired by price index theory.

Two type of properties are analysed. First, a group of properties that have a meaning both at the level of the national contributions and at euro area level. After that, a second group that only makes sense at euro area level. The results for all decompositions presented in this paper are summarized in the next section. Most of the properties are extrapolated to the difference decompositions from the usual index number theory, closely following Diewert (2005). Some additional properties have been adapted from the conventional index number theory, and a few others are created ex-novo for this analysis. A strict hierarchy of the axioms is not presented here; nevertheless, the ordering and later discussion is influenced by the structure by Eichhorn and Voeller as presented in Von der Lippe (2001), and specially by some considerations relative to the particular features of the analysis of interest rates.

3.1 Axiomatic properties at individual level

The axiomatic properties below can be analysed both at levels of country components $\Delta I_{t,t-1}(k)$ or at "summed" level, i.e. euro are level. For the analytical purposes of the properties of various indices, the notion of country component $\Delta I_{t,t-1}(k) = I_t(k) - I_{t-1}(k)$ is introduced, where $I_t(k) = i_t(k)*w_t(k)$ (This notation was already presented when treating the Vartia-type index). This country component includes all types of effects, i.e. interest rate effect **Int(k)**, weight effect **Wgh(k)** and mixed effects **Mix(k)** altogether.

- **(1) Exhaustiveness**: The sum of all components Int(k), Wgh(k) and Mix(k) (mixed component ,if it exists) for a single country should be equal to the country contribution to the change in the euro area interest rate $\Delta I_{t,t-1}(k)$.

  $$\mathbf{\Delta I}(k) = \mathbf{Int}(k) + \mathbf{Wgh}(k) + \mathbf{Mix}(k)$$

  This property means that the decomposition has to contain, in its different components, the whole change in rates from one period to the next, at the level of each country.

- **(2a) Scale (or dimensionality) for rates**: Multiplying a country interest rate in the previous ($i_{t-1}(k)$) and current ($i_t(k)$) period by the same factor z results in a multiplication of the country contribution to the interest rate component by the factor z.

  $$\mathbf{Int}(z * \mathbf{i}(k)_t, \mathbf{w}(k)_t, z * \mathbf{i}(k)_{t-1}, \mathbf{w}(k)_{t-1}) = z * \mathbf{Int}(\mathbf{i}(k)_t, \mathbf{w}(k)_t, \mathbf{i}(k)_{t-1}, \mathbf{w}(k)_{t-1})$$

  This property refers to neutrality in the units used. Irrespective of whether the rates are expressed in percentages or basis points, the result should be the same, just expressed in a different unit.

This property is inspired by the price index dimensionality test. Also Diewert (2005) proposes this axiom as an adaptation of linear homogeneity to the difference indices.

- **(2b) Scale (or dimensionality) for weights**: This property goes along the above lines, just interchanging rates by weights.

$$\mathbf{Wgh}(\mathbf{i}(k)_t, \mathbf{z} * \mathbf{w}(k)_t, \mathbf{i}(k)_{t-1}, \mathbf{z} * \mathbf{w}(k)_{t-1}) = z * \mathbf{Wgh}(\mathbf{i}(k)_t, \mathbf{w}(k)_t, \mathbf{i}(k)_{t-1}, \mathbf{w}(k)_{t-1})$$

- **(3a) (Strong) Identity for rates**: When for a country in both reference periods t-1 and t the interest rates are the same, the country contribution to the interest rate component must be zero.

$$\mathbf{Int}([\mathbf{i}(k)_t = i(k)_{t-1}], \mathbf{w}(k)_t, \mathbf{i}(k)_{t-1}, \mathbf{w}(k)_{t-1}) = 0$$

If $i_{t-1}(k) = i_t(k)$ then the contribution of country k to the interest rate component should be zero. This property reflects the intuitive idea that the interest rate index must be zero when there are no changes in individual rates, and is based on the price index (strong) identity test.

For rates, weak identity can be defined in parallel to the usual price index theory, defined in this case as follows: If there is no change in the interest rate and in the weights, the interest rate aggregate component should be zero. As in the usual price index identity test, strong identity necessarily implies weak identity.

- **(3b) (Strong) Identity for weights**: The same property as above is defined for weights, just interchanging rates by weights.

$$\mathbf{Wgh}(\mathbf{i}(k)_t, [\mathbf{w}(k)_t = w(k)_{t-1}], \mathbf{i}(k)_{t-1}, \mathbf{w}(k)_{t-1}) = 0$$

- **(4a) (Strong) Monotonicity (in both periods) for rates**: Consider two different scenarios for the present period ("a" and "b") and the previous period ("c" and "d"). If for a country the interest rate in the present period in scenario "a" is higher than in scenario "b", $i(k)^a_t > i(k)^b_t$, then the following inequality holds:

$$\mathbf{Int}(\mathbf{i}(k)^a_t, \mathbf{w}(k)_t, \mathbf{i}(k)_{t-1}, \mathbf{w}(k)_{t-1}) > \mathbf{Int}(\mathbf{i}(k)^b_t, \mathbf{w}(k)_t, \mathbf{i}(k)_{t-1}, \mathbf{w}(k)_{t-1})$$

also applying with the opposite inequalities.

If for a country the interest rate in the previous period in scenario "c" is higher than in scenario "d", $i(k)^c_{t-1} > i(k)^d_{t-1}$, then the strong monotonicity test postulates that:

$$\mathbf{Int}(\mathbf{i}(k)_t, \mathbf{w}(k)_t, \mathbf{i}(k)^c_{t-1}, \mathbf{w}(k)_{t-1}) < \mathbf{Int}(\mathbf{i}(k)_t, \mathbf{w}(k)_t, \mathbf{i}(k)^d_{t-1}, \mathbf{w}(k)_{t-1}).$$

also applying with the opposite inequalities.

The intuitive idea is that the interest rate component depends on the change in the individual rates, which in turn is positively related to the rates of the present period and negatively related to

the rates of the previous period. Therefore, an increase in the rates of the present period, makes the interest rate index higher (more positive or less negative, depending on the sign).

- **(4b) (Strong) Monotonicity (in both periods) for weights**: The same as in (4a) applies to weights, switching weights and rates.

- **(5a) Sign consistency for rates:** If the interest rate in a country increases (decreases), the interest rate component must also show an increase (decrease). I.e. if $i(k)_t > i(k)_{t-1}$ then

$$\mathbf{Int}(\mathbf{i}_t(k), \mathbf{w}_t(k), \mathbf{i}_{t-1}(k), \mathbf{w}_{t-1}(k)) > 0.$$

- **(5b) Sign consistency for weights:** If the weight of a country increases (decreases), the weight component must also show an increase (decrease). Thus, if $w(k)_t > w(k)_{t-1}$, then

$$\mathbf{Wgh}(\mathbf{i}_t(k), \mathbf{w}_t(k), \mathbf{i}_{t-1}(k), \mathbf{w}_{t-1}(k)) > 0.$$

- **(6a) Proportionality for rates**: For rates, the interest rate component must increase proportionally to the weight of the country if the weight does not change.

$$\mathbf{Int}([\mathbf{i}(k)_t = i(k)_t + z], [\mathbf{w}(k)_t = \mathbf{w}(k)_{t-1}], i(k)_{t-1}, w(k)_{t-1}) = z * \mathbf{w}(k)_t$$

This property reflects the intuitive idea that the country contribution to the interest rate component must coincide with the interest rate change multiplied with the weight of the country when there is no other effect.

- **(6b) Proportionality for weights**: Interchanging rates by weights, following the lines above, we have:

$$\mathbf{Wgh}([\mathbf{i}(k)_t = \mathbf{i}(k)_{t-1}], [\mathbf{w}(k)_t = w(k)_{t-1} + z], i(k)_{t-1}, w(k)_{t-1}) = z * \mathbf{i}(k)_t$$

- **(7a) (Strong) Translation for rates**: Adding a constant z to the country interest rates in the previous ($i_{t-1}(k)$) and current ($i_t(k)$) period leaves all the terms of the interest rate component Int(k) unchanged.

$$\mathbf{Int}(\mathbf{i}(k)_t + z, \mathbf{w}(k)_t, \mathbf{i}(k)_{t-1} + z, \mathbf{w}(k)_{t-1}) = \mathbf{Int}(\mathbf{i}(k)_t, \mathbf{w}(k)_t, \mathbf{i}(k)_{t-1}, \mathbf{w}(k)_{t-1})$$

The same level shift in the rates of one country (or contributor) in both periods does not modify the change in the interest rate component.

- **(7b) (Strong) Translation for weights**: Adding a constant z to the country <u>interest rates</u> in the previous ($i_{t-1}(k)$) and current ($i_t(k)$) period leaves all the terms of the weight component Wgh(k) unchanged.

$$\mathbf{Wgh}(\mathbf{i}(k)_t + z, \mathbf{w}(k)_t, \mathbf{i}(k)_{t-1} + z, \mathbf{w}(k)_{t-1}) = 0$$

- **(8a) Time reversal for rates**: If we interchange prices and quantities of the two periods, the interest rate component should be the same in absolute terms, with a change in sign.

$$\mathbf{Int}(\mathbf{i}(k)_t \ , \mathbf{w}(k)_t \ , \ \mathbf{i}(k)_{t-1} \ , \mathbf{w}(k)_{t-1}) = -\mathbf{Int}(\mathbf{i}(k)_{t-1} \ , \mathbf{w}(k)_{t-1} \ , \mathbf{i}(k)_t \ , \mathbf{w}(k)_t)$$

The intuition here is that we should get a symmetric indicator by swapping the dates.

- **(8b) Time reversal for quantities**: As above, for the weight component.

$$\mathbf{Wgh}(\mathbf{i}(k)_t \ , \mathbf{w}(k)_t \ , \ \mathbf{i}(k)_{t-1} \ , \mathbf{w}(k)_{t-1}) = -\mathbf{Wgh}(\mathbf{i}(k)_{t-1} \ , \mathbf{w}(k)_{t-1} \ , \mathbf{i}(k)_t \ , \mathbf{w}(k)_t)$$

- **(9a) Rates symmetry**: If weights are interchanged along time (but rates are not) the interest rate component does not change.

$$\mathbf{Int}(\mathbf{i}(k)_t \ , \mathbf{w}(k)_t \ , \ \mathbf{i}(k)_{t-1} \ , \mathbf{w}(k)_{t-1}) = \mathbf{Int}(\mathbf{i}(k)_t \ , \mathbf{w}(k)_{t-1} \ , \mathbf{i}(k)_t \ , \mathbf{w}(k)_t)$$

This property is an adaptation of the symmetry property in usual index number theory. Although it lacks a clear intuitive meaning, it is included here for the sake of completion

- **(9b) Quantity weights symmetry**: If we interchange quantities for the two periods, the weight component should be the same.

$$\mathbf{Wgh}(\mathbf{i}(k)_{t-1} \ , \mathbf{w}(k)_t \ , \ \mathbf{i}(k)_t \ , \mathbf{w}(k)_{t-1}) = \mathbf{Wgh}(\mathbf{i}(k)_t \ , \mathbf{w}(k)_t \ , \mathbf{i}(k)_{t-1} \ , \mathbf{w}(k)_{t-1})$$

(10) Factors reversal: If we interchange rates and weights (for each period), the interest component and weight component would be interchanged.

$$\mathbf{Int}(\mathbf{w}(k)_t \ , \mathbf{i}(k)_t \ , \ \mathbf{w}(k)_{t-1} \ , \mathbf{i}(k)_{t-1}) = \mathbf{Wgh}(\mathbf{i}(k)_t \ , \mathbf{w}(k)_t \ , \mathbf{i}(k)_{t-1} \ , \mathbf{w}(k)_{t-1})$$

As in the case above, this property is an adaptation of the factors reversal property in usual index number theory. Although it lacks a clear intuitive meaning, it is included here for the sake of completion.

(11a) Continuity for rates: The interest rate component must be a continuous function.

- **(11b) Continuity for weights**: Similarly, the weight component must be a continuous function.

- **(12) Transitivity for rates**: Consider three subsequent periods, *t-1*, *t*, *t+1*. The property postulates that the overall change in the aggregate interest rate component (between *t-1* and *t+1*) equals the sum of the changes between the sub-periods ([t-1; t] and [t; t+1]):

$$\mathbf{Int}(\mathbf{i}_{t+1}(k), \mathbf{w}_{t+1}(k), \mathbf{i}_{t-1}(k), \mathbf{w}_{t-1}(k)) = \mathbf{Int}(\mathbf{i}_t(k), \mathbf{w}_t(k), \mathbf{i}_{t-1}(k), \mathbf{w}_{t-1}(k))$$
$$+ \mathbf{Int}(\mathbf{i}_{t+1}(k), \mathbf{w}_{t+1}(k), \mathbf{i}_t(k), \mathbf{w}_t(k))$$

In other words, according to the transitivity property, the interest rate component should be the same when it is directly computed for one longer period as when it is indirectly computed as the result of two shorter periods. This property is based on the price index transitivity test.

A number of properties treated in the usual price index axiomatic theory are not mentioned here because they seem difficult to adapt to a difference index. This the case of the bounding test as proposed by Diewert (2005), because its relevance in the context of the difference index is dubious. The commensurability test by Eichhorn and Voeller does not seem easily adaptable to a non-ratio context. The same holds for the value index preserving test (Voigt 1978) and the value dependence test.. Some other properties that only make sense at the level of the euro area (aggregate) components are presented in section 3.3.

3.2 Individual axiomatic properties at aggregate level

All the above properties have been defined at individual (country) level, referring to the individual rates i(k) or weights w(k) and the effects on the individual components Int(k) and Wgh(k). At aggregate level, some of the properties can be examined from the point of view of the impact of individual rates or weights on the aggregate components Int and Wgh; other properties can be examined by simply substituting individual by aggregate components, e.g. Int (k) by Int. In that respect, it is noted that euro area aggregate components are the sum of individual country components, because euro area interest rates are the weighted sum of country interest rates. Therefore,

$$\mathbf{Int} = \sum \mathbf{Int}(k); \quad \mathbf{Wgh} = \sum \mathbf{Wgh}(k);$$

If a property is fulfilled at individual country level, it is necessarily fulfilled at euro area level. However, the opposite it is not always true.

- **(1) Exhaustiveness**: The sum of the aggregate components Int, Wgh and the Mix (mixed component ,if it exists) should be equal to the change in the euro area interest rate $\Delta I_{t,t-1}$.

  $$\Delta \mathbf{I}_{t,t-1} = \mathbf{Int} + \mathbf{Wgh} + \mathbf{Mix}$$

  This property means that the decomposition has to contain, in its different components, the whole change in rates from one period to other. This property was proposed by Coene (2004b).

  (2a) Scale (or dimensionality) for rates: Multiplying each country interest rates in the previous $(i_{t-1}(k))$ and current $(i_t(k))$ period, <u>for all k</u>, by the same factor z results in the interest rate component being multiplied by the factor z.

  $$\mathbf{Int}(z * \mathbf{i}_t, \mathbf{w}_t, z * \mathbf{i}_{t-1}, \mathbf{w}_{t-1}) = z * \mathbf{Int}(\mathbf{i}_t, \mathbf{w}_t, \mathbf{i}_{t-1}, \mathbf{w}_{t-1})$$

- **(2b) Scale (or dimensionality) for weights**: This property goes along the above lines, just interchanging rates by weights.

$$\mathbf{Wgh}(\mathbf{i}_t, z * \mathbf{w_t}, \mathbf{i}_{t-1}, z * \mathbf{w_{t-1}}) = z * \mathbf{Wgh}(\mathbf{i}_t, \mathbf{w_t}, \mathbf{i}_{t-1}, \mathbf{w_{t-1}})$$

- **(3a) (Strong) Identity for rates**: For rates, on the assumption that <u>for each country</u> in both reference periods t-1 and t interest rates remain unchanged, the interest rate component (or contribution to the interest rate component) must be zero.

$$\mathbf{Int}([\mathbf{i}_t = i_{t-1}], \mathbf{w_t}, \mathbf{i}_{t-1}, \mathbf{w_{t-1}}) = 0$$

- **(3b) (Strong) Identity for weights**: The same property as above is defined for weights, just interchanging rates by weights.

$$\mathbf{Wgh}(\mathbf{i}(k)_t, [\mathbf{w}_t = w_{t-1}], \mathbf{i}_{t-1}, \mathbf{w_{t-1}}) = 0$$

- **(4a) (Strong) Monotonicity (in both periods) for rates**: Consider two different scenarios for the present period ("a" and "b") and the previous period ("c" and "d"). If for a country the interest rate in the present period in scenario "a" is higher than in scenario "b", $i(k)^a_t > i(k)^b_t$, and all <u>other country rates remain unchanged,</u> $i(j)^a_t = i(j)^b_{t-1}$ for all $j \neq k$, then the following inequality holds:

$$\mathbf{Int}(\mathbf{i}^a_t, \mathbf{w}_t, \mathbf{i}_{t-1}, \mathbf{w_{t-1}}) > \mathbf{Int}(\mathbf{i}^b_t, \mathbf{w}_t, \mathbf{i}_{t-1}, \mathbf{w_{t-1}})$$

,also applying with the opposite inequalities.

And if for a country the interest rate in the previous period in scenario "c" is higher than in scenario "d", $i(k)^c_{t-1} > i(k)^d_{t-1}$, and all <u>other country rates remain unchanged,</u> $i(j)^c_t = i(j)^d_{t-1}$ for all $j \neq k$, then the strong monotonicity test postulates that:

$$\mathbf{Int}(\mathbf{i}_t, \mathbf{w}_t, \mathbf{i}^c_{t-1}, \mathbf{w_{t-1}}) < \mathbf{Int}(\mathbf{i}_t, \mathbf{w}_t, \mathbf{i}^d_{t-1}, \mathbf{w_{t-1}}).$$

- **(4b) (Strong) Monotonicity (in both periods) for weights**: The same as in (4a) applies to weights, switching weights and rates.

- **(5a) Sign consistency for rates:** If the interest rate in a country increases (decreases), while it <u>remains constant in all other countries,</u> the interest rate component must also show an increase (decrease). Namely, if $i(k)_t > i(k)_{t-1}$ and $i(j)_t = i(j)_{t-1}$ for all $j \neq k$, then

$$\mathbf{Int}(\mathbf{i}_t, \mathbf{w}_t, \mathbf{i}_{t-1}, \mathbf{w_{t-1}}) > 0.$$

- **(5b) Sign consistency for weights:** This property in its strong form is not directly applicable to aggregate weights, because a change in weight in one country implies a change in at least one other country.

- **(6a) Proportionality for rates**: If the aggregate interest rate increases by the same amount in <u>all</u> countries and the weights remain unchanged, the aggregate interest rate component should increase by precisely that amount.

$$\mathbf{Int}([\mathbf{i}_t = \mathbf{i_t} + \mathbf{z}] , \mathbf{w_t} = \mathbf{w}_{t\text{-}1}, \mathbf{i}_{t-1}, \mathbf{w_{t\text{-}1}}) = z$$

- **(6b) Proportionality for weights**: This property in its strong form is not directly applicable to aggregate weights, because a change in weight in one country implies a change in at least one other country.

- **(7a) (Strong) Translation for rates**: Adding a constant z to <u>all country interest rates</u> in the previous ($i_{t-1}(k)$) and current ($i_t(k)$) period for all k leaves the interest rate component (Int) unchanged.

$$\mathbf{Int}(\mathbf{i}_t + \mathbf{z} , \mathbf{w}_t , \mathbf{i}_{t-1} + \mathbf{z}, \mathbf{w_{t\text{-}1}}) = \mathbf{Int}(\mathbf{i}_t, \mathbf{w}_t, \mathbf{i}_{t-1}, \mathbf{w}_{t-1})$$

- **(7b) (Strong) Translation for weights**: Adding a constant z to <u>all country interest rates</u> in the previous ($i_{t-1}(k)$) and current ($i_t(k)$) period for all k leaves the weight component (Wgh) unchanged.

$$\mathbf{Wgh}(\mathbf{i}_t + \mathbf{z} , \mathbf{w}_t , \mathbf{i}_{t-1} + \mathbf{z}, \mathbf{w_{t\text{-}1}}) = \mathbf{Wgh}(\mathbf{i}_t, \mathbf{w}_t, \mathbf{i}_{t-1}, \mathbf{w}_{t-1})$$

In other words, changes in present interest rates should not affect the weight component.

In the strong version, each country weight components would remain unchanged while in the weak version the aggregate would remain unchanged.

- **(8a) Time reversal for rates**: If we interchange prices and quantities of the two periods <u>in each country</u>, the interest rate component should be the same in absolute terms, with a change in sign.

$$\mathbf{Int}(\mathbf{i}_t , \mathbf{w_t} , \mathbf{i}_{t-1}, \mathbf{w_{t\text{-}1}}) = -\mathbf{Int}(\mathbf{i}_{t-1} , \mathbf{w_{t\text{-}1}}, \mathbf{i}_t , \mathbf{w_t})$$

The intuition here is that we should get the symmetric indicator by swapping the dates.

- **(8b) Time reversal for quantities**: As above, for the weight component.

$$\mathbf{Wgh}(\mathbf{i}_t , \mathbf{w_t} , \mathbf{i}_{t-1}, \mathbf{w_{t\text{-}1}}) = -\mathbf{Wgh}(\mathbf{i}_{t-1} , \mathbf{w_{t\text{-}1}}, \mathbf{i}_t , \mathbf{w_t})$$

- **(9a) Rates symmetry**: If the weights for the two periods are interchanged <u>in each country</u> (but rates are not) the interest rate component does not change.

$$\mathbf{Int}(\mathbf{i}(k)_t , \mathbf{w_t} , \mathbf{i}_{t-1}, \mathbf{w_{t\text{-}1}}) = \mathbf{Int}(\mathbf{i}_t , \mathbf{w_{t\text{-}1}}, \mathbf{i}_t , \mathbf{w_t})$$

- **(9b) Quantity weights symmetry**: If we interchange rates <u>in each country</u> for the two periods, the weight component should be the same.

$$\mathbf{Wgh}(\mathbf{i}_{t-1} , \mathbf{w_t} , \mathbf{i}_t , \mathbf{w_{t\text{-}1}}) = \mathbf{Wgh}(\mathbf{i}_t , \mathbf{w_t} , \mathbf{i}_{t-1}, \mathbf{w_{t\text{-}1}})$$

(10) Factors reversal: If we interchange rates and weights (for each period and country), the interest component and weight component are interchanged.

$$\mathbf{Int}(\mathbf{w}_t\ ,\mathbf{i}_t\ ,\ \mathbf{w}_{t-1}\ ,\mathbf{i}_{t\text{-}1}) = \mathbf{Wgh}(\mathbf{i}_t\ ,\mathbf{w}_t\ ,\mathbf{i}_{t-1}\ ,\mathbf{w}_{t\text{-}1})$$

- **(11a) Continuity for rates**: The interest rate component must be a continuous function.

- **(11b) Continuity for weights**: Similarly, the weight component must be a continuous function.

- **(12a) Transitivity for rates**: Consider three subsequent periods, *t-1*, *t*, *t+1*. The property postulates that the overall change in the aggregate interest rate component (between *t-1* and *t+1*) equals the sum of the changes between the sub-periods ([t-1; t] and [t; t+1]):

$$\mathbf{Int}(\mathbf{i}_{t+1}\ ,\mathbf{w}_{t+1}\ ,\mathbf{i}_{t\text{-}1},\mathbf{w}_{t\text{-}1}) = \mathbf{Int}(\mathbf{i}_t\ ,\mathbf{w}_t\ ,\mathbf{i}_{t-1}\ ,\mathbf{w}_{t\text{-}1}) + \mathbf{Int}(\mathbf{i}_{t+1}\ ,\mathbf{w}_{t+1}\ ,\mathbf{i}_t\ ,\mathbf{w}_t)\ .$$

- **(12b) Transitivity for weights**: Consider three subsequent periods, *t-1*, *t*, *t+1*. The property postulates that the overall change in the aggregate interest rate component (between *t-1* and *t+1*) equals the sum of the changes between the sub-periods ([t-1; t] and [t; t+1]):

$$\mathbf{Wgh}(\mathbf{i}_{t+1}\ ,\mathbf{w}_{t+1}\ ,\mathbf{i}_{t\text{-}1},\mathbf{w}_{t\text{-}1}) = \mathbf{Wgh}(\mathbf{i}_t\ ,\mathbf{w}_t\ ,\mathbf{i}_{t-1}\ ,\mathbf{w}_{t\text{-}1}) + \mathbf{Wgh}(\mathbf{i}_{t+1}\ ,\mathbf{w}_{t+1}\ ,\mathbf{i}_t\ ,\mathbf{w}_t)\ .$$

3.3 Aggregate level axiomatic properties

In addition to the above properties, some properties can only be defined at aggregate level. These are the following:

- **(1) Symmetry across countries**: Uniform permutations across countries have no influence on the aggregate interest component. If vectors $\widetilde{\mathbf{i}}_t$, $\widetilde{\mathbf{w}}_t$, $\widetilde{\mathbf{i}}_{t-1}$ and $\widetilde{\mathbf{w}}_{t\text{-}1}$ result from uniform permutations across countries within the vectors $\mathbf{i}_t$, $\mathbf{w}_t$, $\mathbf{i}_{t-1}$ and $\mathbf{w}_{t\text{-}1}$, then the following equation is satisfied:

$$\mathbf{Int}(\widetilde{\mathbf{i}}_t\ ,\widetilde{\mathbf{w}}_t\ ,\widetilde{\mathbf{i}}_{t-1}\ ,\widetilde{\mathbf{w}}_{t\text{-}1}) = \mathbf{Int}(\mathbf{i}_t\ ,\mathbf{w}_t\ ,\mathbf{i}_{t-1}\ ,\mathbf{w}_{t\text{-}1})$$

This property means that if all the data of two countries in both periods are swapped there is no impact on the interest rate index. Therefore the index is neutral with respect to the order of the data. This property is directly taken from the price index.

- **(2) Inversion**: When the only changes between two periods are swaps of interest rates and weights of two or more countries, no change should be recorded at the level of the aggregate interest rate component.

Let the vectors $\widetilde{\mathbf{i}}_{t-1}$ **and** $\widetilde{\mathbf{w}}_{t\text{-}1}$ be the result from the swap permutation of $\mathbf{i}_{t-1}$ and $\mathbf{w}_{t\text{-}1}$, such that for two given countries k and j rates and weights were swapped, i.e.

$i(j)_{t-1} = \tilde{i}(k)_{t-1}$, $i(k)_{t-1} = \tilde{i}(j)_{t-1}$, $w(j)_{t-1} = \tilde{w}(k)_{t-1}$, $w(k)_{t-1} = \tilde{w}(j)_{t-1}$, where $\tilde{i}(j)_{t-1}$ and $\tilde{i}(k)_{t-1}$ are the country interest rates after the permutation in the past period. For the rest of countries neither rates nor weights are permuted, i.e. for $l \neq j,k$, we have $i(l)_{t-1} = \tilde{i}(l)_{t-1}$ and $w(l)_{t-1} = \tilde{w}(l)_{t-1}$. In the present period, $\mathbf{i}_t$ and $\mathbf{w}_t$ remain unchanged, i.e. the data of two or more countries have only been swapped in the previous period but not in the present period. Then, the inversion test postulates that

$$\mathbf{Int}(\mathbf{i}_t\ ,\mathbf{w}_t\ ,\tilde{\mathbf{i}}_{t-1},\tilde{\mathbf{w}}_{t-1}) = \mathbf{Int}(\mathbf{i}_t\ ,\mathbf{w}_t\ ,\mathbf{i}_{t-1},\mathbf{w}_{t-1})$$

In other words, given that swapping data for two countries in one period (not in the second one) does not produce any difference in the total interest rates and its total change, the decomposition, and in particular the interest rate component, should not be affected by such swap.

- **(3) Mean value:** The interest rate component must be in-between the maximum and minimum changes in the interest rate from one period to another across countries.

  Max $\{i_t(k) - i_{t-1}(k),$ all k$\} < \mathbf{Int}(\mathbf{w}_t\ ,\mathbf{i}_t\ ,\ \mathbf{w}_{t-1},\mathbf{i}_{t-1}) < $ Min $\{i_t(k) - i_{t-1}(k),$ all k$\}$.

  This property is directly taken from the price index mean value test.

- **(4) Consistency in aggregation:** The interest rate component must show the same figures regardless of the number of stages in which it is compiled and the different possible partitions. For example, a country could be divided into two regions whose aggregate weight would sum up to the country. If the two regions are included in the euro area instead of the country, the final result should not change.

  This property permits to apply in an integrated form the decomposition to further levels of analysis, for example to institutions within each country. This property is based on the price index weak consistency in aggregation test.

3.4 Properties beyond mathematical axioms

The axioms in the previous sections can be strictly defined in mathematical terms. However, the above properties do not exhaust the list of possible desirable properties. In particular, other properties may be desirable even though they are more difficult to define in pure mathematical terms. These properties would include the following:

(1) **Simplicity**: Simplicity can be defined in this context by considering a) the number of terms in the decomposition, b) how many variables are used in each term and c) in which form these variables are combined. The simpler a decomposition, the easier to be understood by the users or the public.

(2) Intuitiveness: It would refer to whether the formula of the decomposition is appealing at first sight when explaining or analysing changes in overall interest rates. This property links with the simplicity property above, but goes beyond that, also considering the usefulness of the decomposition formula, for example, in regular publications.

(3) Interpretability: The results of the decomposition (and index) must be easy to be understood and communicated. This property is partially covered by other properties like proportionality or sign consistency. However, it covers also other aspects like helping to identify the country originating a change in the euro area rate.

(4) Relevance: Together with the interpretability, the relevance property analyses whether the decomposition would help in practice for the purpose of analysing changes in euro area interest rates.

Although the assessment of these properties may not be as clear-cut as for the other properties, they should not be undervalued. Actually, they may play a very relevant role in analysing the different alternatives and choosing the preferred decomposition and index.

4. Analysis of results and selection of decomposition

The previous sections have described a number of possible alternative decompositions and a set of axiomatic properties that would be desirable to have in the decomposition, and therefore in the subsequent index to be calculated on the basis of the decomposition interest rate component. This section analyses for each type of index whether these properties apply. The results of the mathematical properties are contained in Tables 1-3.

The results show that all decompositions comply with a majority of the axioms. This is good news, in the sense that whatever of the analysed decompositions is used to calculate the index, the results will be sensible. This is particularly true at aggregate level: all decompositions fulfil the properties that are only applicable at aggregate level (table 3), except for 'consistency in aggregation', which is not passed by the Walsh-type decomposition.

When testing the properties applicable at both country and individual level, the application at euro area level (table 2) reveals some relevant features. Importantly, the Walsh-type decomposition in 3 terms does not comply with the exhaustiveness decomposition in strict sense, because the third term does not have an explicit expression but can only be calculated as the difference between the total interest rate change and the interest rate and weight component. In practice, this implies that a part of the change in the interest rate is not allocated to any component but to a residual which is not easy to analyse. This feature is a strong indication for disregarding the Walsh-type decomposition. Secondly, it is noted that only the

Marshall-Edgeworth-type decomposition complies with the time reversal and symmetry tests. This shows that only this decomposition treats both periods symmetrically, and speaks in favour of its use. In the third place, the transitivity axiom is not accomplished by any decomposition. This would certainly be a desirable feature, however cannot serve here as a deciding criterion.

Regarding the results for the tests applied at individual country level, it is interesting to note that even though the extension of the formulas to include the euro area interest rate in the second component is appealing, it also has its weaknesses. In all cases, it leads to the failure of the exhaustiveness, monotonicity, sign, proportionality, and translation tests. However, the extension has interesting advantages in analysing the weight component, because it provides a good framework for analysing the weight effect in the context of the euro area aggregate. In particular, the extended weight component provides an interesting indication of the impact on the euro area rate of a change in the individual country weight by taking into account the important feature that total euro area weights should in any case sum up to one. Therefore, whenever there is a change in weight in one country, there is a change in the opposite direction in another country or countries. The net impact of these changes is determined by the relative positions of the countries in terms of interest rates in respect of the euro area average. As a consequence, the extended version of the Marshall-Edgeworth index better satisfies the criteria of relevance and intuitiveness when analysing individual country data, which is also shown in the example below.

Example: The effect of the extension in the Marshall-Edgeworth weight (Wgh) component.

Country	1	2	3	4	Total area
Interest rates					
t -1	5	4.5	5.5	3	4.6
t	5	4.5	5.5	3	4.3
Weights					
t-1	0.2	0.3	0.3	0.2	
t	0.2	0.1	0.3	0.4	
$\Delta I_{t, t-1}$					-0.3
Int	0	0	0	0	
1) Wgh (M-E)	0	-0.9	0	0.6	-0.3
2) Wgh (M-E extended)	0	-0.01	0	-0.29	-0.3

The advantages of the extension can be analysed in the above simple situation of an area composed of four countries. From t-1 to t has only occurred a redistribution in the weights, without any change in national interest rates. As a result of the weight in changes, the total area interest rates has reduced in 30 basis points.

1) According to the Marshall-Edgeworth decomposition, the reduction has been caused by the lost of weight of country 2, which has a weight component of -0.9, while the increase in the weight of country 2 results in a weight component of 0.6, resulting in the -0.3 total results.

2) Using the extended version of the Marshall-Edgeworth decomposition, almost all the impact (-0.29) is attributed to country 4, meaning that a country with low interest rates has gained weight at the expense of a country with average interest rates, and this movement explains the decrease in the euro area rates.

In terms of intuitiveness, interpretability and relevance, the extended version of the Marshall-Edgeworth index provides better results when analysing individual country contributions to a certain component. As shown in the example, at total area level the value of the weight component is the same. It is noted that the extension is only done (it can only be done) for the weight component, so it does not affect the interest rate component. .

On the basis of these arguments, it can be concluded that possibly the most useful decomposition is the Marshall-Edgeworth-type decomposition with extended weight effect. To resume, it is the following formula:

$$\Delta I_{t,t-1} = \sum_k \Delta i(k)_{t,t-1} \left(\frac{w(k)_t + w(k)_{t-1}}{2} \right) + \sum_k \Delta w(k)_{t,t-1} \left(\frac{(i(k)_t - I_t)_t + (i(k)_{t-1} - I_{t-1})}{2} \right) \qquad (2.13)$$

This decomposition has several advantages beyond the mathematic axiomatic results. First, it clearly separates into two components the impact of changes in interest rates (first term) and the impact of changes in weights (second term), not containing any intermediate or compound term, which would be more difficult to interpret. Second, it is symmetric between the two components in terms of the period to which they refer; the compound term of other decompositions is allocated to both components in the same way. Third, it permits a more robust analysis of the weight component of each individual country, as the weight effect is calculated by using both the individual (national) interest rate and the aggregated (euro area) rate, giving more relevance to those countries that have an interest rate more distant from the euro area average, i.e. it is analysed in relation to the euro area rate. Fourth, it considers the data of both periods involved, and it is also symmetric with respect to the periods.

However, for the use of the decomposition as an intermediate step in the construction of an index, the Marshall-Edgeworth-type decomposition without extended weight effect

$$\Delta I_{t,t-1} = \sum_k \Delta i(k)_{t,t-1} \left(\frac{w(k)_t + w(k)_{t-1}}{2} \right) + \sum_k \Delta w(k)_{t,t-1} \left(\frac{i(k)_t + i(k)_{t-1}}{2} \right) \qquad [2.12]$$

, seems more appropriate, because it fulfils most of the axiomatic properties and it has a simpler expression, while, importantly, the interest rate component is the same as in the decomposition with extended weight effect. The focus would be in creating an index for the interest rate component, on the basis of this decomposition.

Most of the proposed decompositions show good properties and probably would not give results very different from the one proposed by the ECB and the NBB, at least at aggregate level. It is recognised that the decision depends on the purpose and context of the analysis. In particular, decomposition methods isolating a compound effect provide 'pure' interest rate and weight effects, although it has to be kept in mind that further interest rate and weight effects are combined in this compound effect. In particular, when for some series and reference periods no business is reported at individual level, the impact of those 'gaps' may be identified in the compound effect of a difference Laspeyres-type decomposition. This combined nature may make the compound effect difficult to interpret.

Table 1: Axiomatic results for tests applicable at individual component (country) level

Decomposition / index	1.Exhaus	2.Scale	3.Identit	4.Mono	5.Sign	6.Propo	7.Transl	8.T. rever	9.Symmetry	10. F. rever	11. Con	12. Tran
Laspeyres 3 terms	Y	Y	Y	Y	Y	Y	Y	N	N	Y	Y	N
Laspeyres 3 terms extended	N	Y	Y	Y/N	Y/N	Y/N	Y/N	N	N	N	Y	N
Laspeyres 2 terms	Y	Y	Y	Y	Y	Y	Y	N	N	Y	Y	N
Laspeyres 2 terms extended	N	Y	Y	Y/N	Y/N	Y/N	Y/N	N	N	N	Y	N
Paasche 3 terms	Y	Y	Y	Y	Y	Y	Y	N	N	Y	Y	N
Paasche 3 terms extended	N	Y	Y	Y/N	Y/N	Y/N	Y/N	N	N	N	Y	N
Paasche 2 terms	Y	Y	Y	Y	Y	Y	Y	N	N	Y	Y	N
Paasche 2 terms extended	N	Y	Y	Y/N	Y/N	Y/N	Y/N	N	N	N	Y	N
Marshall-Edgeworth	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N
Marshall-Edgeworth ext.	N	Y	Y	Y/N	Y/N	Y/N	Y/N	Y	Y/N	N	Y	N
Walsh 3 terms	N	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N

Y indicates a positive result, N a negative result.
Y/N indicate a positive result for the interest rate component and a negative result for the weight component.

Table 2: Axiomatic results for test applicable at individual component (country) level –applied at aggregate (euro area) level

Decomposition /index	1.Exhaus	2.Scale	3.Identit	4.Mono	5.Sign	6.Propo	7.Transl	8.T. rever	9.Symmetry	10. F. rever	11. Con	12. Tran
Laspeyres 3 terms	Y	Y	Y	Y	Y	Y	Y	N	N	Y	Y	N
Laspeyres 3 terms extended	Y	Y	Y	Y	Y	Y	Y	N	N	Y	Y	N
Laspeyres 2 terms	Y	Y	Y	Y	Y	Y	Y	N	N	Y	Y	N
Laspeyres 2 terms extended	Y	Y	Y	Y	Y	Y	Y	N	N	Y	Y	N
Paasche 3 terms	Y	Y	Y	Y	Y	Y	Y	N	N	Y	Y	N
Paasche 3 terms extended	Y	Y	Y	Y	Y	Y	Y	N	N	Y	Y	N
Paasche 2 terms	Y	Y	Y	Y	Y	Y	Y	N	N	Y	Y	N
Paasche 2 terms extended	Y	Y	Y	Y	Y	Y	Y	N	N	Y	Y	N
Marshall-Edgeworth	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N
Marshall-Edgeworth ext.	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N
Walsh 3 terms	N	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N

Y indicates a positive result, N a negative result.

Table 3: Axiomatic results for test only applicable at aggregate (euro area) level

Decomposition / index	1.Sym c.	2. Inversion	3. Mean	4. Cons agg.
Laspeyres 3 terms	Y	Y	Y	Y
Laspeyres 3 terms extended	Y	Y	Y	Y
Laspeyres 2 terms	Y	Y	Y	Y
Laspeyres 2 terms extended	Y	Y	Y	Y
Paasche 3 terms	Y	Y	Y	Y
Paasche 3 terms extended	Y	Y	Y	Y
Paasche 2 terms	Y	Y	Y	Y
Paasche 2 terms extended	Y	Y	Y	Y
Marshall-Edgeworth	Y	Y	Y	Y
Marshall-Edgeworth ext.	Y	Y	Y	Y
Walsh 3 terms	Y	Y	Y	N

Y indicates a positive result, N a negative result.

5. From binary to multiple period comparisons

In the previous sections the analysis has focused on the comparison of two consecutive periods, decomposing the movement from one period to the next one into its different components. Each of these components (interest and weight components) is a binary difference index that compares two periods. Similarly to the usual index theory, the difference indices can also be expanded to constitute a series of data that permits different comparisons across time. For doing that, two different aspects must be considered.

Firstly, it must be decided what precise formula will be used for the comparison. This issue has already being discussed in the previous sections and therefore, following the previous results, the Marshall-Edgeworth-type decomposition, will be used. As explained in the previous section, the not-extended version is used for the comparison with multiple periods.

In the second place, it is noted that the components of the decomposition only compare two periods at a time. Therefore, the question arises, when having multiple periods, which period is to be compared with which period.

This section will focus on the different alternatives, selecting different periods to be compared. In pure conceptual terms, the alternatives for multiple comparisons range from comparing each period with a direct[13] index for the whole length of the series to a comparison of each period with the consecutive one and "chaining" the results to form a series, covering intermediate solutions in which the fixed period would change with a certain frequency but not every period, also implying "chaining" at the time of change in the fixed period. For completion, an extension of the "chaining" to continuous time, a Divisia Index, is also examined. All different alternatives are based on the usual index number theory, adapting it to a difference index.

5.1 Direct index

For price indices, a direct index means that the index is calculated for each period by directly comparing the price situation at that month with the situation at a fixed month, usually (although not necessarily) the first reference period in the series. Similarly, in MIR the interest rate index would be constructed as the interest rate component of the decomposition of the changes between each month t and month 0. It is noted that between t and 0 a number of t-1 periods have occurred, with their corresponding rates and weights, which are not considered in the direct index at time t. Previous values to t are considered for the

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[13] Von der Lippe (2001) prefers the use of "direct" to the more common "fixed base" because for a number of indices a direct comparison between the two periods is possible, but there is no fixed weighting, rather, the weights are changing with the period. This is the case in our situation.

value of the index at its corresponding time, resulting in the construction of a series of the index for time 0,1,2 …t , in the same way as for as a for a direct index in usual index number theory.

The total change in interest rate $\Delta I_{t,\,0}$ is decomposed according to the Marshall-Edgeworth decomposition:

$$\Delta I_{t,0} = \sum_{k} \Delta i(k)_{t,0} \left(\frac{w(k)_t + w(k)_0}{2} \right) + \sum_{k} \Delta w(k)_{t,0} \left(\frac{i(k)_t + i(k)_0}{2} \right)$$

Therefore, the difference direct index of interest rates on the basis of the interest rate component of the Marshall-Edgeworth-type decomposition and is calculated as follows:

$$Index\_I = Index_t^{DDME} = \sum_{k} \Delta i(k)_{t,0} \left(\frac{w(k)_t + w(k)_0}{2} \right)$$

where DDME indicates "direct difference Marshall-Edgeworth"-type (index).

As the index is calculated as the sum of the interest rate effects for all countries, to better understand it we can focus on one summand, corresponding to a particular country k.

National contribution of country k : $Index_t^{DDME}(k) = \Delta i(k)_{t,\,0} * [\, w(k)_t + w(k)_0 \,] / 2$

We know that the index is composed of similar contributions for each country. That permits to visualise in graphical terms the calculation of the index, by focusing on one country contribution to the index. The chart below represents a concrete example: At the base period 0 the interest rate is 2% and the weight of the country compared to the total euro area is 0.4, and at period t the interest rate is 4% and the weight is 0.6. The country contribution to the euro area interest rate at each period is determined by the area of the rectangle defined by the 'interest rate' and 'weight' coordinates. In this case the rectangle at time t is larger than at time 0 because both interest rate and weight have increased. The difference between the two rectangles is decomposed into the interest rate effect and the weight effect. The national contribution to the interest rate Marshall-Edgeworth-type difference direct index ($Index_t^{DDME}(k)$) at time t is given by the rectangle that is adjacent to the 'interest rate' axis and lies between the small and the large rectangle, contained on the right hand side of the larger figure.

Figure 2. Weight effect and interest rate index (1)

5.2 Chain index

 In the context of price indices, a chain index means that the index is calculated for each period by comparing the price situation of this period with the previous period, and then linking the results by multiplying (chaining) each individual link with the previous one. The procedure is similar in the case of a difference index, with the difference that each link is chained to the previous one by adding them. Therefore, the difference chain index of interest rates on the basis of the Marshall-Edgeworth-type decomposition is as follows:

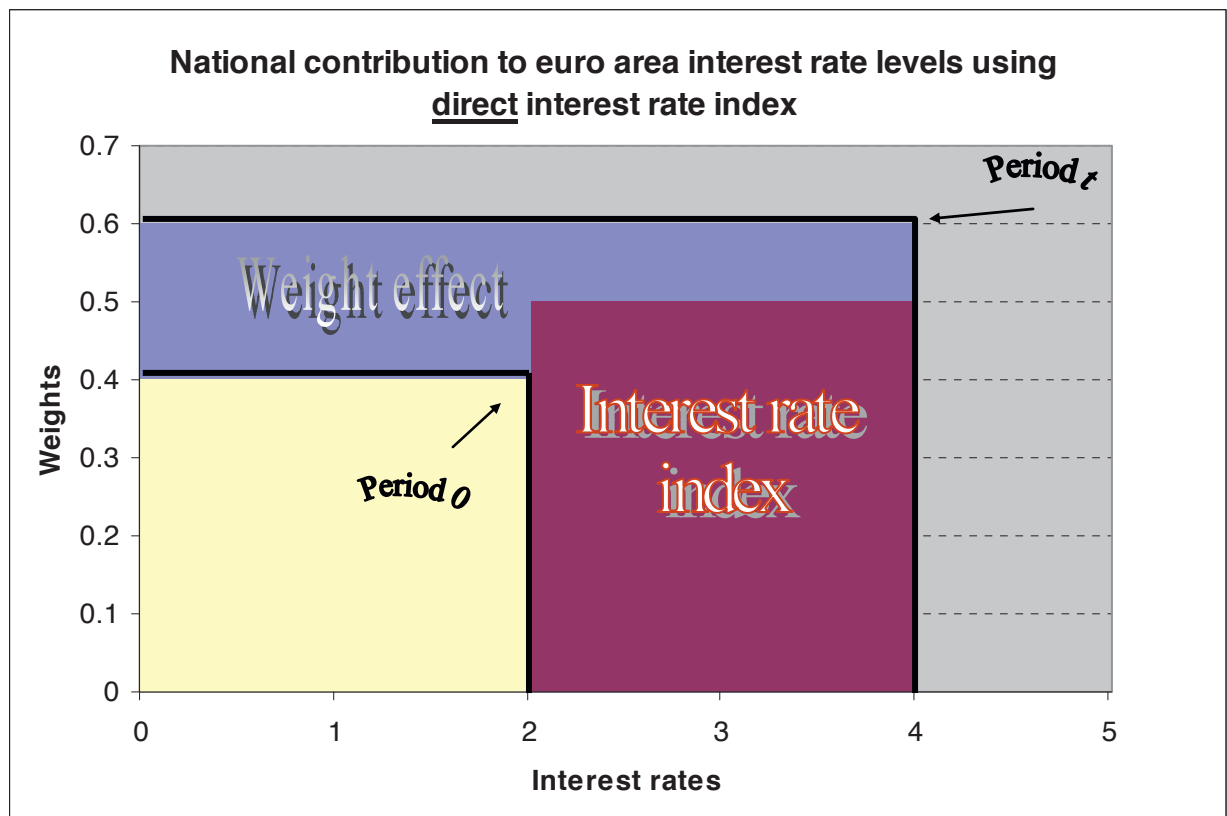$$Index\_II = Index_t^{DCME} = \sum_t \sum_k \Delta i(k)_{t,t-1}\left(\frac{w(k)_t + w(k)_{t-1}}{2}\right)$$

where DCME stands for "chain difference Marshall-Edgeworth-type"(index)

The chain index is obtained as the sum of national contributions, which is shown by just changing the order of the sums:

$$Index_t^{DCME} = \sum_k \sum_t \Delta i(k)_{t,t-1}\left(\frac{w(k)_t + w(k)_{t-1}}{2}\right)$$

As in the case of the direct index, it is possible to separate the national contribution of country k, in this case referred to one of the links in the chain, the one corresponding to period s:

$$Index_{t=s}^{DCME}(k) = \Delta i(k)_{s,s-1} * [\, w(k)_s + w(k)_{s-1} \,] / 2$$

Under the same conditions as before, we can visualise the calculations in a chart. In this case we assume that t=10, so 10 periods have occurred between the initial period and the present period. It is noted that neither rates nor weights are constrained to change linearly, not even to change always with the same sign. In the example below, for the sake of clarity, rates are changing linearly while there is a cyclicality factor in the changes in weights. Even with that linear restriction on the change in interest rates, it is obvious that the chain index does not generally coincide with the direct index. Here the national contribution to the interest rate Marshall-Edgeworth-type difference chain index ($Index_t^{CDME}(k)$) at time t is determined by sum of the rectangles contained on the right hand side of the larger figure.

Figure 2. Weight effect and interest rate index (2)
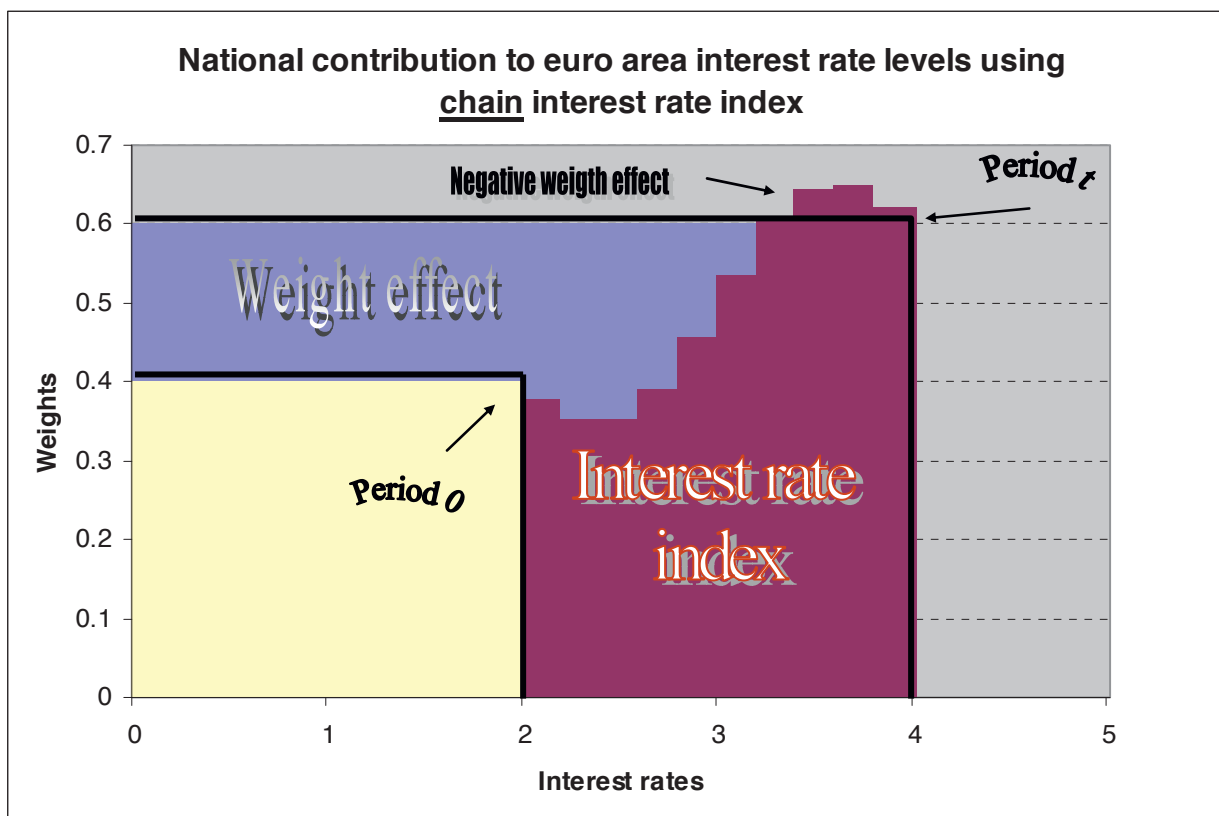
5.3 Divisia index

In theory, a chain index can be translated to a continuous time modelisation. With continuous time, the sum appearing in the chain index formula would be substituted by an integral, as follows:

$$Index_t^{DSME} = \oint_t w(k)_t * \frac{di(k)_t}{dt} * dt$$

where DSME stands for "difference Divisia Marshall-Edgeworth-type" index.

However, this index is not applicable in practice, because we have neither a function that explains weights in terms of time nor a function of interest rates in terms of time. Therefore additional input would be needed to model and estimate the appropriate functions. Furthermore, the functions would need to be re-estimated backwards every time a new observation becomes available.

Using the already used visualisation scheme, an approximation to the Divisia index is presented, also following a linear increase of rates with time and including a cyclical component in the weight function.

Figure 3. Weight effect and interest rate index (3)
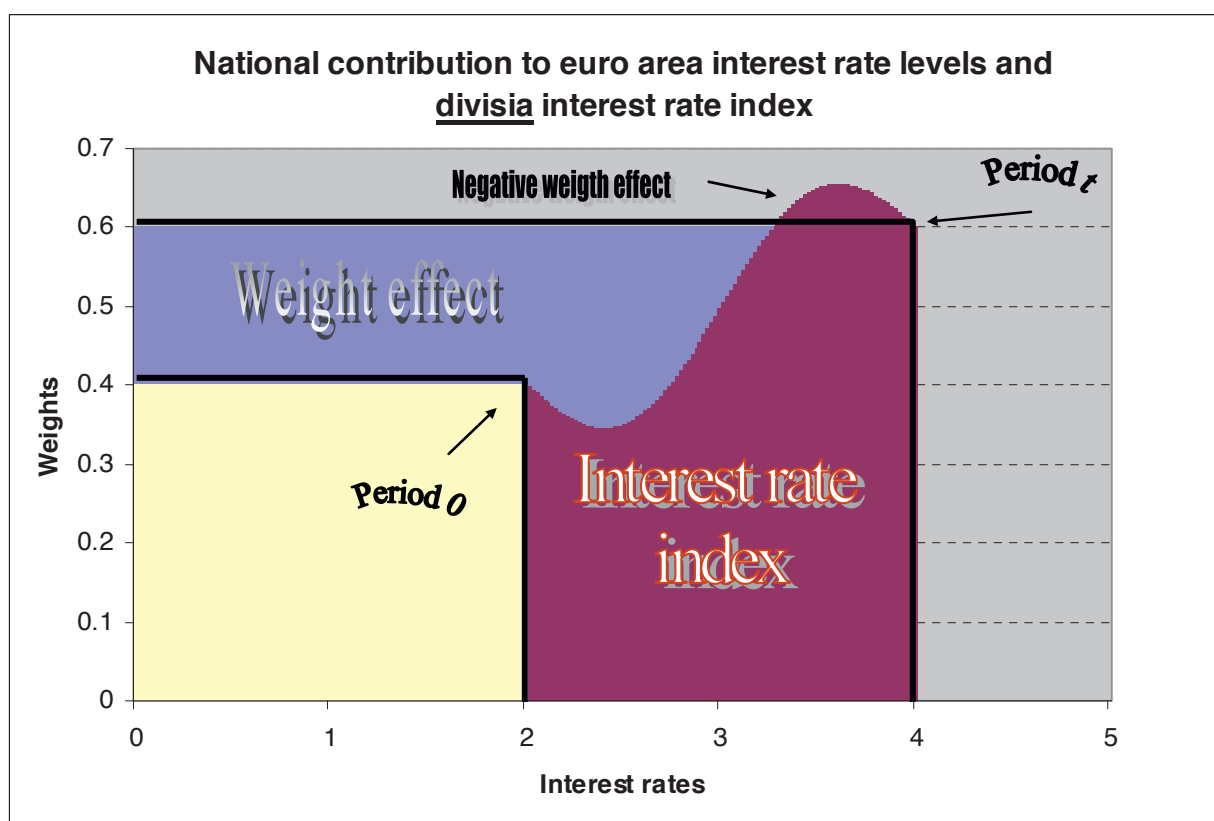
5.4 Re-basable direct Index

A possible compromise between the direct index and the chain index would be a rebasable direct index, consisting in applying the direct index for a limited number of periods and as from one point in time referring the comparisons to a different point in time. The frequency of the change in the reference point can be different; for monthly series, it would possibly be defined in terms of years, e.g. every year or every 5 years. The index formula would be as follows:

$$Index\_III = Index_t^{DRME} = \sum_k \Delta i(k)_{t,s} \left(\frac{w(k)_t + w(k)_s}{2} \right) + Index_s^{DRME}$$ for periods where the last

change in reference period before t happened in s (starting with s=0). In the case of yearly rebasing, s=0,12,24, …, in case of quarterly rebasing s= 0,3,6, 9 … DRME stands for "difference re-basable Marshall-Edgeworth-type"(index).

In terms of the example, we can represent it with a change every 5 periods. Therefore a change in the reference period for comparison would occur once during the period considered, and the index at time t can be visualised as the sum of the rectangles on the right hand side of the chart.

Figure 3. Weight effect and interest rate index (4)
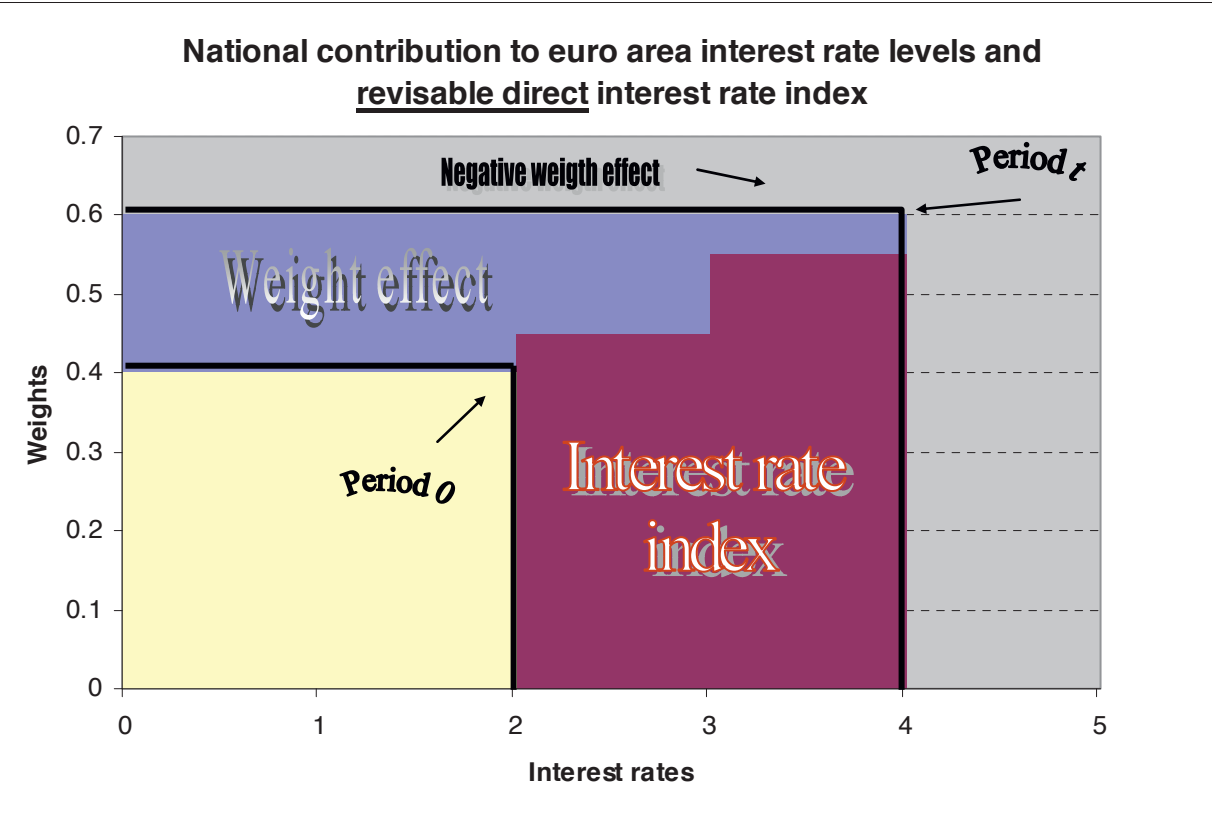
5.5 Direct index versus chain index

As shown in the previous section, the main difference between a chain index and a direct index is that a chain index implicitly reflects in its value the road followed from period 0 to period t, giving different results depending on the road followed from one point to another. A direct index provides the same results for a point in time regardless of the path of intermediate data, because it only considers the weights at the first and last period.

A large part of the literature on index numbers has discussed the relative advantages and disadvantages of a direct index in comparison with chain indices. For example, authors like Stuvel (1989) defend the chain index since that solution solves the "index number problem" of updating the weights; furthermore, this latest approach is recommended in international statistical standards. Conversely, other authors like Von der Lippe (2001), a firm advocate of direct indices, makes clear that "it is the primarily the idea of making pure comparisons, or of comparing 'like with like' which is ignored by chainers, but which is on the other hand a cornerstone of index theory". It is also noted that international statistical standards recommend the use of chain indices.

Without any intention of generalising the discussion to index numbers in general, in the particular case of the propose application of a Marshall-Edgeworth-type difference indices to MIR, the following consideration can be made:

- A clear advantage of a chain index is that the weights are continuously updated. That results in that, when comparing the index at any two points in time, only the weights for the chosen interval have an impact on the changes in the index. It is also noted that, contrary to other data sets for which relative weights may not always be available with the same frequency as prices, that problem does not occur in MIR.

- On the contrary, a direct (Marshall-Edgeworth-type) index implies that the initial weighting has a very strong bearing on the whole index. When trying to find out the evolution of interest rates in isolation of weights between two periods different from the initial period, it seems somehow strange that the weights of the initial period have a bearing on the final result regardless of how distant from the initial period the two periods examined are, and how much may have changed in the meantime.

- Some of the possible advantages of direct indices in usual index theory do not apply to difference indices. In particular, some direct ratio indices (e.g. Laspeyres or Paasche, but not Fisher) can be interpreted as a ratio of expenditures together with a mean of price relatives. This interpretation is not applicable (or easily adaptable) to difference indices, regardless on whether they are direct or chain indices.

- A possible disadvantage of a chain index is that it is unclear whether considering the intermediate steps when comparing euro area interest rates between two points in time would help an analyst, who is possibly more interested in knowing whether the difference in rates between the two separate periods is attributable to changes in national rates or to changes in country weights. On the other hand, the evolution of MIR data is monitored in the ECB on a monthly basis, and the chain index would better help to link the monthly analysis with the longer period comparisons.

- Further disadvantages of a chain and direct re-basable indices, are that they may be more difficult to interpret and there is a general inapplicability or failure in axiomatic considerations when applied to the indices, even though they are applicable to each link of the chain or to segments of the re-basable indices, as seen in previous sections.

- The possible advantages of a rebasable index would be that while the drawbacks of the direct index remain, they are limited in time, depending on the frequency of rebasing, and therefore possibly in size. Of course, the question to be raised is the frequency of the rebasing and on which criteria to decide it. A typical proposal is yearly rebasing, which is equally arbitrary as any other frequency, but possibly more user-friendly. However, this index may show breaks in the series at the time of each re-basement, which would make them difficult to interpret.

- Finally, the advantages of a chain index would be re-enforced if the different indices do not differ very much in practice. In that case, it could be said that in normal circumstances all indices provide similar results, but in case there is any important (isolated) change in weights, it would be taken into account in the index only for the span of periods included in the comparison.

Taking into account this last point, the decision on the preferred index is postpone to the analysis of the data in section 7.

6. Notional interest rates: One step forward?

It is still possible to explore further constructions on the basis of the alternative difference indices. In particular, all indices proposed in the previous section will show positive or negative values expressing changes in interest rates in respect of a previous period. The question has been raised whether it would be possible to have an indicator that expresses the value of aggregate euro area rates in terms of levels of interest rates, from which the effect of changes in weights would have been discounted.

We discuss here whether it is possible to construct a "notional stock", in this case referring to interest rates, on the basis of flows defined as changes in rates once the change of weight is discounted. That is of course mathematically possible by simply building a series on the basis of the successions defined by the previously discussed difference index. For the initial period, the series can take the actual interest rate value, although that is not strictly necessary.

By using this method the notional rates series would be defined as follows

The notional rate based on the direct index:

$$Nr\_I = Nr_t^{DDME} = Nr_{t-1}^{DDME} + Index_t^{DDME} = Nr_{t-1}^{DDME} + \sum_k \Delta i(k)_{t,0}\left(\frac{w(k)_t + w(k)_0}{2}\right)$$

Where Nr_t^{DDME} stands for the notional rate based on direct difference Marshall-Edgeworth-type index. It is noted that at the starting point of the notional index, t=0, the notional index must be calculated as the euro area interest rate, i.e. $Nr_{t=0}^{DDME} = I_0$, where I is the euro area interest rate.

The notional rate based on the chain index:

$$Nr\_II = Nr_t^{DCME} = Nr_{t-1}^{DCME} + Index_t^{DCME} = Nr_{t-1}^{DCME} + \sum_t \sum_k \Delta i(k)_{t,0}\left(\frac{w(k)_t + w(k)_{t-1}}{2}\right)$$

Where Nr_t^{DCME} stands for the notional rate based on "chain difference Marshall-Edgeworth-type"index. As in the previous case, it is noted that at the starting point of the notional index, t=0, the notional index must be calculated as the euro area interest rate, i.e. $Nr_{t=0}^{DCME} = I_0$, where I is the euro area interest rate.

The notional rate based on the rebasable rate index:

$$Nr\_III = Nr_t^{DRME} = Nr_{t-1}^{DRME} + Index_t^{DRME} = Nr_{t-1}^{DCME} + \sum_k \Delta i(k)_{t,s}\left(\frac{w(k)_t + w(k)_s}{2}\right)$$

Where Nr_t^{DRME} stands for the notional rate based on "difference rebasable Marshall-Edgeworth-type"index. As in the previous case, it is noted that at the starting point of the notional index, t=0, the notional index must be calculated as the euro area interest rate, i.e. $Nr_{t=0}^{DRME} = I_0$, where I is the euro area interest rate; in the index s indicates the latest re-basing period.

In the charts below the indices are compared with the stocks for a particular interest rate indicator (NB13. New business. Loans to HHs for consumption, with initial period of rate fixation up to 1 year). The first chart below presents the indices discussed above together with the accumulated change in the euro area interest rates as from January 2003. The second chart presents the notional interest rates calculated on the three indices, together with the original euro area interest rates series. The most important feature to note is that both charts show exactly the same pattern, for both the original series when compared with the accumulated changes and for the indices when compared with the notional stocks, in both cases with a level shift corresponding to the interest rate in January 2007. In other words, the informational content of the index and notional stocks is the same.

Chart 1. Indices applied to MIR indicator NB13

NB13 - New business. Loans to HHs for consumption, with
initial period of interest rate fixation up to 1 year

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing
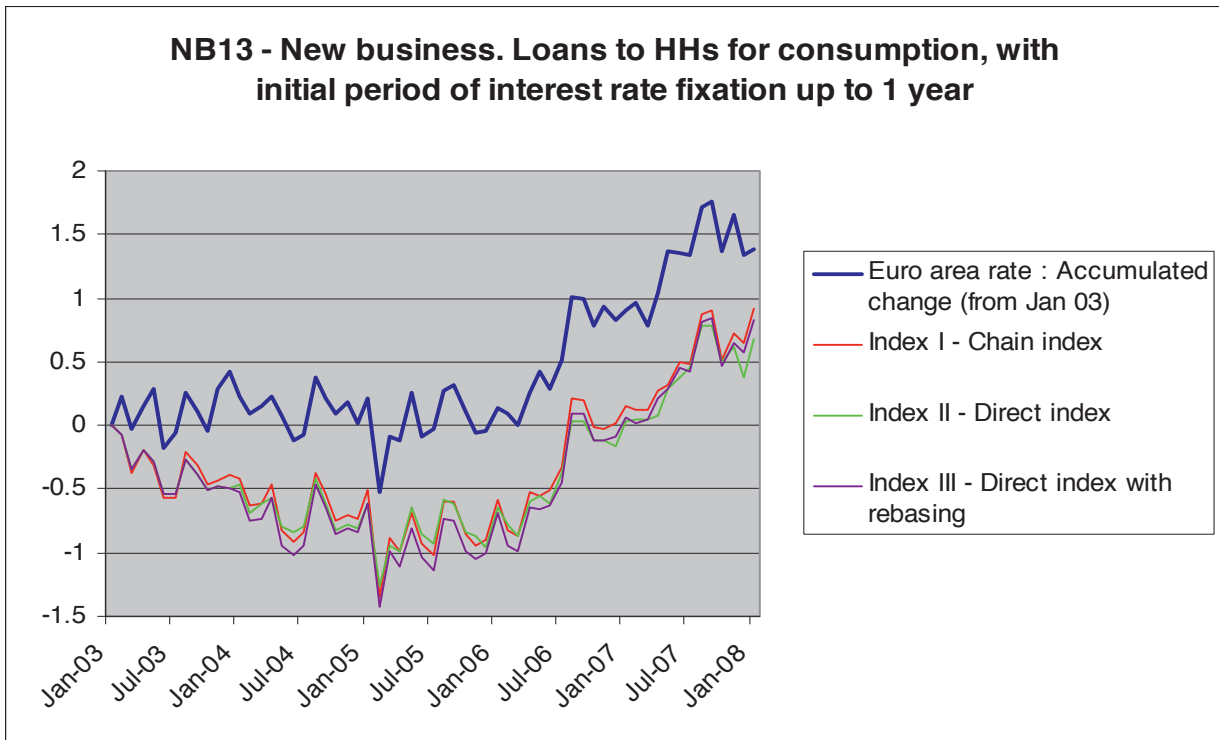
Chart 2. Notional interest rates calculated on MIR indicator NB13

Notional rates - NB13

Legend:
- Euro area rate: levels
- Notional rate I - Based on chain index
- Notional rate II - Based on direct index
- Notional rate III - Based on direct index with rebasing
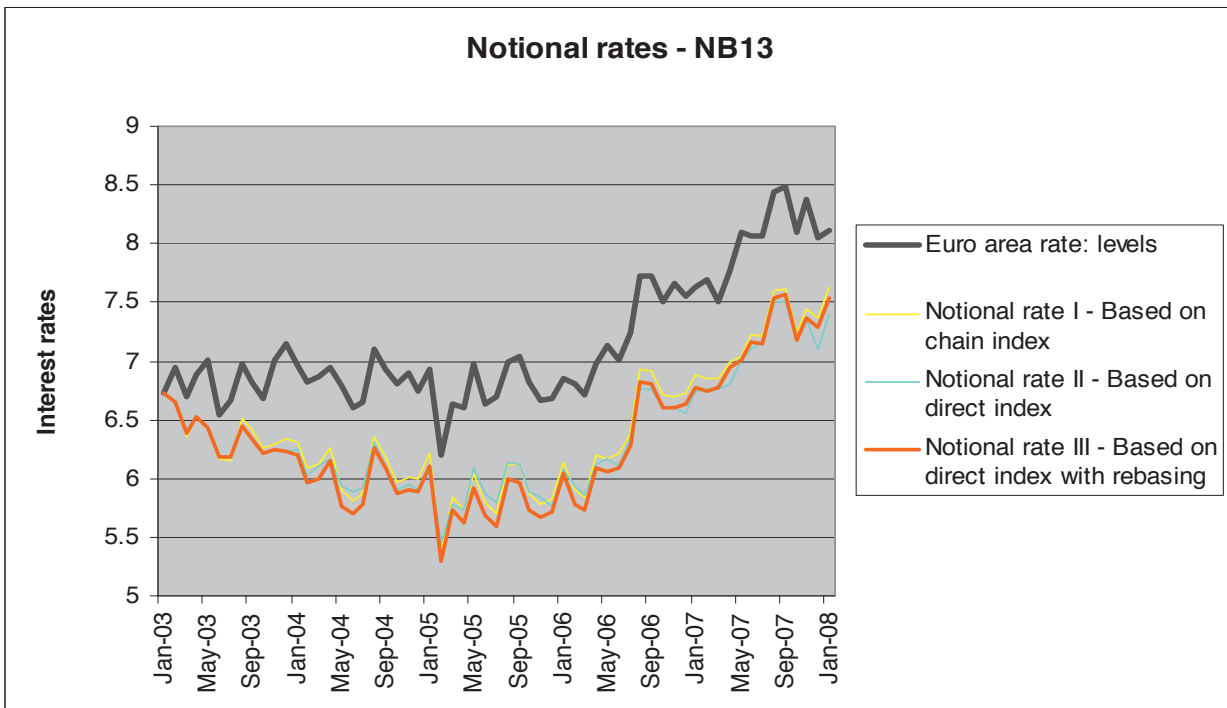
Therefore question at stake is not the mathematical feasibility of the calculation but whether the notional rate would provide additional and/or easy to interpret information. One danger of the presentation of an

indicator like this one in terms of rates is that it may be confused with the true euro area rate. At the same time, the possible value added by the notional rate is in the comparisons of two periods, calculating the difference between them. However, this difference can be expressed and obtained with the index.

Therefore, to use a notional interest rate or an index of interest rates is more a presentational matter than a question of substance. In that sense, for the reasons above, the index seems more appropriate.

7. Application of indices to MIR - January 2003 to January 2008

In order to analyse the possible relevance of the three indices (Index I, Index II, Index III) proposed in section 5, they have been applied to euro area MIR monthy data for the period January 2003 to January 2008. Each index is compared with the accumulated change of the actual euro area aggregate interest rate starting in January 2003. The results are presented in two complementary ways, firstly, in annex 2, tables 1 to 3 show a summary of the values (average deviation, maximum, minimum, closest value to zero, difference at the end of the period analysed) of each index when compared with the accumulated changes in the original series of euro area interest rate levels and tables 4 to 6 shows the differences between the three indices. For each type of index (chain index, direct index, direct index with annual rebasing) and month, the difference between the index and the accumulated changes (i.e. $\Delta I_{t,\,0} - \textbf{Index}_t$) is calculated. In the second way of comparing results, in annex 3, a chart for each MIR indicator shows the evolution of the accumulated changes of the euro area rate and the developments as reflected by the three alternative indices. It is noted that similar comparisons could have been done by using the notional rates rather than the index, and the results would have shown exactly the same figures. Here the indices are chosen after the discussion in the previous section.

A particularity of MIR has been taken into account in the calculations. Whenever no operation has taken place on new business or no outstanding amounts remains for a single category in a country, no figure is reported to the ECB for that country. If this absence of interest rate figure were treated as zero it would result in a spurious impact in the interest rate component. To avoid this, whenever no interest rate was reported for a specific category and month, the latest previously reported interest rate is used to calculate the interest rate effect, resulting in no impact on the interest rate component.[14]

Regarding the summary indicators (annex 2) of the indices when compared with the original series, the average difference is very low for Index I, with the almost only exception of NB13 (annex 2, table 1). The standard deviation of these differences also shows a relatively low value, with the exception of NB10 and NB13. The maximum difference, in absolute values (maximum and minimum of the difference), can nevertheless be higher, reaching 105 bp for NB13; for NB9 the maximum difference is 66 bp, 59 bp for NB10 and significantly lower for other categories. Looking at Index II, similar results are obtained, only

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[14] As suggested by Olivier Coene (National Bank of Belgium) and Magda Gregorova (Czech National Bank)

slightly higher (annex 2, table 2). Again, NB13 and NB10 are almost the only categories for which, as an average, there are relevant differences with the original data. Similar results are repeated for the maximum differences, 108 bp in NB13 and 83 bp in NB10, here with the opposite sign. The differences increase somehow when using Index III, ranging in average from around 0 bp to 85 bp. In terms of maximum at any point, NB13 obtains the maximum value, 116 bp (annex 2, table 3).

In terms of comparing the different indices between them, the highest difference appears when comparing Index I and Index II both in terms of averages and maximum (annex 2, table 5). These differences are particularly visible for averages in categories NB9 and NB10; in terms of maximums, apart from the previous categories, NB4 and NB29 also show some relatively relevant differences. The differences when comparing Index I and III are smaller for some categories but higher for others. The average difference is below 10 basis points for all categories, except for NB9 and NB10 (annex 2, table 5). The maximum differences are somehow higher than the average, but still only relevant, apart from NB9 and NB10, for NB13, NB29 and NB28. When comparing Indices II and III the average difference diminishes in general, the maximum being 8 bp in NB10; however the maximum is higher is some cases, like 65 bp in NB9 and also relevant for NB10 and NB13 (annex 2, table 6).

On the comparison with the help of charts (annex 3), for each MIR category, the accumulated change in levels as from January 2003 is compared with the evolution of Indices I, II and III. It is noted that some charts are scaled differently, according to the evolution of the particular category. In that sense, the charts make more visible the differences proportional to size of change in the original series, while the summary tables commented above, refer to differences in absolute terms. The charts confirm the divergence of the index from the original series in absolute terms for MIR categories NB5, NB9, NB10, NB13, NB28, NB29; a moderate deviation appears in MIR categories NB15 OA4, OA8, OA14. In relative terms differences also arise for other indicators with very small developments in terms of interest rates e.g. NB4, OA2. For the other MIR categories, only small or negligible impact is shown.

The first possible conclusion from this analysis is that all three indices do not deviate much from the original series for most of the MIR categories, indicating that the changes in weight across countries are generally small, having a very limited impact on euro area aggregates for long periods. The second conclusion is that Index I and Index II show a very similar behaviour, staying also closer to the original data for most of the categories. In view of these considerations, further analysis of the indices must concentrate on those categories that show relevant differences. In particular on NB5, NB9, NB10, NB15, NB28, NB29, OA4, OA8, OA14 and especially NB13. Focusing on these categories, it is observed that the three indices behave in a very similar way in OA4, OA8, OA14 and to a large extent in NB13 and NB28. Differences in the developments of the different Indices appear for some periods of NB29, and are more prominent in NB9 and in particular in NB10 (see chart 3). In the latest two categories, the chain index deviates more from the original series for some periods, while at the end of the period examined converge again with the original series; on the contrary, in the OA categories the difference between the

direct index and the other two increases moderately with time. Considering these results there could be some preference for the chain index, which signals in a clearer way the weight changes for some periods.

Chart 3: Indices applied to NB 10 for a selected period (Mar05 to Mar06)

NB10 - New business. Deposits from NFCs with maturity over 2 years

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing
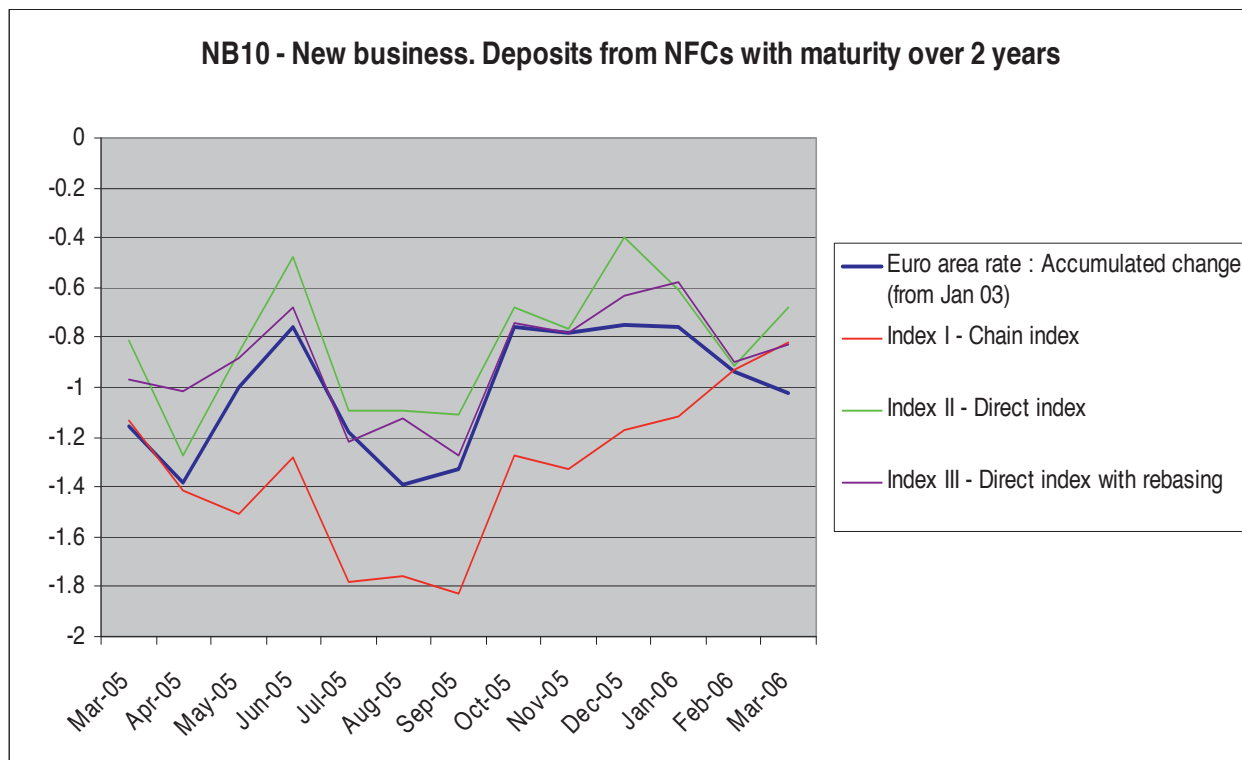
Focusing on the series that shows the higher differences in the development of the indices, the decomposition is an important tool on its own, in addition to being a block of the index. The decomposition with full detail by country has been internally used in the ECB for several years, in order to analyse the main factors influencing euro area rate changes. In this case, it is shown that the decomposition at aggregate level (without breakdowns by country), can also provide relevant insights. In particular annex 4 presents the results for selected periods or bilateral indices) focus on selected periods of the MIR categories NB5, NB10, NB13 and NB29.

NB5 is possibly the most illustrative case on the link between indices and decomposition. For this MIR category, a weight effect occurred in June 2005 (chart 4); this effect caused a drop of the Indices in June while the original series increases (chart 5). For the subsequent periods, the indices and original series behave very much the same way, with just a difference in level coming from that particular weight effect. In fact, the weight effect in June 2005 is caused by a change in the statistical classification of a particular financial instrument in one member state. Therefore the index correctly discounts the weight change caused by this statistical re-classification.[15]

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[15] Other ways of avoiding this type of statistical break, like for example the reporting of pre-break values, are beyond the scope of this paper.

Chart 4: Decomposition to NB 10 for a selected period (Apr05 to Mar06)

NB5 - New business. Deposits from HHs redeemable at notice,
up to 3 month notice.
Decomposition
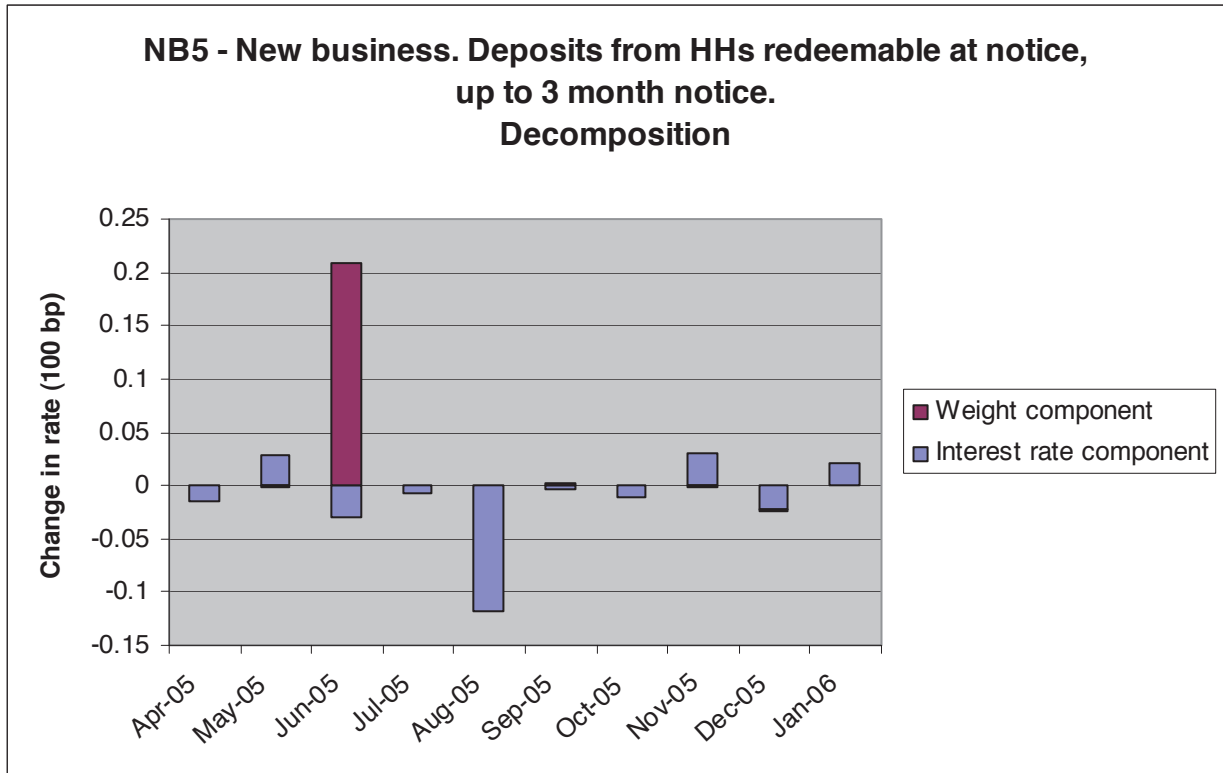
Chart 5: Indices applied to NB 10 for a selected period (Apr05 to Mar06)

NB5 - New business. Deposits from HHs redeemable at notice,
up to 3 month notice
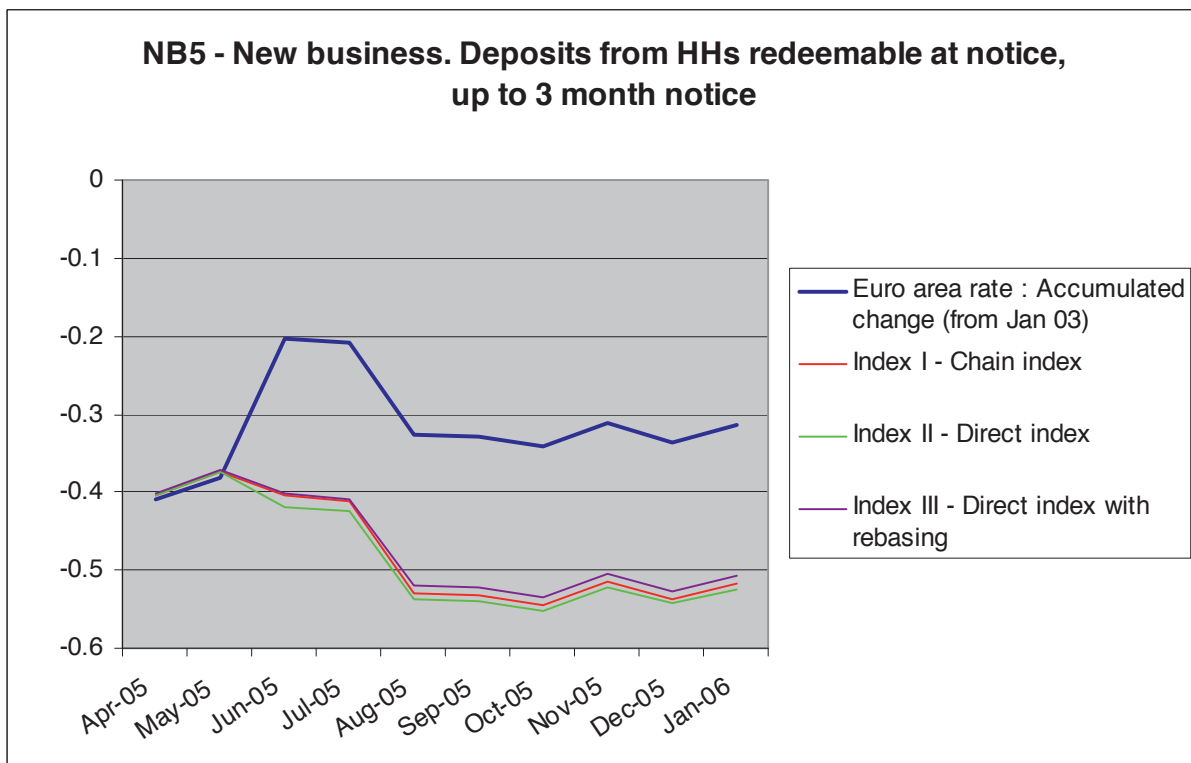
NB13 shows a development along the same line (see chart 1), with a number of important weight effects in 2003 and beginning of 2004; after that period the impact of weight effects considerably reduce, and the indices behave very much like the original series, with a shift level.

For NB29 the weight effects compensate along time, resulting in the indices converging again with the original series at the end of 2004.

Chart 6: Indices applied to NB 10 for a selected period (Apr05 to Mar06)
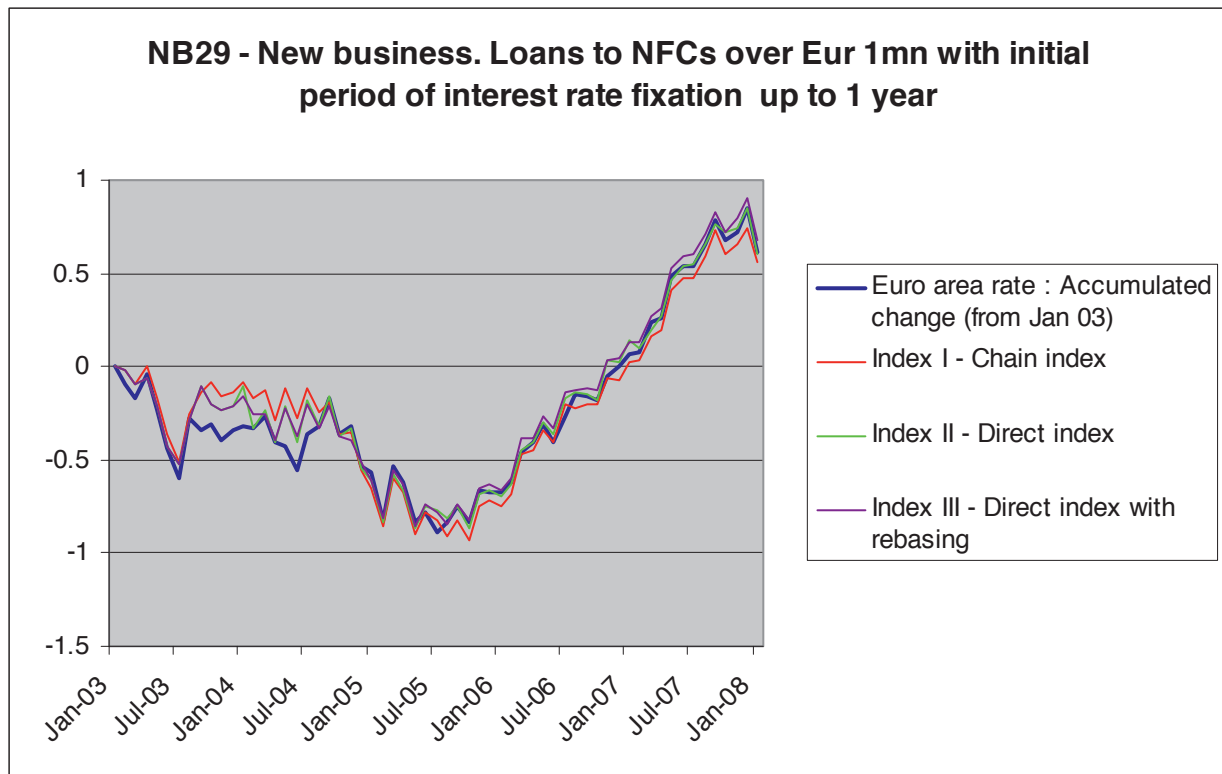
NB10 is an interesting case, with strong weight effects in May 2005, positive, and February 2006 ,negative (chart 7). It seems that the chain index reacts in a most appropriate way to the weight effect in May 2005 by showing a decrease, which corresponds to the negative rate effect of the month, while the other two increase (see chart 3). It is also observed that in February 2006, there is a convergence of indices and original series, because the weight effect has the opposite sign as in May 2005, and intermediate effects also compensate each other.

Chart 7: NB10 Decomposition for a selected period (Mar05 to Jun06)

NB10 - New business. Deposits from NFCs with maturity over 2 years - Decomposition

- Weight component
- Interest rate component
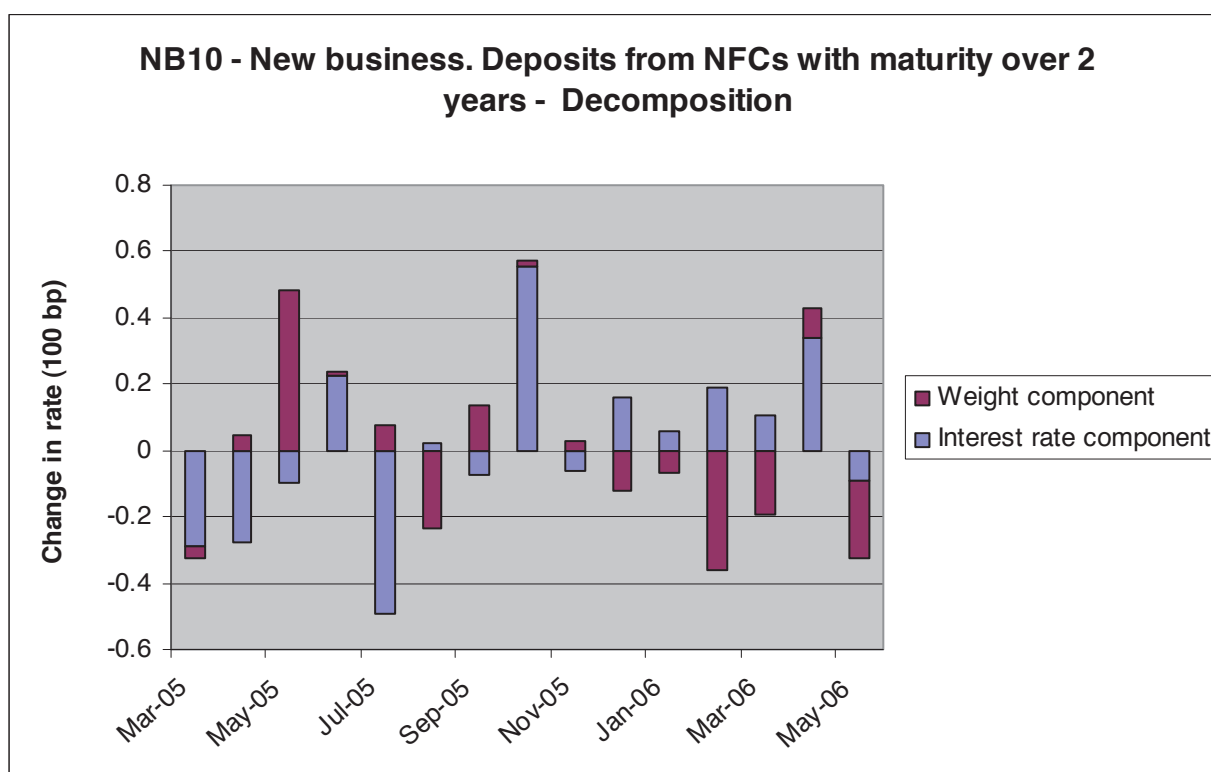
Therefore, this case offers some additional support for the use of the chain index, in addition to the previous theoretical consideration. Finally, this and the previous case strengthen the case for the use of the decomposition in (euro area) aggregate form.

8. Conclusion

This paper is an application of index number theory to MFI interest rates (MIR) statistics, on the basis of the work by Diewert (2005) using differences rather than ratios. This approach seems more appropriate for interest rates, for which changes are usually measured in absolute rather than in relative terms. The paper focuses on the analysis of monthly euro area MIR and how to separately measure the impact of changes of national interest rates and relative country weights.

Following a building blocks approach, it has first examined the possible decompositions applicable to MIR, on the basis of an adaptation of the axiomatic theory of index numbers. Additive decompositions that separate an interest rate component and a weight component were proposed, adapting Laspeyres, Paasche, Marshall-Edgeworth, Walsh, Fischer and Vartia indices to the additive decomposition. Extended versions which measure the weight component taking into account the difference between the country interest rate and the Euro area interest rate were also presented. Then a number of axiomatic properties, adapted from the current index number theory were presented. These properties were examined at both national component and euro area level. On the basis of this analysis, it has been

concluded that the preferred decomposition is the extended Marshall-Edgeworth decomposition, which separates each increase in Euro area aggregate rates into an interest rate component and a weight component, with the peculiarity that the weight component size at national level depends on the difference between the national rate and the euro area rate. This decomposition coincides with the one used at present by ECB to analyse MIR data, as part of the monthly statistical compilation.

As a second step, it proposed three indices constructed on the basis of the interest rate component of the Marshall-Edgeworth decomposition. These indices permit to move from binary to multiple comparisons along time. The three indices proposed consist of a chain index, a direct index, and a direct index with periodic rebasing. As a further step, on the basis of the index, notional interest rates were presented by accumulating the changes shown in the indices.

The indices were then applied to MIR data for the period Jan03 to Jan08. This application showed that for most of MIR categories the indices does not deviates from the accumulated changes in the corresponding MIR original series, meaning that the changes in weights across countries are null or negligible. Furthermore, the three indices analysed behave very much in the same way in most of the MIR categories, with a few exceptions, in particular NB13 (New business. Loans to Households for consumption purposes, with floating rate and rate with initial period of fixation up to 1 year).

The above results confirm that the regular calculation and publication of month-to-month decomposition (binary index) at euro area aggregate level could help analysts to interpret monthly changes in the euro area rates. This decomposition, might be accompanied by an index, which would accumulate all month-to-month decompositions from the starting point of MIR statistics (Jan03). In that sense, Index I (chain index) may have some advantages as it solves the so-called "index number problem", while for the analysed set of data seems to offer better results in some cases. The index would permit to assess the evolution in longer periods in terms of changes in interest rates. It is noted that an index would provide exactly the same information as a notional rate, which however is not recommended since it does not contain any additional information compared to the index, and it could be easily confused with the original rate, creating problems in the communications to the public, and it would be more difficult to interpret. The calculation of the decomposition and/or the index for euro area MIR would be exclusively based on the data regularly reported by NCBs to the ECB as from January 2003. No additional data would be required and the implementation costs at the ECB would be limited. Also, no confidentiality issue would arise, as not individual national data could be derived from the decomposition or index.

In theory, this decomposition and index publication could also be applied at national or reporting institution level, although it is recognised that probably this type of analysis is more useful at euro area level in the context of MIR, given that the banking retail markets remain to a large extent segmented along national borders. It must be remarked again that the decomposition and index do not isolate all

weight effects but only those at the level on the analysis. For example, a decomposition at euro area level isolates the interest and weight effects from the national contributions to MIR but does not take into account the impact on national interest rates from the relative weight of the different institutions at national level or the relative weight of different products included within a single MIR category. In any case, regarding the possible use of these techniques at national level, this note is not prescriptive on the possible applicability and an assessment of its possible usefulness at national level, either for data checking, data analysis or publication, questions which NCBs may consider. It is also not discarded here that a different decomposition and/or index can be better used at national level for some particular purpose.

As a conclusion, the regular publication of the extended Marshall-Edgeworth decomposition would possibly maximise the use of the already existing data at the ECB and increase the value for analysis of the interest rates series. In addition, it may be considered if this decomposition might be accompanied by a difference chain index, although that is not strictly necessary.

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ANNEXES

ANNEX 1 – LIST OF MIR INDICATORS

Code	Content
NB1	New business. Overnight deposits from Households (HHs)
NB2	New business. Deposits with agreed maturity from HHs, up to 1 year maturity
NB3	New business. Deposits with agreed maturity from HHs, over 1 and up to 2 years maturity
NB4	New business. Deposits with agreed maturity from HHs, over 2 years maturity
NB5	New business. Deposits redeemable at notice from HHs, up to 3 months notice
NB6	New business. Deposits redeemable at notice from HHs, over 3 months notice
NB7	New business. Overnight deposits from Non-Financial Corporations (NFCs)
NB8	New business. Deposits with agreed maturity from NFCs, up to 1 year maturity
NB9	New business. Deposits with agreed maturity from NFCs, over 1 and up to 2 years maturity
NB10	New business. Deposits with agreed maturity from NFCs, over 2 years maturity
NB11	New business. Deposits. Repos
NB12	New business. Loans. Bank overdrafts from HHs.
NB13	New business. Loans to HHs for consumption, floating rate and up to 1 year initial rate fixation
NB14	New business. Loans to HHs for consumption, over 1 and up to 5 years initial rate fixation
NB15	New business. Loans to HHs for consumption, over 5 years initial rate fixation
NB16	New business. Loans to HHs for house purchase, floating rate and up to 1 year initial rate fixation
NB17	New business. Loans to HHs for house purchase , over 1 and up to 5 years initial rate fixation
NB18	New business. Loans to HHs for house purchase, over and up to 10 years initial rate fixation
NB19	New business. Loans to HHs for house purchase, over 10 years initial rate fixation
NB20	New business. Loans to HHs for other purposes, floating rate and up to 1 year initial rate fixation
NB21	New business. Loans to HHs for other purposes, over 1 and up to 5 years initial rate fixation
NB22	New business. Loans to HHs for other purposes, over 5 years initial rate fixation
NB23	New business. Loans. Bank overdrafts from NFCs.
NB24	New business. Loans to NFCs up to Eur 1 mn, floating rate and up to 1 year initial rate fixation
NB25	New business. Loans to NFCs up to Eur 1 mn, over 1 and up to 5 years initial rate fixation
NB26	New business. Loans to NFCs up to Eur 1 mn, over 5 years initial rate fixation
NB27	New business. Loans to NFCs over Eur 1 mn, floating rate and up to 1 year initial rate fixation

NB28	New business. Loans to NFCs over Eur 1 mn, over 1 and up to 5 years initial rate fixation
NB29	New business. Loans to NFCs over Eur 1 mn, over 5 years initial rate fixation
NB30	New business (annual percentage rate of charge) Loans to HHs for consumption
NB31	New business (annual percentage rate of charge) Loans to HHs for house purchases
OA1	Outstanding amounts. Deposits with agreed maturity from HHs, up to 2 years maturity
OA2	Outstanding amounts. Deposits with agreed maturity from HHs, over 2 years maturity
OA3	Outstanding amounts. Deposits with agreed maturity from NFCs, up to 2 years maturity
OA4	Outstanding amounts. Deposits with agreed maturity from NFCs, over 2 years maturity
OA5	Outstanding amounts. Deposits. Repos
OA6	Outstanding amounts. Loans to HHs for house purchase, up to 1 year maturity
OA7	Outstanding amounts. Loans to HHs for house purchase, over 1 and up to 5 years maturity
OA8	Outstanding amounts. Loans to HHs for house purchase, over 5 years maturity
OA9	Outstanding amounts. Loans to NFCs for consumer credit and other loans, up to 1 year maturity
OA10	Outstanding amounts. Loans to NFCs for consumer credit and other loans, over 1 and up to 5 years
OA11	Outstanding amounts. Loans to NFCs for consumer credit and other loans, over 5 years maturity
OA12	Outstanding amounts. Loans to NFCs, up to 1 year maturity
OA13	Outstanding amounts. Loans to NFCs, over 1 and up to 5 years maturity
OA14	Outstanding amounts. Loans to NFCs, over 5 years maturity

APRC indicators (NB30, NB31) are compiled according to a slightly different methodology and the indices have not been applied to them.

ANNEX 2 – INDICES APPLIED ON MIR: SUMMARY FOR THE PERIOD JAN03 TO JAN08

Table 1: Index I= Index^{DCME} : Chain index. (Euro area, Jan03-Jan08, basis points /100)

	Average	Maximum	Minimum	Closest zero	SD	Jan 2008
	Comparison of accumulated changes in original series with Index I					
	Accumulated change in original series - Index I					
NB1	-0.0231	0.0055	-0.0446	0.0053	0.0149	-0.0324
NB2	0.0201	0.0744	-0.0072	0.0014	0.0201	0.0744
NB3	-0.0403	0.0982	-0.1582	0.0190	0.0618	0.0776
NB4	-0.1462	0.0061	-0.2756	0.0046	0.0670	0.0046
NB5	0.1054	0.2219	-0.0102	0.0001	0.1039	0.1538
NB6	0.0080	0.0249	-0.0021	0.0028	0.0075	0.0249
NB7	-0.0185	0.0029	-0.0430	0.0007	0.0101	-0.0246
NB8	0.0004	0.0123	-0.0084	0.0015	0.0038	0.0123
NB9	0.0888	0.6654	-0.1545	0.0858	0.1288	0.1221
NB10	0.1013	0.5956	-0.5497	0.0794	0.2477	0.3830
NB11	-0.0052	0.0124	-0.0242	0.0026	0.0068	-0.0225
NB12	0.0066	0.0598	-0.0687	0.0131	0.0261	-0.0637
NB13	0.7593	1.0541	0.3019	0.3019	0.1726	0.4709
NB14	0.0258	0.2741	-0.1357	0.0693	0.0821	0.1947
NB15	-0.1220	0.1277	-0.3719	0.0222	0.1382	-0.2972
NB16	-0.0736	-0.0034	-0.1083	0.0592	0.0210	-0.0636
NB17	-0.0128	0.0921	-0.1056	0.0292	0.0395	-0.0426
NB18	-0.0303	0.0032	-0.0684	0.0048	0.0141	-0.0312
NB19	0.0379	0.1961	-0.0244	0.0010	0.0568	0.1263
NB20	0.0420	0.1215	-0.1162	0.0113	0.0439	0.0113
NB21	0.0464	0.1318	-0.0470	0.0047	0.0377	0.0737
NB22	0.0421	0.1983	-0.0947	0.0234	0.0490	0.0787
NB23	-0.0093	0.0379	-0.0406	0.0020	0.0184	-0.0245
NB24	-0.0216	0.0136	-0.0517	0.0040	0.0154	-0.0224
NB25	0.0652	0.1156	-0.0468	0.0300	0.0367	0.1006
NB26	0.0776	0.1450	-0.0152	0.0107	0.0341	0.1128
NB27	0.0142	0.0424	-0.0212	0.0218	0.0115	0.0218
NB28	0.0639	0.2851	-0.1196	0.0054	0.0855	0.0097
NB29	-0.0163	0.1070	-0.3061	0.0534	0.1081	0.0534
OA1	0.0133	0.1074	-0.0161	0.0032	0.0358	0.1074
OA2	-0.0402	-0.0018	-0.0690	0.0018	0.0217	-0.0449
OA3	-0.0053	0.0040	-0.0105	0.0000	0.0031	-0.0063
OA4	-0.2756	0.0013	-0.3750	0.0013	0.1045	-0.3686
OA5	0.0191	0.0400	-0.0047	0.0047	0.0068	0.0260
OA6	0.0161	0.1049	-0.0737	0.0155	0.0564	-0.0737
OA7	0.0331	0.0681	-0.0118	0.0003	0.0218	0.0123
OA8	-0.1131	-0.0015	-0.1960	0.0015	0.0685	-0.1954
OA9	-0.0290	0.0286	-0.1499	0.0032	0.0432	-0.1204
OA10	0.0196	0.0333	0.0007	0.0010	0.0077	0.0227
OA11	-0.0557	0.0025	-0.0938	0.0025	0.0318	-0.0779
OA12	-0.0461	0.0015	-0.0870	0.0014	0.0318	-0.0748
OA13	-0.0396	0.0028	-0.0712	0.0028	0.0227	-0.0661
OA14	-0.0920	-0.0017	-0.1468	0.0017	0.0481	-0.1404
Maximum	0.7593	1.0541	0.3019	0.3019	0.2477	0.4709
	NB13	NB13	NB13	NB13	NB10	NB13
Minimum	-0.2756	-0.0034	-0.5497	0.0000	0.0031	-0.3686
	OA4	NB16	NB10	OA13	OA3	OA4

Table 2: Index II= Index^DDME: Direct Index. (Euro area, Jan03-Jan08, basis points/100)

	Average	Maximum	Minimum	Closest zero	SD	Jan 2008
		Comparison of accumulated changes in original series with Index II				
		Accumulated change in original series - Index II				
NB1	-0.0257	0.0053	-0.0557	0.0053	0.0173	-0.0477
NB2	0.0189	0.0900	-0.0146	0.0014	0.0240	0.0900
NB3	-0.0320	0.1034	-0.1714	0.0190	0.0478	0.1034
NB4	-0.1201	0.0818	-0.2971	0.0796	0.0857	0.0796
NB5	0.1146	0.2597	-0.0108	0.0001	0.1139	0.1634
NB6	0.0074	0.0203	-0.0065	0.0028	0.0062	0.0203
NB7	-0.0188	0.0036	-0.0428	0.0007	0.0108	-0.0224
NB8	-0.0034	0.0077	-0.0174	0.0015	0.0050	0.0077
NB9	-0.0434	0.2835	-0.3282	0.0858	0.1249	-0.2193
NB10	-0.1879	0.2748	-0.8356	0.0794	0.2094	-0.2254
NB11	-0.0021	0.0176	-0.0200	0.0026	0.0071	-0.0045
NB12	0.0025	0.0614	-0.0841	0.0131	0.0305	-0.0802
NB13	0.8079	1.0815	0.3019	0.3019	0.1833	0.7097
NB14	0.0087	0.2067	-0.1541	0.0693	0.0708	0.1454
NB15	-0.0852	0.1107	-0.2773	0.0222	0.1038	-0.1209
NB16	-0.0732	0.0055	-0.1136	0.0361	0.0242	-0.0361
NB17	-0.0420	0.0423	-0.1199	0.0242	0.0375	-0.0242
NB18	-0.0339	0.0035	-0.0713	0.0028	0.0159	-0.0028
NB19	-0.0217	0.0483	-0.0817	0.0010	0.0235	0.0125
NB20	0.0448	0.1221	-0.0391	0.0198	0.0308	0.0265
NB21	-0.0106	0.0310	-0.0733	0.0047	0.0202	-0.0260
NB22	0.0110	0.1020	-0.1443	0.0234	0.0503	-0.0376
NB23	0.0060	0.0513	-0.0283	0.0020	0.0194	-0.0100
NB24	-0.0196	0.0244	-0.0608	0.0031	0.0209	-0.0031
NB25	0.0669	0.1386	-0.0371	0.0240	0.0409	0.0240
NB26	0.0193	0.1338	-0.1061	0.0107	0.0696	-0.0194
NB27	0.0133	0.0604	-0.0427	0.0245	0.0199	0.0604
NB28	0.1036	0.3709	-0.0842	0.0054	0.0871	0.1070
NB29	-0.0328	0.0422	-0.2367	0.0154	0.0671	0.0154
OA1	0.0101	0.1082	-0.0184	0.0032	0.0344	0.1082
OA2	-0.0324	-0.0018	-0.0561	0.0018	0.0165	-0.0249
OA3	-0.0002	0.0183	-0.0087	0.0000	0.0052	0.0183
OA4	-0.2486	0.0013	-0.3447	0.0013	0.0759	-0.2603
OA5	-0.0006	0.0137	-0.0158	0.0026	0.0066	0.0026
OA6	-0.0162	0.0887	-0.1130	0.0155	0.0632	-0.1130
OA7	-0.0007	0.0456	-0.0401	0.0003	0.0239	-0.0355
OA8	-0.0734	-0.0015	-0.1224	0.0015	0.0375	-0.0650
OA9	-0.0291	0.0224	-0.1292	0.0032	0.0386	-0.1048
OA10	0.0426	0.0895	0.0010	0.0010	0.0293	0.0895
OA11	-0.0282	0.0143	-0.0597	0.0025	0.0211	0.0143
OA12	-0.0293	0.0054	-0.0599	0.0014	0.0199	-0.0281
OA13	-0.0188	0.0028	-0.0342	0.0028	0.0092	-0.0048
OA14	-0.0580	-0.0017	-0.0936	0.0017	0.0252	-0.0349
Maximum	0.8079	1.0815	0.3019	0.3019	0.2094	0.7097
	NB13	NB13	NB13	NB13	NB10	NB13
Minimum	-0.2486	-0.0018	-0.8356	0.0000	0.0050	-0.2603
	OA4	NB18	NB10	NB13	NB10	NB13

Table 3: Index III = Index^DRME: Direct Index with rebasing. (Euro area, Jan03-Jan08, basis points/100)

| | Comparison of accumulated changes in original series with Index III | | | | | |
| | Accumulated change in original series - Index III | | | | | |
	Average	**Maximum**	**Minimum**	**Closest zero**	**SD**	**Jan 2008**
NB1	-0.0246	0.0053	-0.0453	0.0053	0.0151	-0.0354
NB2	0.0149	0.0728	-0.0146	0.0014	0.0190	0.0728
NB3	-0.0493	0.0543	-0.1714	0.0190	0.0404	0.0426
NB4	-0.1636	-0.0366	-0.3100	0.0510	0.0669	-0.0510
NB5	0.0986	0.2068	-0.0117	0.0001	0.0988	0.1375
NB6	0.0081	0.0248	-0.0024	0.0028	0.0053	0.0248
NB7	-0.0191	0.0036	-0.0379	0.0007	0.0094	-0.0210
NB8	-0.0044	0.0102	-0.0152	0.0015	0.0051	0.0102
NB9	-0.0665	0.5050	-0.3282	0.0858	0.1207	-0.1211
NB10	-0.1045	0.3511	-0.8356	0.0794	0.2044	-0.2071
NB11	0.0002	0.0176	-0.0094	0.0026	0.0049	0.0050
NB12	-0.0030	0.0555	-0.0897	0.0131	0.0309	-0.0740
NB13	0.8442	1.1070	0.3019	0.3019	0.2023	0.5638
NB14	0.0148	0.2388	-0.1541	0.0693	0.0760	0.1641
NB15	-0.0208	0.1107	-0.1751	0.0222	0.0637	-0.0519
NB16	-0.0652	-0.0050	-0.1091	0.0122	0.0221	-0.0122
NB17	-0.0457	0.0580	-0.1199	0.0292	0.0356	-0.0463
NB18	-0.0450	-0.0048	-0.0882	0.0048	0.0187	-0.0322
NB19	-0.0003	0.1498	-0.0620	0.0010	0.0519	0.0893
NB20	0.0752	0.1633	-0.0806	0.0198	0.0479	0.0239
NB21	-0.0011	0.0525	-0.0505	0.0047	0.0181	-0.0156
NB22	0.0790	0.1973	-0.0383	0.0234	0.0415	0.1169
NB23	0.0057	0.0478	-0.0206	0.0020	0.0166	-0.0026
NB24	-0.0054	0.0269	-0.0461	0.0034	0.0157	0.0034
NB25	0.1048	0.1495	-0.0371	0.0300	0.0446	0.1381
NB26	0.0607	0.1118	-0.0305	0.0107	0.0311	0.1118
NB27	0.0176	0.0585	-0.0262	0.0245	0.0169	0.0551
NB28	0.1322	0.3611	-0.0842	0.0054	0.0972	0.1899
NB29	-0.0497	0.0797	-0.2367	0.0600	0.0611	-0.0600
OA1	0.0135	0.1158	-0.0161	0.0032	0.0368	0.1158
OA2	-0.0395	-0.0018	-0.0694	0.0018	0.0216	-0.0393
OA3	-0.0011	0.0108	-0.0062	0.0000	0.0029	0.0073
OA4	-0.2872	0.0013	-0.3961	0.0013	0.1033	-0.3930
OA5	-0.0077	0.0076	-0.0270	0.0047	0.0074	-0.0077
OA6	0.0008	0.0916	-0.0888	0.0155	0.0578	-0.0867
OA7	0.0266	0.0539	-0.0068	0.0003	0.0168	0.0092
OA8	-0.1098	-0.0015	-0.1866	0.0015	0.0659	-0.1854
OA9	-0.0357	0.0224	-0.1565	0.0032	0.0453	-0.1168
OA10	0.0192	0.0312	0.0010	0.0010	0.0074	0.0255
OA11	-0.0561	0.0025	-0.0946	0.0025	0.0319	-0.0753
OA12	-0.0449	0.0050	-0.0866	0.0014	0.0322	-0.0667
OA13	-0.0343	0.0028	-0.0659	0.0028	0.0197	-0.0463
OA14	-0.0903	-0.0017	-0.1451	0.0017	0.0475	-0.1369
Maximum	0.8442	1.1070	0.3019	0.3019	0.2044	0.5638
	NB13	NB13	NB13	NB13	NB10	NB13
Minimum	-0.2872	-0.0366	-0.8356	0.0000	0.0029	-0.3930
	OA4	NB4	NB10	OA3	OA3	OA4
	-0.0246	0.0053	-0.0453	0.0053	0.0151	-0.0354

Table 4: Comparison of Indices I and II. (Euro area, Jan03-Jan08, basis points/100)

	Difference between indices (Index I – Index II)					
	Average	**Maximum**	**Minimum**	**Closest zero**	**SD**	**Jan 2008**
NB1	-0.0026	0.0050	-0.0153	0.0000	0.0052	-0.0153
NB2	-0.0012	0.0156	-0.0085	0.0000	0.0050	0.0156
NB3	0.0084	0.0650	-0.0506	0.0000	0.0278	0.0258
NB4	0.0260	0.1631	-0.0586	0.0000	0.0529	0.0750
NB5	0.0092	0.0379	-0.0016	0.0000	0.0118	0.0096
NB6	-0.0006	0.0019	-0.0049	0.0000	0.0017	-0.0046
NB7	-0.0004	0.0021	-0.0032	0.0000	0.0013	0.0021
NB8	-0.0038	0.0040	-0.0161	0.0000	0.0040	-0.0047
NB9	-0.1322	0.0898	-0.8166	0.0000	0.1608	-0.3414
NB10	-0.2892	0.2023	-0.8088	0.0000	0.2437	-0.6083
NB11	0.0030	0.0354	-0.0123	0.0000	0.0061	0.0180
NB12	-0.0041	0.0094	-0.0324	0.0000	0.0096	-0.0164
NB13	0.0486	0.2707	-0.0909	0.0000	0.0782	0.2389
NB14	-0.0172	0.0342	-0.0715	0.0000	0.0235	-0.0492
NB15	0.0368	0.1763	-0.0217	0.0000	0.0471	0.1763
NB16	0.0004	0.0275	-0.0199	0.0000	0.0092	0.0275
NB17	-0.0292	0.0433	-0.0924	0.0000	0.0235	0.0184
NB18	-0.0036	0.0284	-0.0264	0.0000	0.0112	0.0284
NB19	-0.0595	0.0007	-0.1591	0.0000	0.0452	-0.1138
NB20	0.0029	0.0771	-0.0747	0.0000	0.0333	0.0152
NB21	-0.0570	0.0153	-0.2051	0.0000	0.0436	-0.0996
NB22	-0.0311	0.0465	-0.1807	0.0000	0.0629	-0.1163
NB23	0.0153	0.0330	-0.0005	0.0000	0.0078	0.0146
NB24	0.0020	0.0217	-0.0252	0.0000	0.0103	0.0193
NB25	0.0017	0.0491	-0.0766	0.0000	0.0355	-0.0766
NB26	-0.0583	0.0310	-0.1849	0.0000	0.0662	-0.1322
NB27	-0.0009	0.0386	-0.0319	0.0000	0.0124	0.0386
NB28	0.0398	0.1008	-0.0192	0.0000	0.0305	0.0973
NB29	-0.0165	0.1606	-0.1234	0.0000	0.0656	-0.0380
OA1	-0.0033	0.0010	-0.0088	0.0000	0.0030	0.0008
OA2	0.0078	0.0214	0.0000	0.0000	0.0071	0.0200
OA3	0.0051	0.0246	-0.0001	0.0000	0.0048	0.0246
OA4	0.0269	0.1083	-0.0344	0.0000	0.0418	0.1083
OA5	-0.0197	0.0000	-0.0411	0.0000	0.0091	-0.0234
OA6	-0.0322	0.0001	-0.0586	0.0000	0.0201	-0.0393
OA7	-0.0338	0.0050	-0.0586	0.0000	0.0191	-0.0478
OA8	0.0396	0.1304	-0.0001	0.0000	0.0432	0.1304
OA9	-0.0001	0.0207	-0.0109	0.0000	0.0072	0.0156
OA10	0.0230	0.0693	-0.0001	0.0000	0.0263	0.0669
OA11	0.0275	0.0922	-0.0014	0.0000	0.0302	0.0922
OA12	0.0168	0.0468	-0.0001	0.0000	0.0136	0.0468
OA13	0.0208	0.0613	-0.0004	0.0000	0.0188	0.0613
OA14	0.0340	0.1055	-0.0008	0.0000	0.0351	0.1055
Maximum	0.0486	0.2707	0.0000	0.0000	0.2437	0.2389
	NB13	N13	OA2		NB10	NB13
Minimum	-0.2892	0.0000	-0.8166	0.0000	0.0013	-0.6083
	NB10	OA05	NB9		NB7	NB10

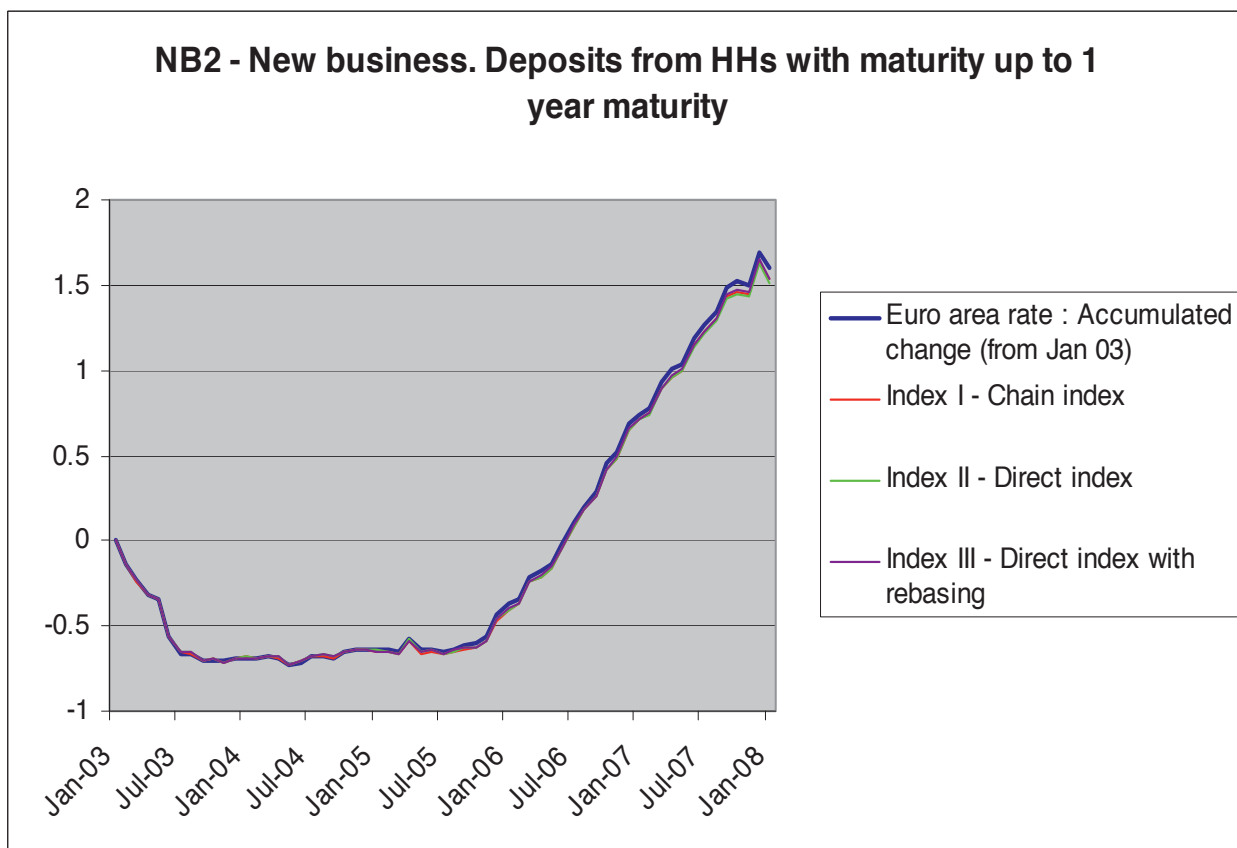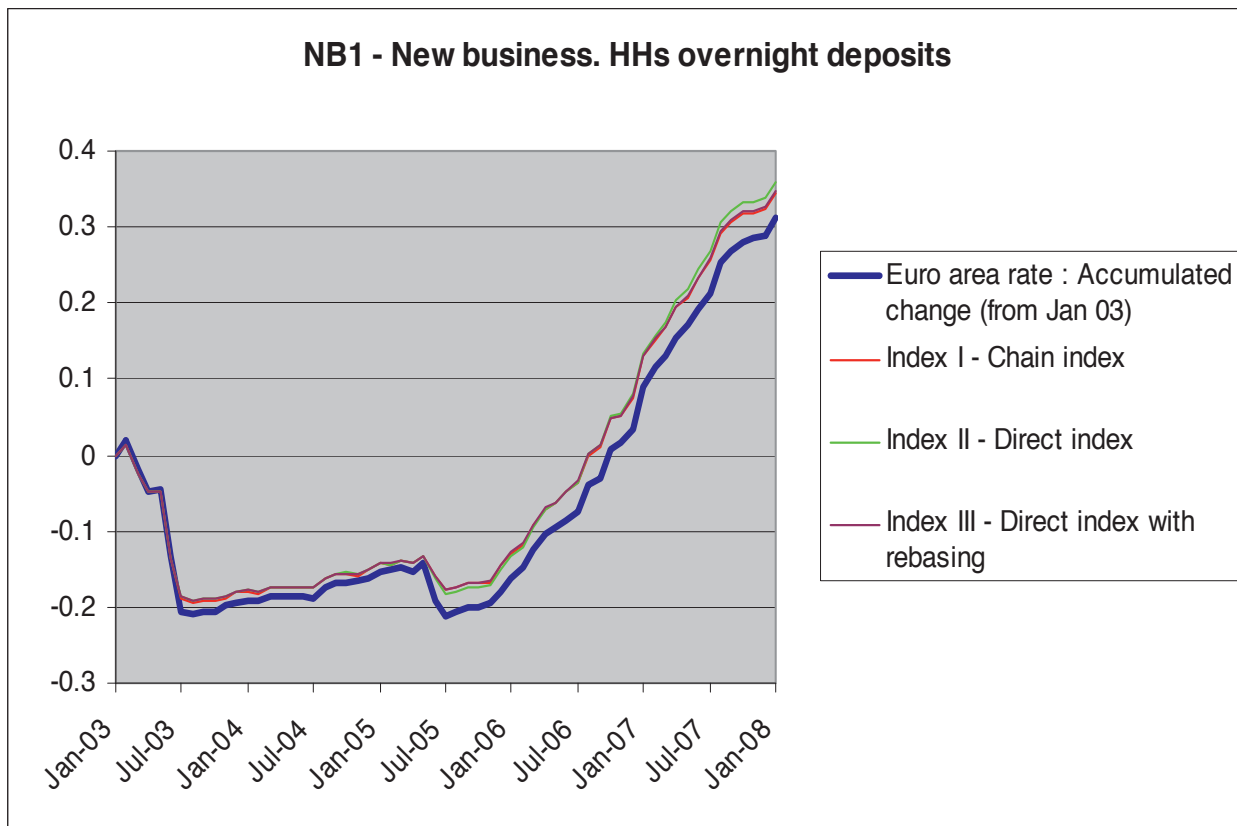**Table 5: Comparison of Indices I and III. (Euro area, Jan03-Jan08, basis points/100)**

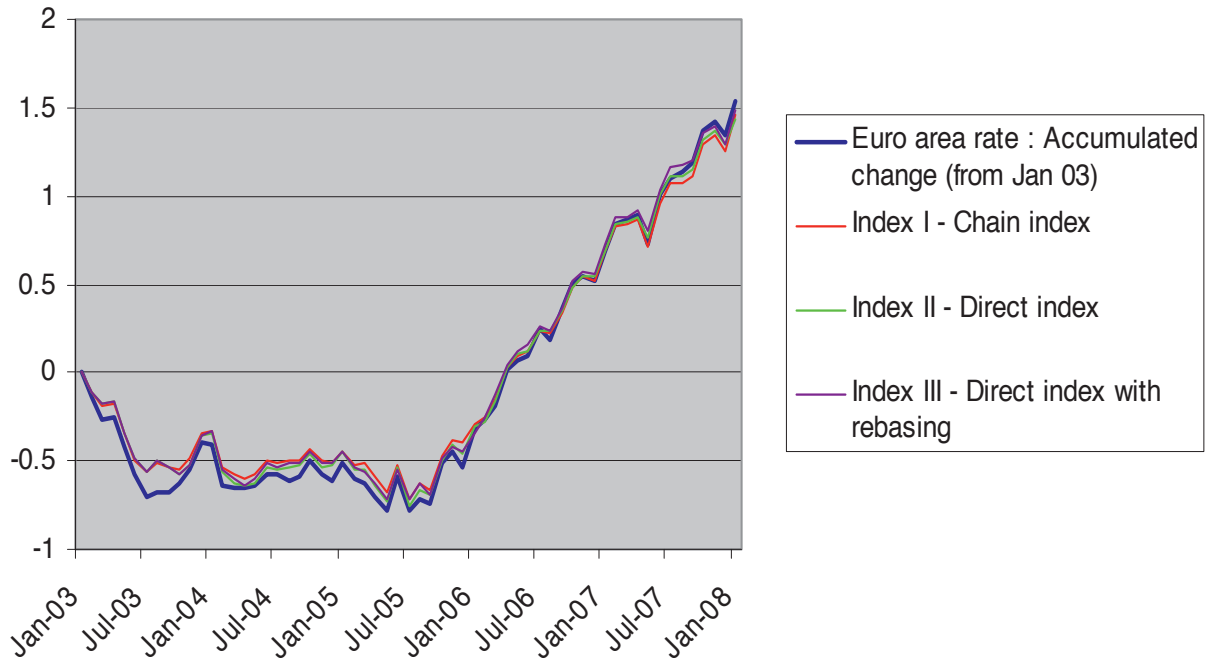| | Difference between indices (Index I – Index III) | | | | | |
	Average	Maximum	Minimum	Closest zero	SD	Dec-06
NB1	-0.0015	0.0000	-0.0038	0.0000	0.0008	-0.0030
NB2	-0.0052	0.0030	-0.0132	0.0000	0.0036	-0.0016
NB3	-0.0090	0.0540	-0.1045	0.0000	0.0377	-0.0350
NB4	-0.0175	0.0653	-0.0687	0.0000	0.0329	-0.0557
NB5	-0.0067	0.0000	-0.0195	0.0000	0.0061	-0.0163
NB6	0.0000	0.0048	-0.0057	0.0000	0.0026	-0.0001
NB7	-0.0006	0.0051	-0.0032	0.0000	0.0020	0.0036
NB8	-0.0048	0.0023	-0.0135	0.0000	0.0035	-0.0022
NB9	-0.1553	0.1168	-0.4660	0.0000	0.1392	-0.2431
NB10	-0.2058	0.1759	-0.6304	0.0000	0.2259	-0.5900
NB11	0.0054	0.0339	-0.0039	0.0000	0.0059	0.0275
NB12	-0.0096	0.0030	-0.0295	0.0000	0.0079	-0.0103
NB13	0.0849	0.1465	-0.0343	0.0000	0.0422	0.0930
NB14	-0.0110	0.0262	-0.0514	0.0000	0.0209	-0.0305
NB15	0.1013	0.2454	-0.0217	0.0000	0.0860	0.2454
NB16	0.0084	0.0514	-0.0082	0.0000	0.0126	0.0514
NB17	-0.0328	0.0163	-0.0631	0.0000	0.0181	-0.0036
NB18	-0.0147	0.0019	-0.0250	0.0000	0.0080	-0.0010
NB19	-0.0381	0.0000	-0.0646	0.0000	0.0121	-0.0371
NB20	0.0332	0.0706	-0.0191	0.0000	0.0188	0.0126
NB21	-0.0475	0.0153	-0.1197	0.0000	0.0351	-0.0893
NB22	0.0370	0.0664	-0.0009	0.0000	0.0155	0.0382
NB23	0.0150	0.0238	-0.0005	0.0000	0.0064	0.0219
NB24	0.0162	0.0298	-0.0042	0.0000	0.0081	0.0258
NB25	0.0396	0.0577	0.0000	0.0000	0.0136	0.0375
NB26	-0.0169	0.0090	-0.0478	0.0000	0.0134	-0.0010
NB27	0.0034	0.0333	-0.0212	0.0000	0.0123	0.0333
NB28	0.0684	0.1802	-0.0128	0.0000	0.0373	0.1802
NB29	-0.0334	0.1369	-0.1615	0.0000	0.0789	-0.1135
OA1	0.0002	0.0084	-0.0025	0.0000	0.0019	0.0084
OA2	0.0007	0.0057	-0.0014	0.0000	0.0018	0.0056
OA3	0.0042	0.0136	-0.0001	0.0000	0.0022	0.0136
OA4	-0.0117	0.0015	-0.0344	0.0000	0.0093	-0.0244
OA5	-0.0267	0.0000	-0.0405	0.0000	0.0089	-0.0338
OA6	-0.0153	0.0001	-0.0247	0.0000	0.0068	-0.0130
OA7	-0.0065	0.0050	-0.0142	0.0000	0.0052	-0.0031
OA8	0.0033	0.0100	-0.0001	0.0000	0.0028	0.0100
OA9	-0.0067	0.0036	-0.0148	0.0000	0.0046	0.0036
OA10	-0.0004	0.0039	-0.0048	0.0000	0.0022	0.0028
OA11	-0.0004	0.0025	-0.0019	0.0000	0.0010	0.0025
OA12	0.0013	0.0082	-0.0016	0.0000	0.0017	0.0082
OA13	0.0052	0.0198	-0.0004	0.0000	0.0039	0.0198
OA14	0.0017	0.0035	-0.0008	0.0000	0.0010	0.0035
Maximum	0.1013	0.2454	0.0000	0.0000	0.2259	0.2454
	NB15	NB15	NB25		NB10	NB15
Minimum	-0.2058	0.0000	-0.6304	0.0000	0.0008	-0.5900
	NB10	NB19	NB10		NB1	NB10

Table 6: Comparison between Indices II and III (Euro area, Jan03-Jan08, basis points/100)

	Difference between indices (Index II – Index III)					
	Average	**Maximum**	**Minimum**	**Closest zero**	**SD**	**Dec-06**
NB1	0.0011	0.0123	-0.0056	0.0000	0.0048	0.0123
NB2	-0.0040	0.0076	-0.0172	0.0000	0.0056	-0.0172
NB3	-0.0174	0.0402	-0.0608	0.0000	0.0189	-0.0608
NB4	-0.0435	0.0198	-0.1417	0.0000	0.0385	-0.1307
NB5	-0.0159	0.0002	-0.0530	0.0000	0.0169	-0.0259
NB6	0.0007	0.0045	-0.0017	0.0000	0.0015	0.0045
NB7	-0.0003	0.0056	-0.0047	0.0000	0.0026	0.0015
NB8	-0.0010	0.0057	-0.0068	0.0000	0.0025	0.0025
NB9	-0.0231	0.6562	-0.3436	0.0000	0.1423	0.0982
NB10	0.0834	0.3577	-0.3131	0.0000	0.1181	0.0183
NB11	0.0023	0.0123	-0.0035	0.0000	0.0034	0.0095
NB12	-0.0055	0.0061	-0.0187	0.0000	0.0057	0.0061
NB13	0.0363	0.2093	-0.1896	0.0000	0.0851	-0.1459
NB14	0.0062	0.0320	-0.0150	0.0000	0.0086	0.0187
NB15	0.0644	0.1523	-0.0148	0.0000	0.0538	0.0691
NB16	0.0080	0.0315	-0.0105	0.0000	0.0107	0.0239
NB17	-0.0036	0.0313	-0.0388	0.0000	0.0161	-0.0221
NB18	-0.0111	0.0085	-0.0396	0.0000	0.0107	-0.0293
NB19	0.0214	0.1154	-0.0297	0.0000	0.0385	0.0767
NB20	0.0303	0.1106	-0.0427	0.0000	0.0339	-0.0026
NB21	0.0095	0.1258	-0.0212	0.0000	0.0188	0.0104
NB22	0.0680	0.1825	-0.0010	0.0000	0.0589	0.1545
NB23	-0.0003	0.0076	-0.0099	0.0000	0.0039	0.0074
NB24	0.0142	0.0426	-0.0025	0.0000	0.0113	0.0065
NB25	0.0379	0.1141	-0.0123	0.0000	0.0373	0.1141
NB26	0.0413	0.1553	-0.0666	0.0000	0.0652	0.1312
NB27	0.0043	0.0276	-0.0238	0.0000	0.0111	-0.0054
NB28	0.0286	0.0968	-0.0404	0.0000	0.0333	0.0829
NB29	-0.0169	0.0678	-0.0755	0.0000	0.0311	-0.0755
OA1	0.0035	0.0090	-0.0006	0.0000	0.0031	0.0076
OA2	-0.0071	0.0000	-0.0188	0.0000	0.0062	-0.0144
OA3	-0.0009	0.0040	-0.0162	0.0000	0.0038	-0.0109
OA4	-0.0386	0.0153	-0.1337	0.0000	0.0423	-0.1327
OA5	-0.0071	0.0073	-0.0264	0.0000	0.0097	-0.0104
OA6	0.0169	0.0393	-0.0069	0.0000	0.0158	0.0263
OA7	0.0273	0.0532	0.0000	0.0000	0.0182	0.0448
OA8	-0.0364	0.0000	-0.1204	0.0000	0.0404	-0.1204
OA9	-0.0067	0.0022	-0.0273	0.0000	0.0077	-0.0120
OA10	-0.0234	0.0002	-0.0671	0.0000	0.0256	-0.0640
OA11	-0.0279	0.0000	-0.0902	0.0000	0.0301	-0.0897
OA12	-0.0155	0.0003	-0.0395	0.0000	0.0137	-0.0386
OA13	-0.0155	0.0002	-0.0442	0.0000	0.0153	-0.0415
OA14	-0.0323	0.0000	-0.1022	0.0000	0.0347	-0.1020
Maximum	0.0834	0.6562	0.0000	0.0000	0.1423	0.1545
	NB10	NB9	OA7		NB9	NB22
Minimum	-0.0435	0.0000	-0.3436	0.0000	0.0015	-0.1459
	NB4	OA8	NB9		NB6	NB13

ANNEX 3: INDICES APPLIED TO EACH MIR CATEGORY: CHARTS

NB1 - New business. HHs overnight deposits

- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

NB2 - New business. Deposits from HHs with maturity up to 1 year maturity

- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

NB3 - New business. Deposits from HHs with maturity over 1 and up to 2 years maturity

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

NB4 - New business. Deposits from HHs with maturity over 2 years

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

NB5 - New business. Deposits from HHs redeemable at notice, up to 3 month notice

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

NB6 New business. Deposits from HHs redeemable at notice, with over 3 month notice

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing
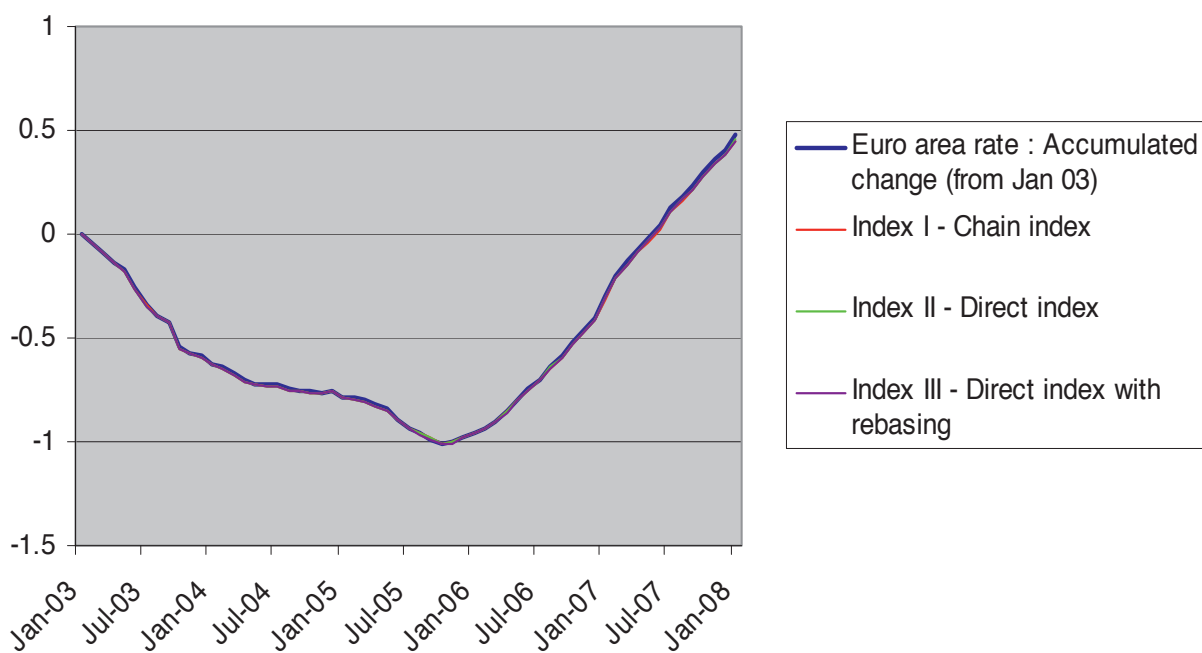
NB7 - New business. NFCs overnight deposits

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

NB8 - New business. Deposits from NFCs with maturity up to 1 year

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

NB9 - New business. Deposits from NFCs with maturity over 1 and up to 2 years maturity

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

NB10 - New business. Deposits from NFCs with maturity over 2 years

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

NB11 - New business. Repos

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

NB12 - New business. HHs bank overdraft

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

NB13 - New business. Loans to HHs for consumption, with initial period of interest rate fixation up to 1 year

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

NB14 - New business. Loans to HHs for consumption with initial period of rate fixation over 1 and up to 5 years

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing
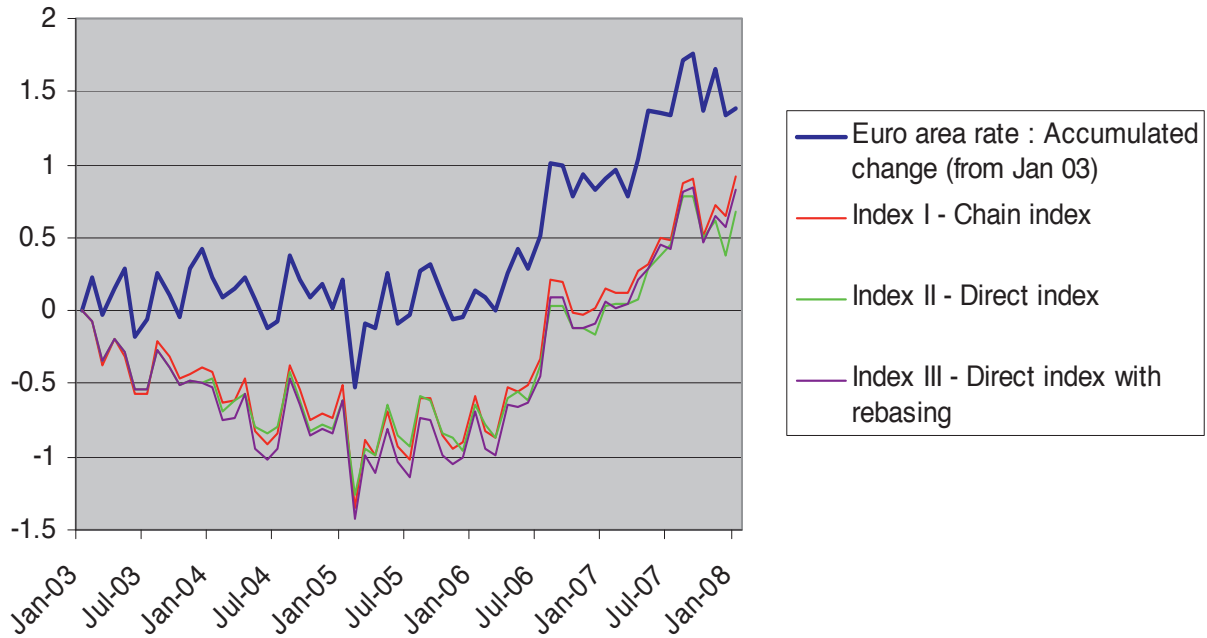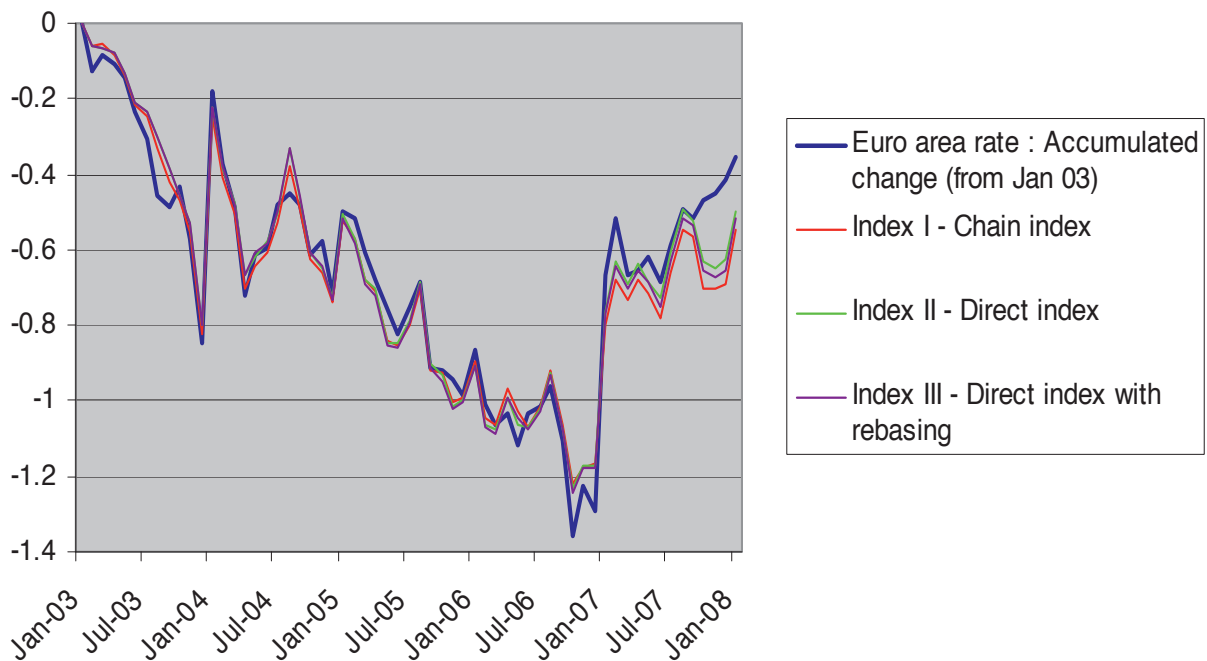
NB15 - New business. Loans to HHs for consumption with initial period of fixation over 5 years
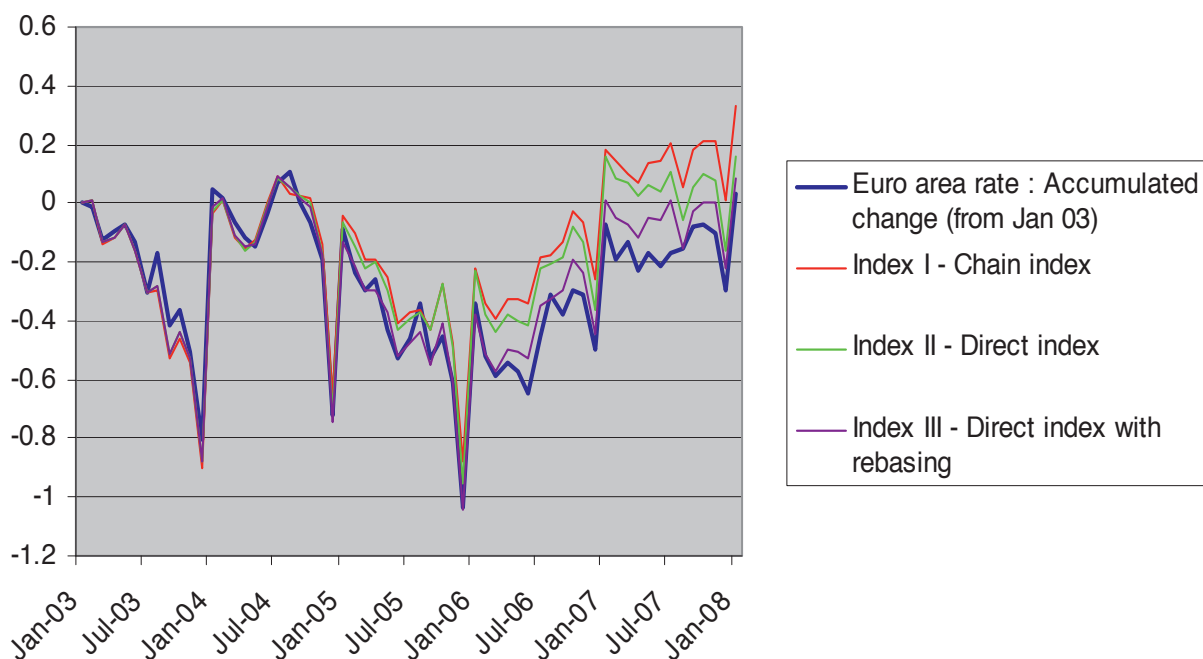
NB16 - New business. Loans to HHs for house purchase with initial period of interest rate fixation up to 1 year

NB17 - New business. Loans to HHs for house purchase with initial period of interest rate fixation over 1 and up to 5 years
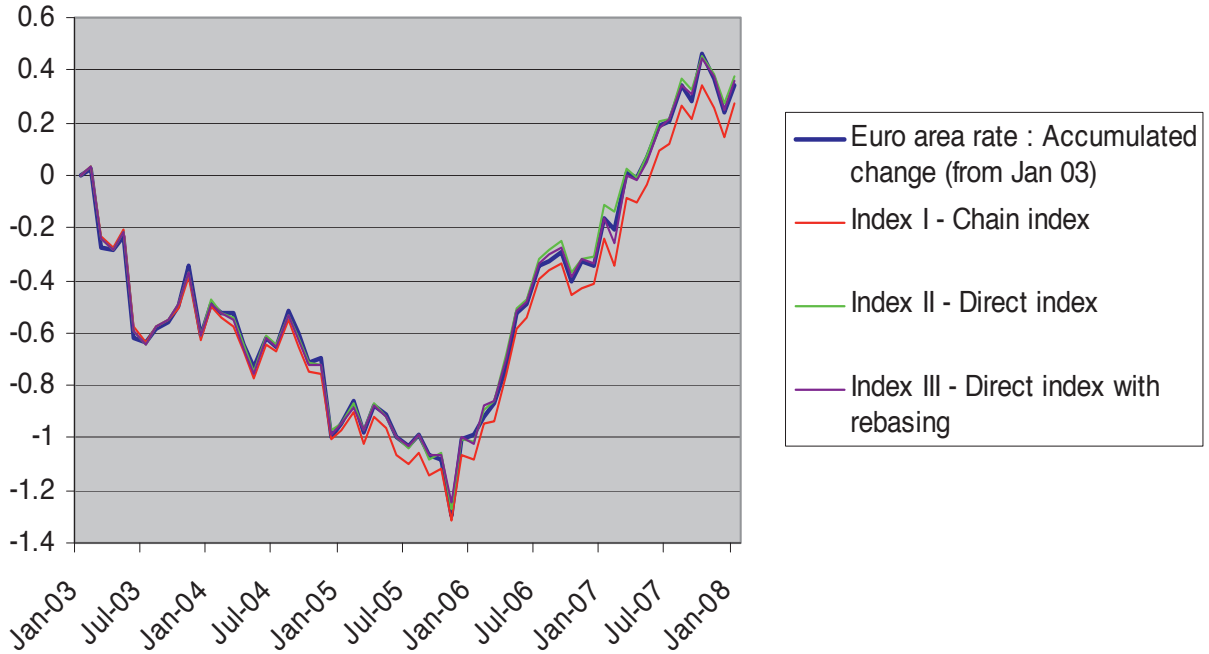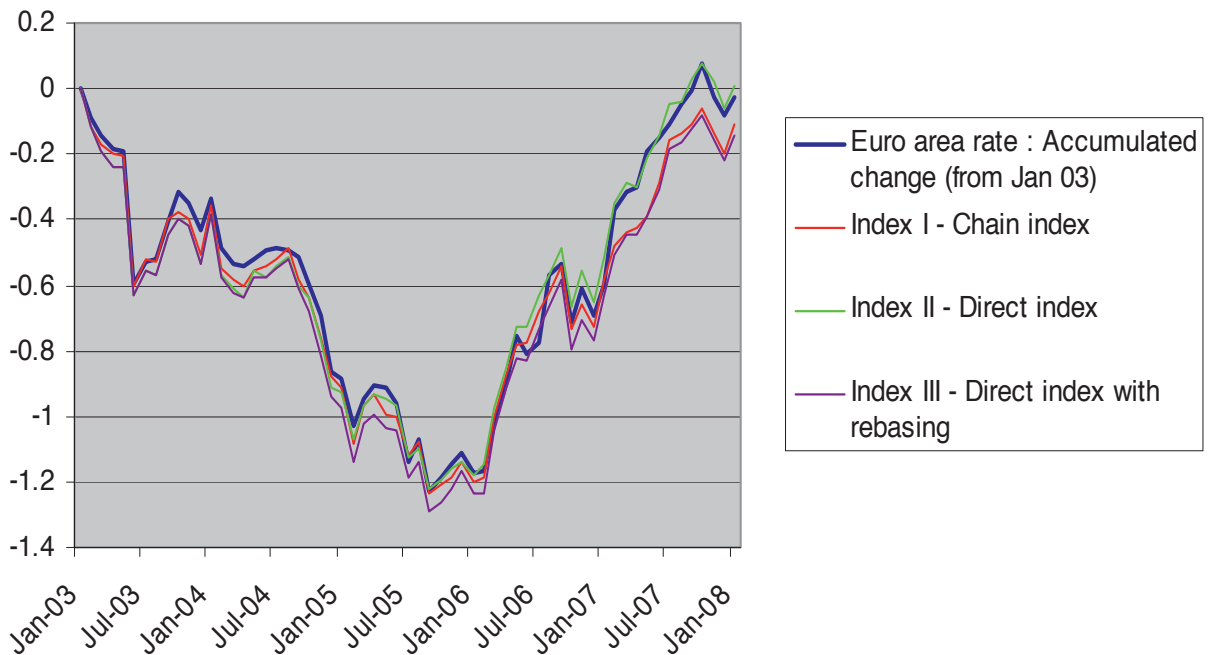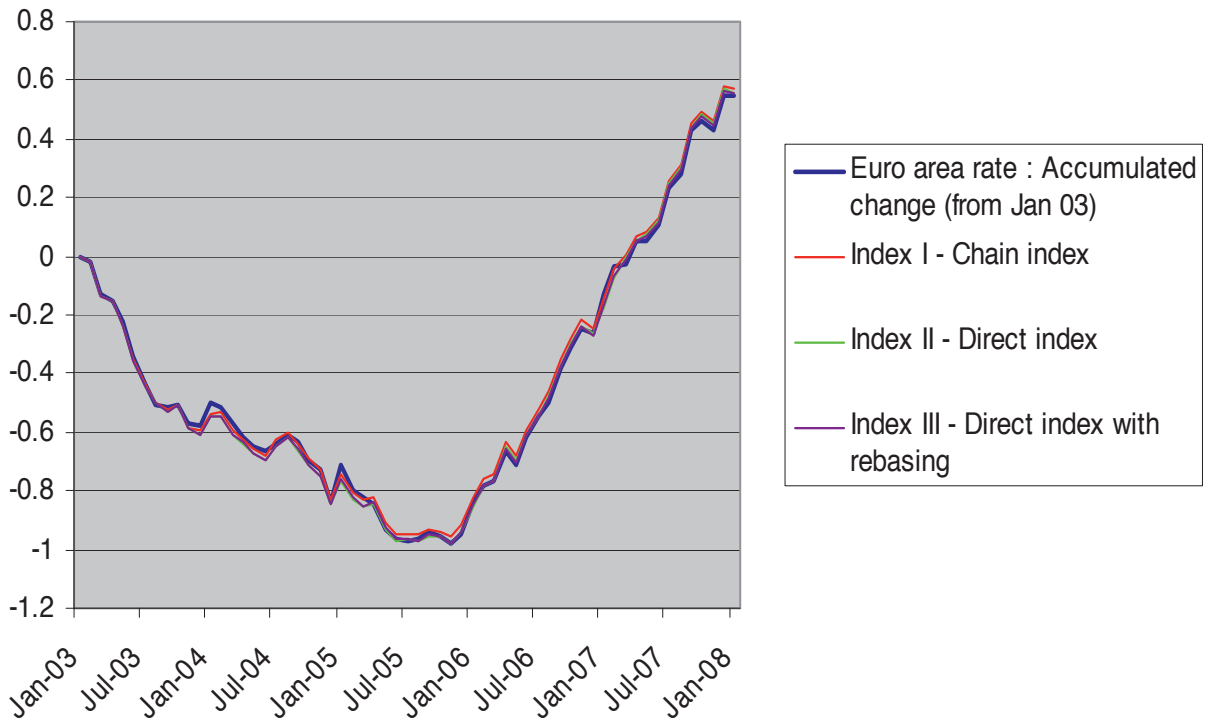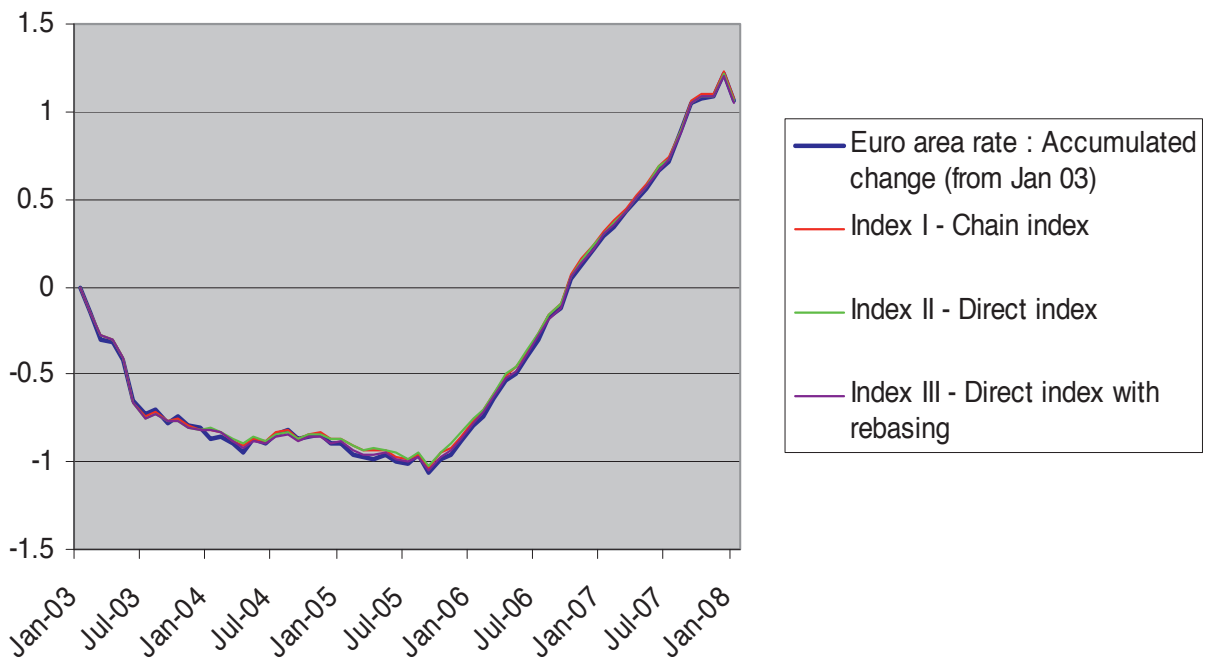

- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing
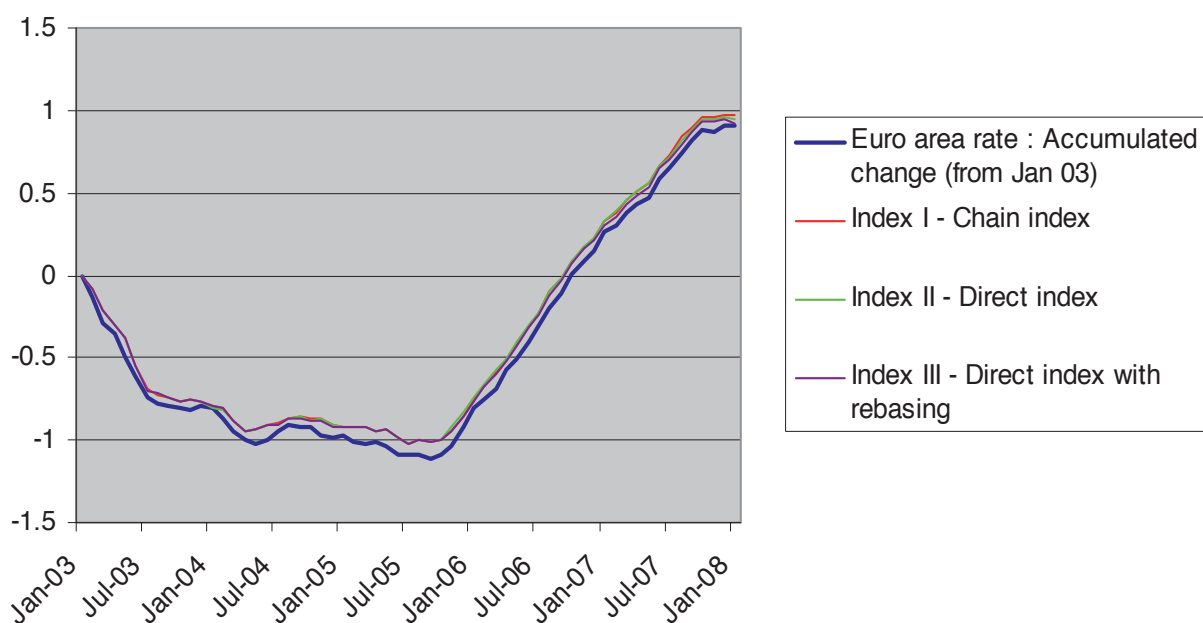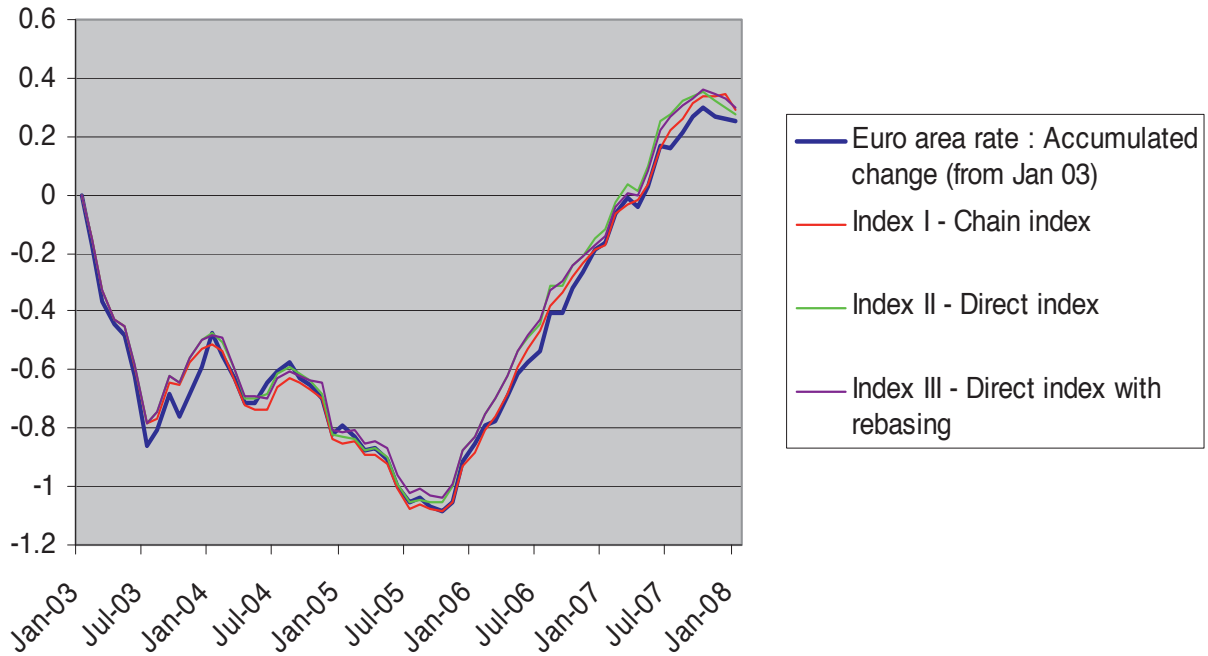
NB18 - New business. Loans to HHs for house purchase with intial period of rate fixation over 5 and up to 10 years
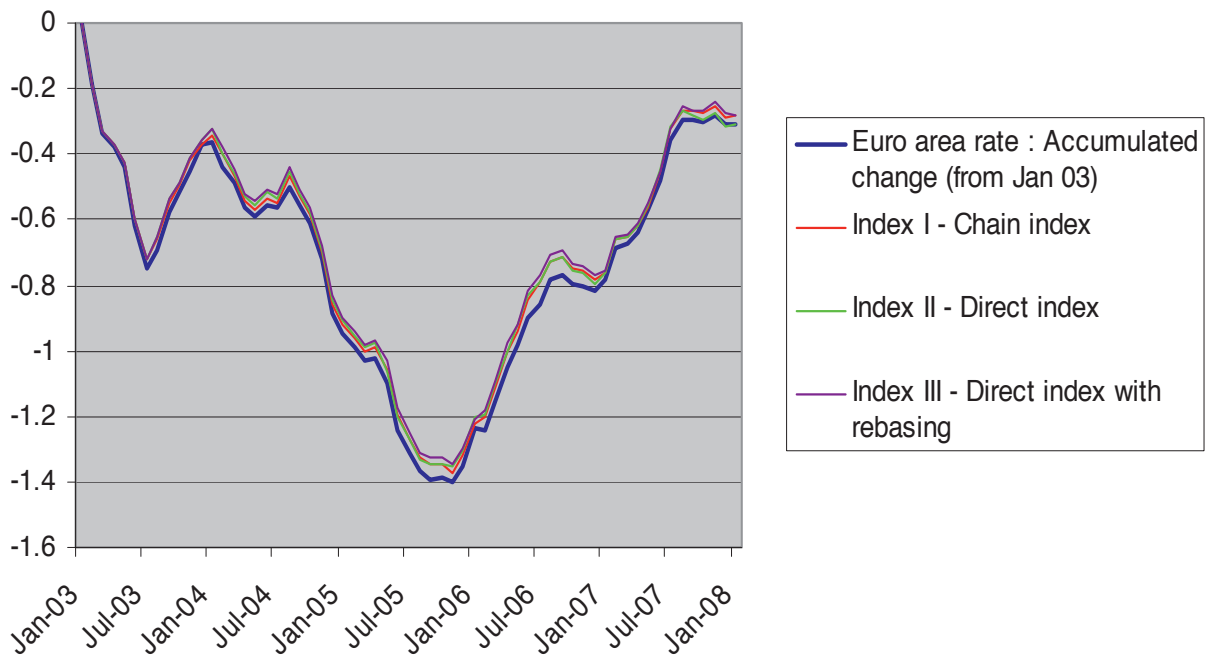
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

NB19 - New business. Loans to HHs for house purchase with initial period of rate fixation over 10 years
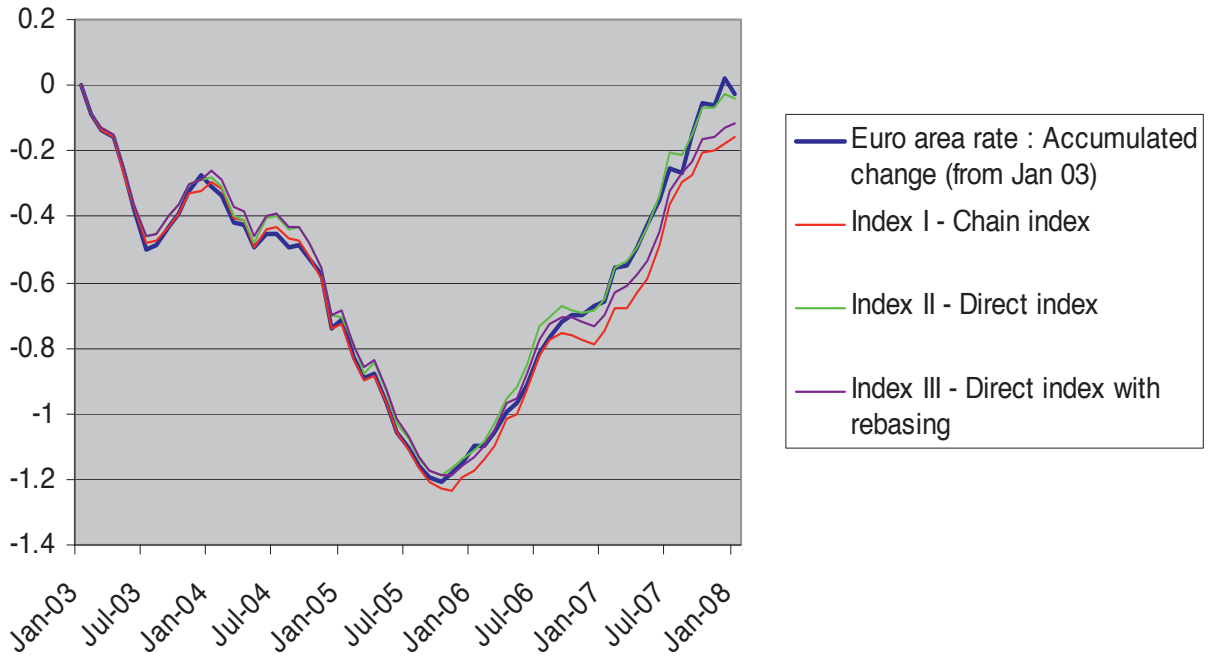
Euro area rate : Accumulated change (from Jan 03)
Index I - Chain index
Index II - Direct index
Index III - Direct index with rebasing

NB20 - New business. Loans to HHs for other purposes, with initial period of interest rate fixation up to 1 year

Euro area rate : Accumulated change (from Jan 03)
Index I - Chain index
Index II - Direct index
Index III - Direct index with rebasing
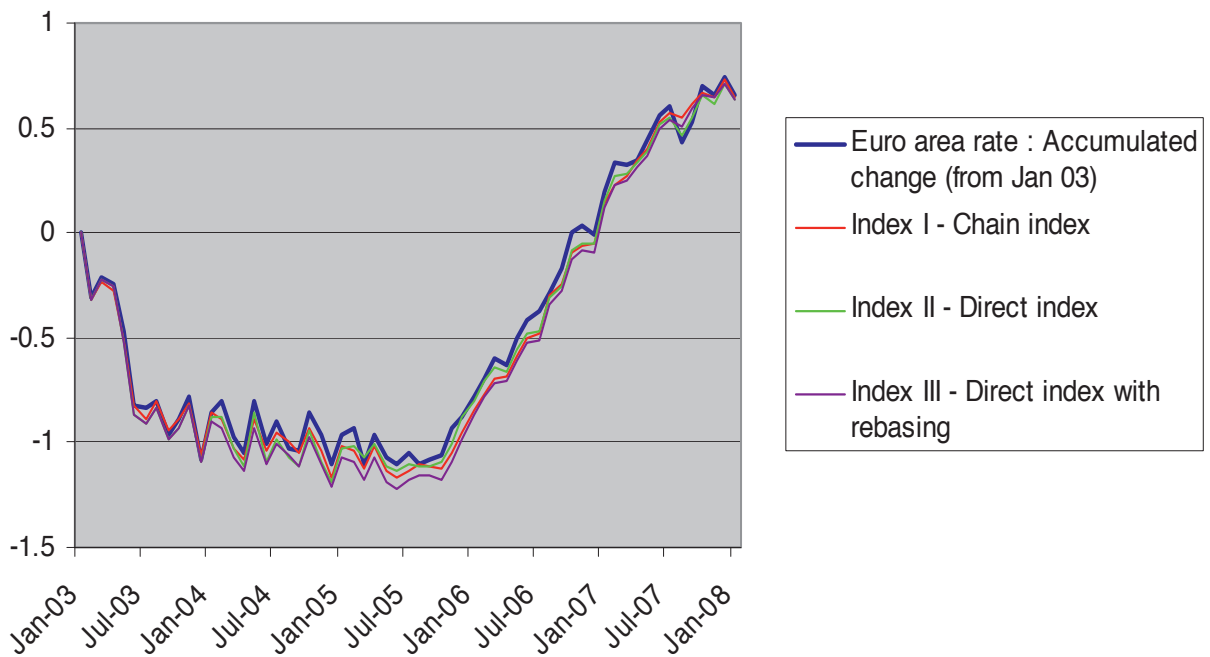
NB21 - New business. Loans to HHs for other purposes with an initial period of interest rate fixation over 1 and up to 5 years

Euro area rate : Accumulated change (from Jan 03)
Index I - Chain index
Index II - Direct index
Index III - Direct index with rebasing

NB22 - New business. Loans to HHs for other purposes with initial period of interest rate fixation over 5 years

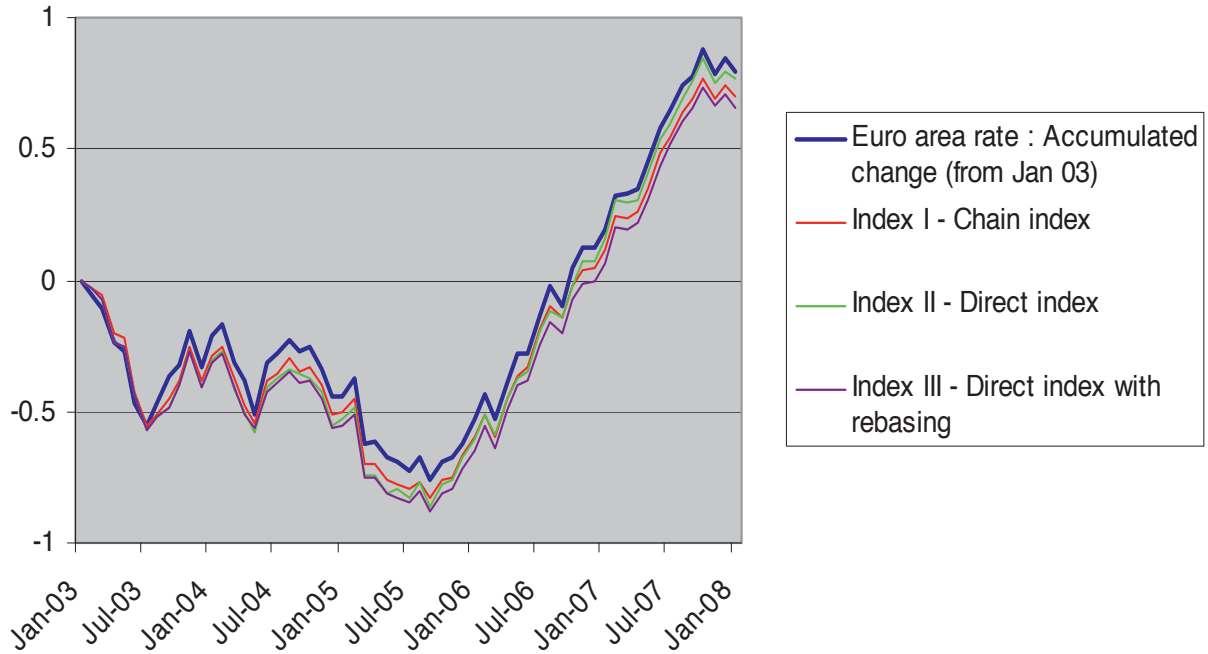Euro area rate : Accumulated change (from Jan 03)
Index I - Chain index
Index II - Direct index
Index III - Direct index with rebasing

NB23 - New business. NFCs bank overdrafts.

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

NB24 - New business. Loans to NFCs, up to Eur 1 mn with initial period of interest rate fixation up to 1 year

Legend:
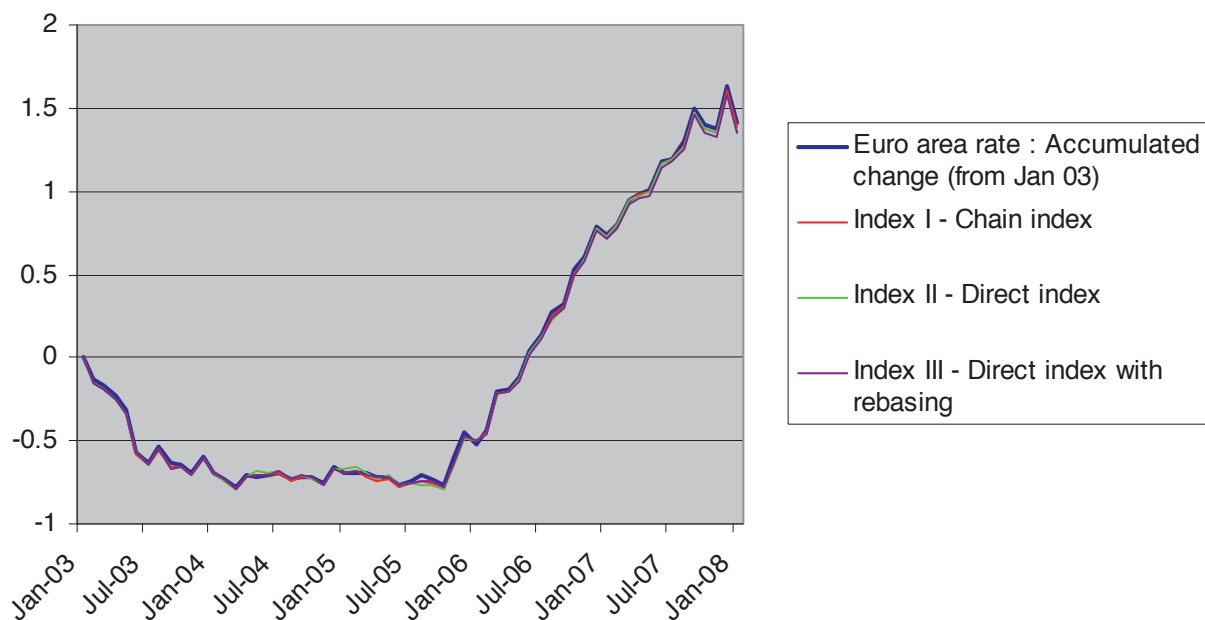- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

NB25 - New business. Loans to NFCs, up to Eur 1 mn with initial period of interest rate fixation over 1 and up to 5 years

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

NB26 - New business. Loans to NFCs, up to Eur 1mn with initial period of interest rate fixation over 5 years

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

NB27 - New business. Loans to NFCs, over Eur 1 mn with initial period of interest rate fixation up to 1 year

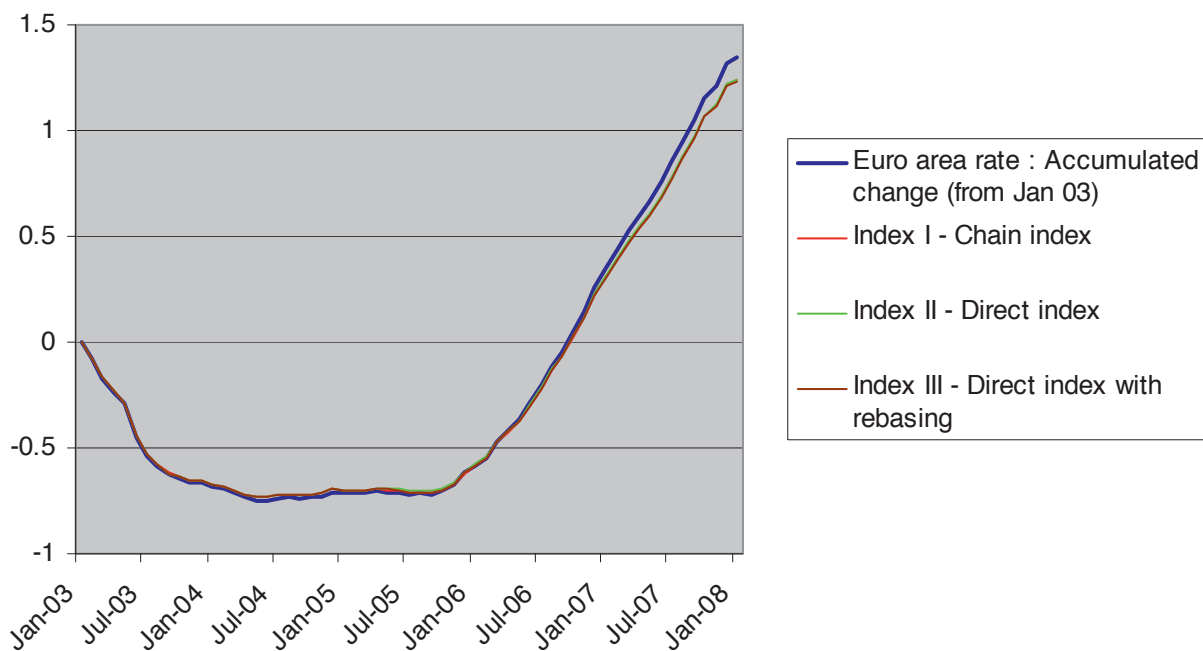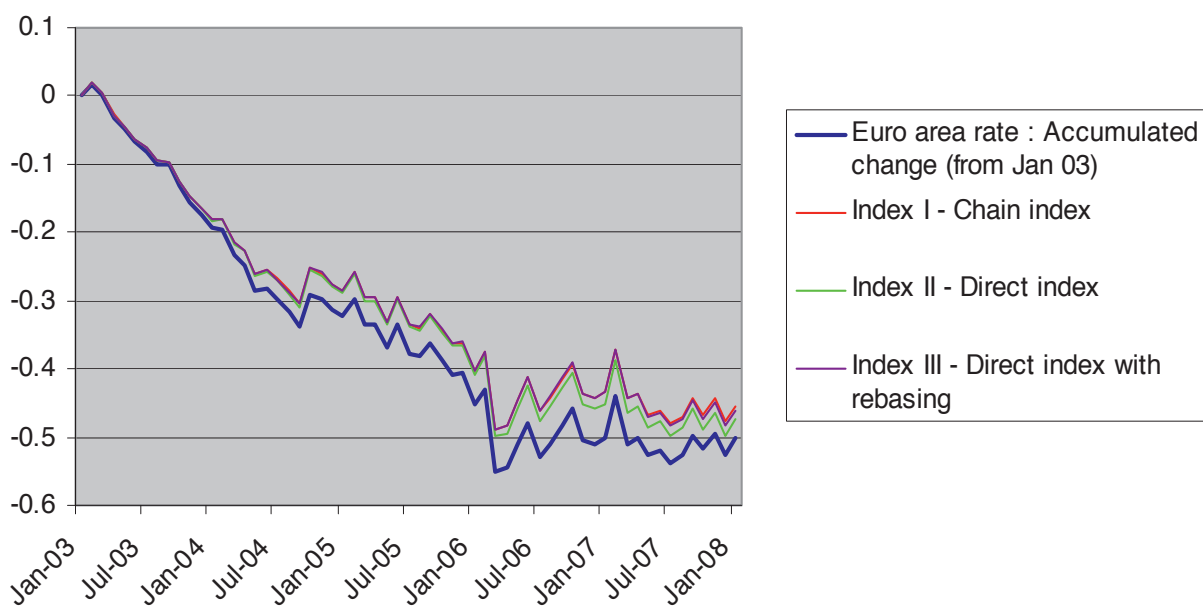- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

NB28 - New business. Loans to NFCs, over Eur 1mn with initial period of interest rate fixation over 1 and up to 5 years

- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

NB29 - New business. Loans to NFCs over Eur 1mn with initial period of interest rate fixation up to 1 year

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

OA1 - Oustanding amounts. Deposits from HHs with maturity up to 2 years

Euro area rate : Accumulated change (from Jan 03)

Index I - Chain index

Index II - Direct index

Index III - Direct index with rebasing

OA2 - Outstanding amounts. Deposits from HHs with maturity over 2 years

Euro area rate : Accumulated change (from Jan 03)

Index I - Chain index

Index II - Direct index

Index III - Direct index with rebasing

OA3 - Outstanding amounts. Deposits from NFCs with maturity up to 2 years

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

OA4 - Outstanding amounts. Deposits from NFCs with maturity over 2 years

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

OA5 - Outstanding amounts. Repos

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

OA6 - Outstanding amounts. Loans to HHs for house purchase with maturity up to 1 year

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

OA7 - Outstanding amounts. Loans to HHs for house purchase with maturity over 1and up to 5 years

Euro area rate : Accumulated change (from Jan 03)

Index I - Chain index

Index II - Direct index

Index III - Direct index with rebasing

OA8 - Outstanding amounts. Loans to HHs for house purchase with maturity over 5 years

Euro area rate : Accumulated change (from Jan 03)

Index I - Chain index

Index II - Direct index

Index III - Direct index with rebasing

OA9 - Outstanding amounts. Loans to HHs, consumer credit and other loans, with maturity up to 1 year

Legend:
- Euro area rate : Accumulated change (from Jan 03)
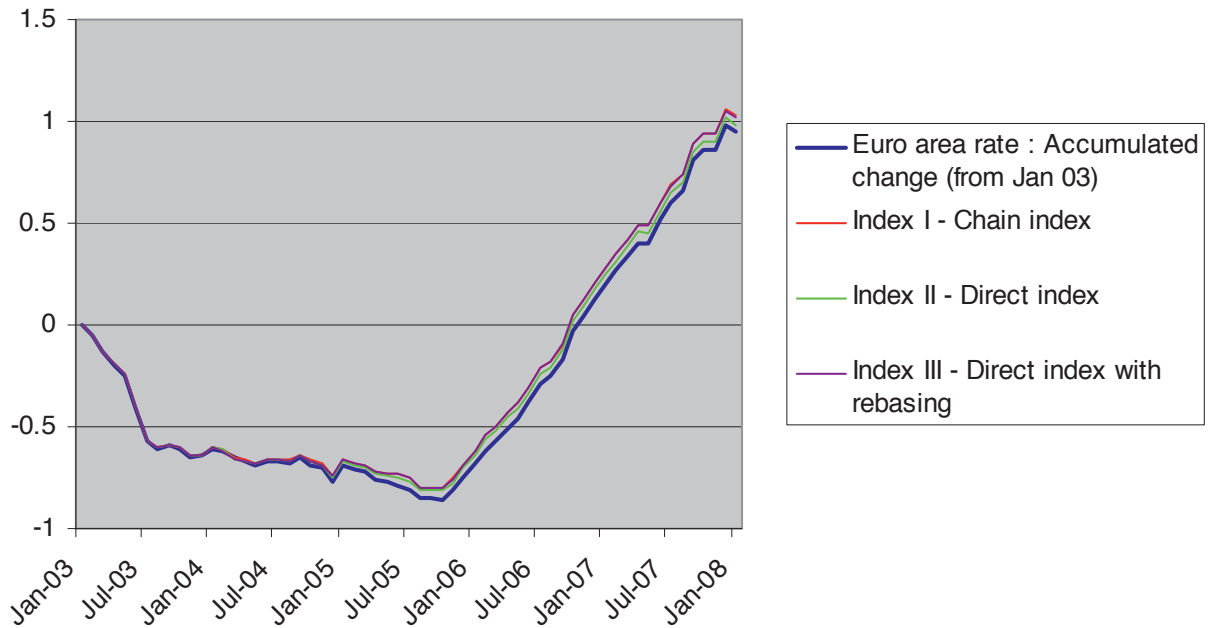- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

OA10 - Outstanding amounts. Loans to HHs, consumer credit and other loans with maturity over 1 and up to 5 years

Legend:
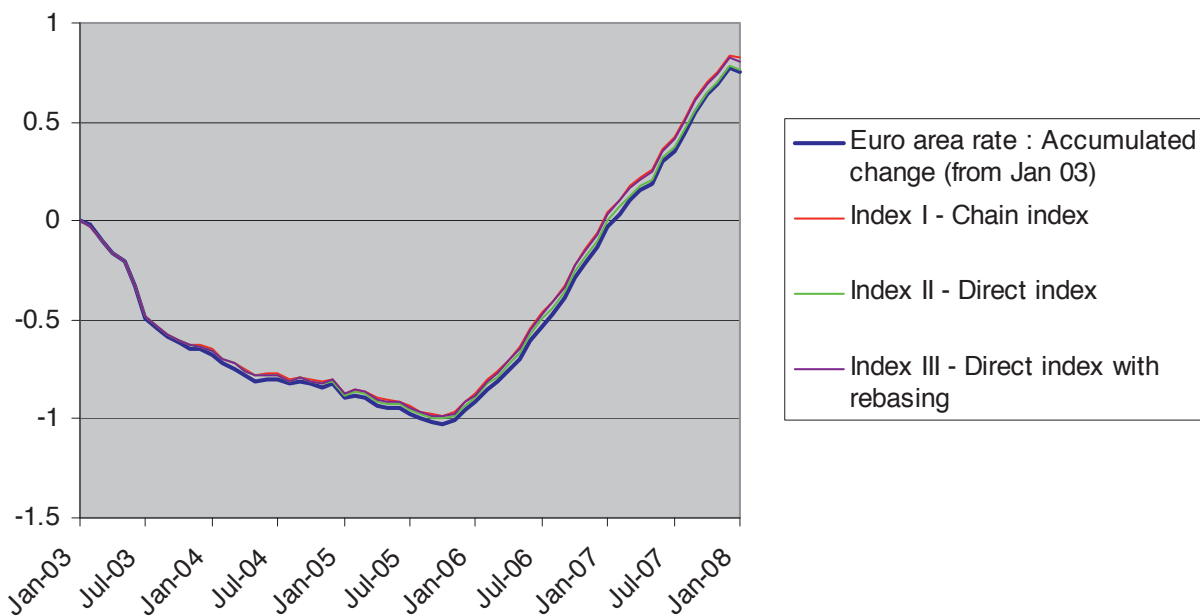- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

OA11 - Outstanding amounts. Loans to HHs, consumer credit and other loans, with maturity over 5 years

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

OA12 - Outstanding amounts. Loans to NFCs with maturity up to 1 year maturity

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

OA13 - Outstanding amounts. Loans to NFCs with maturity over 1 and up to 5 years

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

OA14 - Outstanding amounts. Loans to NFCs with maturity over 5 years

Legend:
- Euro area rate : Accumulated change (from Jan 03)
- Index I - Chain index
- Index II - Direct index
- Index III - Direct index with rebasing

ANNEX 4: DECOMPOSITION ON SELECTED MIR CATEGORIES AND PERIODS
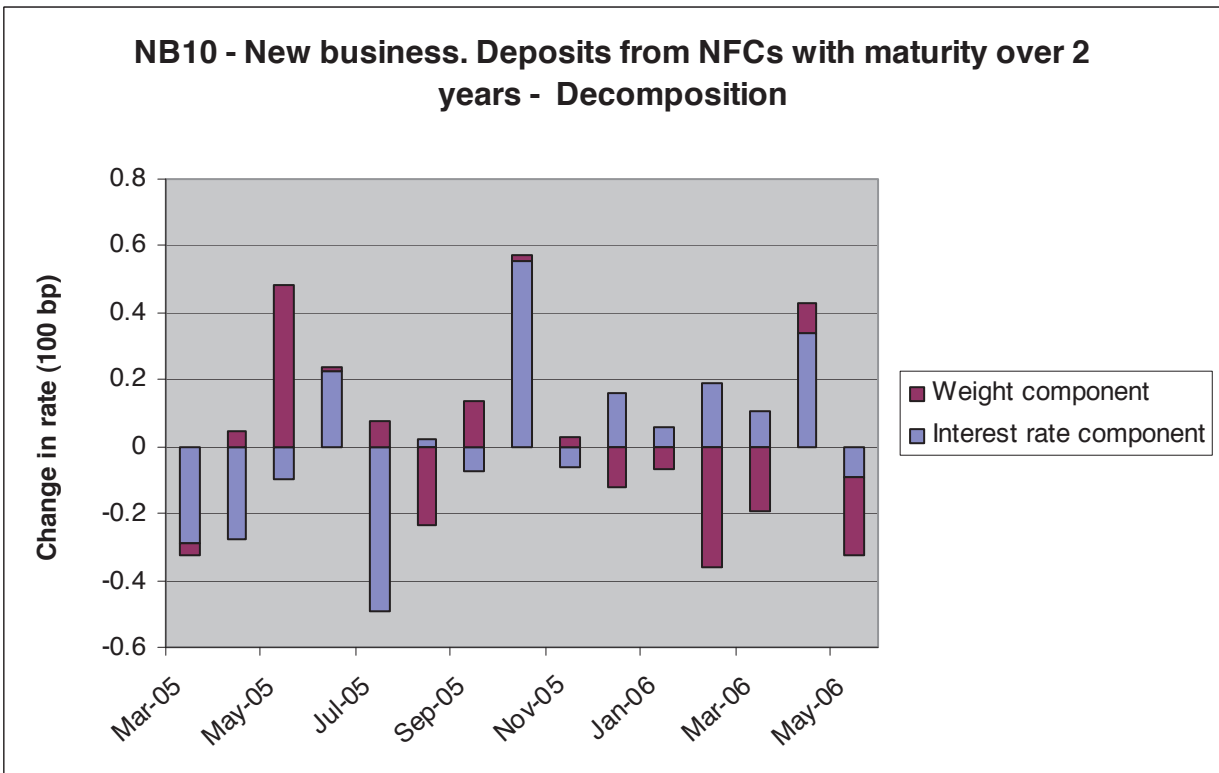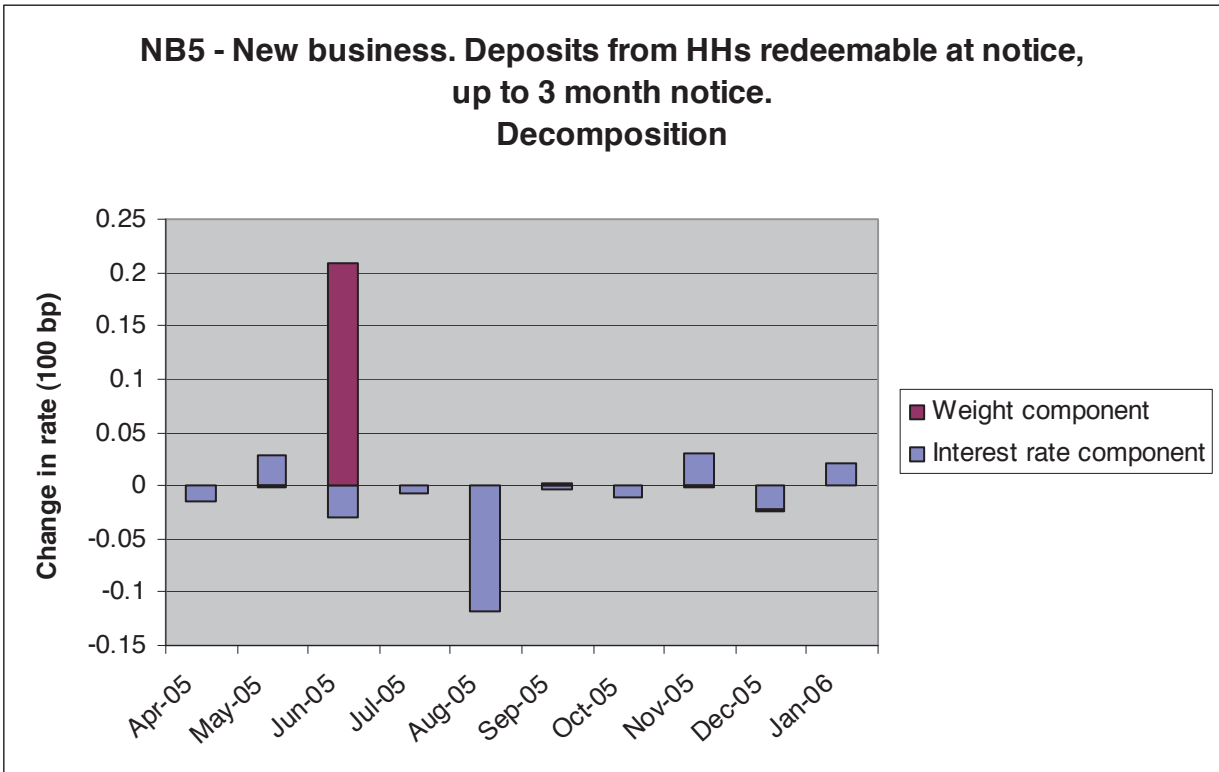
**NB5 - New business. Deposits from HHs redeemable at notice, up to 3 month notice.
Decomposition**

NB10 - New business. Deposits from NFCs with maturity over 2 years - Decomposition

NB13 - New business. Loans to HHs for consumption, with initial period of interest rate fixation up to 1 year Decomposition
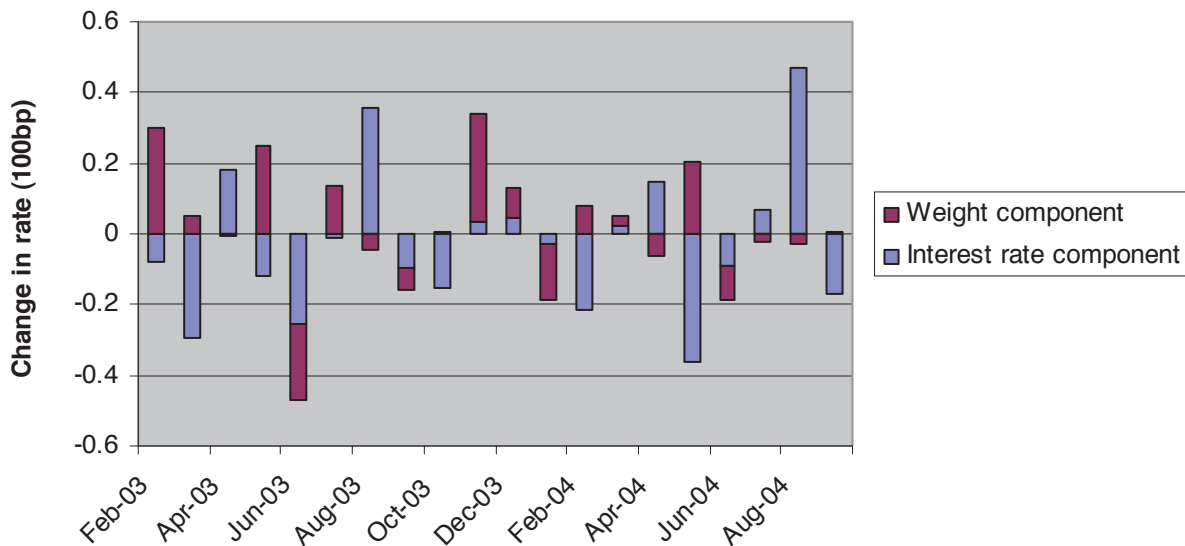
NB29 - New business. Loans to NFCs over Eur 1mn with initial period of interest rate fixation up to 1 year. Decomposition
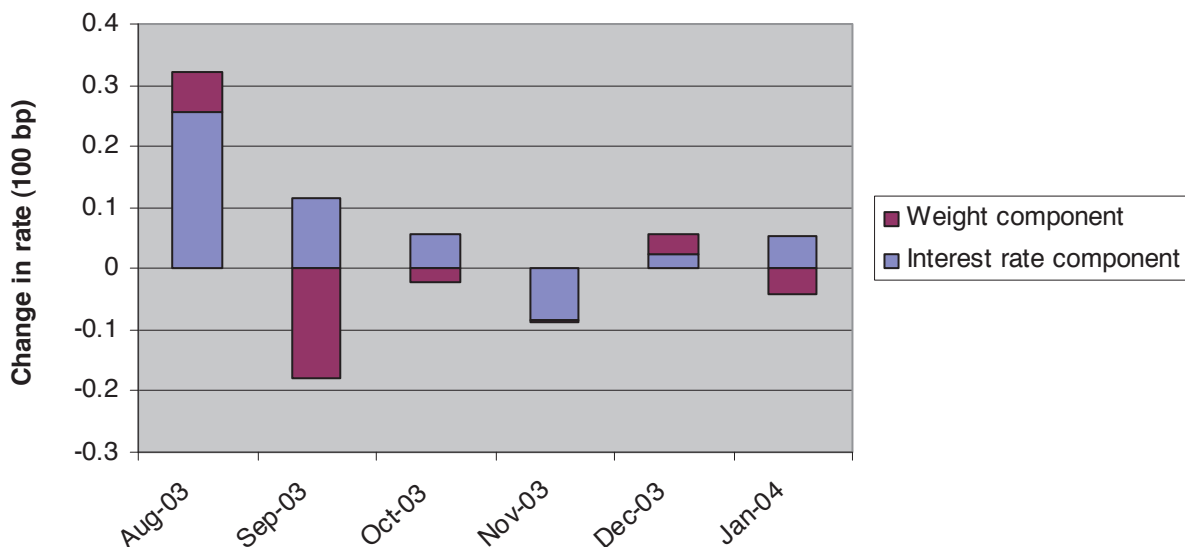
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